Unit commitment by nonlinear mixed variable programming *

G.M. Casolino[†], G. Liuzzi[‡], A. Losi[†]

Abstract

In this paper we consider the unit commitment problem and its solution via a nonlinear mixed variable programming algorithm. Indeed, the natural formulation of the problem involves both integer and continuous variables thus yielding an optimization problem solvable by a mixed variable algorithm. Our formulation of the problem besides taking into account ramp rate and minimum up and down time constraints, handles the size of the operator and the uncertainty related to the selling prices by defining different residual demand curves and using a scenario formulation. The objective function is indeed given by the expected value of the revenue over the different scenarios minus a term which takes into account the risk related to the decision. We report results for an operator managing a single unit and three units at the same time.

Keywords: Unit commitment, mixed variable programming, risk aversion.

1 Introduction

The unit commitment (UC) is the scheduling of power generating units over a daily (or weekly) time frame that optimizes some objective [11]. With a traditional UC the on/off status of the generating units is obtained, while the dispatching (optimal power flow) determines the best production level of the committed generators [5]. Indeed, the two aspects are the two faces of the same problem, which can be comprehensively referred to as the UC problem.
The UC is a mixed-integer nonlinear optimization problem. A huge effort has been devoted to it, since its results strongly affect the economics of the operation of a power system.
Models and methods have usually faced a centralized UC: in regulated markets, where a central authority controls its own generators to serve its loads, and also in deregulated markets, with a centralized unit commitment (assumed or even enforced) in spite of the decentralized ownership/control of generators.
Self-commitment, a decentralized unit commitment, calls now for the attention in liberalized, re-regulated markets; each generation company (GENCO) looks after his own interest, while market rules foster competitiveness and fairness, as well as market equilibrium and price stability [19]. System-wide objective function, such as min production costs or max social welfare, are substituted for individual one, max GENCO's profit, towards the so called profit-based UC [3, 10, 9, 7]. It is driven by the interest of generation companies for an operation as profitable as possible; in contrast, in the past there may have

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†Dipartimento di Ingegneria Industriale, Università degli Studi di Cassino, Via G. Di Biasio 43, 03043 Cassino (FR), Italy, {casolino, losi}@unicas.it.

‡Istituto di Analisi dei Sistemi e Informatica "A. Ruberti", Consiglio Nazionale delle Ricerche, V.le Manzoni 30, 00185 Roma, Italy, liuzzi@iasi.cnr.it.

been little interest in obtaining the ultimate cost saving, since the main focus was on satisfying load supply with adequate reserve margins.

For a system wide unit commitment, the most successful solution approach is based on the Lagrangian relaxation of the constraints that couples the different units (system demand and reserve) [11], adopting dynamic programming to schedule each unit separately, and on the updating of the Lagrange multipliers associated with the relaxed constraints [18]. This has been the mainly used approach, although it generates sub-optimal (near-optimal) solutions. Other approaches have been proposed (augmented Lagrangian relaxation, integer programming, evolutionary algorithms) all intended to overcome the duality gap of the Lagrangian relaxation methods, while the computational burden is kept acceptable [4, 3, 20].

In committing their units, GENCOs face uncertainty; in the following we make specific reference to the GENCO's UC for the day-ahead market. In perfect competition, a GENCO is a price-taker, and its short term commitment decisions depend on the spot price forecast. In oligopolistic markets, a GENCO has to forecast its residual demand curve, which depends on the forecasted load demand and competitors' behavior. The UC is a decision making process which has to consider both the risk of a bid not being accepted and the profit loss that can be incurred into if a low spot price or a low residual demand has been forecasted [1].

A stochastic approach is able to completely handle the uncertainties that characterize the problem and to give evidence to the risk aversion of the decision-maker. The objective function directly accounts for the uncertainties of the problem, by including the expected value of the GENCO's profit over the scenarios chosen to represent the uncertain future; at the same time, it directly includes the risk aversion of the decision-maker, by considering the variance of the profit through an appropriate scaling factor [13, 6].

In the paper, the UC problem is modelled with a stochastic approach. A detailed description of the problem is proposed, to have a complete representation of the actual possibilities of the generating units and of the actual costs incurred in operation. Besides production costs and limits on the operating range of the units, also ramp rate and minimum up and down times constraints are treated, as well as startup and shutdown costs. Revenues have been modelled with reference to a realistic oligopolistic market structure; it determines a coupling between the unit outputs of a multi-unit GENCO, which adds complexity.

To solve the problem, we adopt a recently proposed algorithm, well suited to treat all the above-mentioned characteristics of UC problem [15, 14]. Different cases have been solved: a single unit GENCO, with a small ability to influence the market clearing price, and a multi-unit GENCO whose output has a greater influence on the market price. The results are compared with the ones obtained with a standard local search algorithm [17, 21].

2 The single-unit commitment problem

Let $\mathcal{T} = \{1, \ldots, T\}$ be a set of consecutive time intervals covering some time horizon (e.g. a day or a week). We denote by $\mathcal{T} - 1$ the set $\{0, \ldots, T-1\}$. The problem is that of determining the operating status of a thermal generating unit in each time interval $t \in \mathcal{T}$ so as to maximize the difference between the expected profit over the time horizon and a term related to the risk of the committing.

2.1 Feasible activation and power production schedules

A commitment of the generating unit on the set $\mathcal{T}$ is completely specified by the operating status (active or inactive) and the amount of generated power in each time interval $t \in \mathcal{T}$. Let us denote by the binary variables $z_t \in \{0,1\}$, $t \in \mathcal{T}$, the operating status of the unit. Namely, $z_t = 1$ means that the unit is active in time interval t and $z_t = 0$ that it is inactive. Hence $z \in \{0,1\}^T$ represents an *activation schedule* of the generating unit. Furthermore, let us denote by $q_t \in \Re_+$, $t \in \mathcal{T}$, the amount of power produced in time interval t. Moreover, let $z_0 \in \{0,1\}$, $q_0 \geq 0$, $\hat{y}_0$ and $\tilde{y}_0$ be given constants specifying the initial operating status of the unit at the beginning of the time horizon; $\hat{y}_0$ and $\tilde{y}_0$ represent the number of consecutive time intervals in which the unit has been uncommitted and, respectively, committed at the beginning of the time horizon. More precisely, if $z_0 = 1$ then $\tilde{y}_0 > 0$ and $\hat{y}_0 = 0$, whereas, if $z_0 = 0$ then $\tilde{y}_0 = 0$ and $\hat{y}_0 > 0$.

The commitment of the generating unit is subject to a number of technical constraints so that not all vectors $z \in \{0,1\}^T$ and $q \in \Re_+^T$ are representative of feasible commitment schedules for the generating unit.

To begin with, an activation schedule for the generating unit z must satisfy so-called *minimum up* and *down* constraints. Namely, if the unit has been committed (uncommitted) in time interval t, it must remain committed (uncommitted) until time interval $t + t_{up}$ ($t + t_{dw}$). In order to express these constraints, we need to introduce additional discrete variables to detect changes in the operating status of the unit and to count the number of consecutive up and down time intervals [7].

Let $u_t \in \{-1,0,1\}$, $t \in \mathcal{T} - 1$ be a set of discrete variables indicating a change in the operating status, if any. Namely, $u_t = 1$ when $z_t = 0$ and $z_{t+1} = 1$; $u_t = -1$ when $z_t = 1$ and $z_{t+1} = 0$; $u_t = 0$ otherwise, that is, $z_t = z_{t+1}$. In order to express the aforementioned relationship between the $u(t)$'s and $z(t)$'s variables, the following set of constraints is needed:

$$u(t) = z(t+1) - z(t), \qquad \forall\, t \in \mathcal{T} - 1. \tag{1a}$$

Then, we denote by $\tilde{z}_t \in I\!N_0$, $t \in \mathcal{T}$ the number of consecutive time intervals in which the unit has been committed ($z_t = 1$) up to time interval t. Similarly, let $\hat{z}_t \in I\!N_0$, $t \in \mathcal{T}$ denote the number of consecutive time intervals that the unit has been uncommitted ($z_t = 0$) up to time interval t. In order for $\tilde{z}_t$ and $\hat{z}_t$ to act as we have just described, they must satisfy the following group of constraints.

$$\begin{aligned}
\hat{z}_0 &= \hat{y}_0, && \\
\hat{z}_{t+1} - \hat{z}_t &\leq 1, && \forall\, t \in \mathcal{T} - 1, && \text{(2a)}\\
\hat{z}_t - \hat{z}_{t+1} &\leq M z_{t+1} - 1, && \forall\, t \in \mathcal{T} - 1, && \text{(2b)}\\
\hat{z}_t &\leq M(1 - z_t), && \forall\, t \in \mathcal{T} - 1. && \text{(2c)}
\end{aligned}$$

$$\begin{aligned}
\tilde{z}_0 &= \tilde{y}_0, && \\
\tilde{z}_{t+1} - \tilde{z}_t &\leq 1, && \forall\, t \in \mathcal{T} - 1, && \text{(3a)}\\
\tilde{z}_t - \tilde{z}_{t+1} &\leq M(1 - z_{t+1}) - 1, && \forall\, t \in \mathcal{T} - 1, && \text{(3b)}\\
\tilde{z}_t &\leq M z_t, && \forall\, t \in \mathcal{T} - 1, && \text{(3c)}
\end{aligned}$$

where $M \gg 1$ is a constant such that $M > \max\{\hat{y}_0, \tilde{y}_0\} + T$. We are now in the position to express the minimum up and down constraints, that is,

$$\begin{aligned}
t_{up} - \tilde{z}_t &\leq M(1 + u_t), && \forall\, t \in \mathcal{T}, && \text{(4a)}\\
t_{dw} - \hat{z}_t &\leq M(1 - u_t), && \forall\, t \in \mathcal{T}. && \text{(4b)}
\end{aligned}$$

