SCREENING SOLUTIONS OF MULTIMONOPOLE BY UNIT CHARGE ANTIMONOPOLES*

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ABSTRACT

We would like to show in this paper that there exist a whole range of screening solutions of multimonopole by unit charge antimonopoles in the SU(2) Yang-Mills-Higgs theory. These screening solutions are exact multimonopole-antimonopoles configurations, wh[illegible] the positively charged multimonopole is screened by unit charged antimonopoles located or[illegible] with the multimonopole positioned at the center of the spherical shell. These solutions [illegible] Bogomol'nyi equation but possess infinite energy. Hence they are a different type of BPS solutions. [illegible] these screening solutions possess rotational symmetry about the z-axis.

INTRODUCTION

The SU(2) Yang-Mills-Higgs (YMH) theory, with the Higgs field in the adjoint representation, possesses both the magnetic monopole and multimonopole solutions [1, 2, 3]. The simplest solution with unit magnetic charge is the spherically symmetric 't Hooft-Polyakov monopole [1]. Multimonopole solutions cannot be spherically symmetric and possess at most axial symmetry [3]. In particular, there are no solutions of magnetic charge greater than one with radial symmetry. Analytic monopole and multimonopole solutions [1, 2, 3] have been shown to exist in the Bogomol'nyi-Prasad-Sommerfield (BPS) limit with vanishing Higgs potential. These solutions satisfy the first order Bogomol'nyi equations and they have minimal energies. However, when the Higgs potential is finite, there exist only numerical monopole and multimonopole solutions [3].

Recently, axially symmetric monopoles-antimonopoles chain solutions which do not satisfy the Bogomol'nyi condition were constructed numerically. These non-Bogomol'nyi solutions exist both in the limit of a vanishing Higgs potential as well as in the presence of a finite Higgs potential. BPS axially symmetric vortex rings solutions have also been constructed numerically [4].

We would like to show in this paper there exist a whole range screening solutions by unit charge antimonopoles in the SU(2) YMH model. These screening solutions were actually first reported in ref. [5] and they were labeled as the A1 and the B1 solutions. The screening solutions are exact multimonopole-antimonopoles configurations where the positively charged multimonopole is screened by unit charged antimonopoles located on a spherical shell, with the multimonopole positioned at the center of the shell. The SU(2) YMH model in our work is of vanishing Higgs potential. The solutions are solved from the second order Euler-Lagrange equations and the first order Bogomol'nyi equations $B_i^a \pm D_i\Phi^a = 0$ with the positive sign. The screening solutions satisfy the first order Bogomol'nyi equation but possess infinite energy. Hence they are a different type of BPS solutions.

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* USM Preprint August 2004; Contributed paper presented at the "Persidangan Fizik Kebangsaan", (PERFIK 04), October 5-7, 2004, Palace of the Golden Horses, Kuala Lumpur.

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THE SU(2) YANG-MILLS-HIGGS THEORY

The SU(2) YMH model consist of the Yang-Mills vector fields A_μ^a and the Higgs scalar field Φ^a in 3+1 dimensions. The index a is the SU(2) internal space index. For a given a, Φ^a is a scalar whereas A_μ^a is a vector under Lorentz transformation. The SU(2) Yang-Mills-Higgs Lagrangian in 3+1 dimensions is

$$L = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi_a - \frac{1}{4}\lambda\left(\Phi^a\Phi^a - \frac{\mu^2}{\lambda}\right)^2, \qquad (1)$$

where the constant μ is the mass of the Higgs field, and the constant λ is the strength of the Higgs potential. The vacuum expectation value of the Higgs field is then $\mu/\sqrt{\lambda}$. The Lagrangian from Eq. (1) is invariant under the set of independent SU(2) gauge transformations at each space-time point. The covariant derivative of the Higgs field is

$$D_\mu\Phi^a = \partial_\mu\Phi^a + g\varepsilon^{abc}A_\mu^b\Phi^c, \qquad (2)$$

where A_μ^a is the gauge potential and the gauge field strength tensor is

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon^{abc}A_\mu^b A_\nu^c. \qquad (3)$$

The gauge field coupling constant g can be scaled away, hence it is set to one here without any loss of generality. The metric used is $g_{\mu\nu} = (-+++)$. The SU(2) group indices a, b, c run from 1 to 3 whereas the spatial indices μ, ν, α run from 0 to 3 in Minkowski space. In this paper, we consider only static solutions with $A_0^a = 0$. The spatial indices i, j and k then run from 1 to 3.

