



IMAGE DATA COMPRESSION USING DCT (DISCRETE COSINE  
TRANSFORM) AND INTERPOLATION AND ALLIED TOPICS IN DIGITAL  
IMAGE PROCESSING APPLIED TO SATELLITE IMAGING

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UNIVERSITI SAINS MALAYSIA  
KAMPUS KEJURUTERAAN

2008





# **Laporan Akhir Projek Penyelidikan Jangka Pendek**

## **Image Data Compression using DCT (Discrete Cosine Transform) and Interpolation and Allied Topics in Digital Image Processing Applied to Satellite Imaging**

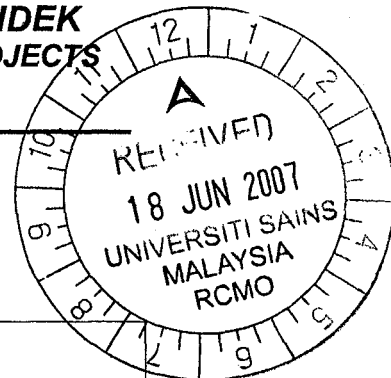
**by**

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**Dr. Aftanasar Md Sahar**

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1) **Nama Ketua Penyelidik :**

*Name of Research Leader :* **Dr. PABBISETTI SATHYANARAYANA**

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**Nama Penyelidik Bersama**

*(Jika berkaitan) :*

*Name/s of Co-Researcher/s  
(if applicable)*

Penyelidik Bersama <i>Co-Researcher</i>	PTJ <i>School/Centre</i>
<b>Dr. AFTANASAR Md SAHAR</b>	<b>Aerospace Engineering/ Engineering campus</b>

2)

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*Title of Project:*

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transform) and interpolation and allied topics in Digital Image  
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### **Abstrak untuk penyelidikan anda**

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#### *Abstract of Research*

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Digital image processing plays an important role in modern scientific endeavors. It has specific uses in satellite imaging, remote sensing, telemetry and medical imaging. Image processing requires huge memory space to store the data, and to process the data in real time high speed computers are required. For transmission, the channel capacity requirement is much more stringent. To circumvent such problem in storage and transmission one of the methods is to compress the data as much as possible before transmission and after reception decompression is to be applied with out loss of much information. The compression process can be carried out by using DCT (Discrete cosine transform), Discrete Hartley transform and also Discrete Fourier transform. Algorithms are developed for all the three methods. Matlab software is used for the compression and decompression process. Numbers of images are tested for the process. Detailed analysis is carried out. Mean square error estimation of the image obtained by compression and decompression process is carried out with respect to the original image. Interpolation is the other technique to recover the original image from the sampled image. Interpolation in two dimensions is the method to recover replica of the original image from the sampled image. There are different methods to implement interpolation. 2-D FFT is used for this and for this method also error estimation is done and compared with other methods.

- 4) Sila sediakan Laporan teknikal lengkap yang menerangkan keseluruhan projek ini.  
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Senaraikan Kata Kunci yang boleh menggambarkan penyelidikan anda :  
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- .....
- Discrete cosine transform
  - Discrete Hartley Transform
  - Image compression
  - Interpolation
  - Mean square error
  - Image processing

5) **Output Dan Faedah Projek**  
*Output and Benefits of Project*

- (a) \* **Penerbitan (termasuk laporan/kertas seminar)**  
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*(Kindly state each type, title, author/editor, publication year and journal/s containing publication)*

1. International Conference: "Image data compression techniques using discrete Hartley type transform and FFTs: A comparison", Pabbiseti Sathyanarayana, Hamid R. Saeedipour, Aftanazar Md. Sahar and Radzuan Razali, International Conference on Robotics, Vision, Information and Signal Processing, (ROVISP-2005) 21-22 July 2005 Pinang, Malaysia

2. International Conference: "Data compression techniques using CAS-CAS transform applied to remotely piloted vehicle (RPV) digital images before transmission to ground station", Pabbiseti Sathyanarayana, Hamid R. Saeedipour, and K.S. Rama Rao 9<sup>th</sup> International Conference on Mechatronics Technology (ICMT-2005) 2005, 5-8 December 2005, Kuala Lumpur, Malaysia.

3. International Conference: "Digital image compression and decompression using three different transforms and comparison of their performance", Pabbiseti Sathyanarayana, and Hamid R. Saeedipour, International Conference on Man-Machine Systems (ICoMMS2006), 15-16 September 2006. Langkawi, Malaysia

- (b) **Faedah-Faedah Lain Seperti Perkembangan Produk, Prospek Komersialisasi Dan Pendaftaran Paten atau impak kepada dasar dan masyarakat.**  
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Different methods of image compression and decompression are tested and merits and demerits of them are also discussed. The algorithm is developed. The implementation in real time using hard ware and software is to be developed as further work in this project.

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## TECHNICAL REPORT

**Project Title:** Image data compression using DCT (Discrete Cosine transform) and interpolation and allied topics in digital image processing applied to satellite imaging

**Project Leader:** Dr. Pabbiseti Sathyanaryana

The project work started in the month of April 2005. The literature survey of the project work is completed after three months of the start of the project. The project work is completed. Digital image processing plays an important role in many modern scientific applications. It has specific applications in satellite imaging, remote sensing and medical imaging. Digital images require huge memory space and larger band width for transmission, since for a reasonable pixel size of 512x 512 of digital image requires about few MBs memory space. To reduce the storage space and burden of transmission of data, compression of data is one of the important applications.

There are different techniques used for data compression. They are

- i. Discrete cosine transform
- ii. Fast Fourier transform
- iii. Fast Hartley Transform
- iv. Interpolation using FFT
- v. Karhunen – Loeve transform

The present work started with analysis of compression and decompression using DCT and Hartley transforms. The Discrete Cosine Transform (DCT) and Fast Hartley type transforms are applied for data compression in two dimensions for digital images. The Hartley transform (HT) was developed as a substitute to Fourier transform (FT) in applications where the data is in real domain. [1] to [16]. The HT has also defined in two dimensions [11] as a separable Hartley type transform, named the CAS-CAS transform (CCT). CAS stands for Cos plus Sine. Algorithms are developed for data compression and decompression using CAS- CAS transform in two dimensions. Comparison of compressed and decompressed image with the original image with respect to mean square error is carried out. Discrete cosine transform method of compression and decompression process is also carried out on the same examples. The Fourier Transform is also used for compression and decompression process by developing an algorithm for the compression. These three transform methods are compared in terms of mean square error with respect to original image.

### I. CAS-CAS Transform method:

Hartley transform (HT) relations in 2-D are [11]

$$H(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad \dots (1)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} H(u, v) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad \dots (2)$$

$\text{Cas } 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right)$  is not a separable Kernel like



Exp  $[j2\pi(\frac{mu}{M} + \frac{nv}{N})]$  for 2-D DFT, hence the row- column decomposition method [13] [14] of computing a 2-D transform from 1-D fast Fourier transform algorithm can not be applied directly in this case of HT. To take advantage of the fast 1-D HT algorithms, a separable Hartley like transform namely CAS-CAS transform is developed.