Now, let $h : \{0,1\}^T \rightarrow \{-1,0,1\}^{T-1} \times I\!N_0^T \times I\!N_0^T \cup \{\infty\}^{3T-1}$ be a vector of discrete-valued functions such that

$$h(z) = \begin{cases} (u^\top, \tilde{z}^\top, \hat{z}^\top)^\top & \text{if } z, u, \tilde{z}, \hat{z} \text{ satisfy (1)–(4)} \\ (\infty, \ldots, \infty)^\top & \text{otherwise,} \end{cases} \tag{5}$$

where symbol $\top$ denotes transposition.
If we denote by $\mathcal{F}_z \subseteq \{0,1\}^T$ the feasible set of activation schedules, that is, the set of $z \in \{0,1\}^T$ such that the minimum up and down time constraints are met, then

$$\mathcal{F}_z = \{z \in \{0,1\}^T : h(z) \neq (\infty, \ldots, \infty)^\top)\}.$$

Now, given a feasible activation schedule $z \in \mathcal{F}_z$, a feasible production schedule q_t, $t \in \mathcal{T}$, is such that when $z_t = 0$, for some $t \in \mathcal{T}$, $q_t = 0$ and, on the contrary, a component q_t strictly positive implies $z_t = 1$. Besides these constraints, when the unit is committed q_t must remain bounded by maximum and minimum output levels, P_{max} and P_{min}, respectively. Moreover, the power output in two consecutive time intervals, say t and $t+1$, is limited by so-called *ramp-up* and *ramp-down* constraints. Hence we get the following group of constraints.

$$P_{min} z_t \leq \quad q_t \quad \leq P_{max} z_t, \qquad \forall\, t \in \mathcal{T}. \tag{6a}$$
$$q_{t+1} - q_t \quad \leq \quad R_{up}, \qquad \forall\, t \in \mathcal{T} - 1, \tag{6b}$$
$$q_{t+1} - q_t \quad \geq \quad -R_{dw}, \qquad \forall\, t \in \mathcal{T} - 1, \tag{6c}$$

where R_{up} and R_{dw} are the maximum ramp-up and ramp-down power rates and we assume that $P_{min} \leq \min\{R_{up}, R_{dw}\}$.
Let us denote by $\mathcal{F}_q(z) \subseteq \Re_+^T$ the feasible set of power levels, that is

$$\mathcal{F}_q(z) = \{q \in \Re_+^T : (q, z) \text{ satisfies (6)}\}.$$

2.2 Objective function

The objective function is the difference between the expected profit over the time horizon and a term related to the risk of the committing. The profit is the difference between the incomes due to selling the power in the day-ahead market and the sum of production costs and startup/shutdown costs.

2.2.1 Costs

Let us first consider the production costs. When the unit is operational and producing power, that is $z_t = 1$ and $q_t \geq P_{min}$, it is subject to a convex cost of the form

$$A q_t^2 + B q_t + C,$$

where A, B, C are positive constants. Besides these production costs, the unit is subject to costs which only depends on $h(z)$ for each $z \in \mathcal{F}_z$. They are:

- The shutdown cost, which is constant and equal to SDC; it is paid for whenever the unit is turned off, that is when $u(t) = -1$.

- The startup cost, which depends nonlinearly on the number of time intervals in which the unit has remained uncommitted and has to be paid whenever $u(t) = 1$. In particular:

$$SUC(\hat{z}_t) = TS + BS \left(1 - \exp - \frac{\hat{z}_t}{\tau}\right).$$

The total cost is therefore given by

$$C_{tot}(z, q, h(z)) = \sum_{t=1}^{T} \left(Aq_t^2 + Bq_t + Cz_t\right) +$$
$$\sum_{t=0}^{T-1} u_t \left(SUC(\hat{z}_t)\frac{u_t + 1}{2} + SDC\frac{u_t - 1}{2}\right).$$

2.2.2 Incomes

As for the income due to selling power produced by the unit, let $\rho(q_t; t)$ be the residual demand function for the considered unit: at each time interval $t \in \mathcal{T}$, $\rho(q_t; t)$ is the price level at which it is possible to sell the electric power q_t.

2.2.3 Profits

Hence, for the profits we have that

$$\mathcal{P}(z, q, h(z)) = \sum_{t=1}^{T} \rho(q_t; t)q_t - C_{tot}(z, q, h(z)).$$

It is not realistic to assume that functions $\rho(\cdot; t)$, $t \in \mathcal{T}$, are completely known; we assume instead they depend on q_t and θ_t, where θ_t is a random variable. Moreover, let each θ_t have only a finite number of different realizations $\theta_t^1, \ldots, \theta_t^{n_t}$ with associated probabilities $\pi_t^1, \ldots, \pi_t^{n_t}$.
Now, since $\rho = \rho(q_t, \theta_t; t)$, profit $\mathcal{P}$ depends on the random vector $\theta = (\theta_1, \ldots, \theta_T)$ too. Hence:

$$\mathcal{P}(z, q, h(z), \theta) = \sum_{t=1}^{T} \rho(q_t, \theta_t; t)q_t - C_{tot}(z, q, h(z)).$$

2.2.4 Objective function

Taking expectation and variance of $\mathcal{P}$ we get:

$$\begin{aligned}
E_\theta[\mathcal{P}(z, q, h(z), \theta)] &= \sum_{t=1}^{T} E_\theta[\rho(q_t, \theta_t; t)]q_t - C_{tot}(z, q, h(z)), \\
\sigma_\theta^2[\mathcal{P}(z, q, h(z), \theta)] &= \sum_{t=1}^{T} \sigma_\theta^2[\rho(q_t, \theta_t; t)]q_t^2 + \sum_{t=1}^{T} \sum_{\ell=1, \ell \neq t}^{T} q_t q_\ell cov_\theta[\rho(q_t, \theta_t; t), \rho(q_\ell, \theta_\ell; \ell)].
\end{aligned}$$

To account for the risk in committing, we consider the following objective function, as in [13]:

$$U(z, q, h(z)) = B_r \sigma_\theta^2[\mathcal{P}(z, q, h(z), \theta)] - E_\theta[\mathcal{P}(z, q, h(z), \theta)],$$

where

$$B_r = \frac{A_r}{2C_{tot}};$$

constant A_r is the so called risk aversion coefficient, that allows to account for the risk attitude of the decision maker. For an average risk averse decision maker, a value $A_r = 3$ has been derived based on historical data [13].

To conclude, the single-unit commitment problem can be thus posed as

$$\begin{array}{ll} \min & U(z, q, h(z)), \\ s.t. & z \in \mathcal{F}_z, \\ & q \in \mathcal{F}_q(z). \end{array} \tag{7}$$

3 The multi-unit commitment problem

In this section we extend the unit commitment problem to the case where possibly more than one unit is to be committed for power production. Let us suppose we have N_u generating units and denote by $q_t^{(i)}$ and $z_t^{(i)}$, $i = 1, \ldots, N_u$, the power production and activation status of unit i at time interval $t \in \mathcal{T}$. The units are assumed to behave "independently", that is, their *technical constraints* (cfr. subsection 2.1) do not interfere with each other. However, even in this setting, it is not possible (nor profitable) to consider N_u distinct optimization problems to find optimal commitment and power production schedules for each unit. It comes from the fact that a GENCO managing all the units faces a global residual demand for each time interval $t \in \mathcal{T}$. Thus, for $t \in \mathcal{T}$,

$$\rho = \rho\left(p_t, \theta_t; t\right),$$

where

$$p_t = \sum_{i=1}^{N_u} q_t^{(i)}.$$

Hence, the multi-unit commitment problem can be posed as

$$\begin{array}{ll} \min & U\left(x, p, H(x), \theta\right), \\ s.t. & x \in \mathcal{F}_x, \\ & p \in \mathcal{F}_p(x), \end{array} \tag{8}$$

where $x = (z^{(1)\top}, \ldots, z^{(N_u)\top})^\top$, $p = (q^{(1)\top}, \ldots, q^{(N_u)\top})^\top$,

$$H : \{0,1\}^{N_u T} \to \prod_{i=1}^{N_u} \left(\{-1,0,1\}^{T-1} \times I\!N_0^T \times I\!N_0^T \cup \{\infty\}^{3T-1}\right),$$

$$H(x) = \begin{pmatrix} h(z^{(1)}) \\ \vdots \\ h(z^{(N_u)}) \end{pmatrix}, \quad \mathcal{F}_x = \prod_{i=1}^{N_u} \mathcal{F}_z^{(i)}, \quad \mathcal{F}_p(x) = \prod_{i=1}^{N_u} \mathcal{F}_q^{(i)}(z^{(i)}),$$

$$\begin{array}{rcl} \mathcal{F}_z^{(i)} & = & \{z^{(i)} \in \{0,1\}^T : h(z^{(i)}) \neq (\infty, \ldots, \infty)^\top\}, \\ \mathcal{F}_q^{(i)}(z^{(i)}) & = & \{q^{(i)} \in \Re_+^T : (q^{(i)}, z^{(i)}) \text{ satisfies (6)}\}. \end{array}$$

4 A neighborhood search approach

When the discrete activation/deactivation variables are held fixed, say $x = \bar{x} \in \mathcal{F}_x$, Problem (8) becomes a standard linearly constrained optimization problem which can be put in the form

$$\begin{array}{ll} \min & f(p) = U(\bar{x}, p, H(\bar{x})), \\ s.t. & p \in \mathcal{F}_p(\bar{x}) \end{array} \tag{9}$$

where $\mathcal{F}_p(\bar{x}) = \{p \in \Re^T : A(\bar{x})p \leq b(\bar{x})\}$.
The main characteristics of Problem (8) is that its objective function nonlinearly depends on the residual demand functions $\rho(p_t, \theta_t; t)$ which is a nonsmooth function. Hence, we choose to solve Problem (9) by the following derivative free optimization method for linearly constrained problems [16], which we report in the following for completeness.

