Textured Renyi Entropy for Image Thresholding

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Abstract

This paper introduces Textured Renyi Entropy for image thresholding based on a novel combination mechanism. The Renyi Entropy is extended by modifying its priori, while still preserving overall functionality. An optional priori is introduced to improve accuracy. The priori modification allows adding of texture information in an efficient way, which results in more accurate thresholding. Furthermore, the proposed mechanism allows adding other features after some normalization. Finally the novel way allows adding multiple features simultaneously using the modified priors.

Keywords--- ***Renyi Entropy, Threshold, Texture.***

1. Introduction

In image thresholding division/segmentation of an image into two or more regions is done by calculating one or more values to act as a boundary. There have been many approaches to image thresholding that has been presented in the literature. A complete review of the different classification for image thresholding can be found in [1]. In this paper, we propose a novel approach to calculate the threshold for global binarisation of gray level images. We call our approach Textured Renyi entropy image thresholding. The proposed method injects the textured feature into Renyi entropy by modifying the entropy prioris.

A gray level image can be represented by an intensity function, which determines the gray level value for each pixel in the image. Specifically, an intensity function F, takes as input a particular pixel from the image, and outputs its gray level value, which is usually in the range of 0 to 255 (*if 256 levels are used*).

Thresholding produces a new image based on the original one represented by **F**. It is basically another function **g**, which produces a new image (i.e. *the thresholded image*). A threshold is calculated for each pixel value. This threshold is compared with the original image (i.e. F) to determine the new value of the current pixel. **g** can be represented by equation 1[2].

$$g(x,y)=\begin{cases}0, \text{ if } F(x,y)\le T\\ 1, \text{ if } F(x,y)>T\end{cases} \quad (1)$$

1.1 Entropy in image processing

Entropy is a term that is difficult to define in a general sense. Simply put, entropy refers to the amount of information that can be obtained from a set of messages [3] and was first introduced in information theory.

When applied to image processing techniques, entropy measures the normality (i.e. normal or abnormal) of a particular gray level distribution of an image. When a whole image is considered, the entropy as defined in Eq.2 will indicate to what extent the intensity distribution is normal. When we extend this concept to image segmentation, i.e. dealing with foreground and background regions in an image, the entropy is calculated for both regions, and the subsequent entropy value provides an indication of the normality of the segmentation. In this case, two equations are need for each region, each of them called priori.

The Renyi Entropy (RE) is one of many types of entropy introduced by Renyi in 1961 [4].

$$H_{\alpha}(P_1, P_2 \ldots\ldots P_n)=\frac{1}{1-\alpha}\ln\left(\sum_{i=1}^{n}P_i^{\alpha}\right) \quad (2)$$

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Over the years, RE has been applied to image thresholding in a number of different ways [5,6,7,8]. Renyi entropy for image thresholding was implemented first using certain modifications and normalization mechanisms [5]. Subsequently, other image features mainly the averaging neighborhood, were used along with RE to increase the thresholding accuracy. The early mechanisms utilized a 1-dimensional histogram that represents the number of pixels at each gray level. Thus, it can be called as 1-dimensional Renyi. The later mechanism [6] utilized a two-dimensional histogram representing the number of the pixels at each gray level and each neighborhood's average value. This is called as 2-dimensional Renyi. More details on the previous work in RE are discussed in Section 2. Section 3 presents our proposed textured Renyi entropy algorithm. An adaptable algorithm is presented to accommodate additional features, whose experimental results are shown in Section 4.

2. Previous work

The different approaches to using RE for image thresholding are based on the law of probabilities. Two priories are at the heart of the RE, each of which represent the foreground and the background. They are viewed as the summation of the probabilities of the pixels' gray levels. The probabilities of the pixels at each gray level are combined using some mathematical operations. And the final threshold is the maximum priori combination value. This is what is referred to as the Maximum Entropy.

