Flood Wave Dynamics Using Lagrangian Block Advection

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ABSTRACT

Flood wave dynamics simulations are determined by a robust and accurate Lagrangian block advection (LBA) scheme that can track the dry-and-wet interface and capture the shocks without using any slope limiter to control the numerical oscillations. Two series of challenging numerical problems are considered using the LBA. First, computations are carried out for water waves in a parabolic bowl. The wetting-and-drying interface on the surface of the bowl is tracked by the LBA method with absolute computational stability. The accuracy of the LBA method is verified by the convergent of the numerical solution to an exact solution. Finally, the LBA method is applied to carry out a series of flood wave simulations, which have closely reproduced the data obtained from the laboratory experiments.

1. INTRODUCTION

The classical computational fluid dynamics are based on the estimate of fluxes on the faces of the finite volume using the truncated series, which have been the source of spurious numerical oscillations. Despite the control of the oscillations using the flux limiters, computational stability is not generally possible. In flood wave simulation, the sudden changes in the water depth and the velocity across the shock waves and at the advancing and recessing fronts between the wet-and-dry interface has led to spurious numerical oscillations, negative water depth and consequent collapse of the numerical computation.

One method to avoid the estimate of the flux is to calculate the mass and momentum transfers using the Lagrangian method. Chu and Altai (2001, 2002) used blocks of fluid and Lagrangian advection of the blocks to conduct turbulence simulations in a stream-function and vorticity formulation. Tan and Chu (2009a, 2009b, 2010) extended the Lagrangian blocks advection (LBA) method for one-dimensional (1D) simulation of the waves in shallow waters using the primitive variable formulation. The present LBA method uses the real fluids as the computational elements. This is to be distinguished with other Lagrangian method such as the particle-in-cell (PIC) methods of Harlow (1964) and the smoothed particle hydrodynamics (SPH) methods of Monaghan (2005). The PIC and SPH methods use the artificial particles while the blocks are real fluid elements. Therefore, the kernel function used to calculate the interaction force between the artificial particles is not required. The extension of the 1D formulation of Tan and Chu to two-dimensional (2D) and application of the 2D method to flood wave simulation are the subjects of this paper.

2. LAGRANGIAN BLOCK ADVECTION

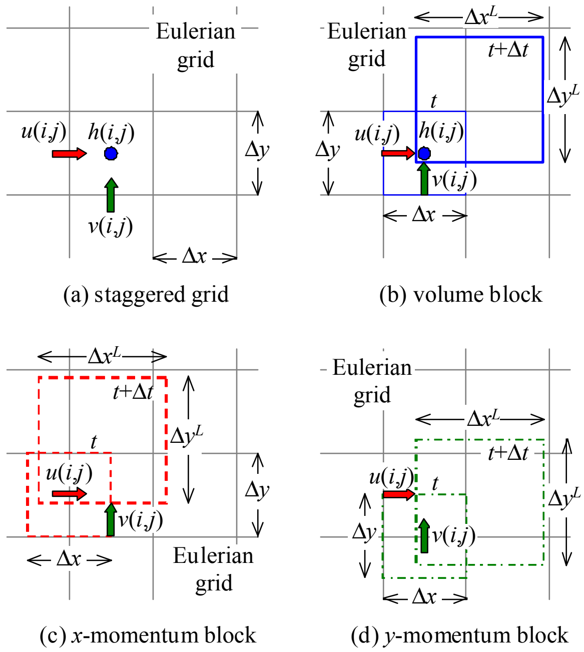

Figure 1. (a) The staggered depth and velocity node on the Eulerian grid, (b) the volume block, (c)-(d) the x- and y-momentum blocks at the beginning and the end of a Lagrangian advection time step.

The Lagrangian blocks are arrays of contiguous fluid elements. The transfer of mass and the momentum in the fluid are carry out in the computation by staggered system of blocks. Figure 1(a) shows the staggered grid and the relative locations of the volume block

($h\Delta x\Delta y$), and the x- and y-momentum blocks [($\rho uh\Delta x$), ($\rho vh\Delta y$)] on the grid. The x- and the y-components of the velocity are defined at a distance of ½ Δx to the west and ½ Δy to the south of the depth node, respectively.