The equations of motion obtained from Eq. (1) are

$$D^\mu F_{\mu\nu}^a = \partial_\mu F_{\mu\nu}^a + \varepsilon^{abc}A_\mu^b F_{\mu\nu}^c = \varepsilon^{abc}\Phi^b D_\nu\Phi^c, \quad D^\mu D_\mu\Phi^a = -\lambda\,\Phi^a\left(\Phi^b\Phi^b - \tfrac{\mu^2}{\lambda}\right). \qquad (4)$$

The electromagnetic field tensor introduced by 't Hooft [1] is

$$F_{\mu\nu} = \hat{\Phi}^a F_{\mu\nu}^a - \varepsilon^{abc}\hat{\Phi}^a D_\mu\hat{\Phi}^b D_\nu\hat{\Phi}^c = \partial_\mu A_\nu - \partial_\nu A_\mu - \varepsilon^{abc}\hat{\Phi}^a\partial_\mu\hat{\Phi}^b\partial_\nu\hat{\Phi}^c, \qquad (5)$$

where $A_\mu = \hat{\Phi}^a A_\mu^a$, the unit vector $\hat{\Phi}^a = \Phi^a/|\Phi|$ and the Higgs field magnitude $|\Phi| = \sqrt{\Phi^a\Phi^a}$. The Abelian electric field is $E_i = F_{0i}$ whereas the Abelian magnetic field is defined as $B_i = -\frac{1}{2}\varepsilon_{ijk}F_{jk}$. The topological magnetic current k_μ is defined to be $k_\mu = \frac{1}{8\pi}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{abc}\partial^\nu\hat{\Phi}^a\partial^\rho\hat{\Phi}^b\partial^\sigma\hat{\Phi}^c$, and the corresponding conserved topological magnetic charge is

$$\begin{aligned} M &= \int d^3x k_0 = \frac{1}{8\pi}\int \varepsilon_{ijk}\varepsilon^{abc}\partial_i\left(\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right)d^3x \\ &= \frac{1}{8\pi}\oint d^2\sigma_i\left(\varepsilon_{ijk}\varepsilon^{abc}\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right) = \frac{1}{4\pi}\oint d^2\sigma_i B_i. \end{aligned} \qquad (6)$$

THE SCREENING SOLUTIONS

We used the ansatz of ref. [5] with the gauge fields and the Higgs field given by

$$A_\mu^a = \frac{1}{r}R(\theta)\left(\hat{\phi}^a\hat{r}_\mu - \hat{r}^a\hat{\phi}_\mu\right) - \frac{1}{r}\psi(r)\left(\hat{\phi}^a\hat{\theta}_\mu - \hat{\theta}^a\hat{\phi}_\mu\right) + \frac{1}{r}G(\theta,\phi)\left(\hat{r}^a\hat{\theta}_\mu - \hat{\theta}^a\hat{r}_\mu\right),$$

$$\Phi^a = \Phi_1\hat{r}^a + \Phi_2\hat{\theta}^a + \Phi_3\hat{\phi}^a, \qquad (7)$$

where $\Phi_1 = \frac{1}{r}\psi(r)$, $\Phi_2 = \frac{1}{r}R(\theta)$, $\Phi_3 = \frac{1}{r}G(\theta,\phi)$, and the spherical coordinate orthonormal unit vectors are

$$\hat{r}^a = \sin\theta\cos\phi\delta_1^a + \sin\theta\sin\phi\delta_2^a + \cos\theta\delta_3^a,$$
$$\hat{\theta}^a = \cos\theta\cos\phi\delta_1^a + \cos\theta\sin\phi\delta_2^a - \sin\theta\delta_3^a,$$
$$\hat{\phi}^a = -\sin\phi\delta_1^a + \cos\phi\delta_2^a. \qquad (8)$$

