Development of Attitude Determination for Student Pico-Satellite INNOSAT

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Abstract: Information of satellite's attitude is very useful in autonomous satellite control system. For a small satellite particularly, attitude sensor cannot be installed in the satellite because of the limited weight and power consumption. So, the attitude should be obtained from other sensors, they are a combination of two satellite's positioning sensors, sun sensor and magnetometer. This paper represents about the development of attitude determination system for INNOSAT by using deterministic and recursive approach. Since given the deterministic method cannot be used when satellite is in eclipse, the recursive method will be used during the eclipse [10][12]. The deterministic attitude determination is calculated by using q-method method. Because of dynamic nonlinearity of the satellite, for recursive estimator was used Extended Kalman filter (EKF). To analyze and develop the attitude determination on ground, the measured attitude data was produced from Satellite Tool Kit (STK). Development of attitude determination is performed by using linear and nonlinear quaternion satellite model for both methods and also observing the effect of attitude shifting from deterministic to recursive method in eclipse problem.

Keywords : Attitude determination, INNOSAT, EKF, q-method(QUEST)

1. Introduction

INNOSAT is a Pico-satellite which was built by four universities in Malaysia and a company engaged in satellite manufacturing. The INNOSAT's structure has been developed mainly by Astronautic Technology Sdn Bhd (ATSB) and experimental payload systems are developed by University Sains Malaysia (USM), Universiti Malaysia Perlis (UNIMAP), Universiti Teknologi Malaysia (UTM), Universiti Kebangsaan Malaysia (UKM) and USM contributes mainly in development of space experimental attitude determination sensors and solar panel sun sensors.

The main mission of INNOSAT is to take photos of Malaysian region from its orbit. Therefore, INNOSAT was carrying a camera as payload that installed in satellite. The captured photos will be stored temporarily in the memory until the satellite enters the communication region, then satellite downlink the photos to ground station. Secondary mission of this satellite is a technology demonstrator for the fourth university in the development of hardware sub-systems and methodologies for attitude determination and control.

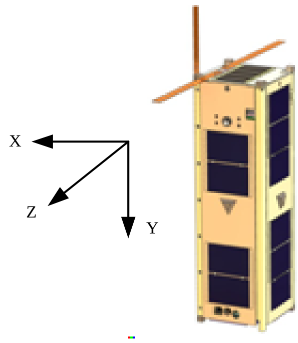

Figure 1. INNOSAT

INNOSAT will orbit the earth at the altitude of 680 km with 1e-5 eccentricity and 9^{o} inclination from equator line. INNOSAT is a small satellite which has a dimension of 10x10x30 cm^3 with 3 Kg of maximum weight and 15W of maximum power. INNOSAT image was shown in Figure 1. Generally small satellite like INNOSAT has those limitations. So the satellite has to be built with inexpensive, lightweight, and low power consumption. With these limitations INNOSAT should be able to carry out mission (taking photos of Malaysia region). Quality of photos was strongly influenced by the attitude, position, and light intensity when the satellite takes the shooting. So attitude determination and control play an important role for achieving the mission subject and sustainability of satellite on its orbit a long it designed[4][7][8][10].

Attitude of the satellite was expressed by the angular position and angular velocity in satellite body frame with relative to local coordinate orbit frame. Accuracy of attitude information is very important in satellite control problem. The problems of accuracy such as noise measurement, bias, and misalignment were affected the attitude estimation. So, in attitude determination was proposed to use two or more sensors as optimal combination to estimate the attitude as a solution to improve the accuracy [4][8][10].

Limitations of mass and power budget for INNOSAT, has been forced to use lightweight hardware, small and low power consumption. There for the use of attitude sensors such as gyroscope is not possible because of the limitations. So INNOSAT attitude are determined and estimated using optimal combination of two position sensors, they are sun sensors and magnetometers. The Attitude cannot be determined by using this technique when the sun sensors are in eclipse phase. This will push to use two methods simultaneously which widely used by various satellite to determine its attitude. They are deterministic method[10][11][12] and Extended Kalman Filter (EKF) [5][6][8][10][12].