$T(u, v)$ , is defined as [11]

$$T(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad \dots(3)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad \dots(4)$$

Where

$\text{Cas}(\alpha) = \cos(\alpha) + \sin(\alpha)$  and

$$H(u, v) = \frac{1}{2} [T(u, v) + T(-u, v) + T(u, -v) - T(-u, -v)] \quad \dots(5)$$

In this way 2-D Discrete Hartley transform can be computed.

### 1.1 The algorithmic steps for compression and decompression using CAS-CAS transform (CCT):

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original digital image matrix A is considered as (M, N).
2. Compute CCT of this matrix A using equation (3) as Matrix B which is of same size A.
3. Modify the CCT coefficients matrix B to reduce its size to (M/2, N/2), to achieve a compression factor of 4.

Construct a new sequence C (u, v) from B (u, v) as follows

$$\begin{aligned} C(u, v) &= B(u, v) && \text{for } u = 0, 1 \dots (M/4) - 1 \\ & && v = 0, 1 \dots (N/4) - 1 \\ &= B(u, v + N/2) && \text{for } u = 0, 1 \dots (M/4) - 1 \\ & && v = (N/4) \dots (N/2) - 1 \\ &= B(u + M/2, v) && \text{for } u = M/4 \dots (M/2) - 1 \\ & && v = 0, 1 \dots N/4 \\ &= B(u + M/2, v + N/2) && \text{for } u = M/4 \dots (M/2) - 1 \\ & && v = N/4 \dots (N/2) - 1 \end{aligned}$$

C (u, v) is matrix of size (M/2, N/2) a compression ratio of 4 with respect to original size of the image matrix A of size (M, N) is achieved.

4. This reduced CCT coefficient matrix is to be transmitted instead of the original matrix, so that burden on the channel can be reduced by a factor of 4.
5. At the receiver, after receiving the modified CCT matrix C (u, v), is appended with zeros to make it to original size as follows.

$$\begin{aligned} D(u, v) &= C(u, v) && \text{for } u = 0, 1 \dots (M/4) - 1 \\ & && v = 0, 1 \dots (N/4) - 1 \end{aligned}$$

$$\begin{aligned}
&= 0 && \text{for } u = 0, 1 \dots (M/4)-1 \\
&&& v = N/4 \dots (3N/4) - 1 \\
&= C(u, v-N/2) && \text{for } u = 0, 1 \dots (M/4)-1 \\
&&& v = 3N/4 \dots (N-1) \\
&= 0 && \text{for } u = M/4 \dots (3M/4)-1 \\
&&& v = 0, 1, 2 \dots (N-1) \\
&= C(u-M/2, v) && \text{for } u = 3M/4 \dots (M-1) \\
&&& v = 0, 1 \dots (N/4)-1 \\
&= 0 && \text{for } u = 3M/4 \dots (M-1) \\
&&& v = N/4 \dots (3N/4)-1 \\
&= C(u-M/2, v-N/2) && \text{for } u = 3M/4 \dots (M-1) \\
&&& v = 3N/4 \dots (N-1)
\end{aligned}$$

6. Multiply each  $D(u, v)$  by the square of the compression factor.

7. Perform the inverse CCT using equation (4) on the sequence  $D(u, v)$  of size  $(M, N)$  to obtain the replica of original image.

## II. Data compression and decompression using discrete cosine transform;

The discrete cosine transform relations in 2-D are

$$X(u, v) = \alpha_u \alpha_v \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N} \quad \dots (6)$$

Where  $m = 0, 1 \dots M-1$  And  $n = 0, 1 \dots N-1$

$$\alpha_u = \frac{1}{\sqrt{M}} \quad \text{for } u = 0 \quad \alpha_u = \sqrt{\frac{2}{M}} \quad \text{for } u = 1, \dots M-1$$

$$\alpha_v = \frac{1}{\sqrt{M}} \quad \text{for } v = 0 \quad \alpha_v = \sqrt{\frac{2}{M}} \quad \text{for } v = 1, \dots M-1$$

And 2-D inverse discrete cosine Transform is

$$X(m, n) = \alpha_u \alpha_v \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N} \quad \dots (7)$$

Where  $u = 0, 1 \dots M-1$

And  $v = 0, 1 \dots N-1$

Equations (6) are forward transform and equation (7) is reverse transform. By using the above two equations discrete cosine transform coefficients and inverse discrete cosine transform coefficients can be computed for the digital image.

## III. Fast Fourier transform method

The Fourier transform relations in 2-D are

$$X(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) e^{-j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (6)$$

Where  $m = 0, 1 \dots M-1$ , and  $n = 0, 1 \dots N-1$

$$X(m, n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) e^{j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (7)$$

Where  $u = 0, 1 \dots M-1$ , and  $v = 0, 1 \dots N-1$

Equation (6) is forward transform and equation (7) is reverse transform. By using the above two equations Fourier transform coefficients and inverse Fourier transform coefficients can be computed for the digital image.

### III.1. The algorithmic steps for compression and decompression using FFT

The procedure is similar to that of Hartley type transform for data compression and decompression using Fast Fourier transform. The algorithmic steps given above for Hartley transform, are to be used for this also by replacing CCT with FFT. FFT magnitude coefficients are also concentrated at the corners as that of CCT.

#### Procedure:

##### 1. CCT method

The above algorithmic steps are applied on the original scanned digital image of size (256,256) pixels shown as figure 1. For this image data CCT coefficients are computed. Then data is reduced by discarding some of the coefficients by using the algorithmic step (3). Most of CCT coefficients are having larger magnitudes in all the four corners of the matrix instead of central region of the matrix. Based on this principle the discarding of negligibly small value data points is done. To bring back to the original size of the matrix, zeros are appended where ever the data points are discarded. Now inverse CCT is applied on this data with a proper multiplier. The resultant replica of the original image is shown as figure 2. These two images figure 1 and figure 2 are look alike, and there is no noticeable loss of information. In this a reduction of CCT coefficients to be transmitted is 1/4th of that of original image data points.

A second example, with a compression ration of 16, is shown as figure 3. The resultant image figure 3 is not up to the mark, and further compression will completely distorts the image. The advantage of CCT is that it is a real transform and hence the storage and transmission is not complex but FFT algorithm of row column decomposition can be applied.

##### 2. DCT method

For the same data (figure 1), equation (6) is applied to get the DCT coefficients. DCT coefficients are concentrated near the origin. To achieve the compression ratio of 4 the DCT coefficient matrix is truncated by retaining (M/4, N/4) coefficients from the origin.