The gauge fixing condition that we used here is the radiation or Coulomb gauge, $\partial^i A_i^a = 0$, $A_0^a = 0$. From the ansatz, Eq. (7), $A_\mu = \hat{\Phi}^a A_\mu^a = 0$. Hence the Abelian electric field E_i is zero and the Abelian magnetic field B_i is independent of the gauge field A_μ^a. To calculate the Abelian magnetic field B_i, we rewrite the Higgs field of Eq. (7) from the spherical coordinate system to the Cartesian coordinate system,

$$\Phi^a = \Phi_1\hat{r}^a + \Phi_2\hat{\theta}^a + \Phi_3\hat{\phi}^a = \tilde{\Phi}_1\delta_1^a + \tilde{\Phi}_2\delta_2^a + \tilde{\Phi}_3\delta_3^a, \qquad (9)$$

where

$$\tilde{\Phi}_1 = \Phi_1\sin\theta\cos\phi + \Phi_2\cos\theta\cos\phi - \Phi_3\sin\phi = |\Phi|\cos\alpha\sin\beta,$$
$$\tilde{\Phi}_2 = \Phi_1\sin\theta\sin\phi + \Phi_2\cos\theta\sin\phi + \Phi_3\cos\phi = |\Phi|\cos\alpha\cos\beta,$$
$$\tilde{\Phi}_3 = \Phi_1\cos\theta + \Phi_2\sin\theta = |\Phi|\sin\alpha. \qquad (10)$$

The Higgs field unit vector is then simplified to

$$\hat{\Phi}^a = \cos\alpha\sin\beta\,\delta_1^a + \cos\alpha\cos\beta\,\delta_2^a + \sin\alpha\,\delta_3^a. \qquad (11)$$

The Abelian magnetic field is derived from Eq. (5) and it is found to be

$$B_i = \frac{1}{r^2\sin(\theta)}\left\{\frac{\partial\beta}{\partial\phi}\frac{\partial\sin\alpha}{\partial\theta} - \frac{\partial\beta}{\partial\theta}\frac{\partial\sin\alpha}{\partial\phi}\right\}\hat{r}_i + \frac{1}{r\sin\theta}\left\{\frac{\partial\beta}{\partial r}\frac{\partial\sin\alpha}{\partial\phi} - \frac{\partial\beta}{\partial\phi}\frac{\partial\sin\alpha}{\partial r}\right\}\hat{\theta}_i$$
$$+\frac{1}{r}\left\{\frac{\partial\beta}{\partial\theta}\frac{\partial\sin\alpha}{\partial r} - \frac{\partial\beta}{\partial r}\frac{\partial\sin\alpha}{\partial\theta}\right\}\hat{\phi}_i, \qquad (12)$$

where

$$\sin\alpha = \frac{\psi\cos\theta - R\sin\theta}{\sqrt{\psi^2 + R^2 + G^2}}, \quad \text{and } \beta = \gamma - \phi, \quad \gamma = \tan^{-1}\left(\frac{\psi\sin\theta + R\cos\theta}{G}\right). \qquad (13)$$

The Abelian field magnetic flux is

$$\Omega = 4\pi M = \iint B_i\left(r^2\sin\theta d\theta\right)\hat{r}_i d\phi \qquad (14)$$

and the magnetic charge enclosed by the sphere centered at $r = 0$ and of fixed radius r_1, is found to be

$$M_{r_1} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left\{\frac{\partial\beta}{\partial\phi}\frac{\partial\sin\alpha}{\partial\theta} - \frac{\partial\beta}{\partial\theta}\frac{\partial\sin\alpha}{\partial\phi}\right\}d\theta d\phi\Bigg|_{r=r_1}. \qquad (15)$$

By substituting the ansatz, Eq. (7) into the equations of motion (4) as well as the first order Bogomol'nyi equations $B_i^a \pm D_i\Phi^a = 0$ with the positive sign, these equations can be simplified to just four ordinary differential equations of first order:

$$r\psi' - \psi^2 + \psi = -p, \qquad (16)$$
$$\dot{R} + R\cot\theta - R^2 = p - b^2\csc^2\theta, \qquad (17)$$
$$\dot{G} + G\cot\theta = 0, \quad G^\phi\csc\theta - G^2 = b^2\csc^2\theta. \qquad (18)$$

where p and b^2 are arbitrary constants, prime means the partial derivative $\partial/\partial r$, dot means the partial derivative $\partial/\partial\theta$, superscript ϕ means the partial derivative $\partial/\partial\phi$.