4.1 A derivative-free method for linearly constrained problems

We begin this subsection by recalling some basic definitions and assumptions which are needed to describe the derivative-free method.
For any $\epsilon > 0$ and $p \in \mathcal{F}_p(\bar{x})$, we define the set of indices of ϵ-active constraints by

$$I(p; \epsilon) = \{j : a_j^\top p \geq b_j - \epsilon\},$$

and the ϵ-approximation of the cone of feasible directions by

$$T(p; \epsilon) = \{d \in \Re^T : a_j^\top d \leq 0, \forall j \in I(p; \epsilon)\}.$$

In order to define a derivative-free method for the solution of Problem (9), we have to associate a suitable set of search directions with each point p^j produced by the algorithm. This set should have the property that the local behavior of the objective function in each direction in the set provides sufficient information to overcome the lack of the gradient. Formally, we introduce the following assumption.

Assumption 4.1 *Let $\{p^j\}$ be a sequence of feasible points and $\{D^j\}$ a sequence of sets of search directions. Then, for all j,*

$$D^j = \{d_i^j : \|d_i^j\| = 1, \quad i = 1, \dots, r^j\},$$

and, for some constant $\bar{\epsilon} > 0$,

$$cone\{D^j \cap T(p^j; \epsilon)\} = T(p^j; \epsilon) \quad \forall \epsilon \in [0, \bar{\epsilon}].$$

Moreover, $\bigcup_{j=0}^{\infty} D^j$ is a finite set and r^j is bounded.

In order to compute sets of search directions D^j satisfying Assumption 4.1, we follow the approach proposed in [12].
Following [16], the derivative-free minimization algorithm for linearly constrained optimization problems can be stated as follows.

Algorithm DFA$(p^1, \alpha_{\text{tol}})$

Data. $\alpha^\circ > 0$.

1. Set $j = 1$.
2. Apply procedure $DF(j, p^j, \alpha^{j-1}, p^{j+1}, \alpha^j)$.
3. If $\alpha^j > \alpha_{\text{tol}}$ then set $j := j + 1$ and go to step 2.
   else return (p^{j+1}, α^j).

Procedure DF$(j, p, \mu^0, \tilde{p}, \mu)$

Data: $\gamma > 0$, $\delta, \delta_1, \theta \in (0, 1)$, $\bar{\epsilon} > 0$.

Step 1: *(Computation of search directions)*

Choose $D^j = \{d_1^j, \dots, d_{r_j}^j\}$ satisfying Assumption 4.1.

Step 2: *(Minimization on the cone$\{D^j\}$)*

Step 2.1: *(Initialization)* Set $i = 1$, $y_i = p$, $\tilde{\alpha}_i = \mu^0$.

Step 2.2: *(Computation of the initial stepsize)*

Compute the maximum steplength $\bar{\alpha}_i$ such that $y_i + \bar{\alpha}_i d_i^j \in \mathcal{F}_p(\bar{x})$
and set $\alpha = \min\{\bar{\alpha}_i, \tilde{\alpha}_i\}$.

Step 2.3: *(Test on the search direction)*

If $\alpha > 0$ and $f(y_i + \alpha d_i^j) \leq f(y_i) - \gamma \alpha^2$, go to Step 2.4
otherwise set $\alpha_i = 0$, $\tilde{\alpha}_{i+1} = \theta \tilde{\alpha}_i$ and go to Step 2.5.

Step 2.4: Let $\hat{\alpha} = \min\{\bar{\alpha}_i, (\alpha/\delta)\}$.

If $\alpha = \bar{\alpha}_i$ or $f(y_i + \hat{\alpha} d_i^j) > f(y_i) - \gamma \hat{\alpha}^2$ Then set $\alpha_i = \alpha$ and go to Step 2.5.
Else set $\alpha = \hat{\alpha}$ and go to Step 2.4.

Step 2.5: *(New point)* Set $y_{i+1} = y_i + \alpha_i d_i^j$.

Step 2.6: *(Test on the minimization on the cone$\{D^j\}$)*

If $i = r^j$, go to Step 3;
otherwise set $i = i + 1$ and go to Step 2.2.

Step 3: *(Main iteration)* Set $\tilde{p} = y_{i+1}$, $\mu = \max_{i=1,\dots,r^j}\{\tilde{\alpha}_i, \delta_1 \mu^0\}$.

Algorithm DFA can be used to solve Problem (9), that is Problem (7) when $x = \bar{x} \in \mathcal{F}_x$.
Obviously, this is not satisfactory in that the computed solution (activation/deactivation schedule plus production levels) $(\bar{x}, p^\star)$ depends on the chosen activation/deactivation schedule $\bar{x}$. Moreover, it is very often the case that, by modifying a little the activation/deactivation schedule $\bar{x}$ by considering in its place $\hat{x} \in \mathcal{F}_x$, the solution of Problem (9) will provide us with a different production plan $p^\dagger$ such that

$$U(\bar{x}, p^\star, h(\bar{x})) > U(\hat{x}, p^\dagger, h(\hat{x})).$$

For this reason, we can try and solve Problem (8) by using some *local search* heuristic.

4.2 Local search approach

Generally speaking, a local search method is completely specified by defining a neighborhood system, that is, a point-to-set map $\mathcal{N}$ that associates any given feasible point $(\bar{x}, \bar{p}) \in \mathcal{F}_x \times \mathcal{F}_p(\bar{x})$ with a subset $\mathcal{N}(\bar{x}, \bar{p}) \subset \mathcal{F}_x \times \mathcal{F}_p(\bar{x})$. In order to simplify this subsection, we report the discrete neighborhood definition in the appendix.

The neighborhood system $\mathcal{N}$ allows us to define the following local search algorithm for the approximate solution of Problem (8).

Local search Algorithm (LSA)

> **Data:** $x^\circ \in \mathcal{F}_x$, $p^\circ \in \mathcal{F}_p(x^\circ)$, $tol > 0$.
>
> Set $k = 0$, $\tilde{x} = x^k$, $\tilde{p} = p^k$, `success=true`
>
> **Repeat**
>
>   **Foreach** $(x, p) \in \mathcal{N}(x^k, p^k)$
>
>   Compute $(q, \alpha) = \text{DFA}(p, tol)$
>
>   **If** $(U(x, q, h(x)) < U(\tilde{x}, \tilde{p}, h(\tilde{x})))$ set `success=true`, $\tilde{x} = x$, $\tilde{p} = q$
>
>   **End Foreach**
>
>   **If** (`success`) **Then**
>
> $x^{k+1} = \tilde{x}$, $p^{k+1} = \tilde{p}$, `success=false`
>
>   **Else** $x^{k+1} = x^k$, $p^{k+1} = p^k$ **End If**
>
>   Set $k = k + 1$
>
> **Until** (`success`)

Even though Algorithm LSA is, in principle, able to find a good local solution of Problem (8), its main limit is the unacceptably large computational time that it requires to get convergence. This is due, on the one hand, to the cardinality of the discrete neighborhoods that get explored during the optimization and, on the other hand, to the fact that every local minimization is completely carried out. Namely, for a given choice of the discrete variables, we solve Problem (9) as best as we can by using Algorithm DFA. In the next section we present a solution technique which is able to handle explicitly the mixed variable nature of Problem (8) thus considerably reducing the computational burden of the solution process.

5 A mixed variable approach

Problem (8) is a mixed variable programming problem. The presence of both continuous and discrete variables requires a suitable definition of local minimum point, which is not immediate. In fact this notion refers to the behavior of the objective function in a "suitable neighborhood" of a given point. While a neighborhood of a continuous variable is well represented by a continuous ball, the neighborhood of a discrete variable must be defined taking into account the structure of the particular problem.

As in [2], we can characterize a local solution (x^*, p^*) of Problem (8) as a point satisfying the following definition:

Definition 5.1 *A feasible point (x^*, p^*) is said to be a local minimizer of Problem (8) with respect to the feasible discrete neighborhood $\mathcal{N}(x^*, p^*)$ if there exists an $\epsilon > 0$ such that $\forall(\hat{x}, \hat{p}) \in \mathcal{N}(x^*, p^*)$*

$$U(x^*, p^*, h(x^*)) \leq U(x, \hat{p}, h(x)) \qquad \forall x \in \mathcal{B}(\hat{x}, \epsilon) \cap \mathcal{F}_p(x), \tag{10}$$

where $\mathcal{N}(x^*, p^*)$ is a finite set of feasible points.