This paper represents the development of attitude determination for INNOSAT. Deterministic method is used as main attitude determination and EKF is used only when the satellite is in eclipse. Satellite dynamics are modeled by using quaternion to avoid singularity when using Euler[1][2][3].To develop and analyzed the attitude determination on ground, the measured sun sensor and magnetometer data was produced from Satellite Tool Kit (STK) software and orbital data of sun vector and Earth magnetic field was produced by sun model and IGRF data. The process was shown in Development of attitude determination is performed by using linear and nonlinear satellite quaternion model for both methods and also observing the effect of attitude shifting from deterministic to EKF during the eclipse problem

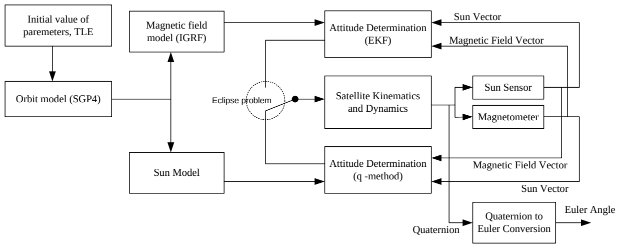

Figure 2. Attitude determination process

2. Attitude Dynamics

2.1. Quaternion Representation

Unit quaternion can be described as,

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T \qquad (1)$$

In term of Euler axis, quaternion can be represented with $\hat{e} = \begin{bmatrix} e_x & e_y & e_z \end{bmatrix}^T$ and angle θ. This vector element are expressed as follow,

$$q_1 = e_x \sin\left(\frac{\theta}{2}\right), q_2 = e_y \sin\left(\frac{\theta}{2}\right)$$
$$q_3 = e_z \sin\left(\frac{\theta}{2}\right), q_4 = \cos\left(\frac{\theta}{2}\right) \qquad (2)$$

Direct cosine matrix of quaternion can be calculated as follows,

$$R_b^o = \left(q_4 - \breve{q}^T \breve{q}\right) I_3 + 2\breve{q}\breve{q}^T - 2q_4 SO_3\left(\breve{q}\right) (3)$$

Which gives R_b^o as,

$$R_b^o = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2-q_3q_4) & 2(q_1q_3+q_2q_4) \\ 2(q_1q_2+q_3q_4) & 1-2(q_1^2+q_3^2) & 2(q_2q_3-q_1q_4) \\ 2(q_1q_3-q_2q_4) & 2(q_1q_4+q_2q_3) & 1-2(q_1^2+q_2^2) \end{bmatrix} \qquad (4)$$

And

$$R_o^b = \left(R_b^o\right)^{-1} = \left(R_b^o\right)^T = \begin{bmatrix} c_1^b & c_2^b & c_3^b \end{bmatrix} \ (5)$$

And attitude conversion from quaternion to Euler becomes,

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\dfrac{2(q_1q_4+q_2q_3)}{1-2(q_1^2+q_2^2)}\right) \\ \arcsin\left(2(q_4q_2-q_1q_3)\right) \\ \arctan\left(\dfrac{2(q_3q_4+q_2q_1)}{1-2(q_2^2+q_3^2)}\right) \end{bmatrix} \qquad (6)$$

2.2. Attitude Kinematic

The kinematic represent the satellite’s orientation in space and found by integration of the angular velocity. Angular velocity of the satellite is described by using quaternion as given by [2],

$$\dot{\boldsymbol{q}}(t)=\begin{bmatrix}\dot{q}_4(t)\\ \dot{q}_{1:3}(t)\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}q_{1:3}{}^T\omega_{ob}^b\\ \frac{1}{2}\left[q_4 I+S\left(q_{1:3}\right)\right]\omega_{ob}^b\end{bmatrix} \quad (7)$$

Where $q_4(t)$ is a part of quaternion vector, $q_{1:3}(t)$ is the scalar part and ω_{ob}^b is angular velocity of body with respect to orbit.

2.3. Attitude Dynamic

The dynamic equation of motion is derived from the change in angular momentum of the satellite and an expression for the change in angular velocity, as a function of the applied torques, is sought. A detailed in [1][9]. The satellite dynamic equation is given by

$$\boldsymbol{I}\dot{\boldsymbol{\omega}}_{ib}^b(t)+\boldsymbol{\omega}_{ib}^b(t)\times\boldsymbol{I}\boldsymbol{\omega}_{ib}^b(t)=\tau_c(t)+\tau_g(t) \quad (8)$$

Where $\boldsymbol{I}$ is the satellite's inertia matrix given in body frame, $\boldsymbol{\omega}_{ib}^b(t)$ is the angular velocity of the body frame relative to the inertial frame given in body frame, $\tau_c(t)$ is the control torque and $\tau_g(t)$ is gravity gradient torque that applied to the satellite.

