AUTOMATIC B-SPLINE PARAMETERISATION AND SCALING ESTIMATION FOR RIGID SUPER-CURVES BASED SKULL REGISTRATION

TAN JOI SAN

UNIVERSITI SAINS MALAYSIA
2010

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by

TAN JOI SAN

Thesis submitted in fulfillment of requirements for the Degree of Master of Science (Computer Science)

JANUARY 2010

DECLARATION

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ACKNOWLEDGEMENT

I would like to thanks to Dr. Iman Yi Liao for giving me the opportunity to take Automatic B-Spline Parameterisation and Scaling Estimation for Rigid Super-Curves based Skull Registration as my research title and had guided me by providing valuable suggestions for my dissertation.

Next, I would also like to thank Assoc. Prof. Dr Bahari Belaton and the Cranio's group members that provide me with suggestions during meetings.

Lastly, I would like to thanks all those that rendered me the assistances, support and distinctive knowledge during the course of my research. Thank you for the help and guidance.

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CT	Computed Tomography
$\mathbf{d_j}$	Control Points
DSM	Downhill Simplex Method
GA	Genetic Algorithm
ICP	Iterative Closest Points
MRI	Magnetic Resonance Imaging
MOM	Measure of Match
RSC	Rigid Super-Curves
SA	Simulated Annealing
SPECT	Single Photon Emission Computed Tomography
SC	Super-Curves
$\mathbf{t_i}$	Measurements of Sampling Points' Distance along B-spline Curves

PEMPARAMETERAN B-SPLINE SECARA AUTOMATIK DAN PENILAIAN PENSKALAAN UNTUK SUPER-LENGKUNG TEGAR BERDASARKAN PEMADANAN TENGKORAK

ABSTRAK

Dalam sistem klinik yang mengamalkan auto atau semi-auto kraniofasial berbantukan komputer, pemadanan pengimbas CT tengkorak yang diambil pada masa yang berbeza bagi pesakit yang sama merupakan suatu prasyarat untuk sebarang analisis lanjutan. Proses ini dinamakan pemadanan tengkorak. Dalam tesis ini, kaedah "super-lengkung tegar" telah dikajikan dan dipertingkatkan secara lanjut dalam dua aspek. Dalam super-lengkung tegar, terdapat dua set lengkung untuk dipadankan di bawah transformasi afin berdasarkan cara lengkung kuasa yang asal, di mana permadanan B-spline merupakan teknik utama bagi permodelan tersebut. Selepas itu, transformasi yang tegar dibaik-pulih dengan dua set lengkung yang hanya mempertimbangkan putaran dan translasi. Pemparameteran B-spline yang digunakan di super-lengkung tegar adalah seragam dimana ia kurang dari segi pengawalan rupa-bentuk dan tidak sesuai untuk lengkung yang mempunyai titik yang tidak seragam. Dengan itu, satu kaedah yang bernama "panjang perentas songsang" telah diubah-suai dan diusulkan dimana ia boleh memperuntukkan lebih banyak titik kawalan di tempat yang mempunyai variasi yang tinggi atau pensampelan yang padat sementara mengekalkan kejituan ralat pemadanan B-spline. Sebelumnya, parameter yang menentukan kedudukan titik kawalan dengan teknik panjang perentas songsang adalah ditentukan secara manual. Jadi, kaedah yang berdasarkan persamaan telah dicadangkan untuk menentukan parameter tersebut secara automatik. Untuk memperbaiki ketepatan pemadanan dengan selanjutnya, penskalaan *seragam dikemukakan di dalam super-lengkung tegar. Skala dihitung dengan menggunakan fungsi analitis. Selain daripada itu, teknik lelaran telah dicadangkan untuk memulihkan penskalaan, putaran dan translasi secara alternatif. Penekaan awal bagi skala telah diperkenalkan untuk mempercepatkan proses pemadanan. Kriteria ditentukan untuk menghentikan lelaran tersebut. Eksperimen untuk data simulasi dan tengkorak tulen menunjukkan keberkesanan teknik yang telah diusul.*

AUTOMATIC B-SPLINE PARAMETERISATION AND SCALING ESTIMATION FOR RIGID SUPER-CURVES BASED SKULL REGISTRATION

ABSTRACT

In auto or semi-auto computer-aided craniofacial clinic systems, matching CT scans of skull from the same patient taken at different times is a prerequisite to any further analysis of the data. This is known as skull registration. In this research, enhancements were done based on 3D Rigid Super-Curves (RSC) that is performed using two sets of curves registered under a transformation based on the Super-curves (SC), where the B-spline fitting is the core technique for the modeling. Then, a rigid transformation is performed between these two sets of curves, where only rotation and translation were considered. B-spline parameterisation utilised in RSC is uniform which lacks shape controllability and is unsuitable for uniform curve samples. Hence, a modified inverse chord length method was proposed where it can assign more control points to high variation or densely sampled area while maintaining its accuracy by a slight increment of B-spline fitting error. Modified inverse chord length was an alteration from inverse chord length. Previously, the parameter of inverse chord length which determines the position of control points was determined manually. Hence, a method which was based on a simple equation was proposed to calculate the parameter automatically. To further improve the registration accuracy, a uniform scaling factor was introduced in RSC for the transformation. The scaling factor was calculated using analytical function. On the other hand, iterations were applied later to recover all the transformations alternatively. Initial guess was introduced to calculate the initial scale factor in order to accelerate the registration process.

