

A CURRENCY FORWARD CONTRACT

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ABSTRACT

The aim of the paper is to present one out of conceivable approaches to problems of foreign exchange rates and subsequently forward contracts on them. Our approach, based on stochastic analysis, provides an untraditional view of the issues. Within the framework of this paper we model the exchange rate on the basis of the geometric Brownian motion. Obtained results entitle the use as a complementary tool when managing the exchange risk.

1. INTRODUCTION

This study investigates a currency forward contract on foreign exchange rates. The issue of forward rates is seen from an investor's point of view. To be able to deal with the problem we need to model the exchange rate. Within the framework of the paper the exchange rate will be perceived as a currency basket. Initially, let us describe the situation of the Malaysian Ringgit (RM) from 2nd January 2001 to 30rd December 2001.

The Central Bank of Malaysia dealt in the market through daily fixing sessions and through interventions in the market. In this study we look trading in two currencies: Singapore Dollar (SGD) and Japan Yen (JPY). The theoretical currency basket rate (IDX) depended on the supply of and demand for foreign currency and on other factors such as the implications of monetary policy of the Central Bank of Malaysia.

Let us now define the term of the currency basket. The definition of the basket listed in table 1 was valid since 2nd January 2001 to 30rd December 2001.

Table 1: RM currency basket.

Currency	SGD	JPY 100
Weight (%)	63	37
Rate against SGD	1.0000	0.6599
MR rate	2.1902	3.3186

For practical purpose it is more suitable to work with an absolute definition of the currency basket:

$$1 \text{ IDX} = a \text{ SGD} + b \text{ JPY}_{100}, \quad \text{-----(1)}$$

where the weights a , b are given as;

$$a = \frac{0.63}{\frac{\text{SGD}}{\text{RM}} \Big|_{2.1.2001}} = 0.287644964 \quad \text{-----(2)}$$

$$b = \frac{0.37}{\frac{\text{JPY}_{100}}{\text{RM}} \Big|_{2.1.2001}} = 0.111492798 \quad \text{-----(3)}$$

Let us now derive the SGD/RM exchange rate. For this purpose we will work with the absolute definition like with an ordinary equation in physics. We divide (1) by variable, "RM", thus we get;

$$\frac{IDX}{RM} = a \frac{SGD}{RM} + b \frac{JPY_100}{RM}, \quad \text{-----(4)}$$

and

$$\frac{IDX}{RM} = \frac{SGD}{RM} \left(a + b \frac{JPY_100}{SGD} \right), \quad \text{-----(5)}$$

and finally we obtain the formula for determination of the exchange rate;

$$\frac{SGD}{RM} = \left(a + b \frac{JPY_100}{SGD} \right)^{-1} \frac{IDX}{RM}, \quad \text{-----(6)}$$

From the just derived equation (6) we may conclude that the percentage weights of SGD and JPY fluctuate with regard of the $\frac{SGD}{JPY_100}$ rate, whereas the absolute formula remains unchanged

until the currency basket is redefined. The percentages of SGD and JPY_100 in the basket are given by;

$$\frac{a}{a + b \frac{JPY_100}{SGD}}, \frac{b \frac{JPY_100}{SGD}}{a + b \frac{JPY_100}{SGD}}. \quad \text{-----(7)}$$

II. THEORY

Let us list some of the assertions we use in our models. Let n one-dimensional Ito processes $X_i(t)$ be given by;

$$dX_i(t) = f_i(t) dt + \sigma_i(t) dW_i(t), \quad i = 1, \dots, n.$$

Suppose that function $u(t, x_1, \dots, x_n) : [0, T] \times R^n \rightarrow R$ has partial derivatives $u_t, u_{x_i}, u_{x_i x_j}$ which are continuous. Then the process $Y(t) = u[t, X_1(t), \dots, X_n(t)]$ is also an Ito process given by;

$$dY(t) = u_t(t) dt + \sum_{i=1}^n u_{x_i} dX_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n u_{x_i x_j} dX_i dX_j,$$

where the product $dX_i X_j$ can be calculated using following multiplication rules:

$$dW_i W_j = \rho_{ij} dt, \quad 1 \leq i, j \leq n,$$

$$dt dW_j = 0, \quad 1 \leq i \leq n,$$

$$dt dt = 0$$

where ρ_{ij} denotes corresponding correlations.

Lemma 1: If $L(Y) = N(\mu, \sigma^2)$, then $Ee^Y = e^{\mu + \frac{\sigma^2}{2}}$, $\text{var } e^Y = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$.

Lemma 2: Suppose the Ito process $(X_t)_{t \geq 0}$ satisfying the stochastic differential equation (SDE);

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t, \quad \text{-----(8)}$$

with the initial condition X_0 and let μ, σ be constant and $(W_t)_{t \geq 0}$ be a Wiener process. Then the solution is given by;

$$X_t = X_0 \exp \left[\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right]$$

The process $(X_t)_{t \geq 0}$ given by (8) is called the geometric Brownian motion.

III. THE DERIVATION OF THE MODELS

Throughout all this work time is considered to be continuous and is measured in trading days.

Before we proceed to the presentation of the models let us denote the processes which we will work with as follows;

$$Z_t \equiv \frac{SGD}{RM}, X_t \equiv \frac{IDX}{RM}, V_t \equiv \frac{RM}{IDX}, Y_t \equiv a + b \frac{JPY - 100}{SGD}, U_t \equiv \frac{JPY - 100}{SGD} \quad (9)$$

As the first step we present the general exchange rate model. Now we may build up the exchange rate model as follows;

$$\begin{aligned} Z_t &= \frac{1}{a + bU_t} X_t, \quad t \leq t_n^* \\ &= Z_{t_n^*} + \alpha \frac{E[\Delta I_t | F_{t_n^*}]}{\Delta t}, \quad t \in (t_n^*, t_n^* + \Delta t) \end{aligned} \quad (10)$$

where $(F_t)_{t \geq 0}$ is the information σ -history and ΔI_t represents an intervention of Central Bank. Coefficient α is the elasticity of exchange rate with respect to intervention ΔI_t . The intervention process I_t can be explained in a few ways. Note that model (10) represents an estimate of the reality given by formula (6).

