Undergraduate Mathematics Enhanced With Graphing Technology

Rosihan M. Ali

School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia
Email: rosihan@cs.usm.my

Kor Liew Kee

[2]Faculty of Information Technology and Quantitative Sciences, MARA University of Technology,
Kedah; Email: korlk@tm.net.my

The School of Mathematical Sciences at University Sains Malaysia has offered a laboratory course on the integration of hand-held technology into the teaching and learning of mathematics since the beginning of the 2001/2002 academic year. This inquiry-based course highlights the explorations and application of mathematics in a data rich modeling environment. In addition, the course addresses several issues related to the effective integration of such technology into the mathematics curriculum. This paper discusses the appropriate use of graphing technology to present mathematical concepts and to support student's understanding in a student-centered learning environment, shares knowledge on the new mathematics that was made possible by hand-held technology, and summarizes student reactions to this innovative learning mode.

1. INTRODUCTION

Research from developed countries has shown that calculators can be used to enhance the understanding of mathematical concepts. It was reported that with proper guidance in using calculator technology, students will not unnecessarily use calculators for simple computations. Incidentally, the results from the Third International Mathematics and Science Study (TIMSS 1999) showed that in most countries where emphasis on calculator use was high, there was a positive association between calculator use and achievement in mathematics. Since 1997 University Sains Malaysia (USM) has

developed an interest in graphic calculators, in particular, the CAS-enabled graphic calculator. The affordability, portability, and varying CAS capabilities of graphic calculators has led it to being utilized more widely as a tool to support and enhance the teaching and learning of senior secondary mathematics in countries such as Austria, Australia, Canada, Denmark, France, Germany and the United States. Beginning in 2002 Singapore will allow the use of non-CAS graphic calculators in its GCE A Level Exams in Further Mathematics. Over the last two years, the Ministry of Education has continuously supplied graphic calculators to several selected schools in Malaysia to explore the calculator efficacy in the teaching and learning of secondary mathematics.

The School of Mathematical Sciences at USM, in collaboration with colleagues from the School of Educational Studies, has been developing a calculator based laboratory course since the beginning of the 2001/2002 academic year. In this course, students are acquainted with the capabilities of the graphic calculator as an instructional tool. In addition, seminars addressing issues related to pedagogical and curricular changes driven by the integration of the new technology into the classroom are also given   in this course. The advent of technology has put at issue teaching pedagogy and strategies. A discussion on the constructivism perspectives that were implemented in the course could be found in Ali et al. (2002). This paper reflects on how graphing technology was used to present mathematical concepts and to support student's understanding in the course. Now in its third year, the course shares knowledge on the new mathematics that was made possible by the technology, and summarizes student reactions to the innovative learning mode.

2. COURSE FEATURES

The laboratory course in graphic calculators seeks to explore the impact of such instructional devices and the perspectives they provide. The course is developed for pre-service teachers and students in mathematics. The course objectives are:

- To acquaint students with the CAS calculators and its capabilities.
- To understand the relevance of calculator technology in the teaching and learning of mathematics and sciences.
- To familiarize students with the issues involved in the use of calculator technology in the classroom.
- To model the effective integration of technology into the mathematics curriculum.
- To teach the development of data rich technology explorations that is designed around the capabilities of calculators.

The course content includes topics from calculus, linear algebra, differential equations,

and statistics. The TI-83Plus graphic calculator was used for statistics in the first half of the semester, while the TI-92Plus was used in the remaining weeks for calculus, linear algebra, and differential equations. Students were not required to purchase graphing calculators; each student had a calculator checked out for the duration of the course. There were 28 class meetings of two hours. The primary teaching mode was an inter-active lecture with in-class exploration activities alternately conducted. Class activities were supported with laboratory assignments that the students completed and turned in for assessment. The course culminated with a group project designed to foster the students' knowledge and critical understanding of principles in mathematics and statistics.

3. THE ROLE OF GRAPHING TECHNOLOGY

A graphic calculator is a powerful tool that can carry out complicated mathematical tasks, thus allowing students to spend more time to work with mathematics at a higher cognitive level. When used effectively, it becomes a tool to help students actively construct their own knowledge bases and skill sets. An important consideration in its use is made by Lim (2002), in that technology should not only affect how we teach because technology makes different approaches possible, technology should also change what we teach because some topics are made obsolete with technology while others are made possible with it.