2.4. Non-Linear Dynamics

By combining the kinematic equation and the dynamic equation, it will produce the nonlinear differential equation. By defining state vector of the system,

$$x(t)=\begin{bmatrix}q_4 & q_1 & q_2 & q_3 & \omega_{ib,x}^b & \omega_{ib,y}^b & \omega_{ib,y}^b\end{bmatrix}^T \quad (9)$$

By inserting eq.(7) to eq. (8), the nonlinear dynamic becomes,

$$\begin{bmatrix}\dot{q}_4\\ \dot{q}_{1:3}\\ \omega_{ib}^b\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}q_{1:3}(t)^T\omega_{ob}^b\\ \frac{1}{2}\left[q_4(t)I+S\left(q_{1:3}(t)\right)\right]\omega_{ob}^b\\ \boldsymbol{I}^{-1}\left(\tau_{ext}-\left(\boldsymbol{\omega}_{ib}^b(t)\times\boldsymbol{I}\boldsymbol{\omega}_{ib}^b(t)\right)\right)\end{bmatrix} \quad (10)$$

However, it is need to represent the attitude of satellite in body frame, so this can be done by exploiting the relation of,

$$\begin{aligned}\boldsymbol{\omega}_{ib}^b&=\boldsymbol{\omega}_{ob}^b+\boldsymbol{\omega}_{io}^b\\ \dot{\boldsymbol{\omega}}_{ib}^b&=\dot{\boldsymbol{\omega}}_{ob}^b+\dot{R}_o^b\boldsymbol{\omega}_{io}^o\\ \dot{\boldsymbol{\omega}}_{ib}^b&=\dot{\boldsymbol{\omega}}_{ob}^b-S\left(\omega_{ob}^b\right)R_o^b\boldsymbol{\omega}_{io}^o\end{aligned} \quad (11)$$

Where $\boldsymbol{\omega}_{io}^o=\begin{bmatrix}0 & -\omega_o & 0\end{bmatrix}^T$ is assume constant by substituting (5) to (11) and (11) to (10), nonlinear dynamic equation becomes,

$$\begin{bmatrix}\dot{\boldsymbol{q}}_4(t)\\ \dot{\boldsymbol{q}}_{1:3}(t)\\ \dot{\boldsymbol{\omega}}_{ob}^b(t)\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}q_{1:3}(t)^T\omega_{ob}^b\\ \frac{1}{2}\left[q_4(t)I+S\left(q_{1:3}(t)\right)\right]\omega_{ob}^b\\ \boldsymbol{I}^{-1}\left(-S\left(\omega_{ob}^b-\omega_o c_2^b\right)I\left(\omega_{ob}^b-\omega_o c_2^b\right)+\tau_g+\tau_c\right)-S\left(\omega_{ob}^b\right)\omega_o c_2^b\end{bmatrix} \quad (12)$$

And the state vector becomes,

$$x(t)=\begin{bmatrix}q_4 & q_1 & q_2 & q_3 & \omega_{ob,x}^b & \omega_{ob,y}^b & \omega_{ob,y}^b\end{bmatrix}^T \quad (13)$$

2.5. Control Torque

Magnetic torque will produce magnetic dipole when current flow through its winding which is proportional with number of windings and cross section area of the coils [1][9][11]. Magnetic dipole will react with the perpendicular Earth's magnetic field to generate torque. The torque generated by the magnetic torque can be modeled by,

$$\tau_m^b=m^b\times B^b \quad (14)$$

Where $B^b=\begin{bmatrix}B_x^b & B_y^b & B_z^b\end{bmatrix}^T$ is the local geomagnetic field vector relative to the satellite body and m^b is magnetic dipole vector which is define by,

$$m^b=m_x^b+m_y^b+m_z^b=\begin{bmatrix}N_x i_x A_x\\ N_y i_y A_y\\ N_z i_z A_z\end{bmatrix}=\begin{bmatrix}m_x\\ m_y\\ m_z\end{bmatrix} \quad (15)$$

Where N_k is number of windings i_k is the coil current, and A_x is the span area of the coil. Based on vector property, equation (14) and (15) generated torque can be representing with,

$$\tau_m^b=S\left(m^b\right)B^b=\begin{bmatrix}B_z^b m_y-B_y^b m_z\\ B_x^b m_z-B_z^b m_x\\ B_y^b m_x-B_x^b m_y\end{bmatrix} \quad (16)$$

Because of local magnetic field vector around the satellite is calculated by using IGRF model, so calculated magnetic field vector is represented in orbit frame. From equation (16), magnetic field vector relative to body $B^b=\begin{bmatrix}B_x^b & B_y^b & B_z^b\end{bmatrix}^T$ will be provided from magnetic field vector relative to orbit $B^o = \begin{bmatrix} B_x^o & B_y^o & B_z^o \end{bmatrix}^T$. So magnetic vector is satellite body become,

$$B^b = R_o^b B^o \tag{17}$$

By substituting (17) to (16), magnetic torque equation becomes,

$$\tau_m^b = S\left(m^b\right) B^b = S\left(m^b\right) R_o^b B^o \tag{18}$$

One of the effects of using magnetic torque is that they will contribute to the measurements of the Earth's magnetic field [1][9][11].