Now we show results which considerably simplify the above listed model. Assuming that X_t is a geometric Brownian motion, by the use of Lemma 2 we may write the process X_t as;

$$X_t = X_0 \exp \left[\left(\mu X - \frac{\sigma_x^2}{2} \right) t + \sigma_x W_t^X \right] \quad (11)$$

We know that the random variable $\left(\left(\mu X - \frac{\sigma_x^2}{2} \right) t + \sigma_x W_t^X \right)$ satisfies;

$$L \left(\left(\mu X - \frac{\sigma_x^2}{2} \right) t + \sigma_x W_t^X \right) = N \left(\left(\mu X - \frac{\sigma_x^2}{2} \right) t, \sigma_x^2 t \right) \quad (12)$$

and its 95 % confidence interval is given by;

$$\left(\left(\mu X - \frac{\sigma_x^2}{2} \right) t - 2\sigma_x \sqrt{t}, \left(\mu X - \frac{\sigma_x^2}{2} \right) t + 2\sigma_x \sqrt{t} \right) \quad (13)$$

From now on we neglect the restriction in model (10) resulting from the use of Markov time t^* and model (10) is simplified with respect to the value of t as;

$$Z_t = \frac{1}{a + bU_t} X_t \quad (14)$$

As the first step we will examine following expressions resulting from the just simplified model (14);

$$Z_t^{-1} = (a + bU_t)V_t = aV_t + b(U_tV_t), \quad \text{-----(15)}$$

since it represents the model which should be the closest to reality model of the set of models we consider. Later we proceed to easier models to show some of their important features. Let assume further that U_t is a geometric Brownian motion and investigate Z_t^{-1} . Its stochastic differential is given by;

$$\begin{aligned} dZ_t^{-1} &= adV_t + bd(U_tV_t) \\ &= \\ &[aV_{tUV} + bU_tV_t(\mu_U + \mu_V + \rho_{UV} \sigma_U \sigma_V)]dt + b\sigma_U U_tV_t dW_t^U + [a\sigma_V V_t + b\sigma_V U_tV_t]dW_t^V \end{aligned}$$

which seems too difficult to solve. As we will understand later, it (15) is even useless for derivation of the forward price. But when we pay more attention to the term $Z_t^{-1} = aV_t + b(U_tV_t)$, we may immediately write;

$$Z_t = \left(aV_0 \exp\left[\left(\mu_V - \frac{\sigma_V^2}{2}\right)t + \sigma_V W_t^V\right] + bU_0V_0 \exp\left[\left(\mu_U + \mu_V - \frac{\sigma_U^2}{2} - \frac{\sigma_V^2}{2}\right)t + \sigma_U W_t^U + \sigma_V W_t^V\right] \right)^{-1}$$

Thus we have a relatively simple term for the process Z_t^{-1} , the sum of two geometric Brownian motions (using Lemma 1 we are able to find its expected value and its variance), but somewhat cumbersome stochastic differential. Let us simplify this situation and proceed to Model 1.

Model 1

This model is based on the following. Let us replace the estimate (14) of $\frac{SGD}{RM}$ by somewhat easier

$$Z_t = \frac{X_t}{Y_t} \quad \text{-----(16)}$$

assuming that X_t, Y_t follow a geometric Brownian motion. Using Ito Theorem we find its stochastic differential as

$$\frac{dZ_t}{Z_t} = [\mu_X - \mu_Y - \sigma_X \sigma_Y \rho_{XY}]dt + [\sigma_X dW_t^X - \sigma_Y dW_t^Y]. \quad \text{-----(17)}$$

Note that $\frac{dZ_t}{Z_t}$ is a random variable and one can compute its expectation and variance. These are given by;

$$E \frac{dZ_t}{Z_t} = [\mu_X - \mu_Y - \sigma_X \sigma_Y \rho_{XY}]dt \quad \text{-----(18)}$$

$$\text{var} \frac{dZ_t}{Z_t} = [\sigma_X^2 + \sigma_Y^2 - 2\sigma_X \sigma_Y \rho_{XY}]dt \quad \text{-----(19)}$$

Again using Lemma 2 provide us

$$Z_t = Z_0 \exp\left[\left(\mu_X - \mu_Y - \frac{\sigma_Y^2}{2} - \frac{\sigma_X^2}{2}\right)t + \sigma_X W_t^X - \sigma_Y W_t^Y\right] \quad \text{-----(20)}$$

and consequently

$$L\left(\ln \frac{Z_T}{Z_t}\right) = N\left(\left(\mu_X - \mu_Y - \frac{\sigma_Y^2}{2} - \frac{\sigma_X^2}{2}\right)(T-t), (\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y\rho_{XY})(T-t)\right) \quad (21)$$

Z_T is the exchange rate of $\frac{SGD}{RM}$ at a future time T , Z_t is the exchange rate of at current time t . (21) says that variable Z_t has a log normal distribution.

Now we have all that we need to derive the forward price F_t at time $t \leq T$ of the forward contracts on the exchange rate $\frac{SGD}{RM}$ maturing at time T . It is known that F_t can be computed as *

$$F_t = Z_t e^{\frac{7}{5}q(T-t)}, \quad q = r_D - r_F \quad (22)$$

where r_F denotes continuously compounded foreign risk-free interest rate per day, r_D the domestic one; t denotes number of trading days. Thus the term $\frac{7}{5}q$ denotes the continuously compounded risk-free rate per trading day.

Ito Theorem and several calculations yield that;

$$\frac{dF_t}{F_t} = \left[-\frac{7}{5}q + \mu_X - \mu_Y - \sigma_X\sigma_Y\rho_{XY}\right]dt + \left[\sigma_X dW_t^X - \sigma_Y dW_t^Y\right] \quad (23)$$

Its expectation and variance are given by;

$$E \frac{dF_t}{F_t} = \left[-\frac{7}{5}q + \mu_X - \mu_Y - \sigma_X\sigma_Y\rho_{XY}\right]dt, \quad (24)$$

$$\text{var} \frac{dF_t}{F_t} = \left[\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y\rho_{XY}\right]dt \quad (25)$$

Let us proceed to the next simplification.

Model 2

Now we suppose Z_t to follow the process given by;

$$\frac{dZ_t}{Z_t} = \mu_Z dt + \sigma_Z dW_t^Z \quad (26)$$

Thus we consider no influence of processes X_t and Y_t . Simply, we consider the process Z_t itself. It is clear that model 2 (see 26) is easier than model 1 (see 16).

Like in the previous case we obtain;

$$L\left(\ln \frac{Z_T}{Z_t}\right) = N\left(\left(\mu_Z - \frac{\sigma_Z^2}{2}\right)(T-t), \sigma_Z^2(T-t)\right) \quad (27)$$

where Z_T is the exchange rate of $\frac{SGD}{RM}$ at future time T , is the exchange rate of $\frac{SGD}{RM}$ at the current time t .

