H^∞ Speed Control for Permanent Magnet Synchronous Motor

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***Abstract*—This paper describes the H_∞ control design for robust speed control for the Permanent Magnet Synchronous Motor. The speed feedback control is used to maintain the speed during load changing. Due to the load changing it will create the disturbance to the motor. To solve this problem the H_∞ control theory has been used to determine the robustness of the controller and to have good tracking performance for the speed . The model and the controller value have been developed in MATLAB/Simulink where the results show the controller able to maintain the speed despite the load is changed.**

***Index Terms*—Permanent Magnet Synchronous Motor, robust control, MATLAB, H_∞**

I. INTRODUCTION

Permanent Magnet Synchronous Motors (PMSMs) have been used widely because of the high torque applications, accurate positioning, lower inertia, large power rate, no spark and lower noise [1], [2], [3] . These advantages can be extracted if the PMSMs have minimum effect to the disturbances due to the changed of the mechanical torque at the input of the PMSM. To meet this condition, uncertainties such as perturbations, disturbances and load changing [2], [4], [5] must be considered when designing the controller. It also must have high sensitivity while at the same time maintain the stability of the system.

As known, speed is a state variable that can be controlled in the PMSM. Due to this variable this paper is focused in speed control design. The control theory which able to response or manipulate this situation is the robust control where the aims are to achieve the robustness performance and to have a close boundary for the differences between the true model and the nominal model [6]. One of the techniques is using the H_∞ or non-linear H_∞ control theory. The standard block diagram for the H_∞ control is shown in Fig.1. It shows that the controller is able responded to the disturbance input in generating the required output.

This paper explains the H_∞ control design for speed control to generate suitable signal for the motor current output. The signal that will be generated is based on the how the load changed will affected the speed. The load is looked as a the disturbance variable (T_m) when designing the controller . A typical PMSM control drive consists of a speed control and the current generator with a closed loop feedback is shown in Fig.2.

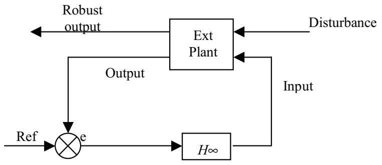

Fig. 1. Standard robust feedback control configuration.

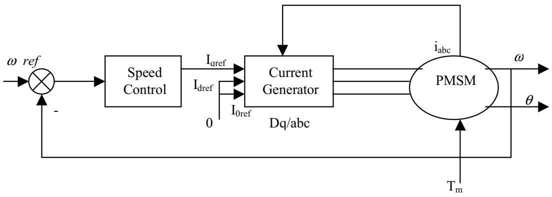

Fig. 2. PMSM control drive structure

II. MODELING OF THE PERMANENT MAGNET SYNCHRONOUS MOTOR

In this section the model of PMSM is explained and referred to [1], [2], [4], [7] . Papers [1], [2], [4], [7] give a clear view to determine the outputs and how the torque is developed inside the PMSM. The model is in *dq* transformation where only the *q* component is appeared while the *d* and *zero* components will not given any significant to the model. The mathematical model of *q* component is expressed in the equation below.

$$\frac{di_q}{dt} = \frac{v_q}{L_q} - \frac{Ri_q}{L_q} - \frac{\omega_i M_f}{L_q} \tag{1}$$

$$\frac{d\omega}{dt} = \frac{T_e}{J} - \frac{T_m}{J} - \frac{B\omega}{J} \tag{2}$$

$$T_e = k_n i_q \tag{3}$$

where
v_q: q axis stator voltage
i_q: q axis stator current
L_q: q axis stator impedance
R: stator resistance
M_f :flux linkage
ω_i: inverter frequency
ω: motor speed
T_e: electrical torque
T_m: mechanical torque
k_n: $\frac{3}{2} \times Pole \times M_f$