2.6. Gravity Gradient Torque

The satellite which orbiting the Earth, will affect by gravitational forces of Earth and the forces will produce torque on satellite body. The torque is derived in [9][11] by assuming that Earth's mass distribution is homogenous, the gravitational torque is derived as,

$$\tau_{grav} = \frac{3\mu}{R_o^3} u_e \times \left(I u_e\right) \tag{19}$$

Where μ is Earth's gravitational coefficient, R_o is distance from Earth's center (m), u_e unit vector toward nadir, and I is inertia matrix. Equation (19) in body will be,

$$\begin{aligned} \tau_{grav} &= 3\omega_o^2 c_3^b \times \left(I c_3^b\right) \\ &= 3\omega_o^2 S\left(c_3^b\right) I c_3^b \end{aligned} \tag{20}$$

Where $\omega_o^2 = \frac{\mu}{R_o^3}$ is angular velocity of orbit and $c_3^b = \begin{bmatrix} c_{13}^b & c_{23}^b & c_{33}^b \end{bmatrix}^T$ is the third column of R_o^b (because only z-axis will be affected to Earth's forces).

2.7. Complete Attitude Dynamic

Complete model is constructed by substituting equation (18) and (20) to (12), by assuming that internal torque is negligible. The nonlinear complete model becomes,

$$\begin{bmatrix} \dot{\boldsymbol{q}}_4(t) \\ \dot{\boldsymbol{q}}_{1:3}(t) \\ \dot{\boldsymbol{\omega}}_{ob}^b(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} q_{1:3}(t)^T \omega_{ob}^b \\ \frac{1}{2}\left[q_4(t) I + S\left(q_{1:3}(t)\right)\right]\omega_{ob}^b \\ \dot{\boldsymbol{\omega}}_{ob}^b(t) = \boldsymbol{I}^{-1}\left(-S\left(\omega_{ob}^b - \omega_o c_2^b\right) I \left(\omega_{ob}^b - \omega_o c_2^b\right) + 3\omega_o^2 S\left(c_3^b\right) I c_3^b + S\left(m^b\right) R_o^b B^o\right) - S\left(\omega_{ob}^b\right)\omega_o c_2^b \end{bmatrix} \tag{21}$$

3. Attitude Determination Method

3.1. q-Method (QUEST)

Wahba's problem is formulated as an eigenvector problem and directly estimates an optimal attitude quaternion minimizing Wahba's problem [10][12]. Given a set of $n \geq 2$ vector observation, a loss function is formulated, known as Wahba's problem given by

$$L\left(\boldsymbol{A}_{ob}^b\right) = \frac{1}{2}\sum_{j=1}^{n} w_j \left\| \boldsymbol{b}_j - \boldsymbol{A}_{ob}^b \boldsymbol{r}_j \right\|^2 \tag{22}$$

Where w_j is the weight of the j[th] vector observation, $\boldsymbol{r}_j$ is vector in the reference frame and $\boldsymbol{A}_{ob}^b$ is the orthonormal rotation matrix, representing the rotation from the reference frame to the body frame, which is sought [10][12]. The loss function is a weighted sum squared of the difference between the measured and the reference vectors in the body frame. By minimizing the loss function, given in equation (22) an optimal attitude may be estimated. Wahba's loss function can also be written as

$$L\left(\boldsymbol{A}_{ob}^b\right) = \sum_{j=1}^{n} w_j - tr\left(\boldsymbol{A}_{ob}^b \boldsymbol{B}^T\right) \tag{23}$$

Where *B* is given by

$$\boldsymbol{B} = \sum_{j=1}^{n} w_j \boldsymbol{b}_j \boldsymbol{r}_j^{\,T} \tag{24}$$

Based on equation (23), the loss function $L\left(\boldsymbol{A}_{ob}^b\right)$ is minimized by maximizing $tr\left(\boldsymbol{A}_{ob}^b \boldsymbol{B}^T\right)$.