In image thresholding, when applying maximum entropy, every gray level value is a candidate to be the threshold value. Each value will be used to classify the pixels into two groups based on their gray levels and their affinity, as less or greater than the threshold value. Formula 3 represents the Renyi threshold formula; the prioris H_1 and H_2 represent the foreground and the background and the chosen threshold value (**T**) has to maximize their combination according to the third equation.

$$H_1(t) = \frac{1}{1-\alpha}\ln\left(\sum_{i=1}^{t} P_i^{\alpha}\right)$$
$$H_2(t) = \frac{1}{1-\alpha}\ln\left(\sum_{j=t+1}^{n} P_j^{\alpha}\right)$$
$$T = \max_t (H_1 + H_2) \tag{3}$$

Sahoo et al [5] proposed a new method which extends the original one [7] by choosing three initial threshold values. The initial threshold values are those that maximize the output T of equation 3 based on three different values for α. The final threshold value is calculated by combining these initial threshold values and the corresponding values of the prioris H_1 and H_2 using the following equation:

$$T = t_1\left(H_1 + \tfrac{1}{4} w \beta_1\right) + \left(\tfrac{1}{4} t_2 w \beta_2\right) + \left(1 - H_1 t_3 + \tfrac{1}{4} w \beta_3\right) \tag{4}$$

The chosen values of α is based on experimental results. Sahoo and Arora [6] work on Kapur et al's difficult question [7] - "*What happens if two different pictures have the same gray-level histogram and thus the same threshold? Will it be suitable for both?*". This question motivated Sahoo and Arora [6] to investigate the two-dimensional Renyi. In fact, our own motivation stems from Sahoo and Arura's [6] previous work in which we wanted to enhance their formulation for 2D Renyi Entropy and increase the accuracy using texture features.

Sahoo and Arora's [6] solution is based on the average value of the neighborhoods as similar to the method proposed by [8] for entropy based thresholding. The two-dimensional entropy considers the averaging of 3x3 blocks of the neighborhoods of each pixel, in so doing, this technique is able to go beyond the simplest threshold technique, but on the other hand it gives the entropy more accurate results. Figure 1 illustrates the two-dimensional histogram.

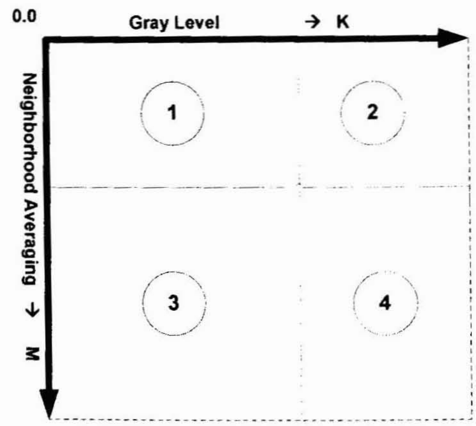

Figure 1: Two-Dimensional Histogram [8]

The background and foreground priories, as proposed by [6], are presented in Equation 5.

$$H_1 = \left(\frac{1}{1-\alpha}\right)\ln\sum_{i=0}^{t}\sum_{j=0}^{s}\left(\frac{P(i,j)}{P_{i,jall}}\right)^{\alpha}$$
$$H_2 = \left(\frac{1}{1-\alpha}\right)\ln\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{P(i,j)}{P_{i,jall}}\right)^{\alpha} \tag{5}$$

Wu et. al. [8] focused on reducing the complexity of the two-dimensional Entropy using fast recurring algorithm. In their approach, the two-dimensional histogram in Figure 2 was divided into four quadrants using demo threshold values for the gray level and the average neighborhood. According to Wu et. al. [8], the second and the fourth quadrant represents noise and unwanted information in the image as it represent high level gray values pixels with low average neighborhood values and vice versa.

The two dimensional entropy also introduces additional accuracy [6,8]. However, this mechanism is still restricted within the local neighborhood information. It is not likely to be used in a fashion similar to global thresholding.

3. Integration of Renyi entropy with texture features

Renyi entropy has been utilized for image thresholding in two ways. It is either used in its original formulation [5] in which the one-dimensional histogram is used or alternately, by integrating the entropy value with the neighborhood average in a two dimensional histogram as in [6,8]. In the latter approach, the integration of entropy with neighborhood averages is mainly to enhance the results of the thresholding process. Apart from that, this integration also provides a solution to the problem posed by Kapur et al [7]. However, the proposed method is strictly limited to the averaging neighborhood since the two dimensional histogram is only able to represent two strongly connected features. Besides, the neighborhood averaging is a local feature which we feel does not optimally contribute towards effective global thresholding because the correlation of the individual pixels in the global sense has been lost.