A few stopping criteria was included for the iteration process. Experiments on both simulated data and real skulls showed the effectiveness of this proposed method.

1 INTRODUCTION

1.1 Background

Skull registration is the basic step in an automatic/semi-automatic clinical analysis system for specialists to do further analysis and measurements on a skull. Skull registration can be viewed as a special 3D image registration. Registration process of the images is performed to calculate the “best” alteration between two acquisitions, or similarly, to determine the point to point correspondence between the images. Simply put, registration is to perform an alignment between two images. These two images are the **references/models** and **test** images. One of the images may show a patient having deformed parts on his/her skull, and another image may contain a perfect skull which acts as a reference for the deformed skull in order to perform a skull reconstruction.

There are many systems that have utilised registration as their component. Some examples include aligning images from different medical modalities for diagnosis, matching a target with a real-time image of a scene, matching stereo images to recover shape for autonomous navigation, and observing global land usage using satellite images [1]. In this research, the focus is on medical images especially on 3D skull data obtained from CT scans.

There are a few existing methods for skull registration – points-based, curve-based, and surface-based. Point-based can also be referred to as feature-based or landmark-based. For this approach, skull images are usually represented by multiple sets of point locations, and each point describes its intensities and orientations. Iterative Closest Points [2] and Active Shape Model are two examples of point-based methods. Most of these methods have a major drawback – locations of the landmarks need to be placed manually by a human in order to get the correspondence between two skull images [3], [4]. Deni Suwardhi *et al* [4] were using surface-based methods and ICP (iterative closest points) in multimodal devices for craniofacial reconstructive surgery. Initially, surface-based registration in the device was used to identify the known features that are common between two skull datasets. After identifying the known features, the application calculated the approximate position of one shell (feature) with respect to another. Then, it overlapped these known features. If two shells aligned exactly with each other, ICP was used to register these two shells. For surface-based methods, segmentation is required to obtain a surface. It is always a high-level task and possibly error-prone. Besides, reducing grey-valued images to surfaces will cause the loss of valuable information [5].

Since there are drawbacks from point-based and surface-based methods, curve-based methods try to compromise between both of them. Curve-based methods do not need to locate landmarks manually, nor do they need to identify known features for registration purposes. Super-curves [6] is a B-spline based method that carried out curve matching and alignment. It is one of the curve-based methods that utilised B-Spline with the advantage of non-ambiguous representation of B-Spline fittings and is intrinsically designs to recover the affine transformation between two 2D images. Later, rigid super-curves [7] is proposed based on SC which is designs to recover only the rotation and translation between two 3D skull images. The curves that are used to perform registration in RSC were extracted from CT-scan data using feature extraction methods. Data points on the curves are generated from various scan systems such as CT, MRI, or SPECT at different times, resolutions and devices. Nevertheless, there are still some problems in RSC. Thus, the intention of this research is to solve the problems. It is to enhance and improve the accuracy of skull registration. In this section, a brief introduction on how super-curve and B-spline work is included. It will be further explained in chapter 3.

The super-curve concept is demonstrated in the following example. Firstly, there are two curves with the assumption that they are affine related, as shown in Figure 1.1-1 (A) & (B), called reference curve and test curve respectively. We will align them together using an affine transformation. Then, a super-curve will be constructed by superimposing these two affine related curves in one coordinate system [6]. After that, from the super-curve, an approximation curve is formed by using B-spline fusion with these two curves, as shown in Figure 1.1-1 (C). It is dissimilar from other techniques as they form two B-spline curves individually using these two curves. In the case of the super-curve technique, only one B-spline is formed. B-spline fitting error is used to calculate the matching between the curves. If the error value is high, it means that they shall not match. Otherwise, a match shall be found. Accuracy and efficiency are achieved at the same time because the curves are superimposed and registered simultaneously.

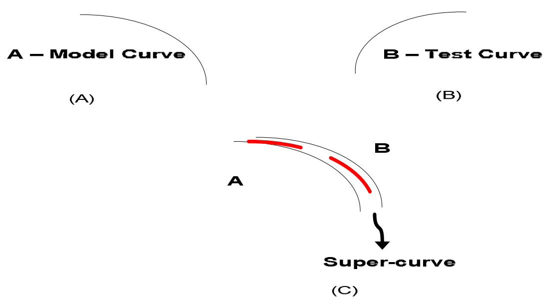

Figure 1.1-1 Example of Super-Curve and B-spline fusion

In summary, this research is intended to further enhancement rigid super-curves. There are many representations and calculations involved which need to handle carefully.