Our course is developed around the capabilities of the technology to enhance the understanding and learning of mathematical concepts and theories. Particular attention was given in the design of laboratory explorations and scientific visualizations to ensure that the graphing technology plays a pivotal role in achieving the learning outcomes. Thus the same content cannot be done without the graphing technology.

The following examples illustrate how graphing technology was used in the course, the different instructional approaches adopted, and the new mathematics brought about by the technology.

Example 1. For the function

$$f(x) = x\sin\left(\tfrac{1}{x}\right), \quad \text{find } \lim_{x\to 0} f(x).$$

Investigate each limit graphically, numerically and symbolically.

In this example, the multiple representation features of the graphic calculator are exploited to study the limit of functions. The graphical, tabular, and symbolic features of the calculator are incorporated to investigate the limit of $f(x) = x\sin(1/x)$ at $x = 0$. Although this is a typical problem in calculus, the oscillation nature of the graph around $x = 0$ is extremely difficult to visualize with chalk and board. The calculator easily

overcomes this difficulty. Changing the window parameters on the graphing calculator allows the student to view the appropriate visualizations. In Figures 1–2, the student gets to see the oscillations around $x = 0$ as well as other properties of the graph such as symmetry. By using the trace feature to explore the functional values around $x = 0$, it is now an immediate step to deduce graphically that the limit is zero.
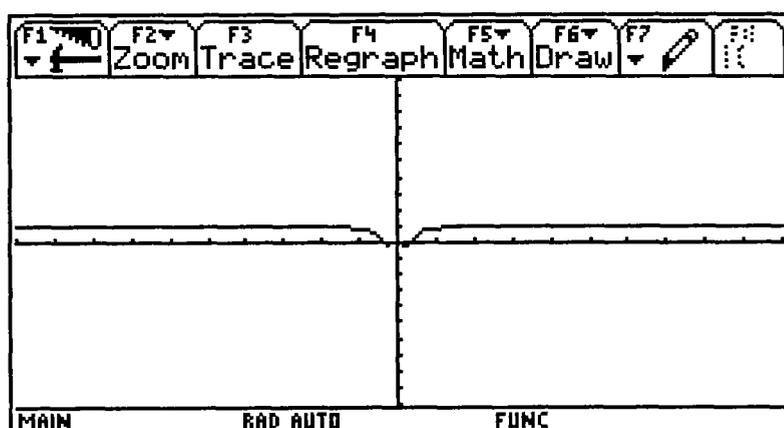

Figure 1. Graph of $f(x) = x \sin(\frac{1}{x})$ on a standard viewing window
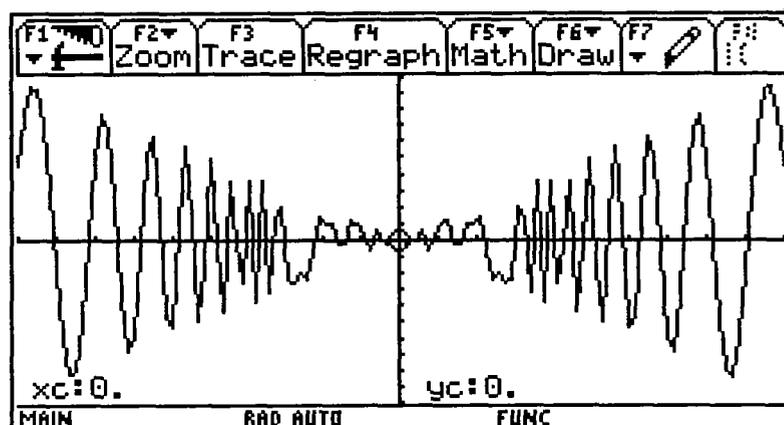

Figure 2. The oscillations of $f(x) = x \sin(\frac{1}{x})$

The investigation of the limit is carried further numerically by creating a table of values. Scrolling through $x = 0$, the table values in Figure 3 support the assertion that

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0.$$

Finally the limit is found symbolically as illustrated in Figure 4.

Figure 3. Table of values for $f(x) = x\sin(\frac{1}{x})$

Figure 4. The limit of $f(x) = x\sin(\frac{1}{x})$ at $x = 0$

It is very important to note that the above efforts are tantamount to an illustration of a result, not a proof. The graphical device gives the student the confidence that the answer is correct, however, the result must be proven rigorously before we may infer it to be true. This is established by an application of the "sandwich" or "squeeze" principle.