Ito Theorem and several calculations yield that;

$$\frac{dF_t}{F_t} = \left[-\frac{7}{5}q + \mu_Z\right]dt + \sigma_Z dW_t^Z \quad (28)$$

Its expectation and variance are given by;

$$E \frac{dF_t}{F_t} = \left[-\frac{7}{5}q + \mu_z \right] dt, \quad \text{-----(29)}$$

$$\text{var} \frac{dF_t}{F_t} = \sigma_z^2 dt \quad \text{-----(30)}$$

Note: To complete, in this section we should list the formula for forward price used by practitioners:

$$F_t = Z_t \left(1 + \frac{7}{5}(r_D - r_F)(T - t) \right), \quad \text{-----(31)}$$

Where T denotes the maturity date in days, r_F denotes foreign risk-free interest rate per day, r_D the domestic one and Z_t the spot exchange rate $\frac{SGD}{RM}$.

For the first sight it is clear that the just listed relation presents an approximation for evaluation of forward contracts.

To conclude this section, we list some of the advantages of stochastic models. First, using stochastic calculus techniques generalizes the deterministic approach and enables us to distinguish many terms which usual techniques do not capture. For instance one can regress linearly $\frac{dZ_t}{Z_t}$ on

$\frac{dX_t}{X_t}, \frac{dY_t}{Y_t}, \frac{dU_t}{U_t}$ and obtain some estimates, but such a model would be a special case of the

listed models. Second, our models provide exhaustive list of factors that can affect $\frac{dZ_t}{Z_t}$ and

consequently $\frac{dF_t}{F_t}$.

IV. THE APPLICATION OF THE MODELS

Let us now proceed to the practical presentation of model (16) and (26) and compare the results obtained through the use of these models.

Daily data was collected from 2nd January 2001 to 30rd May 2001, i.e. 100 samples, for all necessary processes in our models, i.e. $\frac{JPY_{100}}{SGD}$ denoted by U_t , $\frac{IDX}{RM}$ by X_t , $\frac{SGD}{RM}$ by Z_t , $a + b U_t$ by Y_t . We should realize that using a small amount of historical data to estimate the input parameters exposes the model to estimation errors. On the other hand, using too long data increases the possibility of nonstationarity in the parameters.

The following table shows estimated statistics of the daily returns, i.e. sample means, sample standard deviations and sample correlation matrix :

Table 2: Estimated statistics of daily returns.

	$\frac{U_t - U_{t-1}}{U_{t-1}}$	$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$	$\frac{X_t - X_{t-1}}{X_{t-1}}$	$\frac{Z_t - Z_{t-1}}{Z_{t-1}}$
MEAN	$-4.15631 \cdot 10^{-5}$	$-1.99902 \cdot 10^{-5}$	-0.00044812	-0.000429155
SD	0.0062292130	0.002295530	0.003455779	0.002151663
$\frac{U_t - U_{t-1}}{U_{t-1}}$	1	0.999956434	0.793298721	0.208084717
$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$		1	0.793217699	0.207909218
$\frac{X_t - X_{t-1}}{X_{t-1}}$			1	0.760547525
$\frac{Z_t - Z_{t-1}}{Z_{t-1}}$				1

	$\frac{JPY_100}{SGD}$	$a + b \frac{JPY_100}{SGD}$	$\frac{IDX}{RM}$	$\frac{SGD}{RM}$
MEAN	1.479240404	0.452569616	0.968958386	2.1409
SD	0.021338628	0.002379103	0.020929471	0.040926795
2.1. 2001 (t=1)	1.515204091	0.456579308	1.000000000	2.1902
30.5.2001 (t=100)	1.506051653	0.455558877	0.956035859	2.0986

The last two lines show the first and the last observation of the considered time period. Let us remark that from table 2. results that the value $X_{30.5.2001} = 0.956035859$ may be used for the prediction.

Let us proceed to risk-free interest rates. All we need to know about risk-free interest rates r_D, r_F is their subtraction. The subtraction of risk-free interest rates;

$$r_D - r_F$$

can be approximated by the subtraction of corresponding interbank interest rates;

$$KLIBOR - SIBOR$$

Because the risk premiums subtract. This approach we will use further. Statistical properties of interest rates we worked with are listed in table 3.

Table 3: Statistical properties of interest rates.

PER YEAR	MONTH PRIBOR RM (%)	MONTH LIBOR SGD (%)
MEAN	2.9666	2.265455
SD	0.0413	0.222512
30.5.2001	3.00	2.25
p.d	0.008333	0.006250
p.m	0.25	0.1875
p.d..exp	0.0083	0.006231
p.m. exp	0.2231	0.171850

Where p.d. and p.m. denote per day and per month. The fourth line p.d. was obtained from the third one dividing by 360. To recalculate the fifth line to the sixth one we used the formula

$r_M^{\text{exp}} = \ln(1 + r_M)$. Thus the coefficient q at the equation (22) is given by $q = r_D - r_F = 0.000021$ where domestic risk free rate r_D is chosen as MONTH PRIBOR for RM per day and foreign risk free rate r_F MONTH LIBOR for SGD since they represent comparable rates as to time. Note that $\frac{7}{5}q = 0.0000294$. Now we can evaluate Model 1. and Model 2.

Using Lemma 1 and Lemma 2, we can estimate the exchange rate $\frac{SGD}{RM}$ generated by Model 1 for different values of future time T measured in trading days. We suppose to be now at time $T = 0$. The initial value of Z_0 for prediction is set $Z_0 \equiv Z_{30.5.2001} = 2.0986$. Predictions are listed in Appendix 2. The 95 % confidence interval for future value of Z_T was derived from lognormal distribution. The term $F(C)$ denotes the forward price calculated using formula (22) where we substitute for Z_t the real exchange rate at 30.5.2001.

Now we suppose to predict at 30.5.2001, denoted as time $T = 0$, with the initial value of $Z_0 \equiv E Z_t = 2.1409$ in the formula (22). With respect to the low number of observations it is suitable to examine this case. This situation is shown in Appendix 3.

Now let us proceed to the Model 2 (26). In comparison with Model 1, Model 2 is easier. It does not express the impact of the processes $\frac{JPY-100}{SGD}$ and $\frac{IDX}{RM}$ (the process $\frac{SGD}{RM}$ is calculated of them), but considers only the process $\frac{SGD}{RM}$ it self. Since estimates of Z_T are based on its distribution, the estimates in Model 2. do not differ from the estimates in Model 1. To complete this section, we list a comparison of covered T -days forward contracts using formula (31) and formula (22). Predictions in the fourth column of Appendix 4 were calculated using formula (31), in the sixth and seventh columns are listed predictions calculated by formula (22) with the initial value $E Z_t$, resp. $Z_{30.5.2001}$.

Let us emphasize an important result following from figure 1 in Appendix 1. The exchange rate $\frac{SGD}{RM}$ is influenced mainly by $\frac{IDX}{RM}$.

V. CONCLUSION

In this work we presented two models of $\frac{SGD}{RM}$ exchange rate with applications to forward contracts. The $\frac{SGD}{RM}$ exchange rate in the period from 2nd january 2001 to 31st December 2001. was given by formula (6). Our first approach to model $\frac{SGD}{RM}$ was general and dealt with an unknown intervention of Central Bank Malaysia.