1.2 Problem Statements

This research is based on rigid super-curves [7]. There are two main problems faced by the earlier phase of RSC which are B-spline parameterisation and scaling transformation. The control points in B-spline fitting play an important role in the registration process because these control points manage the accuracy and controllability of B-spline curves. And they will be used to calculate the transformation. Accuracy is the fitting between B-spline curves and sampling curves; the controllability is the robustness of B-spline curves which are not affected by noise. Hence, good control points must be obtained. A good B-spline parameterisation method will find ideal sets of control points as it will use the distance between two sampling points to calculate the position of control points along B-spline curves. However, in the earlier phase of RSC, only a simple method called standard uniform was used to calculate the B-spline parameterisation values. This method did not actually provide good accuracy and controllability for B-spline fitting because it is merely calculated using the average distance of all sampling points. Hence, it triggers the first problem – "What B-spline parameterisation methods can be used to obtain ideal sets of control points?"

In later phases, transformation will be carried out. In earlier RSC, only rotation and translation have been applied because supposed that the skull registration technique in this research will only be used in the case where two sets of skulls share the same size but different position. But, in real cases, still registration will be applied to cases where the size of skulls is different from each other. Hence, scaling needs to be included into this research to improve the matching between skulls. “How scaling is carried out to adjust the size of the test skull to be the same as the reference skull?” and “How good is matching when transformation is carried out?” are the second problem that must be solved. To obtain good matching, the test curve has to be properly aligned with the reference curve.

In summary, the problems that need to be solved are finding a good B-spline parameterisation method and scaling transformation for the registration process.

1.3 Research Questions

The first question addresses parameter control in the B-spline function. In what ways can find accurate values of the measurement? Is there a simpler way to find it? How to control the B-spline parameters? Will the accuracy and efficiency of the curve matching be affected if these parameters are not well defined? These are the questions about B-spline that need to be answered in this research.

The second issue is the scaling transformation problem. How to include scaling into transformation? Will scaling improve the results? How to apply it in 3D skull images? Why is affine transformation not considered? Will it affect the results if affine transformation is not applied? These are some of the questions about transformation that need to be answered in this research.

1.4 Scopes

There are three important scopes in this research which are related to B-spline parameterisation, transformation and rigid super-curves. These three scopes are the important elements during the registration process.

1.4.1 B-Splines

B-spline is a spline function that support degree, smoothness and domain partition. In this research, it is used to perform curve fitting and registration between two curves. A curve is represented in a B-spline form. Let [$\mathbf{P}_0$, $\mathbf{P}_1$, …, $\mathbf{P}_{M-1}$] be the M sample points of a curve **P**. It is then divided into **L** segments and approximated by a linear combination of parametric polynomial basis functions [6].

$$\sum_{i=0}^{M-1} \left\| \boldsymbol{P}_i \; - \; \sum_{j=0}^{L+N-1} \boldsymbol{d}_j \, \boldsymbol{B}_j^N \; \left(\frac{\boldsymbol{t}_i - \boldsymbol{u}_j}{\boldsymbol{u}_{j+1} - \boldsymbol{u}_j} \right) \right\|^2 \qquad \textbf{(1.1)}$$

Where

$\boldsymbol{B}_j^N$ = B − spline basis function, $\mathbf{d_j}$ = control points, $\mathbf{j}$ = L + N − 1, $\mathbf{N}$ = B − spline degree′s, $\mathbf{t}_\mathrm{i}$ = measurement of sampling points′ distance along B − spline curves, $\boldsymbol{u}_j$ = knot vectors.

It is also called B-spline fitting of curve **P**. Each of the parameters is represented using letters and they play important roles in this function. They are used to outline the B-spline curves. In order to obtain good B-spline curves, the allocation of control points is vital because they provide good accuracy and controllability for the curves. The number of control points depends on the number of points on sampling curves. The quantity will not increase unless the number of sampling points increases. The only change that can be done on control points is its position. The position of control points along B-spline curves can be determined using the distance between two sampling points. In B-spline, it is possible to adjust the measurement for the distance between two sampling points using different methods. This adjustment is described as B-spline parameterisation, and it is represented as $\mathbf{t}_i$ in the B-spline function for this research as shown in equation 1.1.

In summary, B-spline parameterisation is important because it will determine the position of control points that provide accuracy and controllability of B-spline fitting.

