MODELLING AND CONTROL OF A BALANCING ROBOT USING DIGITAL STATE SPACE APPROACH

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A project report submitted in partial fulfilment of the
requirements for a award of the degree of
Master of Engineering (Electrical-Mechatronics and Automatic Control)

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NOVEMBER, 2005

I declare that this thesis "Modelling and Control a Balancing Robot using Digital State Space Approach" is the result of my own research except for works that have been cited in the reference. The thesis has not been accepted any degree and not concurrently submitted in candidature of any other degree.

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To my dearest mother, father and family for their encouragement and blessing
To my beloved classmate for their support and caring … … …

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ABSTRACT

This thesis is concerned with the problems of modelling a complete mathematical model of a balancing robot and control the system using digital state space approach to pilots the motors so as to keep the system in equilibrium. The research work is undertaken in the following development stages. In order to analyze and design the control system the dynamic of model of the system was first established in discrete-time. Then the difference equation approach is used to obtain the dynamic equations of an actual experimental test-rig. The dynamic of the DC motors as well as chassis and wheels of balancing robot are incorporated in the overall dynamic model, which is in the form of continuous state-space. Two type of controllers, namely pole placement controller and LQR controller are considered in this work. The performance and reliability of both controller will be determined by performing extensive simulation using MATLAB/SIMULINK as the platform.

ABSTRAK

Tesis ini berkenaan dengan masalah untuk memformulasikan model lengkap dinamik robot dan juga kawalan sistem menggunakan ruang digital yang boleh mengawal motor bagi mengekalkan keseimbangan system. Kajian ini telah dibahagikan kepada beberapa peringkat. Bagi menganalisa dan mereka kawalan sistem model dinamik system diambil kira dalam bentuk digital. Selepas itu kaedah persamaan perbezaan digunakan untuk memperolehi dinamik bagi platfom ujian experimen yang sebenar. Dinamik bagi motor DC juga kasis dan tayar bagi robot seimbang itu telah diaplikasikan didalam model dinamik ,dimana ia telah diubah kepada keadaan berterusan dan diubah kepada ruang digital. Dua jenis pengawal yang digunakan adalah pengawal penentuan kutub dan LQR. Prestasi dan kepercayaan bagi kedua-dua pengawal akan ditentukan melalui simulasi secara extensive menggunakan MATLAB/SIMULINK sebagai platform.

CONTENTS

SUBJECT **PAGE**

TITLE i
DECLARATION ii
DEDICATION iii
ACKNOWLEDGEMENT iv
ABSTRACT v
ABSTRAK vi
CONTENTS vii
LIST OF TABLES x
LIST OF FIGURES xi
LIST OF SYMBOLS xiii
LIST OF ABBREVIATIONS xvi

CHAPTER 1 INTRODUCTION 1

1.1 Overview 1
1.2 Objective 3
1.3 Scope of Project 4
1.4 Research Methodology 5
1.6 Literature Review 6
1.7 Layout of Thesis 9

CHAPTER 2 MODELING OF A BALANCING ROBOT 10

2.1 Introduction 10
2.2 Mathematical modeling of the balancing robot 12

CHAPTER 3 DISCRETIZATION OF CONTINOUS-TIME STATE SPACE MODEL 21

3.1 Introduction 21
3.2 Discretization problem 22
    3.2.1 Closed-form discretizing transformation 22
    3.2.2 Open form discrete approximation by optimization 23
3.3 State space discretizing transformation 25
    3.3.1 Conventional method 26
    3.3.2 Extended/Polynomial discrete-time state space Model 27

CHAPTER 4 CONTROL DESIGN FOR BALANCING ROBOT 29

4.1 Introduction 29
4.2 Controllability 29
    4.2.1 Controllability of ZOH Equivalent model 33
4.3 Pole placement by digital state feedback 37
    4.3.1 Pole placement for an artibrary controllable system 39
    4.3.2 Adding a reference input 42
4.4 The Optimal Linear Quadratic Regulator 44

