

MODELLING AND CONTROL OF A BALANCING ROBOT USING
DIGITAL STATE SPACE APPROACH

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SESI PENGAJIAN: 2005/2006

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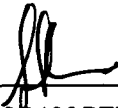
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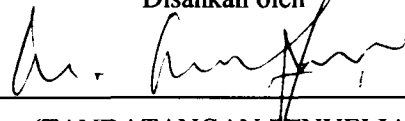
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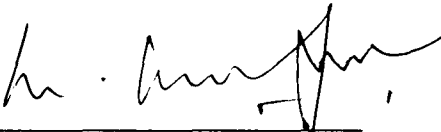
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**MODELLING AND CONTROL OF A BALANCING ROBOT USING
DIGITAL STATE SPACE APPROACH**

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**A project report submitted in partial fulfilment of the
requirements for a award of the degree of
Master of Engineering (Electrical-Mechatronics and Automatic Control)**

**Faculty of Electrical Engineering
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NOVEMBER, 2005

I declare that this thesis “Modelling and Control a Balancing Robot using Digital State Space Approach” is the result of my own research except for works that have been cited in the reference. The thesis has not been accepted any degree and not concurrently submitted in candidature of any other degree.

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To my dearest mother, father and family for their encouragement and blessing
To my beloved classmate for their support and caring

ACKNOWLEDGEMENT

Alhamdullillah, I am grateful to ALLAH SWT on His blessing in completing this project.

I would like to express my gratitude to honourable Associate Professor Dr. Mohamad Noh Ahmad, my supervisor of Master's project. Under his supervision, many aspects regarding on this project has been explored, and with the knowledge, idea and support received from him, this thesis can be presented in the time given.

Finally, I would like to dedicate my gratitude to my parents, my family and friends especially my classmate Fairuz, Adizul, Rafidah, Saleha, Niha, Hafiz, Buvendra and Kumeresan who helped me directly or indirectly in the completion of this project . Their encouragement and guidance mean a lot to me. Their sharing and experience foster my belief in overcoming every obstacle encountered in this project.

ABSTRACT

This thesis is concerned with the problems of modelling a complete mathematical model of a balancing robot and control the system using digital state space approach to pilots the motors so as to keep the system in equilibrium. The research work is undertaken in the following development stages. In order to analyze and design the control system the dynamic of model of the system was first established in discrete-time. Then the difference equation approach is used to obtain the dynamic equations of an actual experimental test-rig. The dynamic of the DC motors as well as chassis and wheels of balancing robot are incorporated in the overall dynamic model, which is in the form of continuous state-space. Two type of controllers, namely pole placement controller and LQR controller are considered in this work. The performance and reliability of both controller will be determined by performing extensive simulation using MATLAB/SIMULINK as the platform.

ABSTRAK

Tesis ini berkenaan dengan masalah untuk memformulasikan model lengkap dinamik robot dan juga kawalan sistem menggunakan ruang digital yang boleh mengawal motor bagi mengekalkan keseimbangan system. Kajian ini telah dibahagikan kepada beberapa peringkat. Bagi menganalisa dan mereka kawalan sistem model dinamik system diambil kira dalam bentuk digital. Selepas itu kaedah persamaan perbezaan digunakan untuk memperolehi dinamik bagi platform ujian eksperimen yang sebenar. Dinamik bagi motor DC juga kasis dan tayar bagi robot seimbang itu telah diaplikasikan didalam model dinamik ,dimana ia telah diubah kepada keadaan berterusan dan diubah kepada ruang digital. Dua jenis pengawal yang digunakan adalah pengawal penentuan kutub dan LQR. Prestasi dan kepercayaan bagi kedua-dua pengawal akan ditentukan melalui simulasi secara extensive menggunakan MATLAB/SIMULINK sebagai platform.

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LIST OF SYMBOLS

x	- Displacement
\dot{x}	- Displacement velocity
ϕ	- Angle
$\dot{\phi}$	- Angular velocity
θ	- Parameters position
ω	- Velocity
V_a	- Applied torque
H_∞	- H-infinity
k_m	- Torque constant
τ_m	- Motor torque
I	- Current that flow through the armature circuit
k_e	- Back emf constant
R	- Lumped armature winding resistance
L	- Self inductance of the armature winding
J_a	- Moment inertia of the armature
k_f	- Frictional constant
R	- Radius of the wheels
θ	- Rotation angle of the wheels which is the same as the rotation angle of the armature
P_R	- Reaction force between the wheel and the chassis of y-component of the force
H_R	- Reaction force between the wheel and the chassis of x-component
H_{fR}	- Friction force between ground and the wheel
M_w	- Mass of the wheel

- J_w - Moment inertia of the wheels
- H_{RR} - Conversion of translational force into rotational force
- G - Gravity 9.81 m/s^2
- J_P - Moment inertia of the robot chassis
- l - Distance between the centre of the wheel and the robot's centre gravity
- M_P - Mass of the robot's chassis

CHAPTER 1

INTRODUCTION

1.1 Overview

Balancing robots are characterised by the ability to balance on its two wheels and spin on the spot similar to inverted pendulum. The inverted pendulum problem is common in the field of control engineering thus the uniqueness and wide application of technology derived from this unstable system has drawn interest from many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems like designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems.