Now zeros are appended to get back the original size of the matrix and with proper multiplier inverse transform equation (7), is applied to get the replica of the original image as shown figure 4. Figure 1 and

figure 4 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. As the compression ratio is increased further say 16, by decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 5.

### 3. FFT method

For the same data (figure 1), equation (6) is applied to get the Fourier coefficients. As in the CCT method some of the coefficients are discarded by following same algorithmic steps as in CCT method. The discarded coefficients are substituted with zeros, and with proper multiplier inverse transform equation (7), are applied to get the replica of the original image as shown figure 6. Figures 1 and figure 6 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. The only difference in this method in comparison with CCT is that the data to be transmitted is complex instead of real and hence effective reduction in memory space is 50 %. As the compression ratio is increased further, decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 7 with a compression ratio of 16. Figure 8 shows the distribution of magnitudes of Fourier coefficients for the figure 1 data points. It shows that the magnitudes are less in the middle region, which are discarded and in the reverse transform they are assumed to be zeros.

### Results and conclusions

CCT and DCT methods are tried for different compression factors. The mean square error estimation with respect to the original image data (figure 1) are presented in table 1.

Serial Number	Compression factor	DCT Method Mean Square error	CCT Method Mean Square Error	FFT Method Mean square error
1	4	0.0019	0.0018	0.0018
2	16	0.0050	0.0048	0.0050

Table 1. Mean square error with CCT and FFT methods for different data compression ratios.

The mean square error in the three methods is almost the same for different compression factors. It shows that CCT method can be used for image compression process as that of DCT method. The advantage of CCT method over DCT method is that the CCT implementation in two dimensions is simpler and well established techniques available for FFT can be used for this. Hence digital image data compression and decompression process can be effectively implemented by using CCT. RPV continuously taking photographs for remote sensing application, and sending them to ground station, the band width requirement is a stringent problem with out compression. By using the compression process the channel bandwidth reduces drastically with out loss of much information and storage space also reduces.



Fig. Original Image



Fig. 3 CCT method compression factor of 16



Fig. 2 CCT Method – Compression Factor 4



Fig.4 DCT method Compression factor of 4



Fig. 5 DCT method compression factor of 16



Fig.6 FFT method Compression factor of 4





Fig. 7 FFT method compression factor of 16

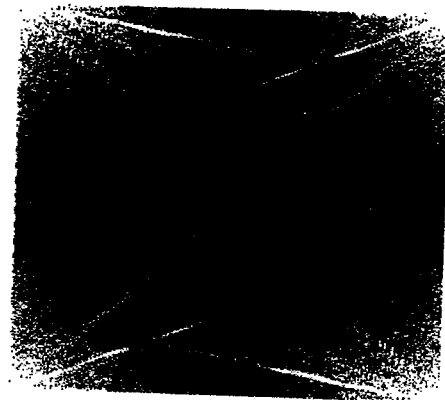


Fig. 8 FFT coefficients magnitudes of Fig. 1

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## Image Data Compression Techniques Using Discrete Hartley Type Transform And Ffts: A Comparison.

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### Abstract

The Image processing plays an important role in modern scientific advancements. It has wide applications in the fields of remote sensing, satellite imaging, medical imaging and telemetry. Digital image processing requires large memory space for storage, high speed computers for processing and wider band widths for transmission. To process these images in real time it will be much more involved. Data compression is an important tool in digital image processing to reduce the burden on the storage and transmission systems. The basic idea of data compression is to reduce the number of the image pixel elements directly, say by sampling or by using transforms and truncate the transformed image coefficients, so that the total number of picture elements or coefficients are reduced. The image information now requires lesser storage and also lesser band width for transmission. When ever the image is to be recovered or received after transmission the image information is to be decompressed i.e. brought back to the original size. There are different methods for compression and decompression process. In this paper discrete Hartley type transforms and Fast Fourier transforms are considered for both data compression and decompression. Algorithms are developed and tested using two dimensional Hartley type transform (HT) and Fast Fourier transform (FFT) for data compression and decompression of the digital image data. The main advantage of Hartley transform is, it is a real transform, hence the storage and the processing of the coefficients require less space and faster operation in comparison with Fast Fourier transforms. Number of digital images are compressed and decompressed and the mean square error estimation is carried out as a comparison with the original images for both the methods.

### Keywords:

Digital Image Processing, Discrete Hartley Transform, Data Compression, Fast Fourier Transform, Remote Sensing,

### Introduction

Active research work is going on, in digital signal processing in recent years. This has good number of applications in satellite imaging, medical imaging and telemetry. Data compression plays an important role in satellite image processing. The satellite image in digital mode requires huge memory space and higher bandwidths for transmission. Data compression before transmission reduces the channel band width requirement. [ 1 ] to [ 8 ] There are different transforms for data compression. There are Karhunen – Loeve transform, Discrete cosine transform and interpolation process. [ 1 ] to [ 16 ]

In this paper discrete Hartley type transform, and fast Fourier transform are studied for data compression in two dimensions, which is directly applicable to digital image processing. The Hartley transform ( HT ) was developed as a substitute to Fourier transform ( FT ) in applications where the data is in real domain. [ 10 ], [ 11 ]. The HT has also defined in two dimensions [ 12 ] as a separable Hartley type transform, named the CAS-CAS transform ( CCT ). Algorithms are developed for data compression and decompression using Hartley type transform in two dimensions. Comparison of compressed and decompressed image with the original image with respect to mean square error is carried out.

FFT is applied for interpolation after sampling in one and two dimensions. [ 13 ], [ 14 ] Interpolation is very useful for reconstruction of the sampled image. Sampled image is a pattern of compressed image. In this paper instead of interpolation process, direct FFT is evaluated, and compression algorithm is applied on the FFT data. In the re-conversion process zeros are substituted to the FFT coefficients where ever they are taken off in the transmission process and inverse FFT is applied on the modified data which presents a replica of original image. This is the process of compression and decompression of the image using FFT. Error analysis is also carried out.

## Data compression and decompression using Hartley transform

Hartley transform relations in 2-D are [ 12 ]

$$H(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad (1)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} H(u, v) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad (2)$$

$\text{Cas } 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right)$  is not a separable Kernel like

$\text{Exp} [ j 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) ]$  for 2-D DFT, hence the row-

column decomposition method [ 14 ] of computing a 2-D transform from 1-D fast Fourier transform algorithm can not be applied directly in this case of HT. To take advantage of the fast 1-D HT algorithms, a separable Hartley like transform namely CAS-CAS transform

$T(u, v)$ , is defined as [ 12 ]

$$T(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad (3)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad (4)$$

Where

$\text{Cas}(\alpha) = \cos(\alpha) + \sin(\alpha)$  and

$$H(u, v) = \frac{1}{2} [ T(u, v) + T(-u, v) + T(u, -v) - T(-u, -v) ] \quad (5)$$

In this way 2-D Discrete Hartley transform can be computed.

## The algorithmic steps for compression and decompression using DHT:

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original image matrix A is considered as (M, N).