In the q-method, the rotation matrix $\boldsymbol{A}_{ob}^b$ is determined as a quaternion by computing the symmetric 4×4 matrix *K*.

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{S} - \sigma \boldsymbol{I} & \boldsymbol{Z} \\ \boldsymbol{Z}^T & \sigma \end{bmatrix} \tag{25}$$

With

$$\boldsymbol{S} = \boldsymbol{B} + \boldsymbol{B}^T \tag{26}$$

$$\boldsymbol{Z} = \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} \tag{27}$$

$$\sigma = tr[\boldsymbol{B}] \quad (28)$$

The eigenvector of K with the largest Eigen value will in this way give the optimal quaternion representing the rotation.

$$\boldsymbol{K}\boldsymbol{q}_{opt} = \lambda_{max}\boldsymbol{q}_{opt} \quad (29)$$

And λ_{max} can be defined as,

$$\lambda_{max} = \sqrt{w_1^2 + w_2^2 + 2w_1 w_2\left[(\boldsymbol{b}_1.\boldsymbol{b}_2)(\boldsymbol{r}_1.\boldsymbol{r}_2) + |\boldsymbol{b}_1 \times \boldsymbol{b}_2||\boldsymbol{r}_1 \times \boldsymbol{r}_2|\right]} \quad (30)$$

The optimal quaternion by using QUEST method is,

$$q_{opt} = \frac{1}{\sqrt{\gamma^2 + |x|^2}}\begin{bmatrix} x \\ \gamma \end{bmatrix} \quad (31)$$

In which γ and x are given by,

$$\gamma = \left(\lambda_{\max}^2 - (trB)^2 + tr(adjS)\right)(\lambda_{\max} + trB) - \det S \quad (32)$$

$$x = \left[\left(\lambda_{\max}^2 - (trB)^2 + tr(adjS)\right)I + (\lambda_{\max} + trB)S + S^2\right]z \quad (33)$$

$$L(A_{opt}) = \lambda_o - \lambda_{\max} \quad (34)$$

As a note, that angular velocity of the satellite cannot be determined by using q-method.

2.8. Extended Kalman Filter (EKF)

The operation of the EKF is illustrated as shown in Figure 3. To apply EKF techniques, it is first necessary to describe the real behavior by a set of nonlinear models. The system can be represented by a nonlinear continuous system model and a nonlinear discrete measurement model in equations (35) and (36), respectively.

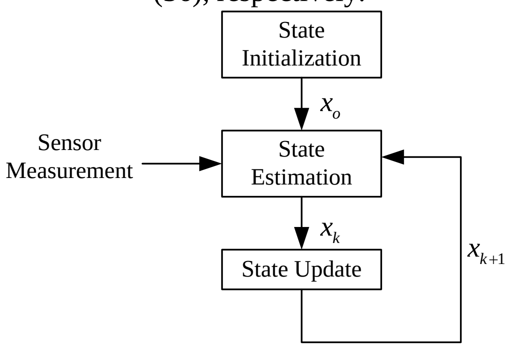

Figure 3. Estimation process of EKF

Where $\boldsymbol{w}(t)$ is zero-mean white process noise and $\boldsymbol{v}_k$ zero-mean measurement noise.

$$\dot{\boldsymbol{x}}(t) = f\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) + \boldsymbol{w}(t) \quad (35)$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k \quad (36)$$

The process noise $\boldsymbol{w}(t)$ is described by equation (37) where $\boldsymbol{Q}(t)$ is the strength of process noise. The discrete noise $\boldsymbol{v}_k$ has the covariance described by equation (38),with R_k representing the measurement noise covariance matrix.

$$E\left[\boldsymbol{w}(t)\boldsymbol{w}^T(t)\right] = \boldsymbol{Q}(t) \quad (37)$$

$$E\left[\boldsymbol{v}_k\boldsymbol{v}_k^T\right] = \boldsymbol{R}_k \quad (38)$$

When a system is described as a nonlinear model equation, the EKF requires linear the equations. The filter linearized the nonlinear functions $f(\boldsymbol{x}, \boldsymbol{u}, t)$ and $h(\boldsymbol{x})$ at a nominal state during estimation process. According to [13] and [14], if there is no deterministic disturbance or control scalar, the discrete model of the continuous system is given by