A desirable image segmentation approach has to maximize the correlation between the pixels in each region and minimize the correlation between the different regions

To preserve the correlation with the different regions using Renyi entropy, we have proposed to use a texture feature. In our approach, we have chosen wavelet coefficients that are calculated for each pixel over three different neighborhoods of 9, 16, and 25 pixels. The pixels with high wavelet coefficient values have low correlation within its neighborhood. Thus, those pixels with wavelet coefficient values are discarded out of any regions.

The proposed Renyi prioris are calculated for the texture features. The mechanism of the prioris work to produce single value that minimize the textured in the distributions, one way of doing this is to through the high textured pixels out the segments. However, the Renyi entropy in specific and the entropy in general have only two prioris which correspond to two regions. Thus, to locate the pixels with high wavelet coefficients, this paper introduces a new priori to represent the area to be binarised. The original and proposed prioris together are aimed to achieve two goals which are:

- Maximize the intra-correlation by minimizing the intra-texture features.
- Minimizing the inter-correlation by maximizing the inter-texture features.

To use texture with Renyi, the two prioris H_1 and H_2 has to be modified in a way that preserves functionality. The prioris are summation of probabilities values which can be used in any type of data as long as it can be represented as probability values.

The texture feature must be represented in the same way as the gray level. Normalization process is used to convert the feature values into probability values allocated with each gray level.

However, the effect of the texture distribution is opposite to the effect of the gray level distribution. The original entropy aim at maximize the uniformity of the distributions by dividing the gray levels into two groups represent the background and the foreground in uniform way. On the other hand, the texture features are undesirable to be uniform within the foreground and the background. Thus, the aim of the entropy has to be modified from maximizing into minimizing the texture within the distributions.

Thus, using the original prioris with the texture will set the effects oppositely and result in totally uncorrelated regions. To set the equations in the proper direction, we have modified the priori equations by converting $(1/1-\alpha)$ into $-1/(1-\alpha)$. Thus, the texture-based priori is constructed as shown in Equation 6.

$$H_1(t) = -1/1-\alpha \ln\left(\sum_{i=1}^{t} TP_i^{\alpha}\right)$$
$$H_2(t) = -1/1-\alpha \ln\left(\sum_{j=t+1}^{n} TP_j^{\alpha}\right) \tag{6}$$

**TP*: The textured value at the gray level i, in a form of probability value.

To calculate the texture within the object, we calculate the texture property at each gray level by aggregating the textures properties for pixels with the same gray value. The texture values are normalized to the range of [0-1] and the total value for texture properties in the image has to be equal to 1, which is equivalent to the gray level probability representation.

However, to preserve the functionality of the original entropy, we have proposed to combine the proposed and the original priori into a single priori. To provide equal weighting for the original and the textured-based prioris, the two priories are combined using an addition operation as indicated in Equation 7.

$$H_1(t)=\frac{1}{1-\alpha}\ln\left(\sum_{i=1}^{t}P_i^{\alpha}\right)+\frac{-1}{1-\alpha}\ln\left(\sum_{i=1}^{t}TP_i^{\alpha}\right)$$

$$H_2(t)=\frac{1}{1-\alpha}\ln\left(\sum_{j=t+1}^{n}P_j^{\alpha}\right)+\frac{-1}{1-\alpha}\ln\left(\sum_{j=t+1}^{n}TP_j^{\alpha}\right)$$

$$T=\max_{t}\left(H_1+H_2\right) \tag{7}$$

Moreover, we have added new priori to the previous two prioris. This new priori calculates the texture property on the threshold region and minimizes the correlation within this region by maximizing the texture between the segments. Thus, in the new priori, called G, the texture property is maximized. The new priori with the modified maximum entropy is represented in Equation 8.

$$G=\frac{1}{1-\alpha}\ln TP_t^{\alpha}$$

$$T=\max_{t}\left(H_1+H_2+G\right) \tag{8}$$

The flexibility of the novel method proposed in the previous section allows us to incorporate various features in the prioris. In a similar manner, this method may be utilized to introduce other features which might enhance the thresholding result.