We suggested two basic models and derived their applications to forward contracts. Model 1. examines the impact of processes $\frac{IDX}{RM}$ (i.e. X_t) and $a + b \frac{JPY-100}{SGD}$ (i.e. Y_t) on the process $\frac{SGD}{RM}$ (i.e. Z_t). Model 2 examines the process Z_t itself and does not consider the influence of processes X_t and Y_t .

To verify the models we calculated the input parameters using daily data from 2nd January 2001 to 30th May 2001, i.e. 100 samples. As to the risk free-rate, we worked with two comparable rates, Month Pribor for RM and Moth Libor for SGD. We needed to know only subtraction of risk-free which we approximated by subtraction of inter bank rates.

The value $X_{30.5.2001} = 0.956035859$ as the initial value could be used for the prediction. We evaluated the models and put their results to tables. We predicted Z_T for different values of T trading days for both models. The last day of our observations 30.5.2001 was chosen as a starting point of our predictions. We choose two initial values $Z_t : Z_{30.5.2001}$ and $E Z_t$. The initial value $E Z_t$ was chosen to investigate the impact of estimation errors in input parameters due to low number of observations.

Applications of Model 1. and Model 2. indicated the following: When predicting it is sufficient to work with Model 2. only. The extra information carried by Model 1. seem to have only negligible influence on predictions. On the basis of the model of the exchange rate we constructed a process of the forward exchange rate F_t . The last model we mentioned the general exchange rate model with Markov times is robust enough to cope with situations like a central bank intervention, speculation attack or abandonment of the fluctuation band. Corresponding figures are listed in the Appendix.

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- Hull, J.C. (1993). *Options, futures and other derivative securities*. Prentice Hall, Englewood Cliffs, New Jersey.

APPENDIX

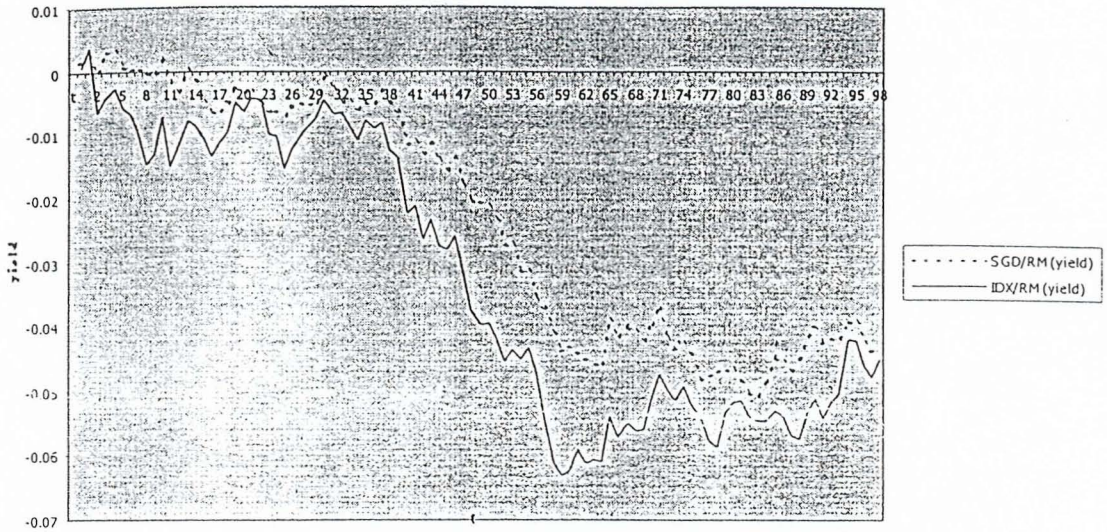
Appendix I. $\frac{SGD}{RM}$ yield and $\frac{IDX}{RM}$

t	SGD	SGD/RM (yield)	IDX/RM	IDX/RM (yield)
0	2.1902		1	
1	2.1931	0.001323204	1.000533139	0.000532997
2	2.1953	0.002325848	1.003763741	0.003756675
3	2.1896	-0.000273985	0.993561517	-0.006459299
4	2.1967	0.00296337	0.995782185	-0.004226735
5	2.1972	0.003190959	0.997486907	-0.002516256
6	2.1918	0.00073026	0.994350426	-0.005665593
7	2.1908	0.00027391	0.993282332	-0.006740333
8	2.1912	0.000456475	0.989974561	-0.010076032
9	2.1896	-0.000273985	0.985333349	-0.01477527
10	2.1891	-0.000502363	0.9871518	-0.012931452
11	2.1940	0.001733498	0.992976375	-0.007048407
12	2.1863	-0.001782247	0.985097674	-0.015014481
13	2.1833	-0.00315537	0.988170435	-0.011900091
14	2.1906	0.000182615	0.992477801	-0.007550634
15	2.1864	-0.001736508	0.991481528	-0.008554961

16	2.1794	-0.004943254	0.989445715	-0.010610376
17	2.1758	-0.006596451	0.986793548	-0.013294433
18	2.1760	-0.006504535	0.989002888	-0.011058028
19	2.1786	-0.005310395	0.990397423	-0.009648979
20	2.1850	-0.002377035	0.995226358	-0.004785073
21	2.1833	-0.00315537	0.993533239	-0.006487761
22	2.1828	-0.003384408	0.995864556	-0.004144018
23	2.1788	-0.005218597	0.995382933	-0.004627758
24	2.1758	-0.006596451	0.989982242	-0.010068274
25	2.1765	-0.006274782	0.98991601	-0.010135177
26	2.1745	-0.007194111	0.984814113	-0.015302373
27	2.1773	-0.005907287	0.988005465	-0.01206705
28	2.1793	-0.00498914	0.989706832	-0.010346509
29	2.1788	-0.005218597	0.991268849	-0.008769491
30	2.1786	-0.005310395	0.992526935	-0.007501128
31	2.1886	-0.000730794	0.995436833	-0.00457361
32	2.1836	-0.003017973	0.993128964	-0.00689475
33	2.1795	-0.004897371	0.99348822	-0.006533074
34	2.1799	-0.00471386	0.99125078	-0.008787719
35	2.1770	-0.006045082	0.989268234	-0.010789767
36	2.1799	-0.00471386	0.992198469	-0.007832122
37	2.1788	-0.005218597	0.990956669	-0.00908447
38	2.1827	-0.003430221	0.991978141	-0.008054207
39	2.1788	-0.005218597	0.98768993	-0.012386466
40	2.1763	-0.006366677	0.986792429	-0.013295566
41	2.1652	-0.011480128	0.977891139	-0.022356925
42	2.1659	-0.011156884	0.979296613	-0.020920707
43	2.1615	-0.013190439	0.974072981	-0.026269049
44	2.1649	-0.011618693	0.976935202	-0.023334953
45	2.1596	-0.014069844	0.973125081	-0.027242653
46	2.1566	-0.015459956	0.972585475	-0.027797315
47	2.1602	-0.013792054	0.974691328	-0.025634445
48	2.1505	-0.018292491	0.968924314	-0.031568777
49	2.1460	-0.02038722	0.963225946	-0.037467267
50	2.1463	-0.020247434	0.961126981	-0.039648744
51	2.1439	-0.021366264	0.961506964	-0.039253471
52	2.1393	-0.023514191	0.958968526	-0.041897025
53	2.1322	-0.026838553	0.955733273	-0.045276408
54	2.1315	-0.027166907	0.957538792	-0.043389045
55	2.1229	-0.031209785	0.955867794	-0.045135666
56	2.1227	-0.031304001	0.957939777	-0.042970366
57	2.1129	-0.035931452	0.954106272	-0.046980217
58	2.1105	-0.037067978	0.947618299	-0.053803496
59	2.1017	-0.041246323	0.940738804	-0.061089751
60	2.0966	-0.043675879	0.938792396	-0.063160915
61	2.0981	-0.042960691	0.939903969	-0.061977569
62	2.0966	-0.043675879	0.942638897	-0.059072
63	2.0942	-0.044821245	0.940588337	-0.061249709