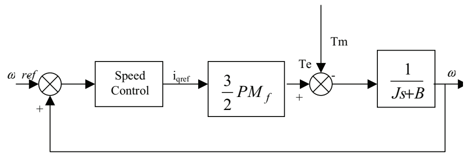

Fig. 3. Block diagram of the PMSM control system

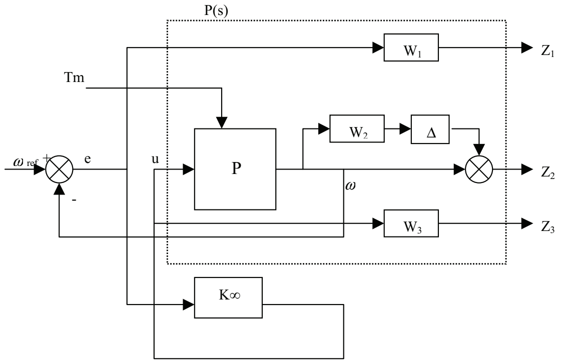

Fig. 4. H_∞structure of the control plant

Eq.1 is used to model the current control for the PMSM and will not been discussed here while Eq.2 is used to model the speed controller with the disturbance affect. Eq.3 shows the generated electrical torque in the PMSM. Fig.3 shows the speed control block diagram that has been modeled using Eq.2 where it consists of two inputs and one output where the inputs are the T_m and i_{qref} while the output is ω. In this model T_m is known as one of the disturbances or the mechanical torque input or load effect. The speed control that will be designed will generate the current reference i_{qref} to the input for ramp comparator for the Pulse Width Modulation (PWM) generation for the motor current.

III. DESIGN OF H_∞ ROBUST CONTROLLER

A robust control design with respect to the plant parameters variations and T_m variations are proposed using H_∞ control theory. The H_∞ control matrix consists of generalized plant $P(s)$ and the new design controller K_∞ where it can be determined refer to Fig.4. The new plant $P(s)$ consists of the weighting functions W_1, W_2,W_3 and the nominal plant P. W_1 is known as error performance, W_2 is the robust performance and W_3 is the input performance constant function to the plant. The Δ is known as the relative plant uncertainty that determined the boundary of W_2 should have. The output of the P is the ω where it should be controlled.

A. Definition of the Generalized Plant

The generalized plant $P(s)$ is shown in Fig.4, where the outputs are z_1,z_2,z_3. These outputs are the new output where have included the weighting functions. The disturbance T_m and the reference signal ω_{ref} are also included in the $P(s)$. The input of P is the output of the K_∞ control transfer function. The state-space equation for the model is given by,

$$\frac{d\omega}{dt} = \left[\frac{-B}{J}\right][\omega] + \left[\begin{array}{cc} \frac{1}{J} & \frac{kn}{J} \end{array}\right]\left[\begin{array}{c} T_m \\ i_q \end{array}\right] \tag{4}$$

$$Y = [1]\,[\omega]$$

The aim of the feedback control is to make sure the speed ω, as the same as the ω_{ref} value when the disturbance is been applied in the nominal plant .The generalized plant $P(s)$ can be written as,

$$\left[\begin{array}{c} z_1 \\ z_2 \\ z_3 \\ e \end{array}\right] = \left[\begin{array}{ccc} W_1 & -W_1P_{11} & -W_1P_{12} \\ 0 & W_2P_{11} & W_2P_{11} \\ 0 & W_3 & 0 \\ 1 & -P_{11} & -P_{12} \end{array}\right]\left[\begin{array}{c} \omega_{ref} \\ \omega \\ T_m \end{array}\right] \tag{5}$$

$$\left[\begin{array}{c} z_1 \\ z_2 \\ z_3 \\ e \end{array}\right] = P(s)\left[\begin{array}{c} \omega_{ref} \\ \omega \\ T_m \end{array}\right]$$

$$P(s) = \left[\begin{array}{cc} P_{11} & P_{12} \\ P_{21} & P_{22} \end{array}\right] = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \end{array}\right] \tag{6}$$

The weighting functions W_1,W_2 and W_3 are the designed parameters to force the closed loop response to meet certain specifications that will be discussed later in this paper. W_1 is chosen to be low pass filter in order to reject the output disturbance [1] and W_2 is to determine the plant relative uncertainty Δ boundary condition [8] . The value of W_3 must be chosen as small as possible, to make sure the D_{12} in generalized plant is full rank, required by the H_∞ control.