2. Compute CCT of this matrix A using equation (3) as Matrix B which is of same size A.

3. Modify the CCT coefficients matrix B to reduce its size to  $(M/2, N/2)$ , to achieve a compression factor of 4.

Construct a new sequence C(u, v) from B(u, v) as follows

$$C(u, v) = B(u, v) \quad \text{for } u = 0, 1, \dots, (M/4)-1$$

$$v = 0, 1, \dots, (N/4)-1$$

$$= B(u, v+N/2) \quad \text{for } u = 0, 1, \dots, (M/4)-1$$

$$v = (N/4), \dots, (N/2)-1$$

$$= B(u+M/2, v) \quad \text{for } u = M/4, \dots, (M/2)-1$$

$$v = 0, 1, \dots, N/4$$

$$= B(u+M/2, v+N/2) \quad \text{for } u = M/4, \dots, (M/2)-1$$

$$v = N/4, \dots, (N/2)-1$$

C(u, v) is matrix of size  $(M/2, N/2)$ , a compression ratio of 4 with respect to original size of the image matrix A of size (M, N) is achieved.

4. This reduced CCT coefficient matrix is to be transmitted instead of the original matrix, so that burden on the channel can be reduced by a factor of 4.

5. At the receiver, after receiving the modified CCT matrix C(u, v), is appended with zeros to make it to original size as follows.

$$D(u, v) = C(u, v) \quad \text{for } u = 0, 1, \dots, (M/4)-1$$

$$v = 0, 1, \dots, (N/4)-1$$

$$= 0 \quad \text{for } u = 0, 1, \dots, (M/4)-1$$

$$v = N/4, \dots, (3N/4)-1$$

$$= C(u, v-N/2) \quad \text{for } u = 0, 1, \dots, (M/4)-1$$

$$v = 3N/4, \dots, (N-1)$$

$$= 0 \quad \text{for } u = M/4, \dots, (3M/4)-1$$

$$v = 0, 1, 2, \dots, (N-1)$$

$$= C(u-M/2, v) \quad \text{for } u = 3M/4, \dots, (M-1)$$

$$v = 0, 1, \dots, (N/4)-1$$

$$= 0 \quad \text{for } u = 3M/4, \dots, (M-1)$$

$$v = N/4, \dots, (3N/4)-1$$

$$= C(u-M/2, v-N/2) \quad \text{for } u = 3M/4, \dots, (M-1)$$

$$v = 3N/4, \dots, (N-1)$$

6. Multiply each D(u, v) by the square of the compression factor.

7. Perform the inverse CCT using equation (4) on the sequence D(u, v) of size (M, N) to obtain the replica of original image.

## Data compression and decompression using fast Fourier transform :

The Fourier transform relations in 2-D are

$$X(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) e^{-j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (6)$$

Where  $m=0,1,\dots M-1$

And  $n=0,1,\dots N-1$

$$X(m, n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) e^{j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (7)$$

Where  $u=0,1,\dots M-1$

And  $v=0,1,\dots N-1$

Equations (6) is forward transform and equation (7) is reverse transform. By using the above two equations Fourier transform coefficients and inverse Fourier transform coefficients can be computed for the digital image.

**The algorithmic steps for compression and decompression using FFT:**

The procedure is similar to that of Hartley transform for data compression and decompression using Fast Fourier transform. In the algorithmic steps given above for Hartley transform, CCT is to be replaced with FFT, for FFT based data compression and decompression process.

**Procedure:**

**1. CCT method**

The above algorithmic steps are applied on the original scanned digital image of size (256,256) pixels shown as figure 1. For this image data CCT coefficients are computed. Then data is reduced by discarding some of the coefficients by using the algorithmic step (3). Most of CCT coefficients are having larger magnitudes in all the four corners of the matrix instead of central region of the matrix. Based on this principle the discarding of small value data points are done. To bring back to the original size matrix, zeros are appended where ever the data points are discarded. Now inverse CCT is applied on this data with a proper multiplier. The resultant replica of the original image is shown as figure 2. These two images figure 1 and figure 2 are looks alike, and there is no noticeable loss of information. In this a reduction of data points to be transmitted is 1/4th of that of original image.

A second example , with a compression ration of 16, is shown as figure 3. The resultant image figure 3, is not up to the mark, and further compression will completely distorts the image. the advantage of CCT is that it is a real transform and hence the storage and transmission is not complex.

**2. FFT method**

For the same data (figure 1), equation (6) is applied to get the Fourier coefficients and as in the CCT method some of the coefficients are discarded and substituted with zeros, and with proper multiplier inverse transform equation (6), is applied to get the replica of the original image as shown figure 4. Figures 1 and figure 4 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. The only difference in this method in comparison with CCT, is that the data to be transmitted is complex instead of real and hence effective reduction is 50 %. As the compression ratio is increased further, decreasing the data points to be transmitted , the recovered image is distorted as shown in figure 5 with a compression ratio of 16. Figure 6 shows the distribution of magnitudes of Fourier coefficients for the figure 1 data points. It shows that the magnitudes are less in the middle region, which are discarded and at the reverse transform they are assumed as zeros.

**Results and conclusions**

CCT and FFT methods are tried for different compression factors . The mean square error estimation with respect to the original image data ( figure 1 ) are presented in table 1.

Serial Number	Compression factor	CCT Method Mean Square error	FFT Method Mean Square Error
1	4	0.0019	0.0018
2	16	0.0050	0.0048

Table 1. Mean square error with CCT and FFT methods for different data compression ratios.

The mean square error in both the methods is almost the same for different compression factors. It shows that CCT method can be used for image compression process as that of FFT method. The advantage of CCT method over FFT method is that the CCT is a real transform and hence the number of multiplications and additions are less than that of FFT and storage also reduces to half that of FFT. The band width required for transmission also reduces with CCT instead of FFT. Hence digital image data compression and decompression process can be effectively implemented using CCT.

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Figure 1. Original Image



Fig.4 FFT method Compression factor of 4



Fig. 2 CCT method Compression factor of 4



Fig. 5 FFT method compression factor of 16



Fig. 3 CCT method compression factor of 16





Fig. 6 FFT coefficients magnitudes of Fig. 1



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# Digital Image Compression and Decompression Using Three Different Transforms and Comparison of Their Performance

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## Abstract

Data compression is an important tool in digital image processing to reduce the burden on the storage and transmission systems. The basic idea of data compression is to reduce the number of the image pixel elements directly, say by sampling, or by using transforms and truncate the transformed image coefficients, so that the total number of picture elements or its coefficients are reduced. The image information now requires lesser storage and also lesser band width for transmission. When ever the image is to be recovered or received after transmission the image information is to be decompressed i.e. brought back to the original size and form. This compression process is essential for images taken by satellites, or unmanned aerial vehicles (UAVs) for remote sensing and weather application. By applying compression algorithm the image data may take one fourth or even less size with out loss of much information. There are different methods for compression and decompression process. In this paper three methods are used for both data compression and decompression process, they are i. Discrete Hartley type transform, ii. Fast Fourier transform (FFT), iii. Discrete cosine transforms (DCT). Algorithms are developed and tested using the three methods on different images. A comparison with respect to mean square error with the original image also presented. The main advantage of discrete Hartley type transform is, it is a real transform. Discrete cosine transform also has similar performance as that of discrete Hartley type transform. An algorithm for compression using FFT method is also presented.