This definition implies that there are not better feasible solutions in the balls centered at the points belonging to the discrete neighborhood of (x^*, p^*). Note that this definition depends on the choice of the discrete neighborhoods, which hence represent a measure of the quality of the solution. In fact, a larger discrete neighborhood $\mathcal{N}(x^*, p^*)$ should give a better local minimizer, but this may increase the computational effort needed to locate the solution, so there is a trade off.
In [15] it has been introduced an algorithm for solving mixed variable programming problems based on the combination of a local search with respect to the continuous variables and of a local search in the discrete neighborhood of the current point. This algorithm has been applied to the solution of Problem (8). In particular, it is based on the idea to alternate two phases:

- an attempt to update the continuous variables by a local continuous search (Phase 1) in $\mathcal{F}_p(x)$,
- an attempt to update the discrete variables by a local search in the discrete neighborhood of the current point (Phase 2).

Phase 1:
Given the current feasible point (x_k, p_k), the discrete variables are fixed to the value x_k and the following continuous optimization problem is considered:

$$\begin{aligned} &\min_p U(x_k, p, h(x_k)) \\ &p \in \mathcal{F}_p(x_k). \end{aligned} \tag{11}$$

Starting from p_k, we perform an iteration of a derivative free local continuous search with the goal of finding a new vector $\tilde{p}_k$ which is, roughly speaking, a better approximation of a stationary point of Problem (11).

Phase 2:
In this phase we try to update the discrete variables by considering the points belonging to the discrete neighborhood $\mathcal{N}(x_k, \tilde{p}_k)$ of the point $(x_k, \tilde{p}_k)$ produced by Phase 1.
First, we simply evaluate the objective function at the points belonging to $\mathcal{N}(x_k, \tilde{p}_k)$. If one of these points produces a sufficient decrease from $U(x_k, \tilde{p}_k, h(x_k))$, then it becomes the current point, and a new iteration is performed.
If none of the points belonging to $\mathcal{N}(x_k, \tilde{p}_k)$ produces a sufficient decrease with respect to $U(x_k, \tilde{p}_k, h(x_k))$, we perform a further investigation, by selecting some of these points which can be considered promising. In particular, we still try to update the discrete variables by selecting some points belonging to $\mathcal{N}(x_k, \tilde{p}_k)$ with objective value not significantly worse than $U(x_k, \tilde{p}_k, h(x_k))$. Starting from each one of these points, we perform a suitable number of local continuous searches with the aim to obtain a point which produces a sufficient decrease from $U(x_k, \tilde{p}_k, h(x_k))$.

The proposed algorithm model is formally stated as follows:

Mixed Integer Variable Algorithm (MIVA)

Data: $x^\circ \in \mathcal{F}_x$, $p^\circ \in \mathcal{F}_p(x^\circ)$, $\xi \geq 0$, $\theta \in (0,1)$, $\eta^0 > 0$, $\mu^{in} > 0$.

Step 0: Set $k = 0$, $\mu^{k,0} = \mu^{in}$.

Step 1: Compute $\tilde{x}^k$ and μ^k by applying Procedure DF$(k, p^k, \mu^{k,0}, \tilde{p}^k, \mu^k)$ to Problem (11) and set $\mu^{k+1,0} = \mu^k$.

Step 2: If there exists a $(\hat{x}^{k+1}, \hat{p}^{k+1}) \in \mathcal{N}(x^k, \tilde{p}^k)$ such that

$$U(\hat{x}^{k+1}, \hat{p}^{k+1}, h(\hat{x}^{k+1})) \leq U(x^k, \tilde{p}^k, h(x^k)) - \eta^k,$$

set $x^{k+1} = \hat{x}^{k+1}$, $p^{k+1} = \hat{p}^{k+1}$, $\eta^{k+1} = \eta^k$, and go to step 5.

Step 3: Define $W^k = \{(x,p) \in \mathcal{N}(x^k, \tilde{p}^k) : U(x,p,h(x)) \leq U(x^k, \tilde{p}^k, h(x^k)) + \xi\}$.

3.1: If $W^k \neq \emptyset$, choose $(x', p') \in W^k$, set $j = 1$, $p^j = p'$, $\mu^{j-1} = \mu^k$.

Otherwise go to Step 4.

3.2: Compute p^{j+1} and μ^j by applying Procedure DF$(j, p^j, \mu^{j-1}, p^{j+1}, \mu^j)$ to Problem (11).

3.3: If $U(x', p^{j+1}, h(x')) \leq U(x^k, \tilde{p}^k, h(x^k)) - \eta^k$, set $p^{k+1} = p^{j+1}$, $x^{k+1} = x'$, $\eta^{k+1} = \eta^k$, and go to step 5.

3.4: If $\mu^j > \mu^k$, set $j = j+1$, and go to 3.2.

Otherwise set $W^k = W^k \setminus \{(x', p')\}$, and go to 3.1.

Step 4: Set $p^{k+1} = \tilde{p}^k$, $x^{k+1} = x^k$.
If $U(x^{k+1}, p^{k+1}, h(x^{k+1})) \leq U(x^k, p^k, h(x^k)) - \eta^k$, set $\eta^{k+1} = \eta^k$.

Otherwise set $\eta^{k+1} = \theta\eta^k$.

Step 5: Set $k = k+1$, and go to step 1.

At Step 1, Phase 1 is performed by applying the local continuous search DF$(k, p^k, \mu^{k,0}, \tilde{p}^k, \mu^k)$. This procedure tries to produce a new point $\tilde{p}^k$, where the objective function is sufficiently decreased. In particular, if the procedure DF is not able to produce a sufficient decrease of the objective function, the point $\tilde{p}^k$ is set equal to p^k.
Phase 2 is performed in Steps 2 and 3. In particular, at Step 2 the objective function is evaluated at each point in $\mathcal{N}(x^k, \tilde{p}^k)$. If one of these points produces a decrease with respect to $U(x^k, \tilde{p}^k, h(x^k))$ greater than or equal to η^k, it becomes the current point and a new iteration is started. Otherwise the discrete neighborhood is further investigated in Step 3. In particular, a set $W^k \subseteq \mathcal{N}(x^k, \tilde{p}^k)$ of points with objective value not significantly worse than $U(x^k, \tilde{p}^k, h(x^k))$ is selected. Each of these points $(x', p') \in W^k$ is considered promising, and the algorithm tries to determine if it is worth replacing x^k with x'. In details, starting from each point $(x', p') \in W^k$, the local continuous search is repeated until

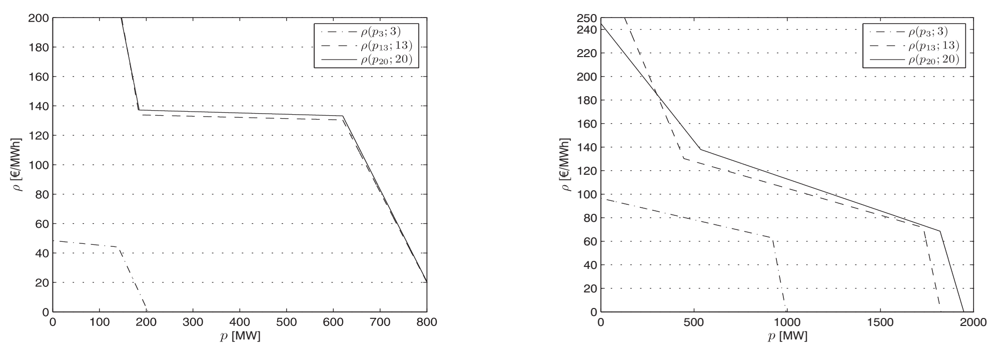
(a) it finds a point significantly better than $(x^k, \tilde{p}^k)$, or

(b) the test at Step 3.4 fails.

In case (a), the new point becomes the current iterate (with new discrete variables x'), and a new iteration is performed. In case (b), we reject the discrete variables x' because a sufficient decrease of the objective function has not been achieved.
At Step 4 the point $(x^k, \tilde{p}^k)$ becomes the new current point and, if neither the local continuous search nor the discrete search have been able to produce a decrease of the objective function greater or equal to η^k, then this parameter is reduced before starting the next iteration.
For an analysis of the theoretical properties of Algorithm MIVA we refer to [15] where global convergence of algorithm MIVA toward a stationary point of the mixed variable problem is proved provided that the objective function is smooth with respect to the continuous variables. Algorithm MIVA has been recently used to solve a difficult problem related to the optimal design of an induction motor [14].

6 Numerical results

In this section we report and analyze the results obtained for both the single- and multi- UC problem. We obtained the residual demand curves $\rho(p_t; t)$ for the case of the Italian electricity market. We considered the historical data freely distributed by the Italian Gestore del Mercato Elettrico (the independent market operator), and we defined $\rho(p_t; t)$ as a piecewise linear function having three segments with different slopes. The first and last segments are the steepest ones and represent situations in which small variations of power output determine large variations in selling prices. On the other hand, the middle segment represents the case of limited dependence of selling prices on the produced power. As an example, in Figure 1(a) and (b) we report, as an example, three residual demand curves at hours $t = 3$ (off-peak hour) and $t = 13, 20$ (peak hours) for the two producers: the first, referred to as *small-GENCO*, has a maximum producing capacity of 600 MW and a limited influence on the market price, while the second, referred to as *large-GENCO*, has a maximum producing capacity of 1800 MW and a significat influence on the market price.