64	2.0922	-0.04577672	0.941161423	-0.06064061
65	2.0925	-0.04563334	0.941091627	-0.060714773
66	2.1064	-0.039012535	0.947486987	-0.053942076
67	2.1008	-0.041674639	0.944660903	-0.056929248
68	2.1041	-0.040105042	0.946647015	-0.054828996
69	2.1032	-0.040532869	0.945384699	-0.056163345
70	2.1000	-0.042055519	0.945724104	-0.055804397
71	2.1053	-0.039534889	0.950236629	-0.051044242
72	2.1102	-0.037210134	0.953942841	-0.047151524
73	2.1006	-0.041769846	0.951560525	-0.049651984
74	2.0968	-0.043580491	0.949954607	-0.051341077
75	2.0994	-0.042341274	0.95216304	-0.049018999
76	2.0954	-0.044248398	0.949262023	-0.052070414
77	2.0873	-0.048121499	0.947233129	-0.054210039
78	2.0885	-0.047546759	0.943887892	-0.057747879
79	2.0896	-0.047020204	0.942955582	-0.0587361
80	2.0902	-0.046733109	0.948267987	-0.05311813
81	2.0902	-0.046733109	0.949739692	-0.051567341
82	2.0867	-0.048408993	0.950093147	-0.05119525
83	2.0833	-0.050039689	0.947598852	-0.053824019
84	2.0820	-0.050663894	0.946935032	-0.054524792
85	2.0868	-0.048361072	0.94704471	-0.054408975
86	2.0947	-0.044582519	0.948358267	-0.053022929
87	2.0929	-0.0454422	0.947639819	-0.053780787
88	2.0896	-0.047020204	0.94483981	-0.056739879
89	2.0939	-0.044964508	0.944315097	-0.057295379
90	2.1014	-0.041389075	0.948122528	-0.053271536
91	2.1035	-0.04039024	0.950242884	-0.051037659
92	2.0988	-0.042627111	0.947207412	-0.05423719
93	2.1008	-0.041674639	0.949711527	-0.051596996
94	2.1001	-0.042007901	0.950914985	-0.050330616
95	2.1061	-0.039154968	0.959073989	-0.041787055
96	2.1058	-0.039297421	0.958742411	-0.042132841
97	2.0993	-0.042388908	0.955144581	-0.045892556
98	2.0962	-0.043866682	0.953160252	-0.047972234
99	2.0986	-0.042722408	0.956035859	-0.044959857

How Much of the SGD/RM yield is caused by the IDX/RM yield



Appendix 2. Model I

Date	T	Real Zt(SGD/RM)	E Zt	var Zt	SD(Zt)	95%(L)	95%(H)	F(C)
31/05/01	1	2.1017	2.0977	2.0360E-05	0.0045	2.0889	2.1065	2.0987
01/06/01	2	2.1014	2.0968	4.0685E-05	0.0064	2.0843	2.1093	2.0987
05/06/01	3	2.0960	2.0959	6.0976E-05	0.0078	2.0806	2.1112	2.0988
06/06/01	4	2.0978	2.0950	8.1231E-05	0.0090	2.0773	2.1127	2.0988
07/06/01	5	2.0992	2.0941	1.0145E-04	0.0101	2.0744	2.1138	2.0989
08/06/01	6	2.0993	2.0932	1.2164E-04	0.0110	2.0716	2.1148	2.0990
11/06/01	7	2.0974	2.0923	1.4179E-04	0.0119	2.0690	2.1156	2.0990
12/06/01	8	2.0957	2.0914	1.6191E-04	0.0127	2.0665	2.1163	2.0991
13/06/01	9	2.0910	2.0905	1.8199E-04	0.0135	2.0641	2.1170	2.0992
14/06/01	10	2.0919	2.0896	2.0204E-04	0.0142	2.0618	2.1175	2.0992
15/06/01	11	2.0950	2.0887	2.2205E-04	0.0149	2.0595	2.1179	2.0993
18/06/01	12	2.0926	2.0878	2.4203E-04	0.0156	2.0573	2.1183	2.0993
19/06/01	13	2.0924	2.0869	2.6198E-04	0.0162	2.0552	2.1186	2.0994
20/06/01	14	2.0908	2.0860	2.8189E-04	0.0168	2.0531	2.1189	2.0995
21/06/01	15	2.0868	2.0851	3.0176E-04	0.0174	2.0511	2.1192	2.0995
22/06/01	16	2.0885	2.0842	3.2161E-04	0.0179	2.0491	2.1194	2.0996
25/06/01	17	2.0885	2.0833	3.4141E-04	0.0185	2.0471	2.1196	2.0996
26/06/01	18	2.0908	2.0825	3.6119E-04	0.0190	2.0452	2.1197	2.0997
27/06/01	19	2.0876	2.0816	3.8093E-04	0.0195	2.0433	2.1198	2.0998
28/06/01	20	2.0848	2.0807	4.0063E-04	0.0200	2.0414	2.1199	2.0998
29/06/01	21	2.0877	2.0798	4.2030E-04	0.0205	2.0396	2.1200	2.0999
02/07/01	22	2.0853	2.0789	4.3994E-04	0.0210	2.0378	2.1200	2.1000
03/07/01	23	2.0853	2.0780	4.5955E-04	0.0214	2.0360	2.1200	2.1000
04/07/01	24	2.0841	2.0771	4.7912E-04	0.0219	2.0342	2.1200	2.1001
05/07/01	25	2.0816	2.0762	4.9865E-04	0.0223	2.0324	2.1200	2.1001