Eq. (16) is exactly solvable for all real values of p and the integration constant can be scaled away by letting $r \to r/c$, where c is the arbitrary integration constant. In order to have solutions of

$\psi(r)$ with $(2m+1)$ powers of r, we write $p=m(m+1)$. Eqs. (18) are also exactly solvable and a general physical solution is $G(\theta,\phi)=b\csc\theta\tan b\phi$, where b is restricted to take half integer values for G to be a single value function. Therefore we can write $b=m\pm s$ where s is a natural number and m can take half-integer values. For all the solutions presented here, b is non-zero. When $b=0$, we have the axially symmetric solutions which is reported in a separate work [6]. Screening solutions arise when we consider $b=m-s$. Hence the solutions for the profile functions $\psi(r)$ and $G(\theta,\phi)$ are

$$\psi(r)=\frac{m+1-mr^{2m+1}}{1+r^{2m+1}},\ G(\theta,\phi)=(m-s)\csc\theta\tan(m-s)\phi, \tag{19}$$

$s=0,1,2,3....$. For $\psi(r)$ to have integer powers of r and $G(\theta,\phi)$ to be a single value function the value of m is restricted to a half integer, where $m\geq s+\frac{1}{2}$. Eq. (19) is a Riccati equation and $R(\theta)$ can be exactly solved for different combinations of p and b. The solution for $R(\theta)$ when $b=m-s$ and $p=m(m+1)$ is given by

$$R(\theta)=(m+1)\cot\theta-(s+1)\csc\theta\frac{P_{m+1}^{m-s}(\cos\theta)}{P_{m}^{m-s}(\cos\theta)}, \tag{20}$$

where $s=0,1,2,3....$, $m=s+\frac{1}{2},s+1,s+1\frac{1}{2},s+2\frac{1}{2},....$ and $P_{m}^{m-s}(\cos\theta)$ is the associated Legendre function of the first kind of degree m and order $m-s$.

The solutions (19) and (20) are the screening solutions of a multimonopole by antimonopoles which we label as the 1_s series. The boundaries conditions of the 1_s series of solutions are:

$$\psi(r)\big|_{r\to\infty}\to -m,\ \psi(r)\big|_{r\to 0}\to(m+1);\ G(\theta,0)=G(\theta,\pi)=0;$$
$$R(\theta)\sin\theta\big|_{\theta\to 0}\to-(m-s),\ R(\theta)\sin\theta\big|_{\theta\to\pi}\to(m-s). \tag{21}$$

We further subdivide the 1_s solutions into the $B1_s$ configurations when s is zero or even and the $A1_s$ configurations when s is odd. It is clear that the B1 and A1 solutions reported in ref. [5] are the cases with $s=0$ and $s=1$ respectively. The difference between the $A1_s$ and $B1_s$ solutions is in the profile function, $R(\theta)$. For the $A1_s$ solutions, $R(\theta)\cos\theta\big|_{\theta=\pi/2}=1$, whereas for the $B1_s$ solutions, $R(\frac{\pi}{2})=0$. Besides in the form of associated Legendre function as in Eq. (20), the profile function for $R(\theta)$ can also be written in trigonometric function, as in Table 1.

The energy of the solutions here is not finite due to the singularity of the solutions at the origin, $r=0$. Unlike the usual BPS solutions, the vacuum expectation value for the Higgs field in our solutions tends to zero at large r. Hence the energy of our solutions is not minimally bounded from below. The net topological charge of the system is given by the integration of the radial component of the Abelian magnetic field over the sphere at infinity,

$$M_{\infty}=\frac{1}{8\pi}\oint d^2\sigma_i\left(\varepsilon_{ijk}\varepsilon^{abc}\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right)\Big|_{r\to\infty}. \tag{22}$$