(a) Residual demand curves for a *small-GENCO* at $t = 3, 13, 20$

(b) Residual demand curves for a *large-GENCO* at $t = 3, 13, 20$

Figure 1: Residual demand curves for a *small-GENCO* and a *large-GENCO*

θ_t	$Pr(\theta_t)$	$\Delta p(\theta_t)$	$\Delta\rho_1(\theta_t)$ (SPV)	$\Delta\rho_2(\theta_t)$ (LPV)
θ_t^1	0.000429117	-100	10	50
θ_t^2	0.000920851	-70	9	45
θ_t^3	0.002480458	-50	8	40
θ_t^4	0.005984882	-35	7	35
θ_t^5	0.012934755	-30	6	30
θ_t^6	0.025040268	-25	5	25
θ_t^7	0.043420952	-20	4	20
θ_t^8	0.067443978	-15	3	15
θ_t^9	0.093837208	-10	2	10
θ_t^{10}	0.116948937	-5	1	5
θ_t^{11}	0.130558596	0	0	0

θ_t	$Pr(\theta_t)$	$\Delta p(\theta_t)$	$\Delta\rho_1(\theta_t)$ (SPV)	$\Delta\rho_2(\theta_t)$ (LPV)
θ_t^{12}	0.130558596	5	-1	-5
θ_t^{13}	0.116948937	10	-2	-10
θ_t^{14}	0.093837208	15	-3	-15
θ_t^{15}	0.067443978	20	-4	-20
θ_t^{16}	0.043420952	25	-5	-25
θ_t^{17}	0.025040268	30	-6	-30
θ_t^{18}	0.012934755	35	-7	-35
θ_t^{19}	0.005984882	50	-8	-40
θ_t^{20}	0.002480458	70	-9	-45
θ_t^{21}	0.001349967	100	-10	-50

Table 1: Scenario realizations

Scenarios. In the function $\rho(p_t; \theta_t; t)$ representing the residual demand curve, we assume every random variable θ_t, $t = 1, \ldots, T$, to have the same discrete probability distribution. Namely, θ_t has $r = 21$ different possible realizations $\theta_t^1, \ldots, \theta_t^r$ with probabilities $\pi^1, \ldots, \pi^r$.
The realizations of prices and quantities in each scenario can be represented, for $s = 1, \ldots, r$, as follows:

$$\rho(p_t; \theta_t^s; t) = \rho(p_t - \Delta p(\theta_t^s); t) + \Delta\rho(\theta_t^s),$$

where $\Delta p(\theta_t^s)$ and $\Delta\rho(\theta_t^s)$ are variations of quantity and price. We examined two cases of price variations, as reported in Table 1. The first case, referred to as "Small Price Variations" (SPV) is represented by $\Delta\rho_1(\theta_t)$ and corresponds to a knowledge of prices less uncertain that the one represented by $\Delta\rho_2(\theta_t)$, referred to as "Large Price Variations" (LPV).

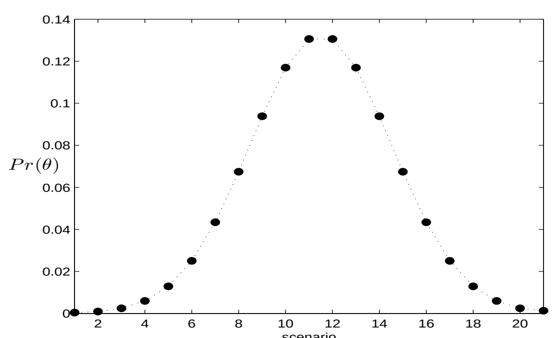


Figure 2: Scenarios' probability distribution

Means and covariances. Means and covariances have been computed as follows. For the sake of readability, let us denote by $\Phi_t = \rho(p_t; \theta_t; t)$ the random variable which gives the price level in the

unit	SDC [€]	τ [h]	TS [€]	BS [€]	A [€/MW^{2}h]	B [€/MWh]	C [€/h]	P_{min} [MW]	P_{max} [MW]
# 1	0	4	585.75	1455.5	$8.0230 \cdot 10^{-4}$	6.406	582.2	300	1000
# 2	0	4	674.50	1491.0	$1.0437 \cdot 10^{-3}$	6.214	426.0	165	600
# 3	0	2	532.50	965.6	$3.6565 \cdot 10^{-3}$	5.008	124.3	50	200

Table 2: Types of unit

interval t at which it is possible to sell the produced power p_t and by $\phi_{t,s}$ the realization of the random variable Φ_t in the s-th scenario. Let

$$E_\theta[\Phi_t] = \bar{\phi}_t = \sum_{s=1}^{21} \phi_{t,s} Pr(\theta_t^s).$$

Given two distinct time intervals $s, r \in \mathcal{T}$, we approximate the covariance between Φ_s and Φ_r by means of the following formula [8]

$$cov_\theta[\Phi_t, \Phi_\ell] = \frac{\sum_{k=1}^{21} \sum_{h=1}^{21} Pr(\theta_t^k) Pr(\theta_\ell^h)(\phi_{t,k} - \bar{\phi}_t)(\phi_{\ell,h} - \bar{\phi}_\ell)}{1 - \sum_{k=1}^{21} \sum_{h=1}^{21} Pr(\theta_t^k) Pr(\theta_\ell^h)}.$$

As concerns the characteristics of the generating units, we consider three different types of unit whose distinguishing cost parameters are reported in Table 2.

6.1 The single-unit case

Let us begin with the case of a single-unit *small-GENCO*. We tried both the local search algorithm LSA and the mixed integer variable algorithm MIVA. We set $\xi = 10^6$, $\theta = 0.9$, $\eta^0 = 10^{-1}$ and $\mu^{in} = 10^2$. Moreover, in order to get comparable results, we chose to stop MIVA whenever parameter μ^k falls below 10^{-4} and set, in LSA, $tol = 10^{-4}$. As concerns the discrete neighborhoods, their definition is rather involved; the interested reader is referred to the appendix for a detailed exposition.
All the results are obtained with an initial configuration in which every unit is active at the beginning of the time period, has been active for the past four time intervals and is producing the minimum allowed power quantity, as reported in the following Table.

unit	z_0	$\tilde{y}_0$	$\hat{y}_0$	q_0
# 1	1	4	0	300
# 2	1	4	0	165
# 3	1	4	0	50

In the single-unit case, both the algorithms performed very similarly, producing the same results reported in the following.
In Figure 3 we report the commitment obtained by considering the three different type of units listed in Table 2, of different size and efficiency. The results refer to the case SPV and moderate risk aversion ($A_r = 3$, see [13]).
As it can be seen, the production levels for the three considered unit types are considerably variable over the time horizon. This is mainly due to GENCO's dependence on the market prices variability. Roughly

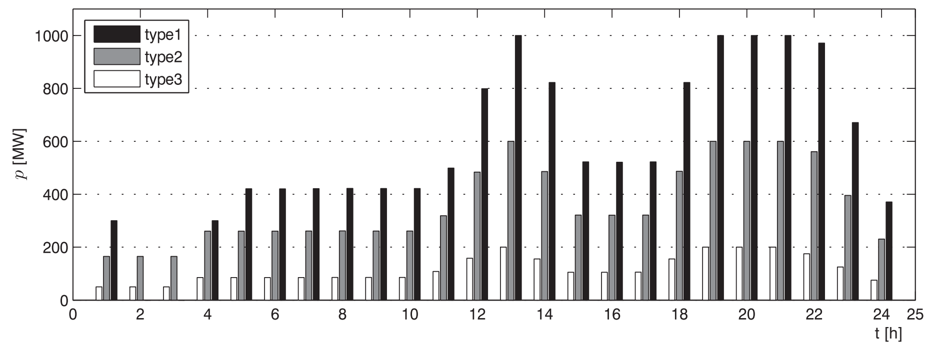


Figure 3: Commitment for *small-GENCO*'s units with SPV ($A_r = 3$)

speaking, we could say that the operator acts as a follower thus trying to adapt its production to the price levels. Moreover, the unit # 1 (i.e. 1000MW maximum capacity) does not produce in hours 2 and 3, whereas units # 2 and # 3 do; this is due to the higher efficiency of units #2 and #3 which allows them to produce even when the selling price is relatively low.
In case of LPV, the results for the commitment of the three unit types are reported in Figure 4 where, again, a moderate risk aversion ($A_r = 3$) has been considered.

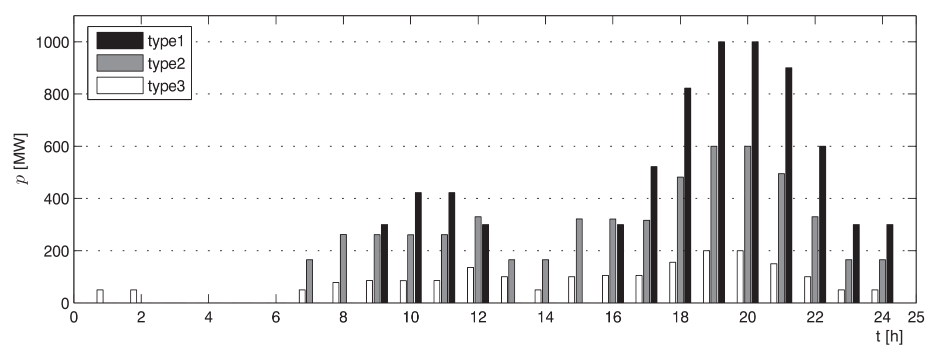


Figure 4: Commitment for *small-GENCO*'s units with LPV ($A_r = 3$)

Comparing the results reported in Figures 3 and 4, it can be noticed that the former case is characterized by smaller production levels for all the unit types and even by shorter up periods. This is due to the fact that in case of LPV there is more uncertainty to be dealt with and this ends up in a more cautious production schedule.
The final part of this subsection is devoted to the comparison of different risk aversions. We ran MIVA with increasing values for the risk aversion coefficients, $A_r = 0.5, 3, 5$. Since the risk aversion coefficient only slightly affects the operator behavior in the case of SPV, we only report results for the case of LPV. Hence, Figure 5 reports the output levels obtained for the three different types of unit in the case of LPV.