B. Weighting functions selection

As known W_1 is used for error tracking performance where it indicates the error from the reference value to the actual value. W_1 is selected base on the Proportional Integrator (PI) which has high gain. This filter will reject the disturbance at the low frequency range [1]. It can be written as

$$W_1 = \frac{0.5s+35}{s+0.0001} \tag{7}$$

W_2 is the robust performance for the speed output which includes the disturbance. In dealing with this the original plant P has been tested under the plant uncertainty Δ condition. This is to make sure the new W_2 responded to the rapid torque changing. The Δ has been chosen to be in range of 80%to 120% from the nominal model [8] and with the time delay response of $0 < t_{sec} < 1$. The general transfer function for W_2 can be written as [9]. In this application the $\triangle$ has been selected to the value of $100\% \approx 1$. The value for W_2 is selected based on the boundary condition [2], [4], [5] where shows in Eq 8. $W_3 = 0.001$ value has been selected to make sure the generalized plant is in full rank .

$$W_2 = 0.000068s + 0.7 \tag{8}$$

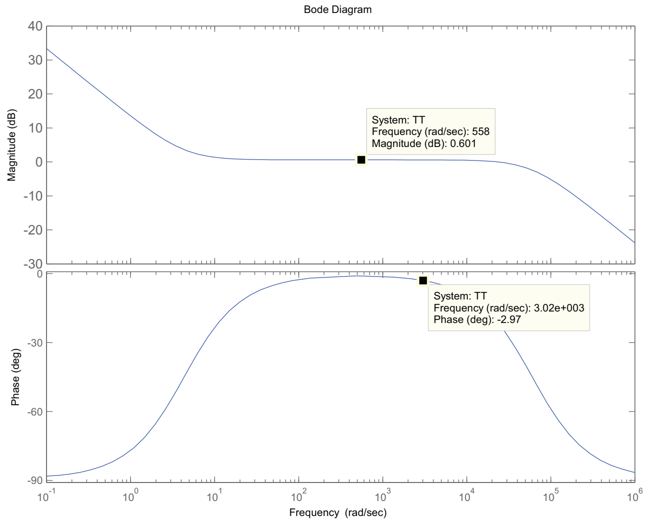

Fig. 5. Controller bode diagram

IV. SIMULATION RESULTS

The simulations were done in MATLAB and the results are the motor current (I_{abc}), speed (ω) and the speed error (e) in which to determine and observe the performance of the controller to the PMSM. The values that were used to model the PMSM is given in Table.I.

TABLE I
PMSM PARAMETERS

Rated Speed	$1000(rpm)$
Torque	$5(Nm)$
Pole Pairs	4
J(moment of inertia)	$1.05 \times 10^{-4}(kg.m^2)$
k_{tn}	$5.336 \times 10^{-1}(N.m/a)$
B(viscous damping coefficient)	$4.5498 \times 10^{-4}(N.m.s/rad)$

The K_∞ controller value has been generated using $hinfsys$ function in MATLAB. The generated H_∞ controller transfer function is given by

$$K_\infty = \frac{2.609(s+345.6)}{s+0.05048} \tag{9}$$

Eq.9 shows the controller is in stable region because all the poles and zeros are at the left hand side of the stability region. The bode diagram of this controller is shown in Fig.5. From the bode diagram the controller responses to the lower input frequency and reject the high frequency.

The complete simulation block diagram is shown in Fig.6. The feedbacks are the i_{abc} and the ω where both of them are used to generate the desire input to the motor. From the block diagram, only q component is considered while the other components are selected to 0 . The results from this simulation are shown in Fig.7.

Fig.7 shows the outputs of the motor which are the motor current and the speed of the motor. As can be seen, at the time of $1sec$ the load is increased and the motor current is dropping. At this stage if there is no speed control the speed will drop. Due to the robustness speed control that has been modeled, the controller is able to maintain the speed before and after the load changing. This is shown in motor speed result in Fig.7. For the ω, the speed is maintained at the target value during the simulation time. It shows that the controller is suitable for the speed control.