1.4.2 Transformation

A transformation is a shifting of locations of points in one image to new locations in another [1]. There are two types of transformations that are usually used to match two images: global and local. A global transformation operates with a single equation which maps the entire image. Examples are the affine, projective, perspective, and polynomial transformations [1]. Local transformations are different from global transformations in the way that the image distinctly depends on the spatial location. Thus, local transformations are much more difficult to apply compared to global transformation.

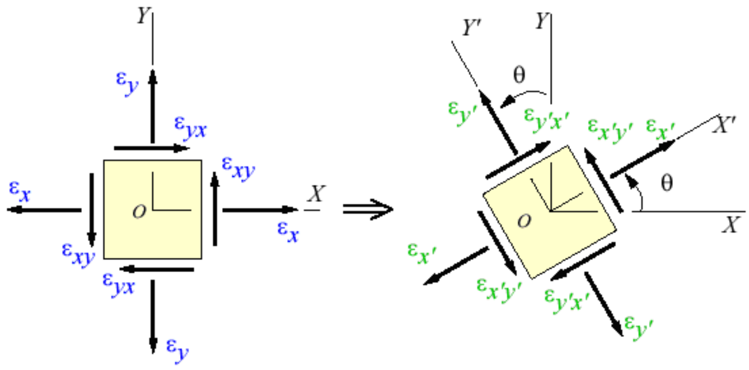

Figure 1.4-1 Examples of transformation taken from efunda

Figure 1.4-1 shows an example of a simple transformation. The box has been rotated to the right side. Hence, the original box's angle has changed.

In this study, global transformation is considered. It is significant to have an optimal transformation so that two curves are related to their corresponding points.

1.4.3 Rigid Super-Curves

Rigid super-curves [7] was proposed based on SC designed to recover only rotation and translation between two 3D skull images. The main difference between RSC and SC is its transformation process. In RSC, a rigid transformation (rotation and translation) between the curves is calculated instead of obtaining the affine transform simultaneously as in the case of SC. For SC, affine transformation is required to recover the matching between two curves. Transformation classes such as translation, rigid body, rotation and horizontal shear are included, as shown in Figure 1.4-2 [1]. In RSC, shearing is not included because it deals with 3D skull images where shearing rarely happens. Rigid transformation is applied because need to have a distinctive separation between rotation, translation and scaling.

With rigid transformation, the shape of the skull is preserved as it will not alter the shape during rotation or translation, as the transformation matrix that is independent. For affine transformation, it is difficult to preserve the shape of the skull. Hence, RSC is implemented for skull registration instead of SC.

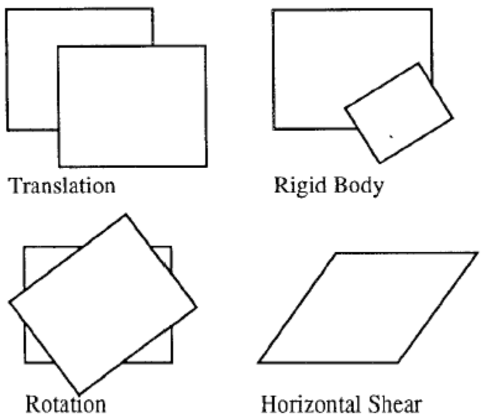

Figure 1.4-2 Examples of typical geometric transformation [1]

1.5 Objectives

The goal of this research is to perform a good matching between reference and test skulls. To obtain good matches, accurate B-spline fitting and scaling must be achieved. In earlier RSC, B-spline parameterisation technique – standard uniform did not provide good accuracy and controllability because it did not assign the control points based on the distance between two sampling points. For scaling, only rotation and translation were applied. Thus, the two main objectives are:

- To propose an effective B-spline parameterisation technique that will provide good accuracy and controllability of B-spline curves.
- To proposed a technique that finds a scale factor for scaling and good matching in transformation.

1.6 Contribution

The main contributions for this research are the proposal of an effective way of generating good B-spline parameterisation values and a technique to find scale factor and perform good matching in rigid super-curves. There are many techniques to find B-spline parameterisation values. However, some of them are burdensome because of excessive number of iterations involved. This causes long computational time and high memory usage. There are other simple techniques but the results are unsatisfying. Thus, a focus on finding an effective B-spline parameterisation that can adjust the measurement of distance between two sampling points that determines the position of control points which have achieved accuracy and controllability. The technique is a modified version of the inverse chord length. Normally, the exponent value which determines the position of control points in inverse chord length is assigned manually and it can only provide controllability for B-spline curves. Hence, an idea is proposed to assign the exponent value automatically and make some changes on the inverse chord length, so that it can achieve accuracy and controllability at the same time. On the other hand, since only rotation and translation were implemented, a scaling process is added in RSC to adjust the size of skulls which are different from each other. Iteration loops for transformation is also included to reduce the registration error. For the iteration loops, an initial guess is proposed and added some stopping criteria. These aspects were used to accelerate the iteration loops for registration.