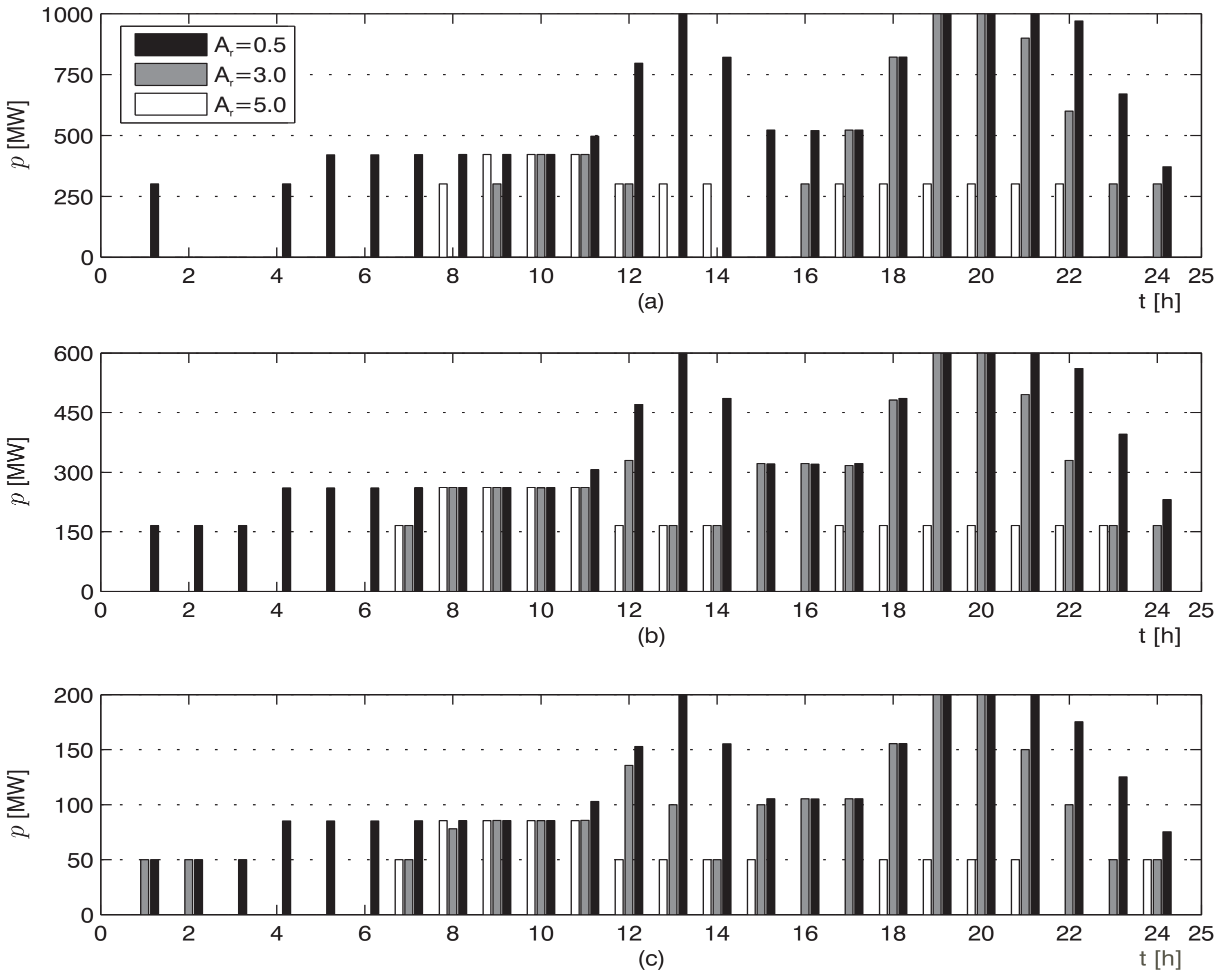


Figure 5: Risk aversion comparison for (a) unit # 1 (b) unit # 2 and (c) unit # 3 in case of LPV

In Figure 5 it can be noticed that the larger the risk aversion the lower the production levels during the time horizon. This is intuitively imputable to the fact that, in uncertain scenarios, the more we produce the bigger is the risk we run. Further, by comparing the results obtained for unit #1 with those obtained for unit # 2 and # 3, we note that unit type 1, which is the one with higher costs, is committed in fewer hours than units # 2 and # 3, at least when $A_r = 3$ and $A_r = 5$. For the sake of completeness, in the following table we report the expected values of the incomes for the three unit types in case of LPV.

	Incomes [€]		
unit	$A_r = 0.5$	$A_r = 3$	$A_r = 5$
# 1	1,284,270	815,407	541,430
# 2	788,089	626,742	359,803
# 3	258,589	210,208	102,523

As it can be noticed, the greater the risk aversion coefficient A_r, the lower the expected value of the incomes due to selling power to the day-ahead market.

6.2 The multi-unit case

Also in this case we used both the algorithm LSA and MIVA. This time they behaved very differently, mainly in terms of total execution time. The LSA did not converge to a solution of the problem within 12 hours of CPU time on an Intel Pentium IV 3.2 GHz processor with 2GB memory; the algorithms were implemented in FORTRAN-90 with a 32-bit library. It is largely due to the cardinality of the discrete neighborhoods visited during the optimization process, which have dimensions ranging from a few thousands elements to a maximum of 15000 elements. Indeed, discrete neighborhoods in the multi-unit case are the cartesian product of the discrete neighborhoods for each one of the N_u units. Since at every iteration LSA might entirely explore the current discrete neighborhood, it is plagued by the dimensionality issue up to the point that it reaches the CPU time limit without converging to a satisfactory solution.

On the contrary, MIVA converged to a solution well within the prescribed limit of 12 hours. This substantial difference is imputable to the fact that MIVA performs the minimizations, with respect to the continuous variables, with a precision that is progressively increased with the number of iterations. This implies that, at the early stages of the optimization process the minimizations only require a few iterations and are thus very rapid. Moreover, partial minimizations are only started from those points of the current discrete neighborhood which are deemed to be promising (see Step 3 of Algorithm MIVA).

In this multi-unit case we consider the case of a *large-GENCO* who owns one unit for every type reported in Table 2, for a total of three units and 1800MW maximum capacity. The commitment of the three units computed by MIVA is reported in Figure 6 for both the cases of SPV and LPV.

As it can be noticed, in the case of SPV, the units # 1 and # 3 (those with largest and smallest capacity) are up only around the peak hours. Off peak hours are covered by using the unit # 2 (the one with intermediate capacity).

In the case of LPV the situation is considerably different. Indeed, even though the largest amount of power is produced around peak hours, unit # 1 is on also in the early hours of the day. This behavior can be explained considering the *large-GENCO*'s ability of influencing the market price. Figure 7 reports the comparison among commitments obtained by considering the case of LPV and running MIVA with increasing values for the risk aversion coefficient, namely $A_r = 0.5, 3, 5$.

Not surprisingly, we can see that the great amount of power is produced in the case $A_r = 0.5$, that is when the operator has lowest risk aversion. Increasing the value of A_r turns in a consequent mostly uniform decrease of power output along the 24 hours of the day. Again, as reported in the following table, it can be noticed that the higher the risk aversion coefficient A_r, the lower the expected value of the income due to selling power.