Figure 1(b) shows the advection of the volume block and Figures 1(c) and 1(d) show the advection of the momentum blocks. A block of water for Lagrangian advection is defined by its water depth $h^L{}_i$ and the block widths $x^L{}_{i+1} - x^L{}_i = \Delta x^L$ and $y^L{}_{i+1} - y^L{}_i = \Delta y^L$. At the beginning of the Lagrangian advection, at time t, the blocks fit the Eulerian mesh, that is $x^L{}_i = x_i$ and $y^L{}_i = y_i$. At the end of the advection step, at time $t + \Delta t$, $\Delta x^L \Delta y^L h^L{}_i = \Delta x_i \Delta y_i h_i$ for volume conservation.

In the present simulation for the shallow water waves, the forces on the blocks are calculated by assuming hydrostatic pressure variation over the depth. The edge positions of the blocks $x^L{}_i$ and $y^L{}_i$ at time $t + \Delta t$ are calculated by Lagrangian integration of the momentum equations

$$\frac{\mathrm{D}u^L{}_i}{\mathrm{D}t} = -g\frac{h_i - h_{i-1}}{\Delta x} - g\left(S_{ox} - S_{fx}\right) \qquad (1)$$

$$\frac{\mathrm{D}v^L{}_i}{\mathrm{D}t} = -g\frac{h_i - h_{i-1}}{\Delta y} - g\left(S_{oy} - S_{fy}\right) \qquad (2)$$

where $u^L{}_i$ = x-component velocity, $v^L{}_i$ = y-component velocity, S_o = bottom elevation, and $S_f = c_f u_i |u_i|/(2gh)$ = friction slope. In the Lagrangian reference frame, the position $x^L{}_i(t)$ and the velocities $u^L{}_i(t)$ and $v^L{}_i(t)$ are functions of time only. To prevent entanglement of Lagrangian paths between adjacent blocks, the mass and the momentum in the blocks are re-casted onto the Eulerian mesh at each computational time step. A numerical solution is possible when the Courant number Co = max[$u\Delta t/\Delta x$, $v\Delta t/\Delta y$] is less than unity. The computational stability of the Lagrangian block advection will be demonstrated by some of the 2D simulations to be presented in this paper. An extensive series of grid refinement studies have been carried out previously by Tan and Chu (2009a, 2009b) to show the convergent of the block advection simulations toward many exact solutions for the dam-break flood waves by Ritter (1892), Stoker (1957), Hogg (2006), Ancey et al. (2008), and for the runup and overtopping of collapsing bores by Shen and Meyer (1963) and Peregrine and Williams (2001).

3. Comparison With Exact Solution

Figure 2 shows a block advection simulation of the wetting-and-drying by water on the surface of a parabolic bowl using a system of relatively coarse blocks. Initiated by a parabolic mound, the water moves up and down in the bowl under the influence of gravity. The wetting and drying on the surface of the bowl is a challenging numerical problem. The numerical oscillations can produce negative water depth in advancing and recessing water layer, and subsequently lead to computation breakdown.

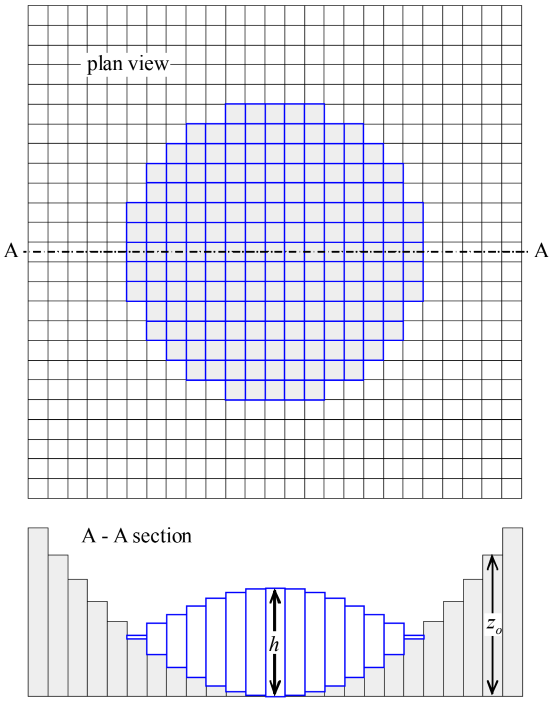

Figure 2. The wetting and drying of water on the surface of a parabolic bowl by block advection. The blocks are $\Delta x = \Delta y = 3.2$ m in a bowl of 40 m radius. The crest of the initial mound of water is $h_o = 0.02$ m.