CHAPTER 5 SIMULATION RESULTS 48

5.1 Introduction 48
5.2 Simulation using pole placement by digital State feedback controller 49
5.2.1 The selection of statefeedbak controller gain 49
5.2.2 The effect of ZOH and sample peroid without Antialising filter and quantizer 54
5.2.3 The effect of antialising filter without quantizer 55
5.2.4 The effect of adding antialising filter with quantizer 59
5.2.5 Comparision of continous-time and discrete-time output response 60
5.3 Simulation using Linear Quadratic Regulator 61
5.3.1 The effect of ZOH and sample peroid without Antialising filter and quantizer 62
5.3.2 The effect of antialising filter without quantizer 63
5.3.3 The effect of adding antialising filter with quantizer 63
5.3.4 Comparision of continous-time and discrete-time output response 65

CHAPTER 6 CONCLUSION & FUTURE WORKS 67

6.1 Conclusion 67
6.2 Recommendation for future work 68

REFERENCES 70

LIST OF TABLES

TABLE NUMBER	TITLE	PAGE
5.1	MATLAB program to compute statefeedback vector	52

LIST OF FIGURES

FIGURE NUMBER	TITLE	PAGE
2.1	Autonomous Balancing Robot	12
2.2	Functional block diagram of balancing robot	12
2.3	Block Diagram of the DC motor	12
2.4	DC motor circuit	12
2.5	The free body diagram :wheeled of balancing robot	15
2.6	The free body diagram of chassis	17
4.1	A regulation system using digital state feedback controller	37
4.2	A ZOH design model for digital statefeedback	38
4.3	Modification of regulator system	44
5.1	Impulse response of the system	49
5.2	Matlab Simulink diagram of statefeedback controller	53
5.3	The system output response with effect of ZOH of sample period	55
5.4	The system output with effect of antialiasing filter	57
5.5	A uniform Quantizer	59
5.6	The system output response with quantizer effect	59
5.7	Comparison of continuous-time and discrete-time	60
	State feedback output response	61
5.8	Matlab Simulink diagram of LQR controller	61
5.9	The system output response with effect of ZOH response and sample period	62

5.10 The system output response with effect of antialising filter 63
5.11 The system output response quantization effect without antialiasing filter 64
5.12 The system output response quantization effect with antialiasing filter 65
5.13 Effect of quantizer without with antialiasing filter system 65
5.14 Comparison of Continuous-time and discrete-time LQR output response 66

LIST OF SYMBOLS

x - Displacement
$\dot{x}$ - Displacement velocity
ϕ - Angle
$\dot{\phi}$ - Angular velocity
θ - Parameters position
ω - Velocity
V_a - Applied torque
H_∞ - H-infinity
k_m - Torque constant
τ_m - Motor torque
I - Current that flow through the armature circuit
k_e - Back emf constant
R - Lumped armature winding resistance
L - Self inductance of the armature winding
J_a - Moment inertia of the armature
k_f - Frictional constant
R - Radius of the wheels
θ - Rotation angle of the wheels which is the same as the rotation angle of the armature
P_R - Reaction force between the wheel and the chassis of y-component of the force
H_R - Reaction force between the wheel and the chassis of x-component
H_{fR} - Friction force between ground and the wheel
M_w - Mass of the wheel

J_w - Moment inertia of the wheels

$H_{fR}r$ - Conversion of translational force into rotational force

G - Gravity 9.81 m/s^2

J_P - Moment inertia of the robot chassis

l - Distance between the centre of the wheel and the robot's centre gravity

M_P - Mass of the robot's chassis

CHAPTER 1

INTRODUCTION

1.1 Overview

Balancing robots are characterised by the ability to balance on its two wheels and spin on the spot similar to inverted pendulum. The inverted pendulum problem is common in the field of control engineering thus the uniqueness and wide application of technology derived from this unstable system has drawn interest from many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems like designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems.

This nonlinear control problem is surprisingly difficult to solve in a methodological approach due to two degrees of freedom, i.e, the balancing robot position and chassis angle using only one control input force. A practical problem with regard to control the balancing robot is similar to the concept designing a controller to swing the inverted pendulum up from a pendant position, achieve inverted stabilization, and simultaneously position.

In this thesis the balancing robots are characterized by the ability to balance on its two wheels and spin on the spot. The robot is composed of a chassis based on a stack of 130mm x 130mm Perspex plates carrying a Faulhaber DC motor, the Mark 4 Eyebot controller running on Robios version 5.2, a HOTEC GY-130 digital rate gyroscope, a SEIKA N3 digital inclinometer as described in (Thomas, 2002). The wheels of balancing robot are directly coupled to the output of the dc motor.