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}_k\boldsymbol{x}_k + B_k u_k + \boldsymbol{w}_k \quad (39)$$

and the measurement model becomes

$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \quad (40)$$

where $\boldsymbol{\Phi}_k$ is the state transition matrix that propagates the states estimates, is derived using forward Euler integration from (35) and defined as,

$$\boldsymbol{\Phi}_k \approx I + \left.\frac{\partial f(x_k, u_k)}{\partial x_k}\right|_{x_k = \hat{x}_k} .T_s \quad (41)$$

where T_s is sample time and $\boldsymbol{H}_k$ is the measurement matrix derive from rotation matrix from orbit to body reference R_o^b, and define with, $\boldsymbol{H}_k = [h_i \quad h_{i+1} \quad \cdots \quad h_{i+n} \quad 0]$ and,

$$h_i = \begin{bmatrix} \dfrac{\partial R_o^b(\hat{q}_k)}{\partial \hat{q}_{i,k}} B_{orb,k+1} \\ \dfrac{\partial R_o^b(\hat{q}_k)}{\partial \hat{q}_{i,k}} S_{orb,k+1} \end{bmatrix},$$

$\hat{q}_{i,k}$ is the i^{th} estimated component of quaternion at time k then $B_{orb,k+1}$ is model magnetic field (IGRF) at time k+1 and $S_{orb,k+1}$ is model sun vector at time k+1.

The EKF algorithm for the system is defined in equation (35) according to [8], [12] and [14] given by,

$$\boldsymbol{K}_k = \bar{\boldsymbol{P}}_k\boldsymbol{H}_k^T\left[\boldsymbol{H}_k\bar{\boldsymbol{P}}_k\boldsymbol{H}_k^T + \boldsymbol{R}\right]^{-1} \quad (42)$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k\left(y_{meas,k}^b - \boldsymbol{H}_k\bar{\boldsymbol{x}}_k\right) \text{ or}$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k\left(y_{meas,k}^b - R_o^b(\hat{q}_k)y_{model,k}^b\right) \quad (43)$$

$$\boldsymbol{P}_k = [\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k]\bar{\boldsymbol{P}}_k [I - K_k H_k]^T + K_k R K_k^T \quad (44)$$
$$\bar{\boldsymbol{x}}_{k+1} = \boldsymbol{\Phi}_k \hat{\boldsymbol{x}}_k$$
$$\bar{\boldsymbol{P}}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{P}_k \boldsymbol{\Phi}_k^T + \boldsymbol{Q} \quad (45)$$

where $\boldsymbol{K}_k$ is the Kalman gain matrix, $\boldsymbol{P}_k$ is the error covariance, and $\bar{\boldsymbol{P}}_{k+1}$ is the error covariance propagation.
The quaternion update can be interpreted as a rotation and the quaternion product given by,

$$\hat{q}_k = \bar{q}_k \otimes \boldsymbol{K}_{q,k} v_k \quad (46)$$

The angular velocity is updated using equation

$$\hat{\boldsymbol{\omega}}^b_{ob,k} = \bar{\boldsymbol{\omega}}^b_{ob,k} + \boldsymbol{K}_{\omega,k} v_k \quad (47)$$

where v_k is the result of innovation process given by,

$$v_k = y^b_{meas,k} - \boldsymbol{H}_k \bar{\boldsymbol{x}}_k \quad (48)$$

which is different between real and the predicted measurement.

4. Simulation Results and Analysis of Attitude Determination

Attitude determination calculation is started by inserting initial values from Table 1 to Table 3 to equations in Section 2.7 and 3.1 for deterministic method. And by inserting initial values in Table 1 to Table 4 to Section 2.7 and 3.2 for EKF. Measurement data of magnetic field and sun vector are provided from Satellite Tool Kit™ (STK) and magnetic field and sun vectors surrounding of satellite are provided from IGRF and sun model which are used in[10]. The simulations were performed for two orbits to determine the effect of the presence of eclipses.

Table 1. Structure parameters of INNOSAT

Parameters	Values	Unit
I_{xx}	32716516.64e-9	Kgm^2
I_{xy}	-518537.85e-9	Kgm^2
I_{xz}	-2774.91e-9	Kgm^2
I_{yy}	4983443.50e-9	Kgm^2
I_{yz}	282033.17e-9	Kgm^2
I_{zz}	33149348.17e-9	Kgm^2
c.g X_m	1.02	mm
c.g Y_m	0.76	mm
c.g Z_m	-2.40	mm

Table 2. Satellite Two Line Elements

Parameters	Values	Unit
Epoch Day	26/10/2008	
Inclination	9	deg
Mean Anomali	0	deg
Eccentricity	1e-007	deg
Argument of perigee	0	deg
RAAN	0	deg
Mean Motion in Revolution	14.62534512	/day