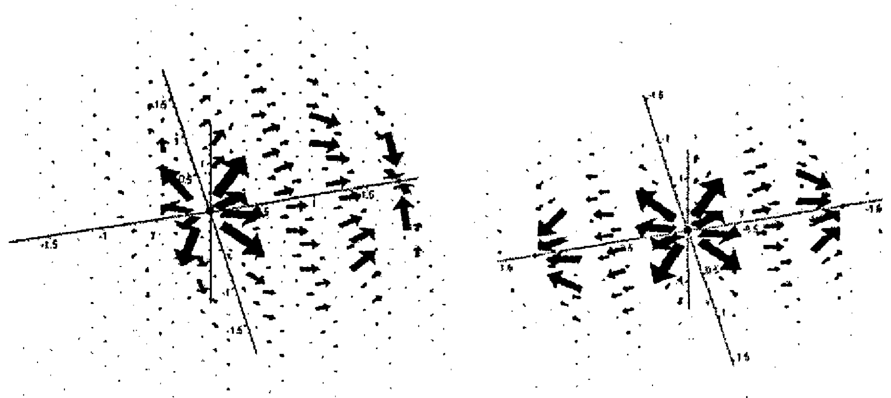
Hence the monopoles and antimonopoles of our solutions here can be associated with the number of point zeros of Φ^a enclosed by the sphere at infinity. The positions of the monopoles and antimonopoles do correspond to the point zeros of the Higgs field but the multimonopole is always located at the origin of the coordinate axes where the Higgs field is singular. The definition for the magnetic charge as given by Eq. (6) and Eq. (22) is not affected by the fact that the magnitude of the Higgs field, $|\Phi|$ vanishes at large r. It only depends on the direction of the unit vector of the Higgs field, $\hat{\Phi}^a$, in internal space.

Table 1: The profile functions $R(\theta)$, $G(\theta,\phi)$ with descending values of b for the series of 1_s screening solutions.

1_s series	b	Function of $R(\theta)$	Function of $G(\theta,\phi)$
Bl_0	m	$-m\cot\theta$	$m\csc\theta\tan m\phi$
Al_1	$m-1$	$\tan\theta-(m-1)\cot\theta$	$(m-1)\csc\theta\tan(m-1)\phi$
Bl_2	$m-2$	$-m\cot\theta+\dfrac{4(m-1)\cot\theta}{2(m-1)\cos^2\theta-\sin^2\theta}$	$(m-2)\csc\theta\tan(m-2)\phi$
Al_3	$m-3$	$\tan\theta-(m-1)\cot\theta+\dfrac{4(m-2)\cot\theta}{2(m-2)\cos^2\theta-3\sin^2\theta}$	$(m-3)\csc\theta\tan(m-3)\phi$

From Eq. (15), the magnetic charge of the 1_s series of solutions at infinity, M_∞ can be exactly calculated using Maple 9, whereas the magnetic charge at the origin, M_0 can be obtained by the use of the approximation methods in the Maple 9 software. For the screening 1_s series, M_0 is the magnetic charge of the multimonopole at $r=0$ whereas M_∞ is the net magnetic charge of the system. We also denote M_A as the net charge of the screening antimonopoles.

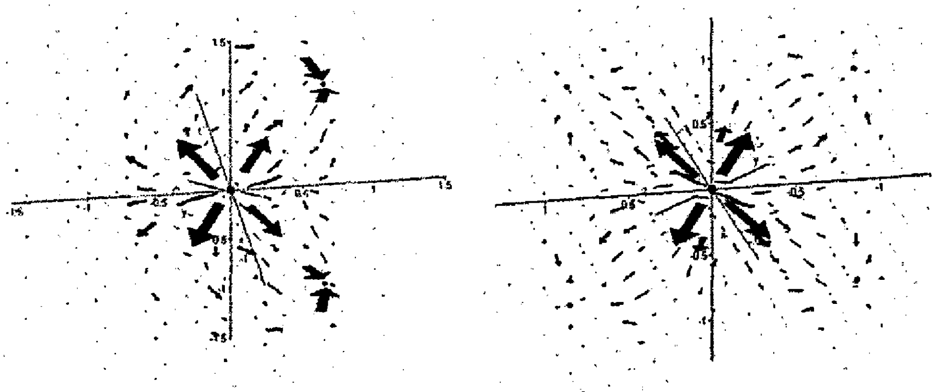
A detailed description of the first two members of the 1_s series, the Bl_0 and Al_1 solutions, is found in ref. [5]. However, in ref. [5], we only consider the solutions for integer values of *m* whereas here we have noticed that *m* can take half-integer values. It turns out that for the Bl_0 solution, the multimonopole can possess odd values of magnetic charge, instead of just even values of magnetic charge as in ref. [5]. Therefore, when $m=\frac{1}{2}$, the Bl_0 configuration is a pair of monopole and antimonopole. The monopole is at the origin, whereas the antimonopole is at the point $(\sqrt{3},0,0)$, as shown in Fig. 1(a). Subsequently when $m=1$, we have the configuration of a 2-monopole, surrounded by two antimonopoles, Fig. 1(b).