Incomes [€]		
$A_r = 0.5$	$A_r = 3$	$A_r = 5$
1,534,645	1,442,479	1,191,857

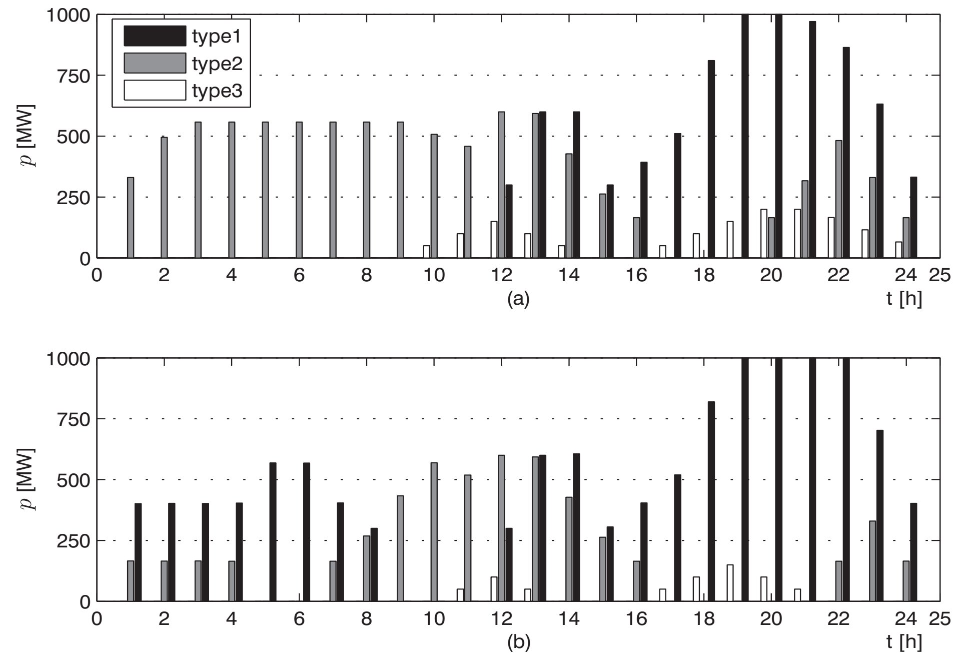


Figure 6: Commitment for a *large-GENCO* managing three units in case of (a) SPV ($A_r = 3$) and (b) LPV ($A_r = 3$)

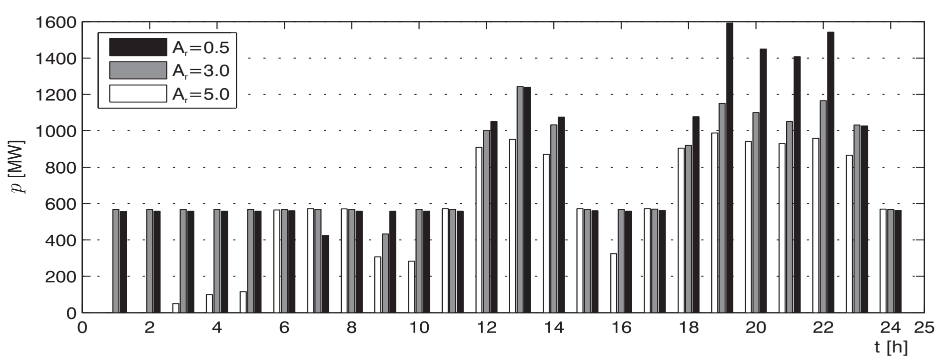


Figure 7: Risk aversion comparison for a *large-GENCO* managing three units in case of LPV

7 Conclusions

We considered the day-ahead UC problem of a GENCO operating in an oligopolistic market. The GENCO's residual demand curves, which give for each hour the price level at which it is possible to

sell power in the day-ahead electricity market, depend on the forecasted load demand and competitors' behavior. Thus, by using different residual demand curves we managed to model different types of GENCO. In particular, we considered two types which differ for their ability to influence the market prices. The first type is representative of a small GENCO (owning a single unit) which has hardly any ability to influence the market price while the second type schematize a larger GENCO (owning three units) with greater ability to influence the market price.
The UC problem is characterized by uncertainty; in oligopolistic markets, it is related to the actual residual demand curve which comes to complete knowledge only after the market clearing. To handle the uncertainty related to selling power into the day-ahead electricity market, we defined some scenarios, each one with an associated probability, and we assumed that the GENCO's residual demand curve varies with the scenarios. Then, according to a stochastic programming approach, we used as objective function the GENCO's expected profit over the scenarios minus a term which accounts for its risk aversion.
We formulated the UC problem by taking into account the relevant minimum up and down time constraints and ramp-up and ramp-down constraints, as well as the limits on the operating range of the units. The effort made in formulating the UC problem allowed us to put evidence on its mixed variable nature. Indeed, the problem we came up with is a mixed-integer nonlinear programming problem. To solve the problem we adopt two methods: a quite standard local search procedure [17, 21] and an algorithm recently proposed [15] to solve mixed variable nonlinear problems named MIVA.
The latter method managed to solved both the single-unit and multi-unit problems within a reasonable amount of time on an Intel Pentium 4 3.2Ghz processor based computer with 2Gb RAM. In fact all the runs terminated within 4 hours of CPU time. On the opposite, the local search strategy failed to solve the multi-unit case within the prescribed limit of 12 hours CPU time.

Further developments can be envisaged, regarding the execution time. At first, we note that by the definition of Algorithm MIVA, the time it requires to get a solution is intimately related to the cardinality of the discrete neighborhoods that it explores during the optimization process. The definition of MIVA promptly allows for a parallel visit of the discrete neighborhood; this, in turn, would imply a consistent reduction in total execution time.
Second, it is possible to study different and more efficient definitions of discrete neighborhoods than that used in the paper. Proposing new way of defining the discrete neighborhoods is not a trivial task. Indeed, a new discrete neighborhood definition should, primarily, reduce the cardinality of the discrete neighborhoods while guaranteeing a sufficient ability to explore the feasible region thus avoiding, as much as possible, the local minima.

Appendix

A Discrete neighborhood definition

In this section we define the discrete neighborhood relative to feasible activation and power production schedules of the N_u generating units. In order to do this, we first define the discrete neighborhood relative to feasible activation and power production schedules for a single unit which, without loss of generality, we assume to be the ℓ-th one. Let $\bar{z} \in \mathcal{F}_z^{(\ell)}$ and $\bar{q} \in \mathcal{F}^{(\ell)}(\bar{z})$ be feasible activation and power production schedules for the ℓ-th unit. We define the discrete neighborhood $\mathcal{H}(\bar{z}, \bar{q})$ relative to $(\bar{z}, \bar{q})$ according to the following rules. We assume $[i, j] = \{i, i+1, \ldots, j-1, j\} \subseteq \mathcal{T}$ and $[i, j] = \emptyset$ whenever $j < i$.

1. *Augmentation, diminution and traslation.* Let i, j be indices such that, $\bar{z}_h = 1$, $h \in [i, j]$. If $i > 1$ then $\bar{z}_{i-1} = 0$ and if $j < T$ then $\bar{z}_{j+1} = 0$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(1,1)}, q^{(1,1)}), (y^{(1,2)}, q^{(1,2)}), (y^{(1,3)}, q^{(1,3)}), (y^{(1,4)}, q^{(1,4)}), (y^{(1,5)}, q^{(1,5)}), (y^{(1,6)}, q^{(1,6)})$ where

$$\begin{aligned} y^{(1,1)} &= \bar{z}, & q^{(1,1)} &= \bar{q}, \\ y^{(1,1)}_{j+1} &= 1, & q^{(1,1)}_{j+1} &= P^{(\ell)}_{min}, & j+1 \leq T, \end{aligned}$$

$$\begin{aligned} y^{(1,2)} &= \bar{z}, & q^{(1,2)} &= \bar{q}, \\ y^{(1,2)}_{i} &= 0, & q^{(1,2)}_{t} &= P^{(\ell)}_{min}, & t \in [i+1, j] \end{aligned}$$

$$\begin{aligned} y^{(1,3)} &= \bar{z}, & q^{(1,3)} &= \bar{q}, \\ y^{(1,3)}_{j+1} &= 1, & y^{(1,3)}_{i} &= 0, & j+1 \leq T, \\ q^{(1,3)}_{t} &= \bar{q}_{t-1}, & t &\in [i+1, j+1] \end{aligned}$$

$$\begin{aligned} y^{(1,4)} &= \bar{z}, & q^{(1,4)} &= \bar{q}, \\ y^{(1,4)}_{i-1} &= 1, & q^{(1,4)}_{i-1} &= P^{(\ell)}_{min}, & i-1 \geq 1, \end{aligned}$$

$$\begin{aligned} y^{(1,5)} &= \bar{z}, & q^{(1,5)} &= \bar{q}, \\ y^{(1,5)}_{j} &= 0, & q^{(1,5)}_{t} &= P^{(\ell)}_{min}, & t \in [i, j-1] \end{aligned}$$

$$\begin{aligned} y^{(1,6)} &= \bar{z}, & q^{(1,6)} &= \bar{q}, \\ y^{(1,6)}_{i-1} &= 1, & y^{(1,6)}_{j} &= 0, & i-1 \geq 1, \\ q^{(1,6)}_{t} &= \bar{q}_{t+1}, & t &\in [i-1, j-1] \end{aligned}$$

provided that $y^{(1,i)} \in \mathcal{F}^{(\ell)}_z$, $q^{(1,i)} \in \mathcal{F}^{(\ell)}_q(y^{(1,i)})$ and $y^{(1,i)} \neq \bar{z}$, $i = 1, \ldots, 6$.

2. *Start-up and fill-up*

   (a) Let $z_0 = 0$, $\bar{z}_h = 0$, $h \in [1, j]$ where j is an index such that, if $j < T$ then $\bar{z}_{j+1} = 1$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (z^{(2,1)}, q^{(2,1)})$, where

   $$\begin{aligned} y^{(2,1)} &= \bar{z}, & q^{(2,1)} &= \bar{q}, \\ y^{(2,1)}_{t} &= 1, & q^{(2,1)}_{t} &= P^{(\ell)}_{min}, & \forall t \in [t_{dw} - \hat{y}_0 + 1, j], \end{aligned}$$

   provided that $y^{(2,1)} \in \mathcal{F}^{(\ell)}_z$, $q^{(2,1)} \in \mathcal{F}^{(\ell)}_q(y^{(2,1)})$. Moreover, if $j \geq 2t_{dw} + t_{up} - \hat{y}_0 + 1$, $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (z^{(2,1+h)}, q^{(2,1+h)})$, for all $h = 1, \ldots, j - 2t_{dw} - t_{up} + \hat{y}_0$, where

   $$\begin{aligned} y^{(2,1+h)} &= \bar{z}, & q^{(2,1+h)} &= \bar{q}, \\ y^{(2,1+h)}_{t} &= 1, & q^{(2,1+h)}_{t} &= P^{(\ell)}_{min}, & \forall t \in [t_{dw} - \hat{y}_0 + h, t_{dw} + t_{up} - \hat{y}_0 + h], \end{aligned}$$

   provided that $y^{(2,1+h)} \in \mathcal{F}^{(\ell)}_z$, $q^{(2,1+h)} \in \mathcal{F}^{(\ell)}_q(y^{(2,1+h)})$ and $y^{(2,1+h)} \neq \bar{z}$.