The most remarkable advantage of the Lagrangian block advection method over other computation methods is the computational stability. The block advection has been able to simulate infinite cycles of advances and recesses of the water on the surface of the parabolic bowl. Figure 3(a) shows the simulated potential and kinetic energies of the water in the parabolic bowl and the rate of energy loss over a period of 20 cycles of water advance and recede. Figure 3(b) shows the water depth contours in the bowl after one revolution. The deviation from the exact solution of Thacker (1981) is determined by evaluating the kinetic and potential energies of water in the bowl. The total (kinetic plus potential) energy E is supposed to be constant but is reduced with time due to computation error as shown in Figure 3(a).

The energy loss over one period T is $(T/E_o)\Delta E/\Delta t$, which is a measure of the simulation error that is plotted against the block/grid size in Figure 3(c). The convergence of the block-advection simulation towards the exact solution is first order with an exponent $p \approx 1.0$ when the order of convergence is determined by the method of Celik et al. (2008).

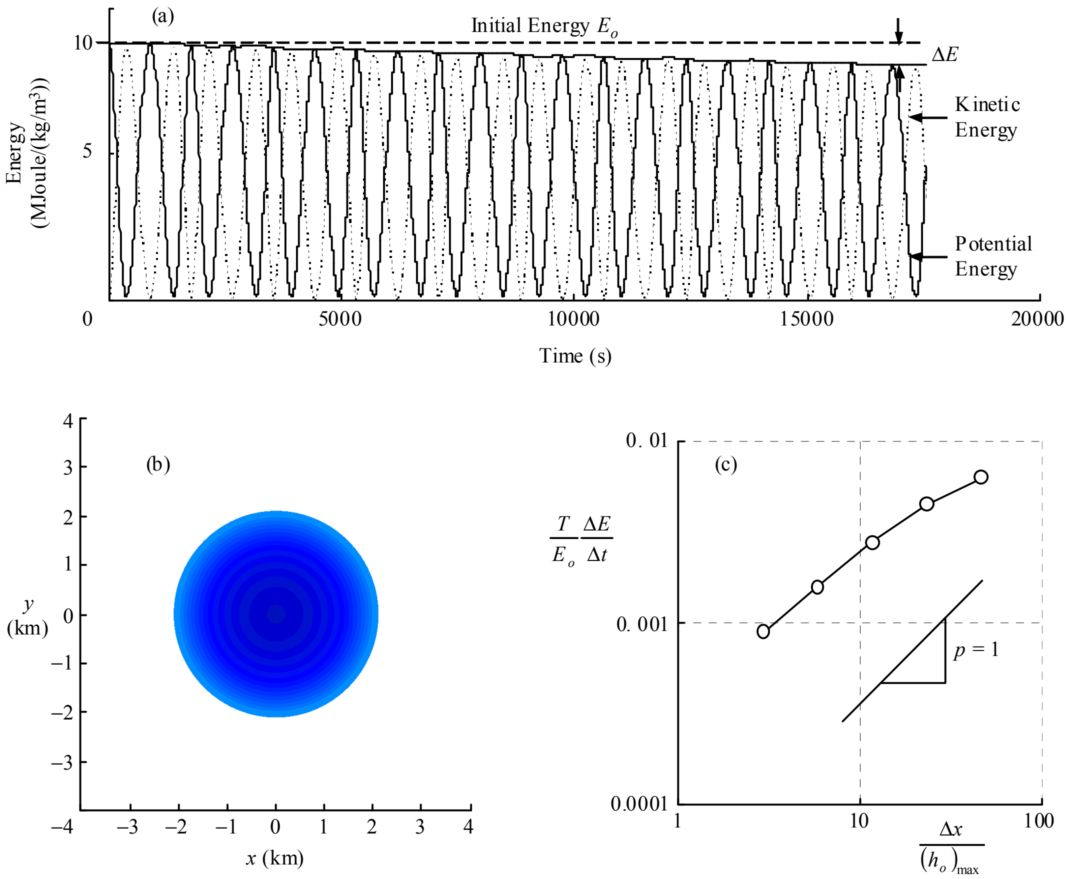

Figure 3. (a) Kinetic and potential energies of the water in the parabolic bowl computed by block advection using $\Delta x = \Delta y = 5$ m, (b) depth contours after one revolution and, (c) the convergence of block advection towards the exact solution using the energy loss over one cycle oscillation as measurement of the error. Calculations for the error were conducted using $\Delta x = 5$ m, 10 m, 20 m 40 m and 80 m.

4. Flood Waves Laboratory Experiments and Computation

A series of 2D flood wave computations are carried out using the Lagrangian block advection method. As shown in Figure 4, the dimensions of the computational domain are the same as the tanks used in the experiment works. The drag coefficient used in the computations is calculated assuming hydro-dynamically smooth surfaces in the laboratory tanks. The drag coefficient c_d is calculated using the formulae for steady flow of Henderson (1966): $c_d = 8/\mathrm{R}$ for $\mathrm{R} < 1189$, $c_d = 0.0395/\mathrm{R}^{0.25}$ for $1189 < \mathrm{R} < 72426$, and $1/c_d^{1/2} = 5.657\log_{10}(\mathrm{R}c_d^{1/2}/0.8874)$ for $\mathrm{R} > 72426$, where $\mathrm{R} = uh/\nu$ = Reynolds number, and ν = kinematic viscosity.