## 1 Introduction

Active research work is going on, in digital signal processing in recent years. This has good number of applications in remote sensing, medical imaging and telemetry. Digital image processing comprises of many sub areas, some of them are image enhancement, image restoration, image encoding, and data compression. Data compression play's an important role in image processing especially in remote sensing using remotely piloted vehicle (RPV). The image taken by the RPV in digital mode requires huge memory space and higher bandwidths for transmission to ground station. Each raw image in digital

form will take few MB (mega bits) hence the transmission and storage in the same form will occupy more bandwidth and larger storage space. Data compression before transmission reduces the channel band width requirement and memory space [1] to [8]. There are different transforms available for data compression. These are Karhunen – Loeve transform, discrete cosine transform, and sampling and interpolation process using fast Fourier transform. [1] to [16].

In this paper Discrete Hartley type transform and Fast Fourier transform are used for data compression in two dimensions, which is directly applicable to digital image processing of RPV photographed images. The performance is compared with the existing discrete cosine transform. The Hartley transform (HT) was developed as a substitute to Fourier transform (FT) in applications where the data is in real domain. [10], [11]. The HT was also defined in two dimensions in which kernel is not separable [12]. Another modified separable Hartley type transform, named the CAS-CAS transform (CCT) was also defined. CAS stands for cos plus sine. And CCT for two dimensional cos plus sine transform. Algorithms are developed for data compression and decompression using the three transforms namely CCT, FFT, and DCT in two dimensions. Comparison of compressed and decompressed image with the original image with respect to mean square error is carried out.

## 2. CCT Method

Discrete Hartley type transform in two dimensions as separable transform is the CAS-CAS transform and is called as CCT.

Hartley transform relations in 2-D in which the kernel is not separable are [12].

$$H(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad (1)$$

And reverse transform is

$$X(m,n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} H(u,v) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad (2)$$

$\text{Cas } 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right)$  is not a separable Kernel.

But for 2-D DFT the kernel is

$\text{Exp} [j2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right)]$  which is separable. Hence the row-column decomposition method [14] of computing 2-D transform from 1-D fast Fourier transform algorithm can be used. In case of 2-D Hartley transform row column can not be applied directly. To take advantage of the fast 1-D HT algorithms, a separable Hartley like transform namely CAS-CAS transform that is CCT is shown.

$T(u, v)$ , is defined as [12]

$$T(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,n) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad (3)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v) \text{cas} \frac{2\pi mu}{M} \text{cas} \frac{2\pi nv}{N} \quad (4)$$

Where

$\text{Cas}(\alpha) = \cos(\alpha) + \sin(\alpha)$  and

$$H(u, v) = \frac{1}{2} [T(u, v) + T(-u, v) + T(u, -v) - T(-u, -v)] \quad (5)$$

In this way 2-D Discrete Hartley transform can be computed. For the compression and decompression process only CCT is used instead of Hartley transform.

## 2.1 The Algorithmic Steps for Compression and De-Compression Using CCT

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original image matrix A is considered as (M, N).

2. Compute CCT of this matrix A using equation (3) as Matrix B which is of same size as A. Important observation is that the CCT coefficient matrix is having larger amplitudes at the four corners and the value diminishes, negligibly small towards central region

3. Based on the above principle, modify the CCT coefficients matrix B to reduce its size to (M/2, N/2), to achieve a compression factor of 4 by neglecting small amplitude coefficients.

Construct a new matrix C (u, v) from B (u, v) which is reduced in size as follows

$$C(u, v) = B(u, v) \quad \text{for } u = 0, 1 \dots (M/4) - 1 \\ v = 0, 1 \dots (N/4) - 1$$

$$= B(u, v+N/2) \quad \text{for } u = 0, 1 \dots (M/4) - 1 \\ v = (N/4) \dots (N/2) - 1$$

$$= B(u+M/2, v) \quad \text{for } u = M/4 \dots (M/2) - 1 \\ v = 0, 1 \dots N/4$$

$$= B(u+M/2, v+N/2) \quad \text{for } u = M/4 \dots (M/2) - 1 \\ v = N/4 \dots (N/2) - 1$$

C (u, v) is matrix of size (M/2, N/2), a compression ratio of 4 with respect to original size of the image matrix A of size (M, N) is achieved.

4. This reduced CCT coefficient matrix is to be transmitted instead of the original matrix, so that memory space is saved for storage and burden on the transmission channel is reduced by a factor of 4.

5. At the receiver, after receiving the modified CCT matrix C (u, v), is appended with zeros at respective regions, to make it to reference image size as follows.

$$D(u, v) = C(u, v) \quad \text{for } u = 0, 1 \dots (M/4) - 1 \\ v = 0, 1 \dots (N/4) - 1$$

$$= 0 \quad \text{for } u = 0, 1 \dots (M/4) - 1 \\ v = N/4 \dots (3N/4) - 1$$

$$= C(u, v+N/2) \quad \text{for } u = 0, 1 \dots (M/4) - 1 \\ v = 3N/4 \dots (N-1)$$

$$= 0 \quad \text{for } u = M/4 \dots (3M/4) - 1 \\ v = 0, 1, 2 \dots (N-1)$$

$$= C(u-M/2, v) \quad \text{for } u = 3M/4 \dots (M-1) \\ v = 0, 1 \dots (N/4) - 1$$

$$= 0 \quad \text{for } u = 3M/4 \dots (M-1) \\ v = N/4 \dots (3N/4) - 1$$

$$= C(u-M/2, v-N/2) \quad \text{for } u = 3M/4 \dots (M-1) \\ v = 3N/4 \dots (N-1)$$

6. Multiply each  $D(u, v)$  by the square of the compression factor.

7. Perform the inverse CCT using equation (4) on the sequence  $D(u, v)$  of size  $(M, N)$  to obtain the replica of original image  $A$ .

### 3. Fast Fourier Transform Method

The Fourier transform relations in 2-D are

$$X(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) e^{-j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (6)$$

Where  $m = 0, 1 \dots M-1$ , and  $n = 0, 1 \dots N-1$

$$X(m, n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) e^{j2\pi(\frac{mu}{M} + \frac{nv}{N})} \quad (7)$$

Where  $u = 0, 1 \dots M-1$ , and  $v = 0, 1 \dots N-1$

Equation (6) is forward transform and equation (7) is reverse transform. By using the above two equations Fourier transform coefficients and inverse Fourier transform coefficients can be computed for the digital image.