(b) Let i, j be indices such that, $\bar{z}_h = 0$, $h \in [i, j]$, $\bar{z}_{i-1} = 1$, $i \geq 0$ (we assume $\bar{z}_0 = z_0$). Furthermore, if $j < T$ then $\bar{z}_{j+1} = 1$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(3,1)}, q^{(3,1)})$ where

$$\begin{aligned} y^{(3,1)} &= \bar{z}, & q^{(3,1)} &= \bar{q}, \\ y_t^{(3,1)} &= 1, & q_t^{(3,1)} &= P_{min}^{(\ell)}, & \forall t \in [i, j], \end{aligned}$$

provided that $y^{(3,1)} \in \mathcal{F}_z^{(\ell)}$, $q^{(3,1)} \in \mathcal{F}_q^{(\ell)}(y^{(3,1)})$. Moreover, $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(3,1+h)}, q^{(3,1+h)})$, for $h = 1, \ldots, j - i - 2t_{dw} - t_{up}$, where

$$\begin{aligned} y^{(3,1+h)} &= \bar{z}, & q^{(3,1+h)} &= \bar{q}, \\ y_t^{(3,1+h)} &= 1, & q_t^{(3,1+h)} &= P_{min}^{(\ell)}, & \forall t \in [i + t_{dw} + h - 1, i + t_{dw} + t_{up} + h - 1], \end{aligned}$$

provided that $y^{(3,1+h)} \in \mathcal{F}_z^{(\ell)}$, $q^{(3,1+h)} \in \mathcal{F}_q^{(\ell)}(y^{(3,1+h)})$ and $y^{(3,1+h)} \neq \bar{z}$.

(c) Let $z_0 = 0, \bar{z} = 0$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(4,1)}, q^{(4,1)})$ where

$$\begin{aligned} y^{(4,1)} &= \bar{z}, & q^{(4,1)} &= \bar{q}, \\ y_t^{(4,1)} &= 1, & q_t^{(4,1)} &= P_{min}^{(\ell)}, & \forall t \in [1 + t_{dw} - \hat{y}_0, T], \end{aligned}$$

provided that $y^{(4,1)} \in \mathcal{F}_z^{(\ell)}$, $q^{(4,1)} \in \mathcal{F}_q^{(\ell)}(y^{(4,1)})$.
Moreover, $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(4,1+h)}, q^{(4,1+h)})$, for all $h = 1, \ldots, T - t_{dw} + \hat{y}_0$, where

$$\begin{aligned} y^{(4,1+h)} &= \bar{z}, & q^{(4,1+h)} &= \bar{q}, \\ y_t^{(4,1+h)} &= 1, & q^{(4,1+h)} &= P_{min}^{(\ell)}, & \forall t \in [t_{dw} - \hat{y}_0 + h, \min\{T, t_{dw} + t_{up} - \hat{y}_0 + h], \end{aligned}$$

provided that $y^{(4,1+h)} \in \mathcal{F}_z^{(\ell)}$, $q^{(4,1+h)} \in \mathcal{F}_q^{(\ell)}(y^{(4,1+h)})$ and $y^{(4,1+h)} \neq \bar{z}$.

3. *Shut-down and clear-down*

(a) Let $z_0 = 1$, $\bar{z}_h = 1$, $h \in [1, j]$ where j is an index such that, if $j < T$ then $\bar{z}_{j+1} = 0$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(5,1)}, q^{(5,1)})$ where

$$\begin{aligned} y^{(5,1)} &= \bar{z}, & q^{(5,1)} &= P_{min}^{(\ell)} \\ y_t^{(5,1)} &= 0, & q_t^{(5,1)} &= 0, & \forall t \in [t_{up} - \tilde{y}_0 + 1, j - 1], \end{aligned}$$

provided that $y^{(5,1)} \in \mathcal{F}_z^{(\ell)}$, $q^{(5,1)} \in \mathcal{F}_q^{(\ell)}(y^{(5,1)})$.
Moreover, if $j \geq 2t_{up} + t_{dw} - \tilde{y}_0 + 1$, then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(5,1+h)}, q^{(5,1+h)})$, for all $h = 1, \ldots, j - 2t_{up} - t_{dw} + \tilde{y}_0$, where

$$\begin{aligned} y^{(5,1+h)} &= \bar{z}, & q^{(5,1+h)} &= P_{min}^{(\ell)}, \\ y_t^{(5,1+h)} &= 0, & q_t^{(5,1+h)} &= 0, & \forall t \in [t_{up} - \tilde{y}_0 + h, t_{dw} + t_{up} - \tilde{y}_0 + h], \end{aligned}$$

provided that $y^{(5,1+h)} \in \mathcal{F}_z^{(\ell)}$, $q^{(5,1+h)} \in \mathcal{F}_q^{(\ell)}(y^{(5,1+h)})$ and $y^{(5,1+h)} \neq \bar{z}$.

(b) Let i, j be indices such that, $\bar{z}_h = 1$, $h \in [i, j]$, $\bar{z}_{i-1} = 0$, $i \geq 0$ (we assume $\bar{z}_0 = z_0$). Furthermore, if $j < T$ then $\bar{z}_{j+1} = 0$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(6,1)}, q^{(6,1)})$ where

$$\begin{aligned} y^{(6,1)} &= \bar{z}, & q^{(6,1)} &= P^{(\ell)}_{min}, \\ y^{(6,1)}_t &= 0, & q^{(6,1)}_t &= 0, \qquad \forall t \in [i, j], \end{aligned}$$

provided that $y^{(6,1)} \in \mathcal{F}^{(\ell)}_z$, $q^{(6,1)} \in \mathcal{F}^{(\ell)}_q(y^{(6,1)})$.
Moreover, $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(6,1+h)}, q^{(6,1+h)})$, for $h = 1, \ldots, j - i - 2t_{up} - t_{dw}$, where

$$\begin{aligned} y^{(6,1+h)} &= \bar{z}, & q^{(6,1+h)} &= P^{(\ell)}_{min}, \\ y^{(6,1+h)}_t &= 0, & q^{(6,1+h)}_t &= 0, \qquad \forall t \in [i + t_{up} + h - 1, i + t_{dw} + t_{up} + h - 1], \end{aligned}$$

provided that $y^{(6,1+h)} \in \mathcal{F}^{(\ell)}_z$, $q^{(6,1+h)} \in \mathcal{F}^{(\ell)}_q(y^{(6,1+h)})$ and $y^{(6,1+h)} \neq \bar{z}$.

(c) Let $z_0 = 1, \bar{z} = 1$. Then $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(7,1)}, q^{(7,1)})$ where

$$\begin{aligned} y^{(7,1)} &= \bar{z}, & q^{(7,1)} &= P^{(\ell)}_{min}, \\ y^{(7,1)}_t &= 0, & q^{(7,1)}_t &= 0, \qquad \forall t \in [1 + t_{up} - \tilde{y}_0, T], \end{aligned}$$

provided that $y^{(7,1)} \in \mathcal{F}^{(\ell)}_z$, $q^{(7,1)} \in \mathcal{F}^{(\ell)}_q(y^{(7,1)})$.
Moreover, $\mathcal{H}^{(\ell)}(\bar{z}, \bar{q}) \ni (y^{(7,1+h)}, q^{(7,1+h)})$, for all $h = 1, \ldots, T - t_{up} + \tilde{y}_0$, where

$$\begin{aligned} y^{(7,1+h)} &= \bar{z}, & q^{(7,1+h)} &= P^{(\ell)}_{min}, \\ y^{(7,1+h)}_t &= 0, & q^{(7,1+h)} &= 0, \qquad \forall t \in [t_{up} - \tilde{y}_0 + h, \min\{T, t_{dw} + t_{up} - \tilde{y}_0 + h\}], \end{aligned}$$

provided that $y^{(7,1+h)} \in \mathcal{F}^{(\ell)}_z$, $q^{(7,1+h)} \in \mathcal{F}^{(\ell)}_q(y^{(7,1+h)})$ and $y^{(7,1+h)} \neq \bar{z}$.

Thus, given $\bar{z}^{(\ell)}$, $\bar{q}^{(\ell)}$, $\ell = 1, \ldots, N_u$, and defined $\mathcal{H}^{(\ell)}(\bar{z}^{(\ell)}, \bar{q}^{(\ell)})$, $\ell = 1, \ldots, N_u$, $\mathcal{N}(\bar{x}, \bar{p})$ is given by

$$\mathcal{N}(\bar{x}, \bar{p}) = \prod_{\ell=1}^{N_u} \mathcal{H}^{(\ell)}(\bar{z}^{(\ell)}, \bar{q}^{(\ell)}).$$

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