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Time structure properties of $1/f$ noise and the applications of noise analysis

PhD dissertation theses

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SZEGED
2008

Introduction

Randomness is a fundamental property of the majority of the processes occurring in nature. The future behaviour of such processes cannot be predicted precisely, we can only determine certain probabilities at best. One reason for this unpredictability is the fact that systems are often too complicated to be described accurately. The other reason was proposed by quantum-mechanics, namely, that truly random phenomena can occur in nature; there are laws which yield not concrete values but probabilities.

As a result of these random phenomena, quantities measured on a physical system are fluctuating as well. This fact is often viewed as a harmful factor that limits the precision of measurements. However, recently the use of noise as a source of information and their constructive role emerge more and more often. Fluctuations coming from the system offer information about the system itself. Random signals can be used in the analysis of systems, for example the measurement of the transfer function. In certain cases, the constructive role of noise also arises, as in the case of dithering or stochastic resonance.

In order to exploit the possibilities inherent to the analysis of noise, a thorough knowledge of noises is necessary. We have to be familiar with the fundamental characteristics of noises and the behaviour of fluctuations in non-linear systems. Moreover, we have to devise suitable methods for the measurement and processing of noise.

In one part of my research I studied the time-structure properties of $1/f$ noises. $1/f$ noise is very common in several natural and artificial systems and it can be observed in numerous processes. However, no universal model has been proposed which would either describe its origins or predict its properties. In the present paper I examine the properties of the level-crossing statistics of $1/f^{\alpha}$

noise. Some research has been carried out in this field but only for specific cases. My aim was to examine the level-crossing statistics of different $1/f^{\alpha}$ ($0 \leq \alpha \leq 2$) coloured noises, the correlation between successive level-crossing intervals and to supplement theoretical results. The results have applications in numerous areas, like the theoretical analysis of stochastic resonance or even system diagnostics.

During my research I carried out several numerical simulations both in LabVIEW development system and using programs written in JAVA. Naturally, I have also carried out real measurements, on the one hand to support the results of the simulations, on the other hand to examine phenomena that would have been difficult to examine by means of numerical simulations. I used different noise generators for both the simulations and the real measurements, thus my work was also concerned with the development and examination of such devices. There are various ways to create random signals with certain characteristics. If we need analogue signals we can use, for example, noises occurring in semiconductors and employ filters to achieve the required spectral shape. One disadvantage of these methods is that it is difficult to adjust them to our needs. We can devise more flexible signal generators by using digital techniques. In my paper, I describe the model of an analogue noise generator based on a digital signal processor, I analyse the problems which may arise and propose solutions to them.

Stochastic resonance is a field in which we can directly observe and precisely measure the constructive role of noise. In these systems the adequate amount of additional noise can optimise the SNR measured on the output of the system. Stochastic resonance has been observed in numerous systems, for example, biological and technical systems; moreover, it can also be used to account for the alternation of ice ages. Earlier research was carried out mostly with periodical signals; a considerably low number of articles discuss aperiodical excitation. My work focused on the latter, as I used aperiodical signals for the study of non-linear systems. In order to determine the SNR, I used cross power spectral density and cross-correlation based analysis.

The constructive role of noise emerged in an unexpected area as well. With my colleagues we have developed a device used to synchronise the impulses of an excimer laser precisely. Our task was to keep the total delay of the laser system constant, so as to compensate the slow drift of the laser delay. The system contains delay lines to measure and control the laser delay and a microcontroller that runs our control algorithm. We encountered difficulties at two points: the relatively wide time detecting window of the delay detection and the random jitter of the laser. However, it was later realised that the latter may play a beneficial role. I developed the control algorithm of the system, analysed its behaviour and examined how the jitter noise influences the performance of the control.

New scientific results

1. With my colleagues we have developed a DSP (Digital Signal Processor) based $1/f^{\alpha}$ noise generator. The time series corresponding to the desired noise was generated by means of properly configured digital filters; then we get the analogue signal by using D/A converters. In my work I pointed out what effects modify the spectral shape of the generated noise. One of the most important of these is the asymmetric placement of the filters at the beginning and the end of the filter chain. These deviations can be handled by modifying the amplitude

of certain filters. I introduced a Monte-Carlo based optimisation procedure in order to obtain the ideal parameters for the filters. I optimised the parameters of the filters by means of numerical simulations in order to exploit the numerical range of the 16 bit fixed-point DSP to the maximum. I also examined in what ways the fixed-point calculation influenced the properties of the generated noise. I found that the generated noise met our requirements. We realised the noise generator: in the case of $1/f$ noise it followed the desired spectral shape over 4 decades, and the maximal sampling frequency reached 300 kHz. [1, 9].