1.7 Motivation

With the questions in **section 1.3**, it motivates me to research on curve matching using super-curve and B-spline. An answer must be answered whether scaling will affect the results of curve matching. Besides, questions on “What method is good in finding B-spline parameterisation value?” and “Does it actually enhance the efficiency and accuracy of curve matching?” are motivating us to do this research.

1.8 Research Organisation

There are eight chapters in this report. Brief description will be given based on each chapter.

1 Introduction

This chapter briefly explains the research background, problems, questions, objectives, scopes, contribution and motivation. This chapter provides an overview of this research.

2 Literature review

This chapter provides details on the existing or related work on curve matching. Existing methods for skull registration and curve matching are reviewed. A few existing techniques on finding B-spline parameterisation and adjustment for scale factor are also reviewed in this chapter.

3 Theoretical background

This chapter provides a detailed explanation of how each B-spline parameterisation technique works. Equations are provided for each technique. Besides, the difference between super-curves and rigid super-curves is explained, together with the explanation of the concept of rigid super-curves.

4 Research methodology

In this chapter, the proposed techniques for B-spline parameterisation and scaling transformation are explained. Iterations to obtain optimised results are explained in this chapter.

5 Implementation

This chapter describes in detail how to implement the proposed techniques. Pseudo code is provided in order to give a clear view on the methods used for this research.

6 Experiments and results

This chapter provides a detailed explanation of how the experiments are done. The evaluation of these methods is explained in details. Results of the experiments are shown by figures, graphs and diagrams.

7 Discussion

Discussion is done based on the analysis of graphs and diagrams in chapter 6. Comparison between the methods and previous works are discussed.

8 Conclusions and future works

Conclusion is made based on the overall proposed techniques and their performance. An inspection on whether the objectives of this research have been achieved in this research. Future works to enhance are provided.

2 LITERATURE REVIEW

This chapter is divided into three parts. For the first part, discussion on the existing methods for skull registration is carried out. The next part is about scaling transformation techniques, and the last part is about determining the measurement for B-spline.

2.1 3D surface registration methods

There are a few existing methods for skull registration – point-based, surface-based and curve-based.

2.1.1 Point-based registration methods

Point-based methods can also be referred to as feature-based or landmark-based methods. In this approach, skull images are usually represented with multiple sets of point locations with each point describing its intensities and orientations. Iterative Closest Points [2] and Active Shape Model are two methods of the point-based approach. Paul J. Besl and Neil D. Mckay [2] proposed iterative-closest point to determine the closest pair of points and computed the transformation from these pairs.

Deni Suwardhi *et al* [4] used surface-based and ICP (iterative-closest points) in multimodal devices for craniofacial reconstructive surgery. Initially, surface-based registration in the device is used to identify the known features that are common between two skull datasets. After identifying these known features, the application calculates the approximate position of one shell (feature) with respect to another. Then, it overlaps with the corresponding known feature identified earlier. If two shells align exactly with each other, ICP is used to register these two shells.

Most of these methods have a major drawback – locations of the landmarks need to be placed manually in order to relate the correspondence between two skull images [3],[4].

2.1.2 Surface-based registration methods

In surface-based methods, interest surfaces are determined from reference and test images initially. Later, minimum distance between these corresponding surfaces is measured. Segmentation is required to obtain the surface by slices. These surfaces are represented with a large number of points that are connected triangularly. The difference between surface and landmark is that points in landmark are determined manually while points in surface are determined automatically [8].

The Active Surface, Euclidean distance and closest surface points are the common approaches. George K. Matsopoulos *et* al [8] has studied on CT-MRI automatic surface-based registration schemes combining global and local optimisation techniques. Initially, rigid transformation is carried out. Later, Measure of Match (MOM) is used to define the average Euclidean distance between CT and MRI surfaces. In order to obtain accurate result, Downhill Simplex Method (DSM), Genetic Algorithm (GA), and Simulated Annealing (SA) are used to optimise the parameters in MOM. Slyvain Jaume *et al* [9] has performed multi-resolution surface matching. In this method, multi resolution reference surface of an object is obtained and this surface is used to match with the target image using deformable surface-like algorithm. J. Zhang *et al* [10] has done research on surface registration of 3D volumes using neural network approach. Patient's specific surface model is constructed using multilayer perceptron neural network. A function representing the surface is derived and used for intra-operative registration.

Surface-based method is a high-level task and possibly error-prone. Besides, reducing grey-valued images to surfaces will cause the loss of valuable information [5].