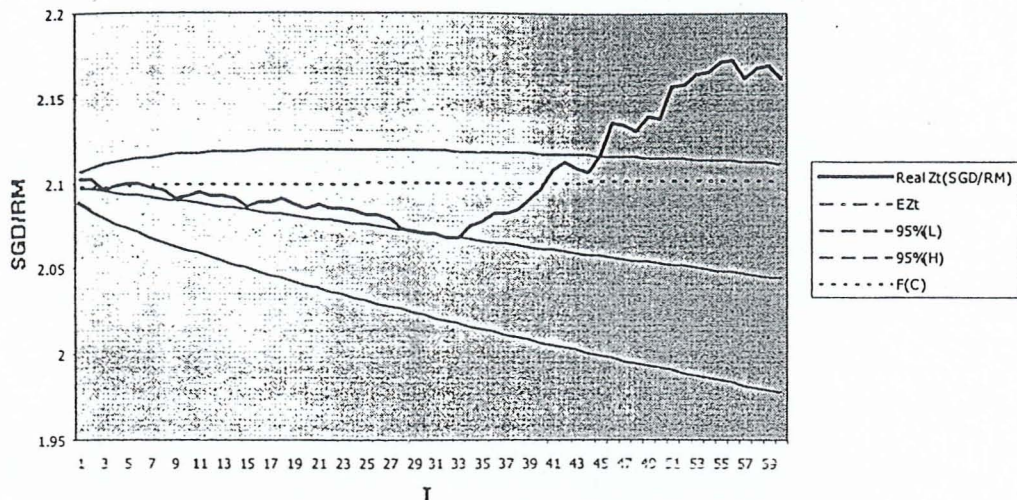
06/07/01	26	2.0816	2.0753	5.1815E-04	0.0228	2.0307	2.1199	2.1002
09/07/01	27	2.0793	2.0744	5.3762E-04	0.0232	2.0290	2.1199	2.1003
10/07/01	28	2.0728	2.0735	5.5706E-04	0.0236	2.0273	2.1198	2.1003
11/07/01	29	2.0720	2.0726	5.7646E-04	0.0240	2.0256	2.1197	2.1004
12/07/01	30	2.0708	2.0718	5.9583E-04	0.0244	2.0239	2.1196	2.1005
13/07/01	31	2.0708	2.0709	6.1516E-04	0.0248	2.0223	2.1195	2.1005
16/07/01	32	2.0686	2.0700	6.3446E-04	0.0252	2.0206	2.1193	2.1006
17/07/01	33	2.0688	2.0691	6.5373E-04	0.0256	2.0190	2.1192	2.1006
18/07/01	34	2.0759	2.0682	6.7296E-04	0.0259	2.0174	2.1190	2.1007
19/07/01	35	2.0776	2.0673	6.9216E-04	0.0263	2.0157	2.1189	2.1008
20/07/01	36	2.0832	2.0664	7.1133E-04	0.0267	2.0142	2.1187	2.1008
23/07/01	37	2.0832	2.0655	7.3046E-04	0.0270	2.0126	2.1185	2.1009
24/07/01	38	2.0850	2.0647	7.4956E-04	0.0274	2.0110	2.1183	2.1009
25/07/01	39	2.091	2.0638	7.6863E-04	0.0277	2.0094	2.1181	2.1010
26/07/01	40	2.0977	2.0629	7.8766E-04	0.0281	2.0079	2.1179	2.1011
27/07/01	41	2.1078	2.0620	8.0667E-04	0.0284	2.0063	2.1177	2.1011
30/07/01	42	2.1123	2.0611	8.2563E-04	0.0287	2.0048	2.1174	2.1012
31/07/01	43	2.1094	2.0602	8.4457E-04	0.0291	2.0033	2.1172	2.1013
01/08/01	44	2.1065	2.0593	8.6347E-04	0.0294	2.0018	2.1169	2.1013
02/08/01	45	2.1157	2.0585	8.8234E-04	0.0297	2.0002	2.1167	2.1014
03/08/01	46	2.1360	2.0576	9.0117E-04	0.0300	1.9987	2.1164	2.1014
06/08/01	47	2.1340	2.0567	9.1998E-04	0.0303	1.9972	2.1161	2.1015
07/08/01	48	2.1302	2.0558	9.3875E-04	0.0306	1.9958	2.1159	2.1016
08/08/01	49	2.1384	2.0549	9.5748E-04	0.0309	1.9943	2.1156	2.1016
09/08/01	50	2.1378	2.0540	9.7619E-04	0.0312	1.9928	2.1153	2.1017
10/08/01	51	2.1573	2.0532	9.9486E-04	0.0315	1.9913	2.1150	2.1017
13/08/01	52	2.1585	2.0523	1.0135E-03	0.0318	1.9899	2.1147	2.1018
14/08/01	53	2.1646	2.0514	1.0321E-03	0.0321	1.9884	2.1144	2.1019
15/08/01	54	2.1649	2.0505	1.0507E-03	0.0324	1.9870	2.1141	2.1019
16/08/01	55	2.1717	2.0496	1.0692E-03	0.0327	1.9856	2.1137	2.1020
17/08/01	56	2.1723	2.0488	1.0877E-03	0.0330	1.9841	2.1134	2.1021
20/08/01	57	2.1615	2.0479	1.1062E-03	0.0333	1.9827	2.1131	2.1021
21/08/01	58	2.1676	2.0470	1.1247E-03	0.0335	1.9813	2.1127	2.1022
22/08/01	59	2.1689	2.0461	1.1431E-03	0.0338	1.9799	2.1124	2.1022
23/08/01	60	2.1622	2.0453	1.1614E-03	0.0341	1.9785	2.1120	2.1023

$$Z_0 = Z_{30/05/01} = 2.0986$$

$$\text{Mean } \ln(ZT/Z_0) = -0.000431466$$

$$\text{var } \ln(ZT/Z_0) = 4.62692E-06$$

Model 1A



Model 1 b:

Date	T	Real Zt(SGD/RM)	E Zt	var Zt	SD(Zt)	95%(L)	95%(H)	F(C)
31/05/01	1	2.1017	2.1400	2.1189E-05	0.0046	2.1310	2.1490	2.1410
01/06/01	2	2.1014	2.1391	4.2342E-05	0.0065	2.1263	2.1518	2.1410
05/06/01	3	2.0960	2.1381	6.3459E-05	0.0080	2.1225	2.1538	2.1411
06/06/01	4	2.0978	2.1372	8.4539E-05	0.0092	2.1192	2.1552	2.1412
07/06/01	5	2.0992	2.1363	1.0558E-04	0.0103	2.1162	2.1565	2.1412
08/06/01	6	2.0993	2.1354	1.2659E-04	0.0113	2.1133	2.1574	2.1413
11/06/01	7	2.0974	2.1345	1.4756E-04	0.0121	2.1107	2.1583	2.1413
12/06/01	8	2.0957	2.1336	1.6850E-04	0.0130	2.1081	2.1590	2.1414
13/06/01	9	2.0910	2.1326	1.8940E-04	0.0138	2.1057	2.1596	2.1415
14/06/01	10	2.0919	2.1317	2.1027E-04	0.0145	2.1033	2.1602	2.1415
15/06/01	11	2.0950	2.1308	2.3109E-04	0.0152	2.1010	2.1606	2.1416
18/06/01	12	2.0926	2.1299	2.5189E-04	0.0159	2.0988	2.1610	2.1417
19/06/01	13	2.0924	2.1290	2.7264E-04	0.0165	2.0966	2.1614	2.1417
20/06/01	14	2.0908	2.1281	2.9336E-04	0.0171	2.0945	2.1616	2.1418
21/06/01	15	2.0868	2.1272	3.1405E-04	0.0177	2.0924	2.1619	2.1418
22/06/01	16	2.0885	2.1262	3.3470E-04	0.0183	2.0904	2.1621	2.1419
25/06/01	17	2.0885	2.1253	3.5532E-04	0.0188	2.0884	2.1623	2.1420
26/06/01	18	2.0908	2.1244	3.7589E-04	0.0194	2.0864	2.1624	2.1420
27/06/01	19	2.0876	2.1235	3.9644E-04	0.0199	2.0845	2.1625	2.1421
28/06/01	20	2.0848	2.1226	4.1695E-04	0.0204	2.0826	2.1626	2.1422
29/06/01	21	2.0877	2.1217	4.3742E-04	0.0209	2.0807	2.1627	2.1422
02/07/01	22	2.0853	2.1208	4.5786E-04	0.0214	2.0788	2.1627	2.1423
03/07/01	23	2.0853	2.1199	4.7826E-04	0.0219	2.0770	2.1627	2.1423
04/07/01	24	2.0841	2.1190	4.9862E-04	0.0223	2.0752	2.1627	2.1424
05/07/01	25	2.0816	2.1181	5.1896E-04	0.0228	2.0734	2.1627	2.1425
06/07/01	26	2.0816	2.1171	5.3925E-04	0.0232	2.0716	2.1627	2.1425
09/07/01	27	2.0793	2.1162	5.5951E-04	0.0237	2.0699	2.1626	2.1426
10/07/01	28	2.0728	2.1153	5.7974E-04	0.0241	2.0681	2.1625	2.1427

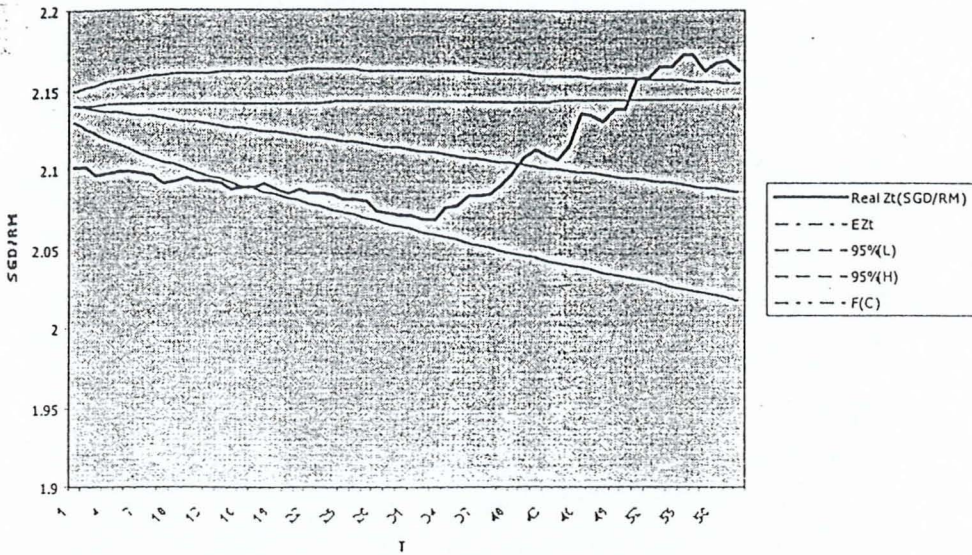
11/07/01	29	2.0720	2.1144	5.9993E-04	0.0245	2.0664	2.1624	2.1427
12/07/01	30	2.0708	2.1135	6.2009E-04	0.0249	2.0647	2.1623	2.1428
13/07/01	31	2.0708	2.1126	6.4021E-04	0.0253	2.0630	2.1622	2.1429
16/07/01	32	2.0686	2.1117	6.6030E-04	0.0257	2.0613	2.1621	2.1429
17/07/01	33	2.0688	2.1108	6.8035E-04	0.0261	2.0597	2.1619	2.1430
18/07/01	34	2.0759	2.1099	7.0036E-04	0.0265	2.0580	2.1618	2.1430
19/07/01	35	2.0776	2.1090	7.2035E-04	0.0268	2.0564	2.1616	2.1431
20/07/01	36	2.0832	2.1081	7.4029E-04	0.0272	2.0548	2.1614	2.1432
23/07/01	37	2.0832	2.1072	7.6021E-04	0.0276	2.0531	2.1612	2.1432
24/07/01	38	2.0850	2.1063	7.8008E-04	0.0279	2.0515	2.1610	2.1433
25/07/01	39	2.091	2.1054	7.9993E-04	0.0283	2.0499	2.1608	2.1434
26/07/01	40	2.0977	2.1045	8.1974E-04	0.0286	2.0483	2.1606	2.1434
27/07/01	41	2.1078	2.1036	8.3951E-04	0.0290	2.0468	2.1603	2.1435
30/07/01	42	2.1123	2.1027	8.5925E-04	0.0293	2.0452	2.1601	2.1435
31/07/01	43	2.1094	2.1018	8.7896E-04	0.0296	2.0436	2.1599	2.1436
01/08/01	44	2.1065	2.1009	8.9863E-04	0.0300	2.0421	2.1596	2.1437
02/08/01	45	2.1157	2.1000	9.1827E-04	0.0303	2.0406	2.1593	2.1437
03/08/01	46	2.1360	2.0991	9.3787E-04	0.0306	2.0390	2.1591	2.1438
06/08/01	47	2.1340	2.0982	9.5744E-04	0.0309	2.0375	2.1588	2.1439
07/08/01	48	2.1302	2.0973	9.7697E-04	0.0313	2.0360	2.1585	2.1439
08/08/01	49	2.1384	2.0964	9.9647E-04	0.0316	2.0345	2.1582	2.1440
09/08/01	50	2.1378	2.0955	1.0159E-03	0.0319	2.0330	2.1579	2.1440
10/08/01	51	2.1573	2.0946	1.0354E-03	0.0322	2.0315	2.1576	2.1441
13/08/01	52	2.1585	2.0937	1.0548E-03	0.0325	2.0300	2.1573	2.1442
14/08/01	53	2.1646	2.0928	1.0741E-03	0.0328	2.0285	2.1570	2.1442
15/08/01	54	2.1649	2.0919	1.0935E-03	0.0331	2.0270	2.1567	2.1443
16/08/01	55	2.1717	2.0910	1.1128E-03	0.0334	2.0256	2.1563	2.1444
17/08/01	56	2.1723	2.0901	1.1320E-03	0.0336	2.0241	2.1560	2.1444
20/08/01	57	2.1615	2.0892	1.1513E-03	0.0339	2.0227	2.1557	2.1445
21/08/01	58	2.1676	2.0883	1.1704E-03	0.0342	2.0212	2.1553	2.1446
22/08/01	59	2.1689	2.0874	1.1896E-03	0.0345	2.0198	2.1550	2.1446
23/08/01	60	2.1622	2.0865	1.2087E-03	0.0348	2.0183	2.1546	2.1447