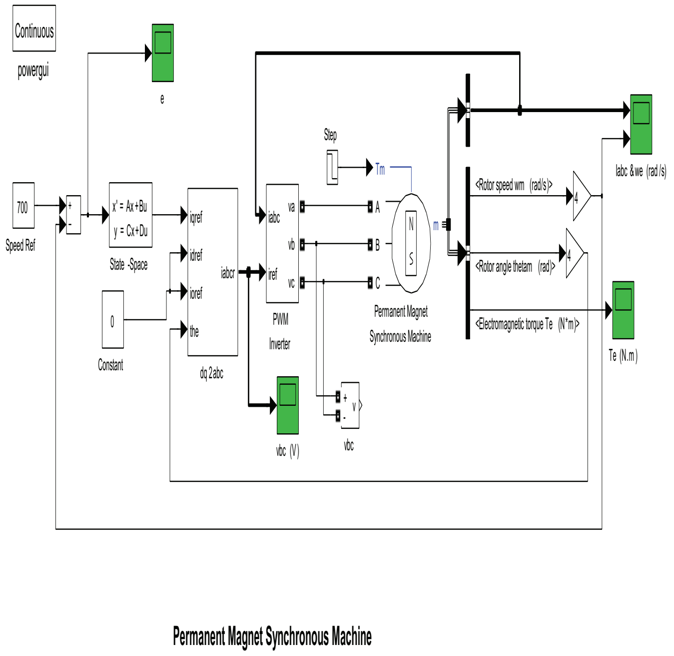

Fig. 6. Simulation Block Diagram

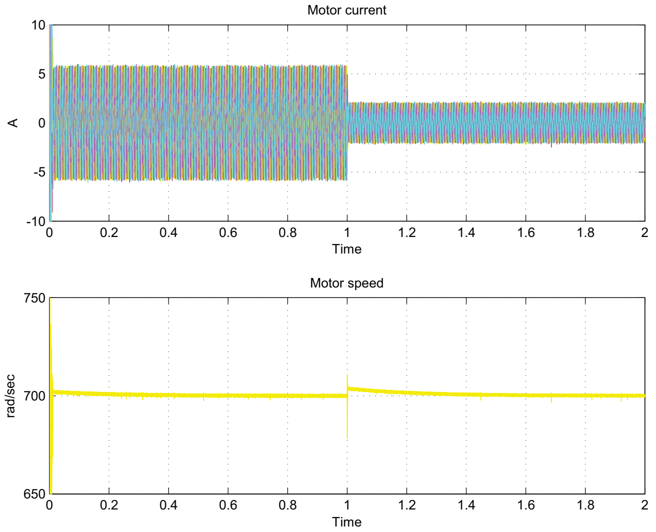

Fig. 7. Outputs simulation

Fig.8 shows the speed error where it graph has been limited from -3 to 2. It indicates that when the load changed at $1sec$ point, the controller needs $0.2sec$ to restore back to the normal value. It shows that the controller is robust enough to response in quick time when the disturbance affected the PMSM.

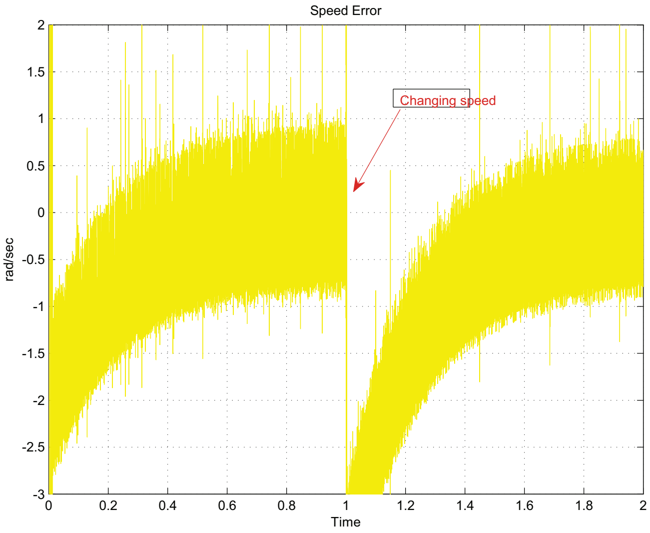

Fig. 8. Speed error $(\omega_{ref} - \omega)$

V. CONCLUSION

As a conclusion, this paper shows that the speed controller for PMSM can be designed and modeled with the helps of H_∞ control theory where it is included the disturbance effect or load changing to generate the desire signals in maintain the speed of the motor. As a result the robustness speed controller that has been proposed can be implemented in the motor control application without any additional controller to the PMSM.

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