2.1.3 Curve-based registration methods

Since there are drawbacks faced by point-based and surface-based methods, curve-based try to compromise between both of them. Curve-based approach does not need to locate landmarks manually and identify known features for registration. It will use the curves extracted by feature extraction methods to perform registration. This method requires local structure information rather than images or points intensities. Active contour algorithm [11] and 3D-multimodality brain image registration algorithm [12] are methods based on curve matching. However, B-spline based methods are more favored by researchers. Choong-Gyoo Lim [13],[14],[15],[16],[17] used b-spline fitting to perform curve matching.

Super-Curves is based on Xia and Liu's [6] Super-Curves idea and further investigated by Iman *et al* [7]. SC is a B-Spline based method that has the advantage of non-ambiguous representation of B-Spline fittings and it is intrinsically designed to recover the affine transformation between two 2D images. Affine transformation in SC is not suitable to be applied for skull registration because it is difficult to preserve the shape of the skull. Later, rigid super-curves [7] is proposed based on SC which was designed to recover only the rotation and translation between two 3D skull images. There are still two problems in RSC. The first problem is the determination of B-spline parameterisation. It is an important parameter in B-spline that measures the distance between sampling points along B-spline curves. The second is the scaling transformation. In earlier research by Iman *et al* [7], only rotation and translation have been considered. In transformation, scaling should be included in order to obtain good results.

2.2 Scaling Transformation

For scaling, the scaling factor for transformation needs to be determined. Basically, there are two types of scaling - uniform scaling and non-uniform scaling. Generally, non-uniform scaling does not apply on skull because the alteration for the 3 dimensions – x, y and z must be applied simultaneously with same scale factor in order to attain the shape of the skull. Hence, in this research, uniform scaling is applied.

It is important to have accurate scaling factor. In uniform scaling, there is only one scaling factor that needs to be concerned as it will scale images in all axes (x, y, and z) with the same value. It is also called isotopic scaling. Many researchers have tried to find the best scaling factor that can be adjusted for CT scan data. According to Terry S. Yoo [18], scaling factor that is usually used is rarely below 0.5 or over 2.0 unless there is a need to register infant to adult. It mostly remains around 1.0. Optimisation process [18] is carried out in order to alter scale values. Scale factors are altered by calculating the gradient of cost function in the parameter space which is divided with step length $\boldsymbol{\lambda}$. A new scaling factor is adjusted with every transformation. This function provides a proper way to adjust scale factor automatically instead of adjusting it manually by guessing the values. However, at the same moment, $\boldsymbol{\lambda}$ also need to be adjusted so that the scale factor was uniform. Philippe *et al* [19] had carried out experiments to test on λ using various values. The range of the values was from 0.500 to 1.000. From the experiments, they showed that the smaller the values of λ, more quality was lost. Hence, 1.000 is the best value to provide a better scale factor. Terry S. Yoo also mentioned that the value is always close to 1.000 in most of the medical image registration. Ma Jian-Lin *et al* [20] also assigned λ as 1.000. Darius *et al* [21] had used eigen-value to calculate scaling factor. However, to calculate eigen-value, it requires to collect non-collinear points from camera.

2.3 B-spline

Many researches had been done to find B-spline parameterisation value. B-spline parameterisation is represented as $\mathbf{t}_i$ in this research and it is the distance measurement of sampling points along B-spline curves. It is very important because it is used to calculate control points, **d**. Methods such as equidistant, chordal, centripetal, and Foley [13] are used to find $\mathbf{t_i}$ for B-spline. Equidistant method is a simple method that only calculates the distance between $\mathbf{t_i}$ using average distance between sampling points. It does not make any proper adjustment between $\mathbf{t_i}$ according to the distance traveled between data points. Hence, it is a poor technique that provides bad results as control points are scattered unevenly along B-spline curves. As for the Foley method, it is an effective method that can be applied to determine $\mathbf{t}_i$. It calculates the distance between two consecutive sampling points and also the angle (θ) between them. Using this method, control points are distributed evenly between knot values. However, it requires a lot of calculation that burden the whole registration process and it is only suitable to be used up to curves of order 4.

Minghui Xia *et al* [6] used centripetal parameterisation to initialise $\mathbf{t}_\mathbf{i}$. Later, $\mathbf{t}_\mathbf{i}$ is required to calculate control points. But, this initial set of $\mathbf{t}_\mathbf{i}$ is not accurate enough as the distance between two control points is not the closest to each other. In order to obtain a better $\mathbf{t}_\mathbf{i}$, $\mathbf{t}_\mathbf{i}$ needed to be updated by using a set of control points that are generated initially. After the updating process, a new $\mathbf{t}_\mathbf{i}$ is obtained and is used to calculate new control points. Later, $\mathbf{t}_\mathbf{i}$ is updated again by using a new set of control points. This iterative process continues until fine $\mathbf{t}_\mathbf{i}$ and control points are obtained. It is a good method to gain control point and $\mathbf{t}_\mathbf{i}$ because the iterative process to update these values will definitely provide good results at the end. However, this might not be efficient in terms of time factor because it involves an iterative process. Besides, it is difficult to determine its stopping criterion because it does not know what control points and $\mathbf{t}_\mathbf{i}$ are the finest.