Table 3. Initial attitude of INNOSAT

Parameters	Values	Unit
q_{10}	0.45652	
q_{20}	0.53956	
q_{30}	0.455703	
q_{40}	0.541109	
$\omega^b_{ob,x}$	0.000701	deg/s
$\omega^b_{ob,y}$	0.06101	deg/s
$\omega^b_{ob,z}$	0	deg/s

Table 4. EKF initial parameters

Parameters	Values
P_k	P_0=*diag(1e-3 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3)*
Q	*Q=diag*(6.25e-1 6.25e-1 6.25e-1 6.25e-1 6.25e-1 6.25e-1 6.25e-1)
R	*R=diag*(8.5e-4 8.5e-4 8.5e-4 8.5e-4 8.5e-4 8.5e-4)

Estimation outputs of q-method and EKF are attitude in quaternion, as stated in eq.(13), then the outputs are converted to Euler angle (φ, θ, ψ) by using eq.(6). The Euler outputs from q-method and EKF are compared with Euler from STK software which called as true attitude.

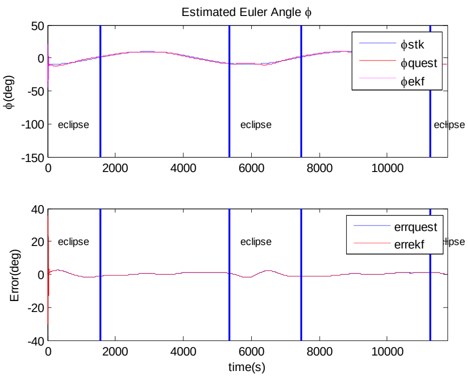

Figure 4. Estimated roll (φ)attitude

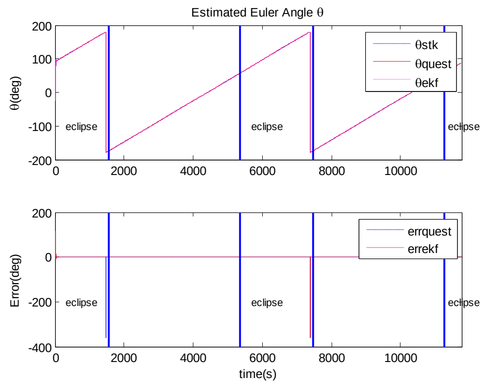

Figure 5. Estimated pitch (θ) attitude

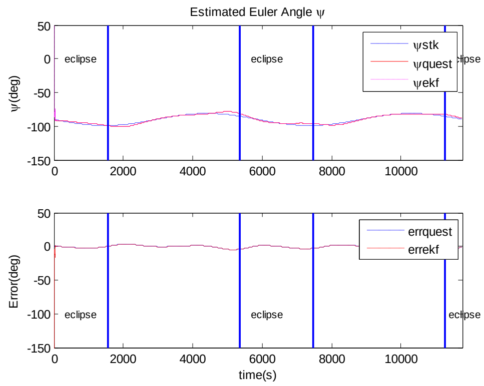

Figure 6. Estimated yaw (ψ) attitude

Figure 4 shows the difference between true and estimated roll (φ) attitude. It's about 2.5 deg of steady state attitude error can be made when applying QUEST and EKF. Maximum errors from both methods were relatively large. This is due to the differences in the measurement data of the sun's and magnetic field vector with the actual values (Figure 7). This happen particularly from EKF. This also could be caused by difference parameters of mathematical model and physical model. Inertial moment of INNOSAT is coupled, so the Euler attitude on each axis is influenced by the attitude of the other axis, so it caused difficulty in obtaining pure attitude estimation on each axis.

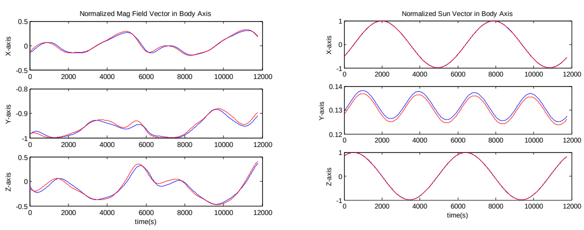

Figure 7. Normalized magnetic field and sun vector

Figure 4 to Figure 6 shows the estimated attitude of the satellite and the analysis shows that the maximum error between QUEST and EKF methods is 0.01. This means the problems in switching estimated attitude data from QUEST to EKF or vice versa at the eclipse will produce insignificant effect on the satellite dynamics. The same analysis was done for Figure 5 and Figure 6, the maximum error is 0.28 degree and 4.8 degree respectively.