#### 3.1. The Algorithmic Steps for Compression and Decompression Using FFT

The procedure is similar to that of Hartley type transform for data compression and decompression using Fast Fourier transform. The algorithmic steps given above for Hartley transform, are to be used for this also by replacing CCT with FFT. FFT magnitude coefficients are also concentrated at the corners as that of CCT.

### 4. Discrete Cosine Transform Method

The discrete cosine transform relations in 2-D are

$$X(u, v) =$$

$$\alpha_u \alpha_v \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N} \quad (8)$$

Where  $m = 0, 1 \dots M-1$ ; and  $n = 0, 1 \dots N-1$

$$\alpha_u = \frac{1}{\sqrt{M}} \quad \text{For } u = 0$$

$$\alpha_u = \sqrt{\frac{2}{M}} \quad \text{For } u = 1 \dots M-1$$

$$\alpha_v = \frac{1}{\sqrt{M}} \quad \text{For } v = 0$$

$$\alpha_v = \sqrt{\frac{2}{M}} \quad \text{For } v = 1 \dots M-1$$

And 2-D inverse discrete cosine Transform is

$$X(m, n) =$$

$$\alpha_u \alpha_v \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N} \quad (9)$$

Where  $u = 0, 1 \dots M-1$ , and  $v = 0, 1 \dots N-1$

Equation (8) is forward transform and equation (9) is reverse transform. By using the above two equations discrete cosine transform coefficients and inverse discrete cosine transform coefficients can be computed for the digital image.

#### 4.1. The Algorithmic Steps for Compression and Decompression Using DCT

1. Scan the analog Image to get digital image with a size of pixels  $(256 \times 256)$  as reference data matrix  $A$ . Instead of particular size of the digital image data, general size of the original image matrix  $A$  is considered as  $(M, N)$ .

2. Compute DCT of this matrix  $A$  using equation (8) as Matrix  $B$  which is of same size as  $A$ . The DCT coefficient matrix is having larger magnitudes near the origin and is

diminishing as the  $(m, n)$  increases.

3. Based on this principle truncate the Matrix B keeping the DCT coefficients for  $m=0$  to  $(M/2)-1$ , and  $n = 0$  to  $(N/2)-1$  so that the size of the matrix formulated C is  $(M/2, N/2)$  a compression ratio of 4 is achieved.

4. The matrix c is transmitted and received. Now decompression process is applied by appending the C matrix zeros to get back the size of the original matrix A.

5. Compute inverse discrete cosine transform on this modified matrix using equation (9) to get back the replica of the original digital image.

## 5. Procedure

### 5.1. CCT Method

The above algorithmic steps referred in chapter 2.1 are applied on the original scanned digital image of size  $(256,256)$  pixels shown as figure 1. For this image data CCT coefficients are computed. Then data size is reduced by discarding some of the coefficients by using the algorithmic step (3). Most of CCT coefficients are having larger magnitudes in all the four corners of the matrix instead of central region of the matrix. Based on this principle the discarding of small value data points is achieved. To bring back to the original size matrix, zeros are appended where ever the data points are discarded. Now inverse CCT is applied on this data with a proper multiplier. The resultant replica of the original image is shown as figure 2. These two images figure 1 and figure 2 are looks alike, and there is no noticeable loss of information. In this a reduction of data points to be transmitted is 1/4th of that of original image.

A second example, with a compression ratio of 16 for the same image is applied and the resultant image is shown as figure 3. Figure 3 is not up to the mark, and further compression will completely distorts the image. The advantage of CCT is that it is a real transform and hence the storage and transmission is not complex and FFT algorithm of row column decomposition can be applied.

### 5.2. FFT Method

For the same data (figure 1), equation (6) is applied to get the Fourier coefficients. As in the CCT method some of the coefficients are discarded by following same algorithmic steps as in CCT method. The discarded coefficients are substituted with zeros, and with proper multiplier inverse transform equation (7), are applied to get the replica of the original image as shown figure 4. Figures 1 and figure 4 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. The only difference in this method in comparison with CCT is that the data to be transmitted is complex instead of real and hence effective reduction is 50 %. As the compression ratio is increased further, decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 5 with a compression ratio of 16. Figure 6 shows the distribution of magnitudes of Fourier coefficients for the

figure 1 data points. It shows that the magnitudes are less in the middle region, which are discarded and in the reverse transform they are assumed to be zeros.

### 5.3. DCT Method

For the same data (figure 1), equation (8) is applied to get the DCT coefficients. DCT coefficients are concentrated near the origin. To achieve the compression ratio of 4 the DCT coefficient matrix is truncated by retaining  $(M/2, N/2)$  from the origin.

Now zeros are appended to get back the original size of the matrix and with proper multiplier inverse transform equation (9), is applied to get back the replica of the original image. Hence there is a compression ratio of 4 is achieved with out distortion. As the compression ratio is increased further say 16, by decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 7.

## 6. Results and Conclusions

Three methods are tried for different compression factors. The mean square error estimation of resultant images with respect to the original image data (figure 1) are presented in table 1.

Table 1. Mean square error with the three methods for different data compression ratios.

Serial No	Compression factor	CCT Method Mean Square error	FFT Method Mean Square Error	DCT Method Mean Square Error
1	4	0.0019	0.0018	.0019
2	16	0.0050	0.0048	.0050

The mean square error for all the methods is almost the same for different compression factors. It shows that CCT method can be used for image compression process as that of FFT method. The advantage of CCT method over FFT method is that the CCT is a real transform and hence the number of multiplications and additions are less than that of FFT and storage also reduces to half that of FFT. The band width required for transmission also reduces with CCT instead of FFT. Hence digital image data compression and decompression process can be effectively implemented using CCT. DCT method also gives similar performance as that of CCT. Either DCT or CCT can be implemented for any practical system.