A series of flood routings in urban settings is presented as application example taking advantage of the computational stability of the block advection method. The experiments of flood waves through idealised city have been carried out by Soares-Frazão and Zech (2008). Two idealised city layouts are considered as shown in Figure 5. The idealised city is represented by group of 5 × 5 buildings with the streets in between. Buildings are represented by 0.3 m × 0.3 m square blocks and the streets are 0.1 m wide. The initial upstream and downstream water depths are $h_o = 0.4$ m and $h_d = 0.11$ m, respectively. Water depth h and velocity u are measured along section B-B for square city and section C-C for oblique city as shown in Figure 4. The comparison of the simulated profiles of the water depth and velocity with the experimental data obtained along the section C-C is shown in Figure 5 for time t = 4 s, 5 s, 6 s, 8 s, and 10 s. The computed results are denoted by solid lines and the circles denote the experiment data. Figure 6 shows the water depth h and the vorticity ω contours of the flood waves around the idealised oblique city. The agreements between the computation results and the observation data are very good given the effect of the surface tension in the laboratory experiment.

5. Conclusions

The Lagrangian block advection has been able to track the flood wave front where water meets the dry land and to capture the sudden jumps in water depth and velocity with absolute computational stability. The problem of the spurious numerical oscillations has been eliminated when the block advection is used to transfer mass and momentum of the flood waves in shallow waters. Beside the usual Courant stability condition, absolute computational stability is attained as a consequence. The wet-and-dry interface of the water waves on the surface of a parabolic bowl has been determined directly by the Lagrangian block advection without using any frontal tracking procedure. The sudden jumps in depth and velocity across the hydraulic jumps are reproduced by the Lagrangian block advection without using any flux or slope limiter, since the control of the numerical oscillation is no longer required in absence of the numerical oscillations. A total of six series of flood wave simulations have been conducted without ever encountering any computational instability. The computation of the waves in the parabolic bowl approaches the exact solution. The flood wave simulation results reproduced closely the experimental data.

The laboratory model and the numerical simulations have been carried out for the idealized floods. Nevertheless, the results presented in this paper have demonstrated clearly the capability of the Lagrangian block advection method to tackle the real flood wave simulation problems with absolute computational stability.