Chordal or chord length method [13], [14], [15], [16], [17] is a common method used by researchers to locate $\mathbf{t}_\mathbf{i}$. Centripetal parameterisation is the modification from chord length. For chordal parameterisation, the $\mathbf{t}_i$ spacing is proportional distance between points, while with centripetal, it is proportional to the square root of the distance between points. Both methods are similar where they are calculating the fraction of the distance traveled between two data points relative to total distance of data points. Hyungjun Park *et al* [22] used general exponent method which has the same concept as chord length, where it make sure that each of the knot spans will contain at least one $\mathbf{t_i}$. Tet Toe *et al* [15] had used chord length parameterisation to estimate $\mathbf{t_i}$. General exponent and chord length has the common features where it equally splits the $\mathbf{t_i}$ value between two data points along the curves. These methods are performed well on uniformly sampled data because the traveling speed and distance between data points are constant. It is easier to distribute $\mathbf{t}_i$ along the curves with uniform sampling data. However, they cannot be applied on the curves where curvatures change rapidly, as shown in Figure 2.3-2, because more control points are needed for the curvatures. In high curvatures areas, it is better to have more control points in order to mark the shape of the curvatures. It also requires the best affine transformation between reference and test curves because it will fail to register if test curves are stretching in some directions. The main problem is it suffers for non-uniformly sampled data and non-uniform distributed noise. It is only suitable for uniformly sampled data. For non-uniformly sampled data, dense and sparse areas happened on one curve, as shown in Figure 2.3-1. If chordal is applied, every curve segments will obtain the same number of control points, and information loss happens in dense areas as not many control points are assigned. Hence, chordal is not suitable to be used to find $\mathbf{t}_i$ since traveling distance between data points in non-uniform data is not uniform. More control points are needed to allocate to densely sampled areas.

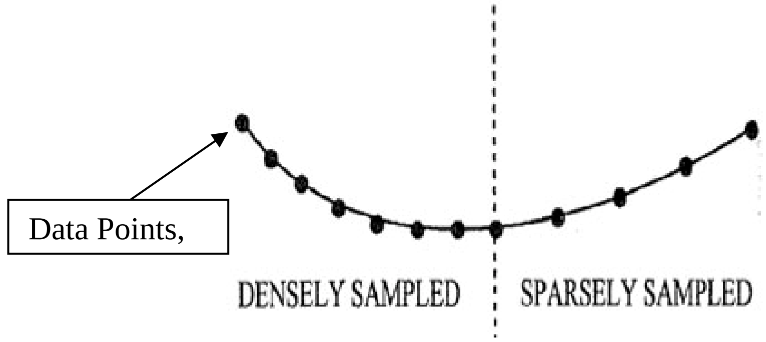

Figure 2.3-1 densely and sparsely sampled areas

Since chord length parameterisation has such problems, Zhaohui Huang *et al* [17] had used inverse chord length method to overcome the problems. Inverse chord length is able to assigned more control points to dense areas. In densely sampled area, distance between the neighbouring control points must be shorter compared to sparsely sampled area. Since the distance is shorter, more control points can be assigned to dense areas. Hence, the traveling speed will be slower in dense areas. This can be achieved by making the speed of traveling between two data points longer in dense areas compared to sparse areas. To achieve slower speed, Zhaohui Huang *et al* [17] make the traveling time between two points inversely proportional to the chord length between them. It has good ability to control shape variation of curves, as shown in Figure 2.3-2. It will apply more control points to high variation areas compared to low variation areas.

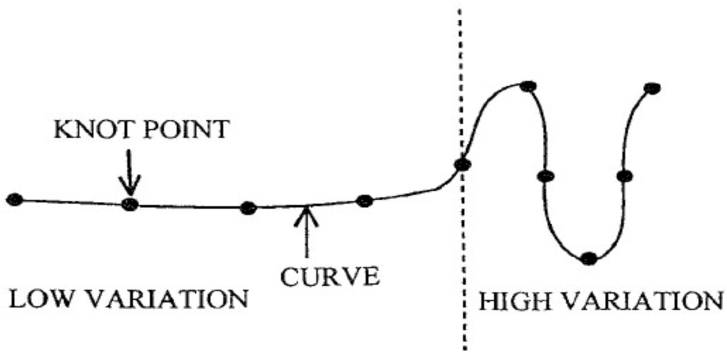

Figure 2.3-2 High variation and low variation areas

For this method, the exponent value must be determined manually. The characteristics of the data need to be known before a proper exponent value is assigned. It will cause inconvenience for researchers.