4. Experimental results

This section discusses the results obtained using the proposed Texture Renyi Entropy method. The discussion will focus on sample images, while the complete set of images will be provided in Appendix A.

Figure 2 and 3 shows the result of thresholding using the original entropy and the proposed method respectively. The original entropy resulted in a threshold value of (187) while the proposed method resulted in a better result as illustrated in Figure 3. We have realized that the proposed method here mimics the edge detection techniques where in the edges in the image normally have a high texture values.

Figure 4 shows another binarised image. Again, the introduced G priori has played a major role in the effective binarisation of the image.

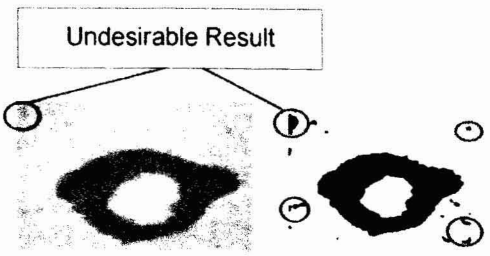

Figure 2: Threshold using the original entropy

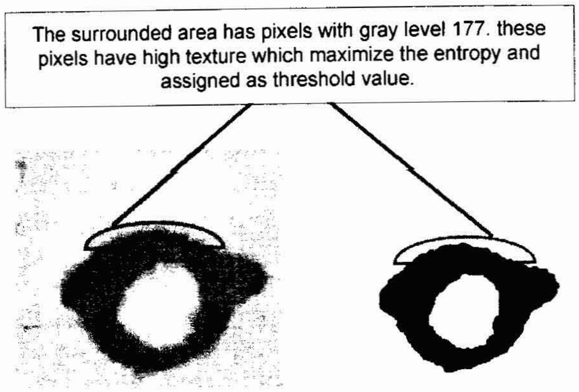

Figure 3: Threshold using the textured renyi entropy

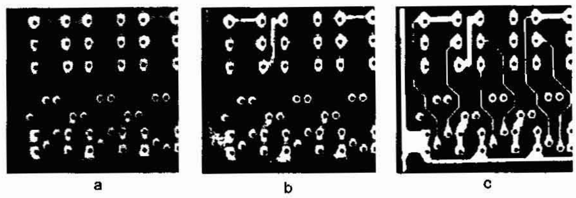

Figure 4: Threshold using the original entropy, a. The original image, b. Thresholded image using the original entropy, c. The thresholded image using textured renyi entropy

Figure 5 shows two thresholded images using Texture Renyi method. The original images are provided in the appendix.

The image in 5a has noisy characters, while the second one is considerably less noisy. The thresholding operation using the proposed method for these images resulted in the same threshold value. This is because the high value for the texture located on the character edges. The proposed method does not add any improvement in such case, wherein the noise has affected the features badly.

More comparison results with the error rates are provided in Appendix A. The error rate is defined as the number of the misclassified pixels divided by the total number of pixels. The experimental results show that the proposed method improves the result by reducing the error rate by 1/2 to 2/3.

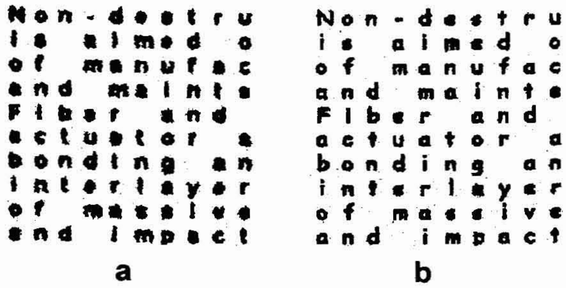

Figure 5: Segmentation using textured renyi entropy for different two images

5. Conclusion

In this paper, Textured Renyi Entropy mechanism for image thresholding has been developed using novel method. The Textured Renyi method was developed by introducing texture feature and modifying the original prioris. Besides, the proposed method adds one more priori. The experimental results show that the proposed method enhances the thresholding result and reduce the error rate by ½ to 2/3. Finally, this paper discussed how to add other features to the entropy prioris.