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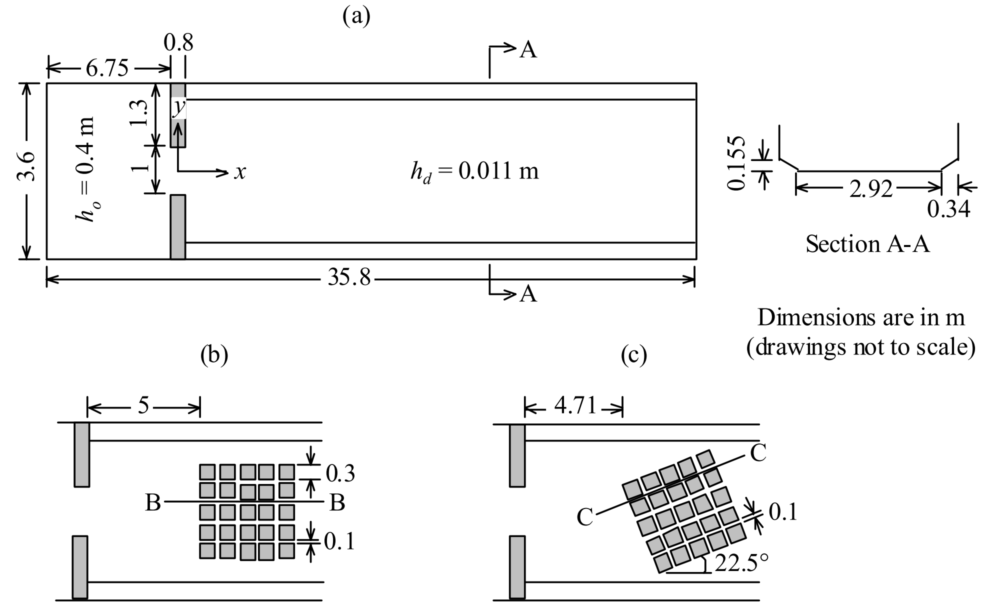

Figure 4. Channel dimensions for experiment of flood waves through the idealized city of two layouts: (a) buildings oriented in a direction normal to the flood waves, and (b) buildings in an oblique direction to the flood waves.

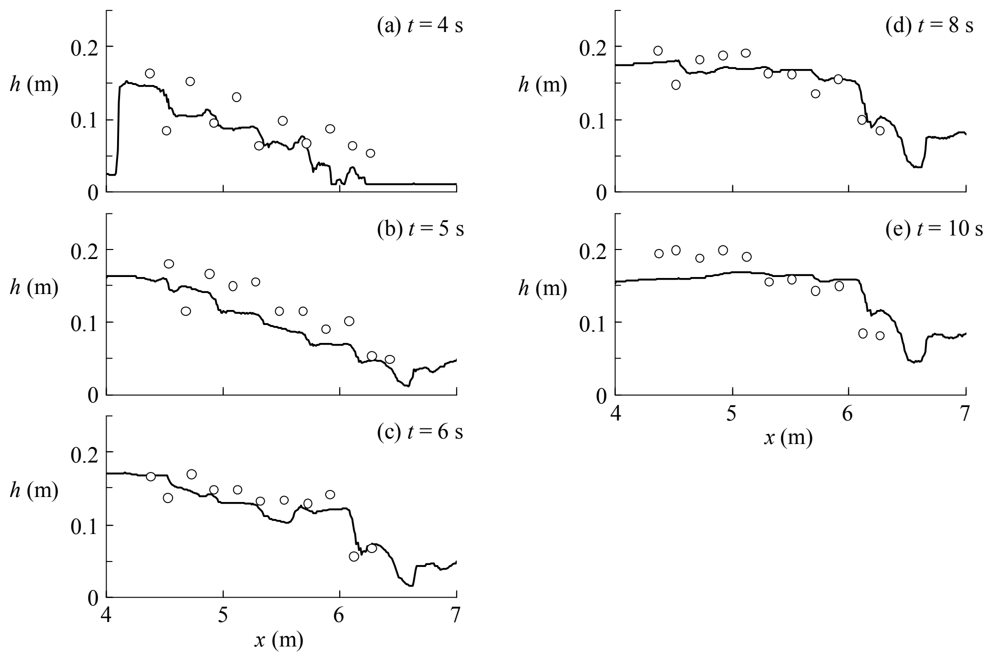

Figure 5. Water depth h and surface velocity u profiles of flood waves through section C-C of the idealized oblique city at time $t = 4$ s, 5 s, 6 s, 8 s and 10 s. The block advection computation results denoted by the solid lines are obtained using block size of $\Delta x = \Delta y = 0.01$ m. The circles are the experiment data.