2.4 Summary

In summary, there are many techniques used for image registration including curve matching via B-spline. In B-spline, $\mathbf{t_i}$ is a very important parameter. Many researchers have tried many techniques to find an excellent $\mathbf{t_i}$ which does not require excessive time but returns an accurate value. Hence, in this research, an effective technique to find $\mathbf{t_i}$ is chosen. For scaling, appropriate techniques need to be applied to either uniform or non-uniform data. Appropriate decision on adjusting scaling values need to be made. This is to ensure that accuracy is achieved during the matching procedure. Table below shows comparison of techniques in finding $\mathbf{t_i}$.

	Advantages	Disadvantages
Uniform Parameterisation	- Simple and fast.	- Did not make any proper adjustment between $\mathbf{t_i}$. - Data points were scattered unevenly along B-spline curves.
Chord length Parameterisation	- Simple and fast. - Suitable for uniform data.	- Cannot apply on the curves where curvatures change rapidly. - Required the best affine transformation between reference and test curves because it will fail to register if test curves are stretching in some directions. - Only suitable for uniform B-spline curves of degree 3. - It suffers for non-uniform sampling data and non-uniform distributed noise. - Non-sampling data that used chord length will cause information loss in dense areas.
Centripetal Parameterisation	- Accurate. - Simple and fast.	- Requires updating of the parameter distance between two knots in order to calculate $\mathbf{t_i}$. - Does not take sharp corners into consideration.
Inverse Chord length parameterisation	- Able to handle difficult data. - Suitable for non-sampling data. - Able to handle noise very well. - More control points are assigned to densely areas. - Good handling of high and low variation area. - Might be suitable for higher order.	- Need to define exponent value manually.
Foley	- It presented good results because it had divided $\mathbf{t_i}$ wisely using θ. - Data points were distributed evenly between knot values.	- It required a lot of calculations that burden the whole registration process. - It is only suitable to be used up to curves of order 4. - Order > 4 that used Foley will achieve deviate results.

Table 2.4-1 Comparison of Techniques to find t

3 THEORETICAL BACKGROUND

In this chapter, a few methods of B-spline parameterisation, the concept of super-curves and rigid super-curves are briefly introduced.

3.1 Methods of B-spline parameterisation

B-spline is a robust technique that possesses local propagation ability that controls the shape of the curves using a few neighbouring control points. It can be in any degree without affecting the number of control points [23]. Control points are considered as important elements because they control the shape of the curves. It does not propagate the entire curve. Besides, it is suitable for fitting non-uniform sampling data and noise. Other prominent properties of B-spline are spatial uniqueness, boundedness and continuity, and invariance to affine transformation. It is utilised to form a single B-spline curve and register between two curves concurrently. In B-spline representation, each sample point has a parameter, measuring its distance from the starting point along the B-spline. Suppose [$\mathbf{P}_0$, …, $\mathbf{P}_{N-1}$] are the sample points of a curve. Their corresponding B-spline parameters are denoted as $\mathbf{t}$ = [$\mathbf{t}_0$, …, $\mathbf{t}_{N-1}$]. $\mathbf{t}$ is important as it affects the position of control points. Hence, the shape of the curve needs to be handled carefully. There are a few methods used to determine $\mathbf{t_i}$ value – standard uniform, uniform [13], [24], chordal [13], [14], [15], [22], [24], [25], centripetal [6], [13],[14], [22], [24], and inverse chord length [16], [17] which will be explained in detail. $\mathbf{t_1}$ for reference curve and $\mathbf{t_2}$ for test curve are denoted respectively.

3.1.1 Uniform

It is a simple method that only calculates the average distance between $\mathbf{t_i}$ based on the number of sampling points. It does not make any proper adjustment between $\mathbf{t_i}$ according to the distance traveled between data points.

$$To\ find\ t_1\ \&\ t_2\ with\ uniform \quad \longrightarrow \quad t_i \;=\; t_{i-1} \;+\; \frac{|P_i \,-\, P_{i-1}|^0}{\sum_{s=1}^{j-1}|P_{s+1} \,-\, P_s|^0}, \qquad \textbf{(3.1)}$$

$$with\ \ 2\ \le i\ \le j$$

3.1.2 Standard uniform

This technique is formulated by calculating the uniform distance between reference and test curves for each data points. To explain,

- Firstly, assume that reference and test curves are the same curves with same length.
- Next, obtain **t** for these two curves using uniform parameterisation, then re-adjust t_2 so that it fits t_1. This is to ensure that it will have the same length as reference curve.
- Since both curves have the same length, assume that they have the same first point and last point.
- From a new set of t_2, the first t_1 value will be the same as the reference curve.
- For the rest of it, it is necessary to adjust them according to the fraction between test and reference curves.

For example, in Figure 3.1-1, there are two curves, reference (red) and test (blue) curves. They are in different length to fit between test curve and reference curve, adjust the t_2 on test curve to fit reference curve.