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Appendix A

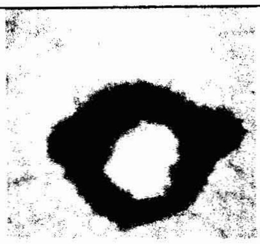
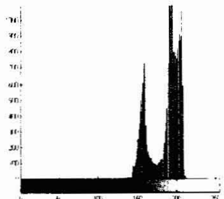
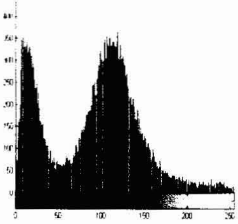
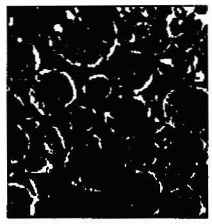
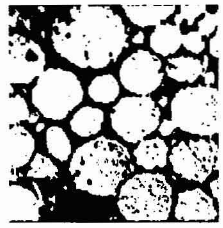
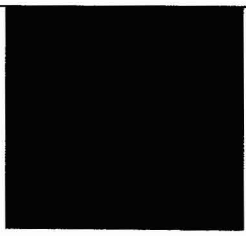
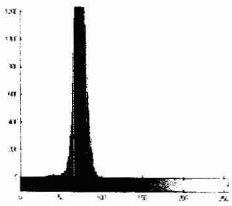
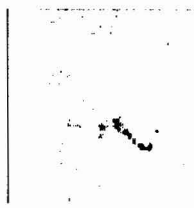
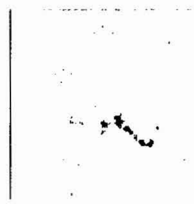
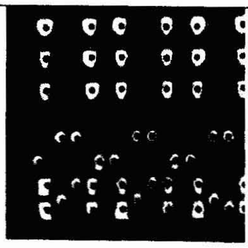
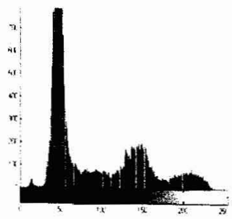
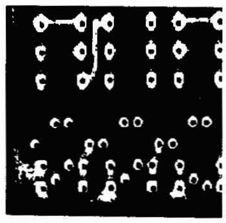
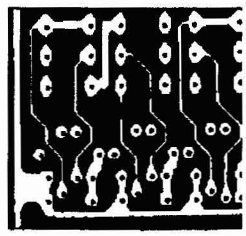
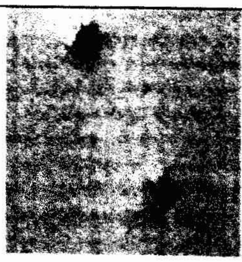
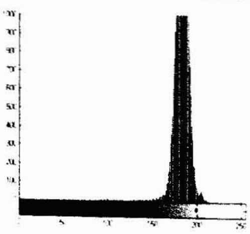
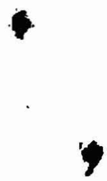
	Image	Histogram	Threshold using the Original Entropy	Threshold using Textured Renyi
1			 T=187	 T=177, S=0.09
2			 T=155	 T=86, S=0.10
3			 T=55	 T=55, S=0.12
4			 T=154	 T=74, S=0.11
5			 T=153	 T=149, S=0.01