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Figure 1. Original Image



Fig. 2 CCT method  
Compression factor of 4



Fig. 3 CCT method compression  
Factor of 16



Fig.4 FFT method Compression factor of 4

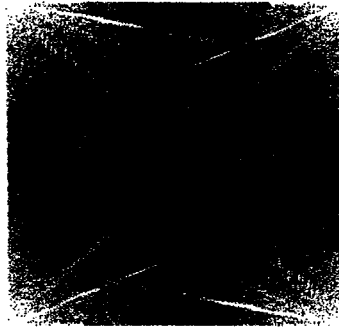


Fig. 6 FFT coefficients  
magnitudes of Fig. 1



Fig. 5 FFT method compression factor of 16



Fig. 7 DCT method compression  
factor of 16

# Data compression technique using CAS-CAS transform applied to remotely piloted vehicle (RPV) digital images before transmission to ground station

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## Abstract

Remotely piloted vehicle plays an important role in modern scientific advancements. It has wide applications in the field of remote sensing, and military applications. One of the important tasks of remotely piloted vehicle (RPV) is to take images continuously and send them to ground station for further processing. If the raw images are to be sent directly to ground station wider band widths are required for data transmission. To reduce the band width requirement images are to be compressed before transmission. Data compression is an important tool in digital image processing to reduce the burden on the storage and transmission systems. The basic idea of data compression is to reduce the number of the image pixel elements directly, say by sampling or by using transforms and truncate the transformed image coefficients, so that the total number of picture elements or coefficients are reduced. The image information now requires lesser storage and also lesser band width for transmission. When ever the image is to be recovered or received after transmission the image information is to be decompressed i.e. brought back to the original size. There are different methods for compression and decompression process. In this paper CAS-CAS transform is used for both data compression and decompression process. Algorithms are developed and tested using two dimensional CAS-CAS transform. A comparison is also presented with this transform to that of discrete cosine transform (DCT). The main advantage of CAS-CAS transform is, it is a real transform, and is similar to that of fast Fourier transform (FFT). The fast algorithms of FFT can be applied directly to this. Row column decomposition method can be used. Number of digital images is compressed and decompressed and the mean square error estimation is carried out.

## Keywords:

Digital Image Processing, CAS-CAS transform, Data Compression, Remotely piloted vehicle, Remote Sensing and Discrete cosine transform.

## 1. Introduction

Active research work is going on, in remotely piloted vehicles in recent years. This has good number of applications in remote sensing, and telemetry and military applications. Data compression plays an important role in remotely piloted vehicle image processing. The RPV image in digital mode requires huge memory space and higher bandwidths for transmission to ground station. Data compression before transmission reduces the channel band width requirement [1] - [8]. There are different transforms available for data compression. These are Karhunen - Loeve transform discrete cosine transform and interpolation process. [1] - [16]

In this paper CAS- CAS transform and DCT are studied for data compression in two dimensions, which is directly applicable to digital image processing of RPV photographed images. The Hartley transform (HT) was developed as a substitute to Fourier transform (FT) in applications where the data is in real domain. [10], [11]. The HT was also defined in two dimensions [12] as a separable Hartley type transform, named the CAS-CAS transform (CCT). CAS stands for cos +Sin. Algorithms are developed for data compression and decompression using CAS- CAS transform in two dimensions. Comparison of compressed and decompressed image with the original image with respect to mean square error is carried out.

Discrete cosine transform method of compression and decompression process is also carried out on the same examples. These two methods are compared in terms of mean square error.

## 2. CAS-CAS Transform method:

Hartley transform relations in 2-D are [12]

$$H(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \text{cas} 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad \dots (1)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} H(u, v) \cos 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \quad (2)$$

$\cos 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right)$  is not a separable Kernel like

$\exp \left[ j 2\pi \left( \frac{mu}{M} + \frac{nv}{N} \right) \right]$  for 2-D DFT, hence the row-column decomposition method [14] of computing a 2-D transform from 1-D fast Fourier transform algorithm can not be applied directly in this case of HT. To take advantage of the fast 1-D HT algorithms, a separable Hartley like transform namely CAS-CAS transform  $T(u, v)$ , is defined as [12]

$$T(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \cos \frac{2\pi mu}{M} \cos \frac{2\pi nv}{N} \quad (3)$$

$$X(m, n) = \frac{1}{M \times N} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) \cos \frac{2\pi mu}{M} \cos \frac{2\pi nv}{N} \quad (4)$$

Where

$$\text{Cas}(\alpha) = \cos(\alpha) + \sin(\alpha) \quad \text{and}$$

$$H(u, v) = \frac{1}{2} [T(u, v) + T(-u, v) + T(u, -v) - T(-u, -v)] \quad (5)$$

In this way 2-D Discrete Hartley transform can be computed.

## 2.1. The algorithmic steps for compression and decompression using CAS-CAS transform (CCT):

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original image matrix A is considered as (M, N).

2. Compute CCT of this matrix A using equation (3) as Matrix B which is of same size A.

3. Modify the CCT coefficients matrix B to reduce its size to (M/2, N/2), to achieve a compression factor of 4.

Construct a new sequence C (u, v) from B (u, v) as follows

$$\begin{aligned} C(u, v) &= B(u, v) & \text{for } u &= 0, 1 \dots (M/4)-1 \\ & & v &= 0, 1 \dots (N/4)-1 \\ &= B(u, v+N/2) & \text{for } u &= 0, 1 \dots (M/4)-1 \\ & & v &= (N/4) \dots (N/2)-1 \end{aligned}$$

$$\begin{aligned} &= B(u+M/2, v) & \text{for } u &= M/4 \dots (M/2)-1 \\ & & v &= 0, 1 \dots N/4 \\ &= B(u+M/2, v+N/2) & \text{for } u &= M/4 \dots (M/2)-1 \\ & & v &= N/4 \dots (N/2)-1 \end{aligned}$$

C (u, v) is matrix of size (M/2, N/2); a compression ratio of 4 with respect to original size of the image matrix A of size (M, N) is achieved.

4. This reduced CCT coefficient matrix is to be transmitted instead of the original matrix, so that burden on the channel can be reduced by a factor of 4.

5. At the receiver, after receiving the modified CCT matrix C (u, v), is appended with zeros to make it to original size as follows.

$$\begin{aligned} D(u, v) &= C(u, v) & \text{for } u &= 0, 1 \dots (M/4)-1 \\ & & v &= 0, 1 \dots (N/4)-1 \\ &= 0 & \text{for } u &= 0, 1 \dots (M/4)-1 \\ & & v &= N/4 \dots (3N/4)-1 \\ &= C(u, v-N/2) & \text{for } u &= 0, 1 \dots (M/4)-1 \\ & & v &= 3N/4 \dots (N-1) \\ &= 0 & \text{for } u &= M/4 \dots (3M/4)-1 \\ & & v &= 0, 1, 2 \dots (N-1) \\ &= C(u-M/2, v) & \text{for } u &= 3M/4 \dots (M-1) \\ & & v &= 0, 1 \dots (N/4)-1 \\ &= 0 & \text{for } u &= 3M/4 \dots (M-1) \\ & & v &= N/4 \dots (3N/4)-1 \\ &= C(u-M/2, v-N/2) & \text{for } u &= 3M/4 \dots (M-1) \\ & & v &= 3N/4 \dots (N-1) \end{aligned}$$

6. Multiply each D (u, v) by the square of the compression factor.

7. Perform the inverse CCT using equation (4) on the sequence D (u, v) of size (M, N) to obtain the replica of original image.