$$Z_0 = E Z_t = 2.1409$$

$$\text{Mean } \ln(Z_t/Z_0) = -0.000431466$$

$$\text{var } \ln(Z_t/Z_0) = 4.62692E-06$$

Model 1B



Appendix 3. Model 2.

Date	T	Real Zt(SGD/RM)	CSOB,Z 30/05/01	CSOB,Ezt	F(C),Z 30/05/01	F(C),Ezt
31/05/01	1	2.1017	2.0987	2.1410	2.0987	2.1410
01/06/01	2	2.1014	2.0987	2.1410	2.0987	2.1410
05/06/01	3	2.0960	2.0988	2.1411	2.0988	2.1411
06/06/01	4	2.0978	2.0988	2.1412	2.0988	2.1412
07/06/01	5	2.0992	2.0989	2.1412	2.0989	2.1412
08/06/01	6	2.0993	2.0990	2.1413	2.0990	2.1413
11/06/01	7	2.0974	2.0990	2.1413	2.0990	2.1413
12/06/01	8	2.0957	2.0991	2.1414	2.0991	2.1414
13/06/01	9	2.0910	2.0992	2.1415	2.0992	2.1415
14/06/01	10	2.0919	2.0992	2.1415	2.0992	2.1415
15/06/01	11	2.0950	2.0993	2.1416	2.0993	2.1416
18/06/01	12	2.0926	2.0993	2.1417	2.0993	2.1417
19/06/01	13	2.0924	2.0994	2.1417	2.0994	2.1417
20/06/01	14	2.0908	2.0995	2.1418	2.0995	2.1418
21/06/01	15	2.0868	2.0995	2.1418	2.0995	2.1418
22/06/01	16	2.0885	2.0996	2.1419	2.0996	2.1419
25/06/01	17	2.0885	2.0996	2.1420	2.0996	2.1420
26/06/01	18	2.0908	2.0997	2.1420	2.0997	2.1420
27/06/01	19	2.0876	2.0998	2.1421	2.0998	2.1421
28/06/01	20	2.0848	2.0998	2.1422	2.0998	2.1422
29/06/01	21	2.0877	2.0999	2.1422	2.0999	2.1422
02/07/01	22	2.0853	2.1000	2.1423	2.1000	2.1423
03/07/01	23	2.0853	2.1000	2.1423	2.1000	2.1423
04/07/01	24	2.0841	2.1001	2.1424	2.1001	2.1424
05/07/01	25	2.0816	2.1001	2.1425	2.1001	2.1425
06/07/01	26	2.0816	2.1002	2.1425	2.1002	2.1425

09/07/01	27	2.0793	2.1003	2.1426	2.1003	2.1426
10/07/01	28	2.0728	2.1003	2.1427	2.1003	2.1427
11/07/01	29	2.0720	2.1004	2.1427	2.1004	2.1427
12/07/01	30	2.0708	2.1005	2.1428	2.1005	2.1428
13/07/01	31	2.0708	2.1005	2.1429	2.1005	2.1429
16/07/01	32	2.0686	2.1006	2.1429	2.1006	2.1429
17/07/01	33	2.0688	2.1006	2.1430	2.1006	2.1430
18/07/01	34	2.0759	2.1007	2.1430	2.1007	2.1430
19/07/01	35	2.0776	2.1008	2.1431	2.1008	2.1431
20/07/01	36	2.0832	2.1008	2.1432	2.1008	2.1432
23/07/01	37	2.0832	2.1009	2.1432	2.1009	2.1432
24/07/01	38	2.0850	2.1009	2.1433	2.1009	2.1433
25/07/01	39	2.091	2.1010	2.1434	2.1010	2.1434
26/07/01	40	2.0977	2.1011	2.1434	2.1011	2.1434
27/07/01	41	2.1078	2.1011	2.1435	2.1011	2.1435
30/07/01	42	2.1123	2.1012	2.1435	2.1012	2.1435
31/07/01	43	2.1094	2.1013	2.1436	2.1013	2.1436
01/08/01	44	2.1065	2.1013	2.1437	2.1013	2.1437
02/08/01	45	2.1157	2.1014	2.1437	2.1014	2.1437
03/08/01	46	2.1360	2.1014	2.1438	2.1014	2.1438
06/08/01	47	2.1340	2.1015	2.1439	2.1015	2.1439
07/08/01	48	2.1302	2.1016	2.1439	2.1016	2.1439
08/08/01	49	2.1384	2.1016	2.1440	2.1016	2.1440
09/08/01	50	2.1378	2.1017	2.1440	2.1017	2.1440
10/08/01	51	2.1573	2.1017	2.1441	2.1017	2.1441
13/08/01	52	2.1585	2.1018	2.1442	2.1018	2.1442
14/08/01	53	2.1646	2.1019	2.1442	2.1019	2.1442
15/08/01	54	2.1649	2.1019	2.1443	2.1019	2.1443
16/08/01	55	2.1717	2.1020	2.1444	2.1020	2.1444
17/08/01	56	2.1723	2.1021	2.1444	2.1021	2.1444
20/08/01	57	2.1615	2.1021	2.1445	2.1021	2.1445
21/08/01	58	2.1676	2.1022	2.1446	2.1022	2.1446
22/08/01	59	2.1689	2.1022	2.1446	2.1022	2.1446
23/08/01	60	2.1622	2.1023	2.1447	2.1023	2.1447