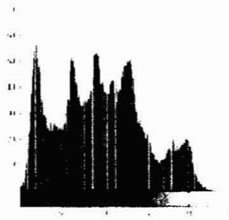
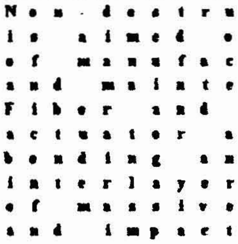
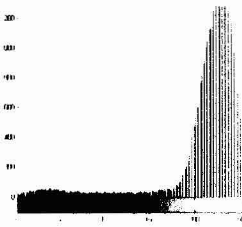
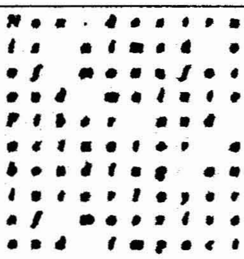
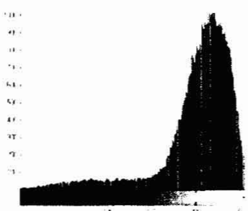
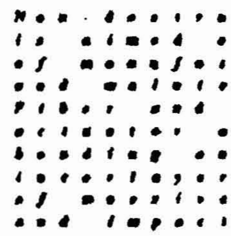
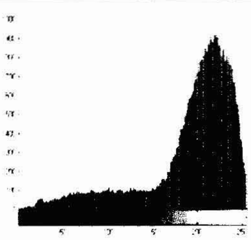
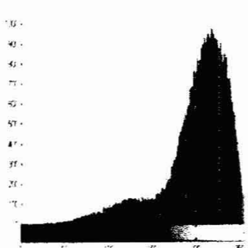
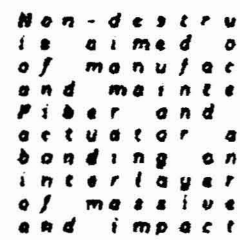
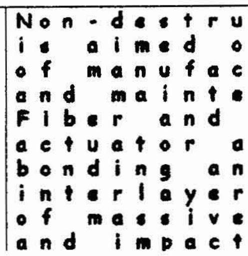
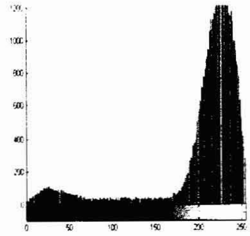
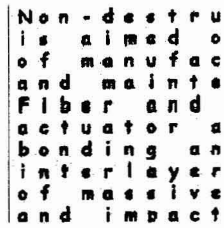
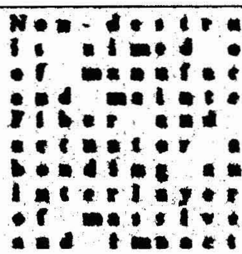
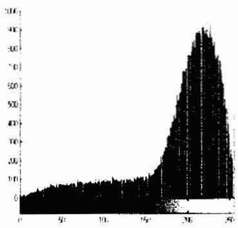
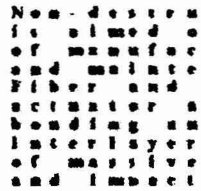
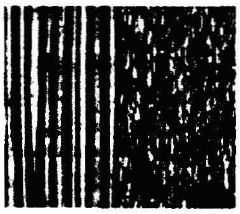
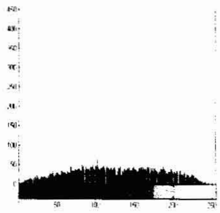
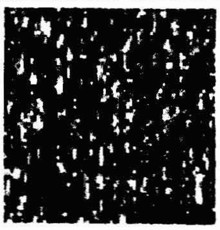
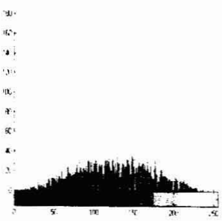
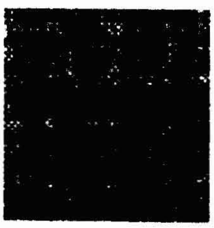
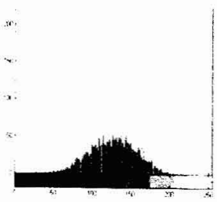
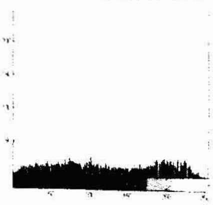
	Image	Histogram	Threshold using the Original Entropy	Threshold using Textured Renyi
6			 T=133	 T=133, S=0.09
7			 T=178	 T=172, S=0.02
8			 T=157	 T=154 ,S=0.04
9			 T=158	 T=141, S=0.07
10			 T=160	 T=153, S=0.06

	Image	Histogram	Threshold using the Original Entropy	Threshold using Textured Renyi
11			 T=180	 T=179, S=0.03
12			 T=158	 T=147, S=0.01
13			 T=120	 T=126, S=0.01
14			 T=115	 T=115, S=0.01
15			 T=108	 T=111, S=0.01
16			 T=115	 T=123, S=0.01

* S is the error rate.