This nonlinear control problem is surprisingly difficult to solve in a methodological approach due to two degrees of freedom, i.e, the balancing robot position and chassis angle using only one control input force. A practical problem with regard to control the balancing robot is similar to the concept designing a controller to

swing the inverted pendulum up from a pendant position, achieve inverted stabilization, and simultaneously position.

In this thesis the balancing robots are characterized by the ability to balance on its two wheels and spin on the spot. The robot is composed of a chassis based on a stack of 130mm x 130mm Perspex plates carrying a Faulhaber DC motor, the Mark 4 Eyebot controller running on Robios version 5.2, a HOTEK GY-130 digital rate gyroscope, a SEIKA N3 digital inclinometer as described in (Thomas, 2002). The wheels of balancing robot are directly coupled to the output of the dc motor.

The balancing robot chassis is constructed from a single sheet of aluminium, drilled with holes for the easy mounting of motors, controller, sensors and battery pack. A pair of Faulhaber DC motors drive the robot's wheels. Each motor has a gear reduction of 54.2:1 and a torque constant of $6.9203 \times 10^{-4} \text{ kg}^{\text{m}}/\text{A}$. These motors have encapsulated encoders, and can be used to measure displacement and velocity of the robot. The robot is controlled by an EyeBot. A Mark 4 Eyebot controller running on Robios version 5.2 is used as the 'brain' of the balancing robot system.

The controller consists of a powerful 32-Bit microcontroller running at 33MHz, there is 512k ROM and 2048k RAM on board. The gyroscope modifies a servo control signal by an amount proportional to its measure of angular velocity. Instead of using the gyro to control a servo, we read back the modified servo signal to obtain a measurement of angular velocity. An estimate of angular displacement is obtained by integrating the velocity signal over time. The Inclinometer outputs an analogue signal, proportional to the angular displacement of the sensor (Braunl, 2002).

The balancing robot with two degree of freedom (DOF), is able to move along x, y axes describe by displacement, x and displacement velocity, \dot{x} and chassis angle corresponding the angle, ϕ and angular velocity, $\dot{\phi}$. These four state space variable fully describe the dynamics of the 2 DOF system.

The balancing robot balance the load with its wheels while dragging the weight around on a pivot in a regular differential drive robot. This thesis will delve into the suitability and performance analysis of Pole Placement and LQR controllers in balancing the balancing robot in discrete-time environment.

1.2 Objective

The objectives of this research are as follows:

1. To formulate the complete mathematical dynamic model of the Balancing Robot using differential equation method.
2. To establish the state space model of the Balancing Robot using Digital State Space Approach.
3. To show mathematically that the Balancing Robot system is controllable and observable in discrete-time.
4. To design digital state feedback regulators for the Balancing Robot using pole placement approach and Optimal Controller (LQR).

5. To simulate the Balancing Robot continuous system and hybrid system using MATLAB-SIMULINK.
6. To demonstrate that the digital state space approach is as accurate as the continuous state space approach.

1.3 Scope of Project

The work undertaken in this project are limited to the following aspects:

1. Balancing Robot as described by Thomas Braunl (2002).
2. Digital State Space Approach as described by Richard J. Vaccaro (1995)
3. State feedback with Pole Placement Approach and Linear Quadratic Regulator.
4. Simulation on MATLAB-SIMULINK.

1.4 Research Methodology

The research work is undertaken in the following nine developmental stages:

1. Formulate the complete mathematical dynamic model using differential equation method.
2. Establish continuous state space mathematical model.
3. Linearization: Nonlinear equations of motion are linearized around the operating point .
4. Choose Sampling Interval.
5. Discretize the linearized continuous state space model to digital state space model.
6. Check the controllability and observability of ZOH Equivalent Models.
7. Design continuous-time and discrete-time state feedback controller using the pole placement method and LQR.
8. Verify the controller design of the balancing robot simulated on MATLAB SIMULINK.
9. Evaluate results

1.5 Literature Review

The research on balancing robot has gained momentum over the last decade. This is due to the nonlinear and inherent unstable dynamics of the system. The balancing problem extensively studied by numerous researchers (Mori, 1976). An understanding of how to control such a system will allow us to easily solve the other related control problems, such as single-link flexible manipulators (Yeung et al., 1990) and stabilization of a rocket booster by its own thrust vector. Below is several research that has been done by researches.

Yangsheng Xu (2004), developed a dynamic model for the single wheel robot and verified it through simulations and experiments. Using the linearization method, a linear state feedback approach to stabilize the robot at any desired lean angle was developed. This feedback provides means for controlling the steering velocity of the robot. Line following controller is developed for tracking any desired straight line while keeping balance. The controller is composed of two parts: the velocity control law and the torque control law. In the velocity control law, the velocity input (steering velocity) is designed for ensuring the continuity of the path curvature. Then, the robot can be stabilized for tracking a lean angle trajectory in which the steering velocity is identical to the desired value.

Henrik Niemann (2003) has derived a linear model of a double inverted pendulum system together with a description of the model uncertainties. For the double inverted pendulum system the trade-off between robust stability and performance is quite limited. There is not much space for reduction of the robustness to increase the performance of the system. The reason is the nonlinearities in the system together with the limitations/saturations in the system. The limitations in the system are e.g. maximal