(a) (b)

Figure 1: Magnetic field plot of the Bl_0 solution when (a) $m=\frac{1}{2}$ and (b) $m=1$.

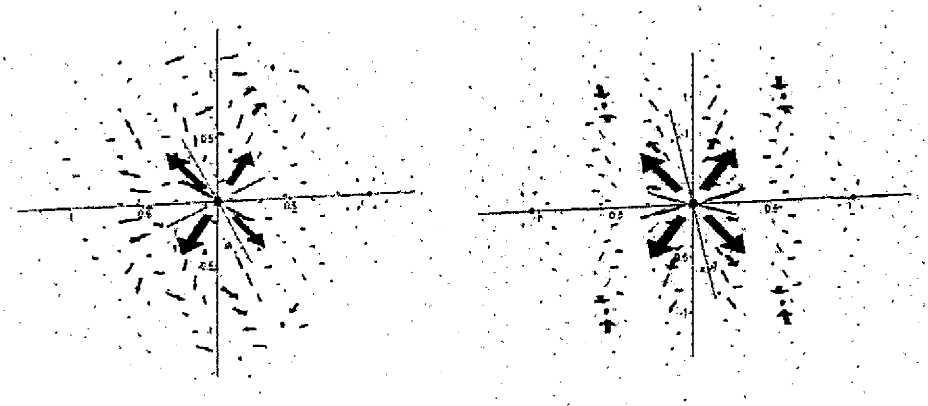
Similarly we consider the cases of half-integer values of *m* for the Al_1 solution. The first member of the Al_1 solution when $m=1\frac{1}{2}$ has $M_0=+2\frac{1}{2}$ and $M_A=-2$, hence there is a multimonopole of charge $+2\frac{1}{2}$ at $r=0$ and partially screened by two antimonopoles located at the *x-z* plane, as shown in Fig. 2(a). The next member when $m=2$ has $M_0 = 4$ and $M_A = -4$. Hence the net magnetic charge is zero and the configuration is a 4-monopole at $r=0$, surrounded by four antimonopoles, Fig. 2(b). As m increases in steps of one-half, M_0 increases by $1\frac{1}{2}$ monopole charge and the screening antimonopoles increases by two. Therefore the multimonopole in the $\mathrm{A1}_1$ solution can take half-integer values of monopole charge when m is a half-odd integer.



(a) (b)

Figure 2: Magnetic field plot of the $\mathrm{A1}_1$ solution when (a) $m = 1\frac{1}{2}$ and (b) $m = 2$.

The third member of the 1_s series is the $\mathrm{B1}_2$ solution with $s=2$. The value for b is $m-2$ and m increases in steps of half, starting from $2\frac{1}{2}$. The first member when $m = 2\frac{1}{2}$ has $M_0 = +4$, $M_A = -3$, hence there is a 4-monopole at the origin, partially screened by three antimonopoles at constant r along the x-z plane, Fig. 3(a). When $m=3$, $M_0 = +6$ and $M_A = -6$, hence there is a 6-monopole at the origin, surrounded by six antimonopoles along the x-z plane, Fig. 3(b). By induction, the values of M_0 and M_∞ is given by $(4m-6)$ and $(6-2m)$ respectively. Hence, we conclude that the $\mathrm{B1}_2$ solution consists of a $(4m-6)$-monopole at the origin, surrounded by $6(m-2)$ antimonopoles at finite and constant r. As m increases in steps of one-half, M_0 increases by two monopole charge and the screening antimonopoles increases by three, Table 2.



(a) (b)

Figure 3: Magnetic field plot of the $\mathrm{B1}_2$ solution when (a) $m = 2\frac{1}{2}$ and (b) $m = 3$.