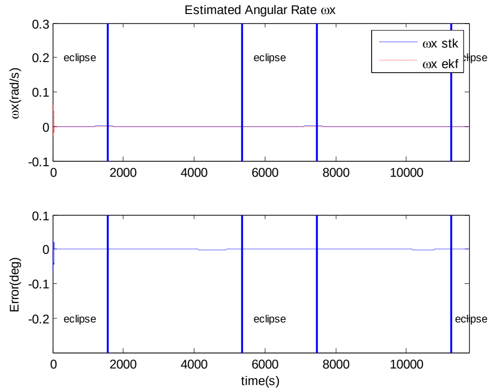

Figure 8. Estimated ω_x

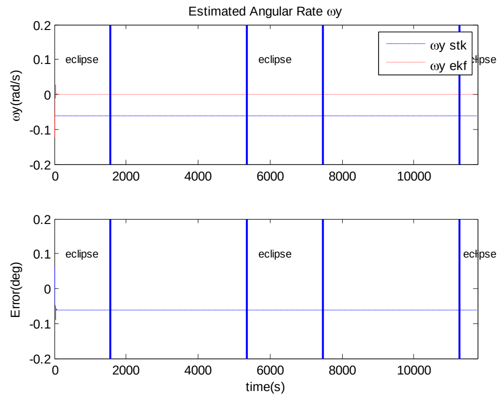

Figure 9. Estimated ω_y

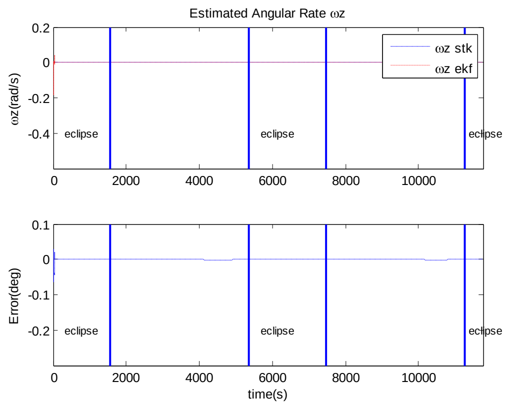

Figure 10. Estimated ω_z

The QUEST method cannot be used to estimate the angular velocity of satellite. In this case the EKF method is used instead. Figure 8 and Figure 10 shows the angular velocity for ω_x and ω_z which is estimated using the EKF method and this is possible because it has steady state error of 0.07

rad/s and 1.7e-5 rad/s respectively. However, estimation of ω_y using EKF is not adequate because the steady state error value is 0.4 rad/s. Specifically for ω_y, estimation is obtained using a differential EKF calculation of estimated pitch (θ) from Figure 5 using equation (49) and obtained the results in Figure 11.

$$\frac{\partial\theta}{\partial t} = \frac{\theta(t+1) - \theta(t)}{\Delta t} \tag{49}$$

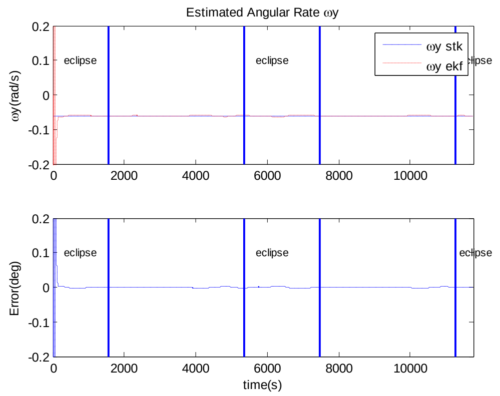

Figure 11. Estimate ω_y by using differential equation

5. Conclusion and Further Works

Attitude determination for INNOSAT has been developed by using deterministic q-Method (QUEST) and recursive using EKF. It has been shown that attitude of INNOSAT can be determined using two position sensors. The maximum error which can be obtained using the QUEST and the EKF is 4.8 deg for yaw attitude and 2.5 deg for roll and only 0.28 deg of error for pitch attitude. Estimated angular velocity in X and Z-axis are acceptable since using EKF but for Y-axis is only possible calculated using difference of estimated pitch angle.

For further works, its need to attempt using the Euler dynamic model for the EKF to know better possibility of estimated results. Continuous EKF is also possible to use to reduce computational load. Improvement of inertial moment is also possible to attempt in order to reduce attitude coupling.

6. Acknowledgement

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