2. I investigated the level crossing properties of $1/f^{\alpha}$ noises by means of numerical simulations. I analysed how the noise power exponent and the value of the crossed level influence the distribution of level crossing intervals. Furthermore, I examined if the level crossing statistics depend on the method of noise generation. In addition, I compared the results with real measurements. The results show that the level crossing statistics depend primarily on the power exponent and the crossed level, and not on the source of the noise. I investigated the effect of the noise bandwidth on the distribution of level crossing times, and concluded that both the lower and the higher cut-off frequency have a significant impact on the level crossing properties of $1/f$ noise. [4, 7]

3. Using numerical simulations, I examined the correlation between the successive level crossing intervals. The results showed that, in the case of $1/f$ noise, correlation is significantly high, and this points to the prominent role of $1/f$ noise. If we try to reconstruct the noise by using the level crossing statistics, even considering the correlation between successive level

crossings we do not get $1/f$ noise. In conclusion, the correlation between level crossings is unambiguously connected to the properties of $1/f$ noise and it is not a possible deficiency of the noise generators used. [4]

4. As regards white noise and $1/f^2$ noise, I supported the results of the measurements with a theoretical background, as the level crossing statistics of the measurements exactly matched the ones predicted. In the case of white noise, we find an exponentially decaying distribution, whereas for $1/f^2$ noise we find a power function. Although earlier research predicts a power function for $1/f$ noise, experimental results deviate significantly from this prediction. Considering the results of the measurements, I propose that one source of this difference is the low and high cut-off frequency of the noise.

   I fitted a formula with two parameters on the experimental results, and determined the value of this parameter as a function of power exponent α. The outstanding role of the $1/f$ noise can be observed here as well. [7, 8]

5. Based on research carried out by L. B. Kish, I examined the possibility of stochastic resonance in the case of non-periodic excitations. I studied three systems: the Schmitt trigger and the level crossing detector with numerical simulations, and the double well system with mixed signal simulation. Exciting the system with periodical signals, I compared the behaviour of the traditional signal-to-noise ratio definitions with newly proposed cross-spectrum and cross-correlation based methods. Both the simulations and measurements showed that cross-spectral analysis can be used without limitations for the characterisation of stochastic resonance, while cross-correlation offers useful insights on the operation of the system.

Using these recently proposed methods for the separation of signals and noise content, I used an aperiodic pulse train and band-limited noise as excitation signals. The results show that stochastic resonance occurs in all three systems, even with aperiodical excitation. Moreover, with an adequate amount of noise, we can get SNR gain values above 1.

According to generally accepted views, significant SNR gain mainly occurs with spiky signals near the threshold level. However, during my research I found that significant SNR gain is possible for several types of signals. In addition, approaching threshold level is not a necessary requirement. According to these results, we can conclude that earlier strict conditions can be linked to the traditional definition of SNR, rather than stochastic resonance itself. [2, 5, 6]

6. I have devised an algorithm which was based on adaptive averaging for a device used for the precise synchronisation of laser impulses. The created device was able to compensate reliably the drift of the laser it controlled, even in the presence of erroneous impulses. Using numerical simulations, I determined the performance of the control algorithm as a function the amount of the laser jitter, and I pointed out that the algorithm displayed a stochastic resonance-like behaviour. In the presence of a proper amount of jitter noise the error of the control was far below the detection window length. While the window length was 6 ns the error could be as small as 0.25 ns. [3, 10]

Papers on which the thesis is based

[1] R Mingesz – P Bara – Z Gingl – P Makra: Digital Signal Processor (DSP) based $1/f^\alpha$ noise generator. *Fluctuation and Noise Letters* , vol 4 (2004) L605–L616. p.

[2] R Mingesz – Z Gingl – P Makra: Marked signal improvement by stochastic resonance for aperiodic signals in the double-well system. *European Physical Journal B,* vol 50 (2006) 33-9-344. p.

[3] R Mingesz – Z Gingl – G Almási – A Csengeri – P Makra: Utilising jitter noise in the precise synchronization of laser pulses. *Fluctuation and Noise Letters* , vol 8 (2008) L41–L49. p.