Table 2: The magnetic charge at the origin, M_0, the net charge of antimonopoles, M_A, and the magnetic charge at infinity, M_∞, of the B1_2 solution for different values of m.

Values of m	M_0	M_A	M_∞
$2\frac{1}{2}$	+4	-3	1
3	+6	-6	0
$3\frac{1}{2}$ . . . m	+8 . . . $4m-(3\times 2)$	-9 . . . $-6(m-2)$	-1 . . . $-2(m-3)$

The subsequent 1_s series has $M_0=(s+2)m-(s+1)s$, $M_\infty=s(s+1-m)$ and $M_A=-2(s+1)(m-s)$, Table 3. The A1_s series possesses half-integer multimonopole charge when the parameter m is a half-odd-integer, whereas the B1_s screening series possesses only integer values of multimonopole charge, Table 3. In the screening solutions, the positively charged multimonopole is screened by unit charge antimonopoles located on a spherical shell of radius $r=\sqrt[2m+1]{\frac{m+1}{m}}$, with the multimonopole positioned at the center of the shell. However, one can also describe the position of the screening antimonopoles as located on the horizontal circles of the spherical shell. In particular, the antimonopoles in the B1_0 solution are located on just one horizontal circle, along the *x-y* plane. For the A1_1 solution, the antimonopoles are located on two horizontal circles of the spherical shells, whereas the antimonopoles in the B1_2 solution are located on three horizontal circles and so on. Hence the number of horizontal circles increases as $s+1$.

Table 3: The 1_s series of screening solutions. Here $b=m-s$ where $s\in\{0,1,2,3,...\}$. Each series starts with $m=s+\frac{1}{2}$ and m increases in steps of one-half. When *s* is odd, we have the A1_s series and when *s* is zero or even we have the B1_s series.

1_s series	s	M_0	M_∞	M_A	No. of horizontal circles
B1_0	0	$2m$	0	$-2m$	1
A1_1	1	$3m-(2\times 1)$	$1(2-m)$	$-4(m-1)$	2
B1_2	2	$4m-(3\times 2)$	$2(3-m)$	$-6(m-2)$	3
A1_3	3 . . .	$5m-(4\times 3)$ . .	$3(4-m)$ . .	$-8(m-3)$ . .	4 . . .
1_s	s	$(s+2)m-(s+1)s$	$s(s+1-m)$	$-2(s+1)(m-s)$	$s+1$

CONCLUSIONS AND COMMENTS

We have shown the existence of screening solutions, the 1_s series, where s is a natural number and $m \geq s+\frac{1}{2}$, increases in steps of one-half. The 1_s series can further be subdivided into the $A1_s$ solutions when s is odd, and the $B1_s$ solutions when s is zero or even. All the members of the screening solutions have a multimonopole at $r=0$ and surrounded by antimonopoles located on a spherical shell. However, there is a difference between the $A1_s$ and $B1_s$ solutions. The magnetic charge of the multimonopole at $r=0$ of the $A1_s$ solutions can possess half-integer values of monopole charge when m is a half-odd integer, whereas the magnetic charge of the multimonopole of the $B1_s$ solutions are always of integer values. The screening antimonopoles possess magnetic charge -1 and there are no screening antimonopoles with fractional magnetic charge.

We would also like to note that there exist another series of solutions when the constants of Eq. (17) are $p=m(m+1)$ and $b=m+s$, where $s \in \{1,2,3,...\}$. In these series, there exist only monopoles or multimonopole with no antimonopoles located at finite r and we labeled them as the 2_s series. The first and second members of the 2_s series were also reported in ref. [5] and they are the A2 and B2 solutions. The A2 solution consists of only a multimonopole at the origin whereas the B2 solution consists of finitely separated monopoles located on a circle along the horizontal plane. The 2_s series are reported in ref. [7].

Besides the 1_s series and the 2_s series, the ansatz (7) actually supports more static monopoles solutions. In particular, there always exists an anti-configuration for every monopoles solution of the ansatz, where the magnetic charge of the "poles" and the direction of the magnetic field changed sign [8].

The authors would like to acknowledge the School of Physics and Universiti Sains Malaysia for the short term grant (Account No: 304/PFIZIK/634039).

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