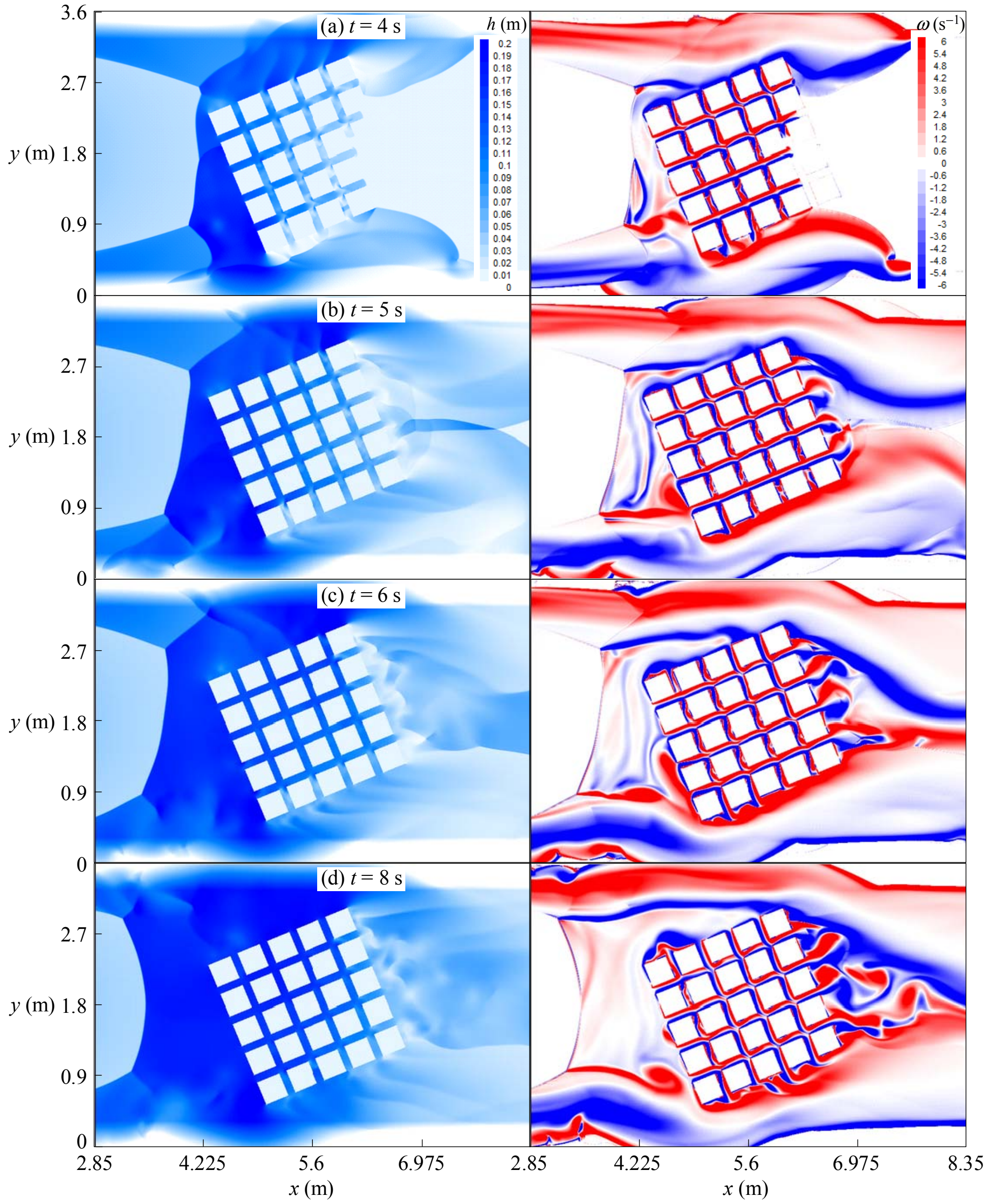

Figure 6. Water depth h and vorticity ω contours of the flood waves impinges against an oblique group of buildings in an idealized city at time t = 4 s, 5 s, 6 s and 8 s. The block advection computations are carried out using block size of $\Delta x = \Delta y = 0.01$ m.