## 3. Data compression and decompression using discrete cosine transform:

The discrete cosine transform relations in 2-D are

$$X(u, v) =$$

$$\alpha_u \alpha_v \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N}$$

## 4.1. CCT method

The above algorithmic steps are applied on the original scanned digital image of size (256,256) pixels shown as figure 1. For this image data CCT coefficients are computed. Then data is reduced by discarding some of the coefficients by using the algorithmic step (3). Most of CCT coefficients are having larger magnitudes in all the four corners of the matrix instead of central region of the matrix. Based on this principle the discarding of small value data points is done. To bring back to the original size matrix, zeros are appended where ever the data points are discarded. Now inverse CCT is applied on this data with a proper multiplier. The resultant replica of the original image is shown as figure 2. These two images figure 1 and figure 2 are looks alike, and there is no noticeable loss of information. In this a reduction of data points to be transmitted is 1/4th of that of original image.

A second example, with a compression ration of 16, is shown as figure 3. The resultant image figure 3 is not up to the mark, and further compression will completely distorts the image. The advantage of CCT is that it is a real transform and hence the storage and transmission is not complex but FFT algorithm of row column decomposition can be applied.

## 4.2. DCT method

For the same data (figure 1), equation (6) is applied to get the DCT coefficients. DCT coefficients are concentrated near the origin. To achieve the compression ratio of 4 the DCT coefficient matrix is truncated by retaining (M/4, N/4) from the origin.

Now zeros are appended to get back the original size of the matrix and with proper multiplier inverse transform equation (7), is applied to get the replica of the original image as shown figure 4. Figures 1 and figure 4 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. As the compression ratio is increased further say 16, by decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 5. Figure 6 shows the distribution of magnitudes of CCT coefficients for the figure 1 data points. It shows that the magnitudes are negligibly small in the middle region, which are discarded and zeros are appended for the reverse transform

## 5. Results and conclusions

CCT and DCT methods are tried for different compression factors. The mean square error estimation with respect to the original image data (Figure 1) are presented in table 1.

Where  $m = 0, 1 \dots M-1$

And  $n = 0, 1 \dots N-1$

$$\alpha_u = \frac{1}{\sqrt{M}} \quad \text{for } u = 0$$

$$\alpha_u = \sqrt{\frac{2}{M}} \quad \text{for } u = 1 \dots M-1$$

$$\alpha_v = \frac{1}{\sqrt{M}} \quad \text{for } v = 0$$

$$\alpha_v = \sqrt{\frac{2}{M}} \quad \text{for } v = 1 \dots M-1$$

And 2-D inverse discrete cosine Transform is

$X(m, n) =$

$$\alpha_u \alpha_v \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} X(u, v) \cos \frac{\pi(2m+1)u}{2M} \cos \frac{\pi(2n+1)v}{2N} \dots (7)$$

Where  $u = 0, 1 \dots M-1$

And  $v = 0, 1 \dots N-1$

Equations (6) are forward transform and equation (7) is reverse transform. By using the above two equations discrete cosine transform coefficients and inverse discrete cosine transform coefficients can be computed for the digital image.

### 3. 1. The algorithmic steps for compression and decompression using DCT:

The procedure is similar to that of CAS-CAS transform for data compression and decompression using discrete cosine transform. In the algorithmic steps given above for CCT is to be replaced with DCT, for DCT based data compression and decompression process.



Table 1. Mean square error with CCT and FFT methods for different data compression ratios.

Serial Number	Compression factor	DCT Method Mean Square error	CCT Method Mean Square Error
1	4	0.0019	0.0018
2	16	0.0050	0.0048

The mean square error in both the methods is almost the same for different compression factors. It shows that CCT method can be used for image compression process as that of DCT method. The advantage of CCT method over DCT method is that the CCT implementation in two dimensions is simpler and well established techniques available for FFT can be used for this.. Hence digital image data compression and decompression process can be effectively implemented by using CCT. RPV continuously taking photographs for remote sensing application, and sending them to ground station, the band width requirement is a stringent problem with out compression. By using the compression process the channel bandwidth reduces drastically with out loss of much information and storage space also reduces.

6. Acknowledgements

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Figure 1. Original Image



Fig. 2 CCT method Compression factor of 4



Fig. 3 CCT method compression factor of 16



Fig.4 DCT method Compression factor of 4



Fig. 5 DCT method compression factor of 16

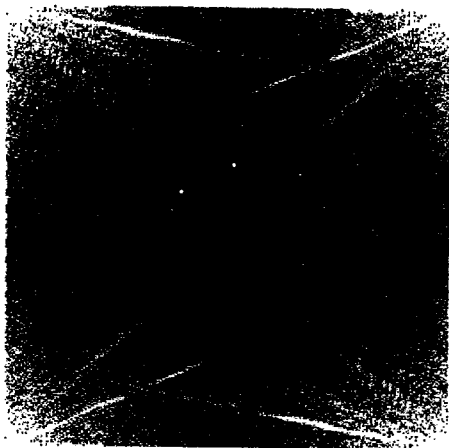


Fig. 6 CCT coefficients magnitudes of Fig. 1

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PANEL :- J/PENDEK

PENAJA :- JANGKA PENDEK

UNIT KUMPULAN WANG AMANAH  
UNIVERSITI SAINS MALAYSIA  
KAMPUS KEJURUTERAAN  
SERI AMPANGAN

**PENYATA KUMPULAN WANG**

TEMPOH BERAKHIR 30 APRIL 2007

Tempoh Projek:15/04/2005 - 14/04/2007

IMAGE DATA COMPORESSION USING DCT AND INTERPOLATION AND ALLIED TOPIC IN DIGITAL

<u>Vot</u>	Peruntukan (a)	Perbelanjaan sehingga 31/12/2006 (b)	Tanggungan semasa 2007 (c)	Perbelanjaan Semasa 2007 (d)	Jumlah Perbelanjaan 2007 (c + d)	Jumlah Perbelanjaan Terkumpul (b+c+d)	Baki Peruntukan Semasa 2007 (a-(b+c+d))
11000 GAJI KAKITANGAN AWAM	450.00	0.00	0.00	0.00	0.00	0.00	450.00
21000 PERBELANJAAN PERJALANAN DAN SARA	4,000.00	2,126.35	0.00	0.00	0.00	2,126.35	1,873.65
23000 PERHUBUNGAN DAN UTILITI	394.00	0.00	0.00	0.00	0.00	0.00	394.00
24000 SEWAAN	0.00	500.00	0.00	0.00	0.00	500.00	(500.00)
27000 BEKALAN DAN ALAT PAKAI HABIS	3,000.00	1,947.00	0.00	0.00	0.00	1,947.00	1,053.00
29000 PERKHIDMATAN IKTISAS & HOSPITALITI	3,350.00	2,500.00	0.00	0.00	0.00	2,500.00	850.00
	<u>11,194.00</u>	<u>7,073.35</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>7,073.35</u>	<u>4,120.65</u>
Jumlah Besar	<u>11,194.00</u>	<u>7,073.35</u>	<u>0.00</u>	<u>0.00</u>	<u>0.00</u>	<u>7,073.35</u>	<u>4,120.65</u>