The balancing robot chasis is constructed from a single sheet of aluminium, drilled with holes for the easy mounting of motors, controller, sensors and battery pack. A pair of Faulhaber DC motors drive the robot's wheels. Each motor has a gear reduction of 54.2:1 and a torque constant of 6.9203 x 10^{-4} kg^{-m}/A. These motors have encapsulated encoders, and can be used to measure displacement and velocity of the robot. The robot is controlled by an EyeBot. A Mark 4 Eyebot controller running on Robios version 5.2 is used as the 'brain' of the balancing robot system.

The controller consists of a powerful 32-Bit microcontroller running at 33MHz, there is 512k ROM and 2048k RAM on board. The gyroscope modifies a servo control signal by an amount proportional to its measure of angular velocity. Instead of using the gyro to control a servo, we read back the modified servo signal to obtain a measurement of angular velocity. An estimate of angular displacement is obtained by integrating the velocity signal over time. The Inclinometer outputs an analogue signal, proportional to the angular displacement of the sensor (Braunl, 2002).

The balancing robot with two degree of freedom (DOF), is able to move along x, y axes describe by displacement, x and displacement velocity, $\dot{x}$ and chassis angle corresponding the angle, ϕ and angular velocity, $\dot{\phi}$. These four state space variable fully describe the dynamics of the 2 DOF system.

The balancing robot balance the load with its wheels while dragging the weight around on a pivot in a regular differential drive robot. This thesis will delve into the suitability and performance analysis of Pole Placement and LQR controllers in balancing the balancing robot in discrete-time environment.

1.2 Objective

The objectives of this research are as follows:

1. To formulate the complete mathematical dynamic model of the Balancing Robot using differential equation method.
2. To establish the state space model of the Balancing Robot using Digital State Space Approach.
3. To show mathematically that the Balancing Robot system is controllable and observable in discrete-time.
4. To design digital state feedback regulators for the Balancing Robot using pole placement approach and Optimal Controller (LQR).

5. To simulate the Balancing Robot continuous system and hybrid system using MATLAB-SIMULINK.

6. To demonstrate that the digital state space approach is as accurate as the continuous state space approach.

1.3 Scope of Project

The work undertaken in this project are limited to the following aspects:

1. Balancing Robot as described by Thomas Braunl (2002).

2. Digital State Space Approach as described by Richard J. Vaccaro (1995)

3. State feedback with Pole Placement Approach and Linear Quadratic Regulator.

4. Simulation on MATLAB-SIMULINK.

1.4 Research Methodology

The research work is undertaken in the following nine developmental stages:

1. Formulate the complete mathematical dynamic model using differential equation method.
2. Establish continuous state space mathematical model.
3. Linearization: Nonlinear equations of motion are linearized around the operating point .
4. Choose Sampling Interval.
5. Discretize the linearized continuous state space model to digital state space model.
6. Check the controllability and observability of ZOH Equivalent Models.
7. Design continuous-time and discrete-time state feedback controller using the pole placement method and LQR.
8. Verify the controller design of the balancing robot simulated on MATLAB SIMULINK.
9. Evaluate results

1.5 Literature Review

The research on balancing robot has gained momentum over the last decade. This is due to the nonlinear and inherent unstable dynamics of the system. The balancing problem extensively studied by numerous researchers (Mori, 1976). An understanding of how to control such a system will allow us to easily solve the other related control problems, such as single-link flexible manipulators (Yeung et al., 1990) and stabilization of a rocket booster by its own thrust vector. Below is several research that has been done by researches.

Yangsheng Xu (2004), developed a dynamic model for the single wheel robot and verified it through simulations and experiments. Using the linearization method, a linear state feedback approach to stabilize the robot at any desired lean angle was developed. This feedback provides means for controlling the steering velocity of the robot. Line following controller is developed for tracking any desired straight line while keeping balance. The controller is composed of two parts: the velocity control law and the torque control law. In the velocity control law, the velocity input (steering velocity) is designed for ensuring the continuity of the path curvature. Then, the robot can be stabilized for tracking a lean angle trajectory in which the steering velocity is identical to the desired value.

Henrik Niemann (2003) has derived a linear model of a double inverted pendulum system together with a description of the model uncertainties. For the double inverted pendulum system the trade-off between robust stability and performance is quite limited. There is not much space for reduction of the robustness to increase the performance of the system. The reason is the nonlinearities in the system together with the limitations/saturations in the system. The limitations in the system are e.g. maximal