[4] Z Gingl – R Mingesz – P Makra: On the amplitude and time-structure properties of $1/f^\alpha$ noises. *Third International Conference on Unsolved Problems of Noise and Fluctuation in Physics, Biology and High Technology (UPoN)*. Washington DC, USA, 2–6 September 2002. In bezrukov, s m (ed): *Proceedings of the Third International Conference on Unsolved Problems of Noise and Fluctuations in Physics, Biology and High Technology (AIP Conference Proceedings 665)*. Melville, 2003, American Institute of Physics, 578–583. p.

[5] R Mingesz – P Makra – Z Gingl: Cross-spectral analysis of signal improvement by stochastic resonance in bistable systems. *Fluctuations and Noise 2005*. Austin, Texas, USA, 24–26 May 2005. In kish, l b – lindenberg, k – gingl z (eds): *Noise in Complex Systems and Stochastic Dynamics III (Proceedings of SPIE Vol 5845)*. Bellingham, 2005, SPIE, 283–292. p.

[6] R Mingesz – Z Gingl – P Makra: Marked signal improvement by stochastic resonance for aperiodic signals in the double-well system. *News, Expectations and Trends in Statistical Physics, NEXT-SigmaPhi 3rd International Conference*. Kolymbari, Crete, 13–18 August 2005.

[7] R Mingesz – Z Gingl – P Makra: Level-crossing time statistics of Gaussian $1/f^{\alpha}$ noises. *Fluctuations and Noise*. Santa Fe, New Mexico, USA, 1–4 June 2003. In sikula, j (ed): *Proceedings of SPIE volume 5110: Fluctuations and Noise in Biological, Biophysical, and Biomedical Systems, edited by Sergey M Bezrukov &al.* Bellingham, 2003, SPIE, 312-319. p.

[8] R Mingesz – Z Gingl – P Makra: Level-crossing time statistics of Gaussian $1/f^{\alpha}$ noises. *17th International Conference on Noise and Fluctuations*. Prague, Czech Republic, 18–22 August 2003. In sikula, j (ed): *Proceedings of the 17th International Conference on Noise and Fluctuations.* Brno, 2003, CNRL, 505–508. p.

[9] R Mingesz – P Bara – Z Gingl – P Makra: Digital Signal Processor (DSP) based $1/f^{\alpha}$ noise generator. *Fluctuations and Noise 2004*. Maspalomas, Gran Canaria, Spain, 26–28 May 2004. In white, l b (ed): *Noise in Communication (Proceedings of SPIE Vol 5473).* Bellingham, 2004, SPIE, 213–221. p.

[10] R Mingesz – Z Gingl – G Almási – P Makra: Utilising jitter noise in the precise synchronization of laser pulses. *Fluctuations and Noise 2007*. Florence, Italy, 21–24 May 2007. In massimo macucci *&al*(ed): *Noise and fluctuations in circuits, devices and materials (Proceedings of SPIE Vol 6600).* Bellingham, 2007, SPIE, 6600 0Z. p.

Other papers and conference talks

[11] L B Kish – R Mingesz: Totally secure classical networks with multipoint telecloning (teleportation) of classical bits through loops with Johnson-like noise. *Fluctuation and Noise Letters* , vol 6 (2006) C9–C21. p.

[12] P Makra – Z Gingl – R Mingesz: Signal-to-noise ratio gain by stochastic resonance and its possible applications. *International Workshop on Stochastic Resonance: New Horizons in Physics and Engineering*. Drezda, Németország, 2004. október 4–7.

[13] L B Kish – R Mingesz – Z Gingl: Thermal noise informatics: totally secure communication via a wire, zero-power communication and thermal noise driven computing. *Fluctuations and Noise 2007*. Firenze, Olarszország, 2007. május 21–24. In massimo macucci *&al*(ed): *Noise and fluctuations in circuits, devices and materials (Proceedings of SPIE Vol 6600).* , 2007, SPIE, 6600 03. p.

[14] Z Gingl – P Makra – T Fülei – R Vajtai – R Mingesz: Colored noise driven stochastic resonance in a double well and in a FitzHugh-Nagumo neuronal model. *16th International Conference on Noise in Physical Systems and 1/f fluctuations (ICNF)*. Gainesville, USA, 2001. október 22–25. In bosman, g (ed): *Proceedings of the 16th International Conference on Noise in Physical Systems and 1/f fluctuations.* 2001, World Scientific, 420–423. p.