Example 2. Let

$$u_1 = \frac{1}{4}, \quad u_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{4 \cdot 6 \cdot 8 \cdots (2n+2)} = \frac{2n-1}{2n+2} u_{n-1}, \quad n \geq 2. \text{ Investigate } \lim_{n \to \infty} u_n.$$

This is an example on an application of the Bounded Monotonic Convergence Theorem (BMCT) for a sequence of terms. The sequence $\{u_n\}$ defined recursively is first investigated graphically (Figures 5–6) and numerically (Figure 7).
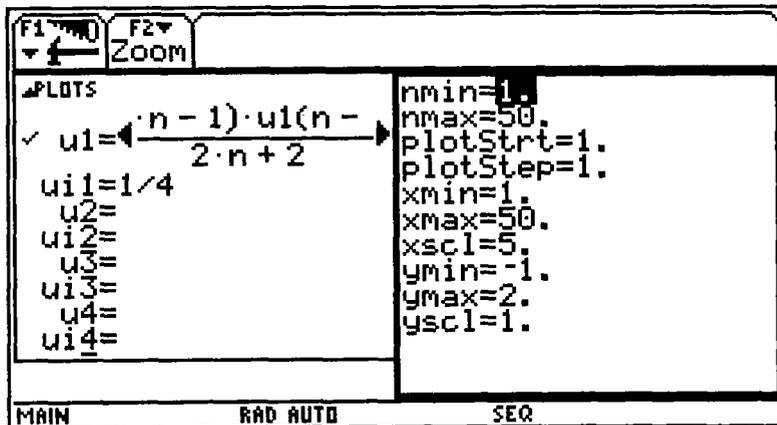
Figure 5. A split screen of two editors
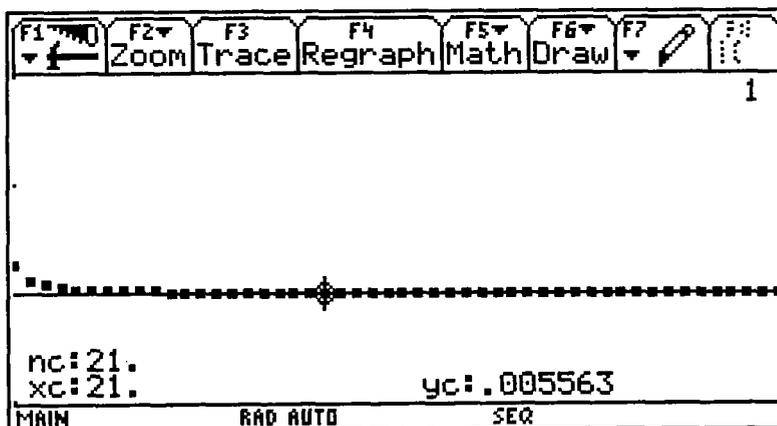
Figure 6. Graph of the sequence $\{u_n\}$

From the visualization and by using the trace feature, the graph suggests that

- $u_n \geq 0$,
- $\{u_n\}$ is decreasing, and
- $\lim\limits_{n \to \infty} u_n = 0$.

The table (Figure 7) of values also supports these observations.

The explorations give valuable information about the sequence $\{u_n\}$. It gives the students concrete feedback about the accuracy of their ideas, and draws them to apply the BMCT in proving the existence of the limit. However the assertion that the limit is zero is still a formidable task to show since it cannot be obtained from the BMCT. The student is required to use the Raabe's test for a positive-term series to establish this fact.

Figure 7. Table of values for the sequence $\{u_n\}$

Example 3. The economic performance of a country may be measured by its gross domestic product GDP. The outcome variable GDP depends on several components. Table 1 provides data on the Malaysian GDP (Y) and the explanatory variables-growth in GNP X_1, agricultural growth X_2, industrial growth X_3, population growth X_4, export growth X_5, and import growth X_6. Assuming the regression function of Y on $X_1, X_2, X_3, X_4, X_5, X_6$ is linear, find the estimated regression function.

Table 1. Economic performance of Malaysia from 1985-1997

Item Number	GDP Y (RM Millions)	Growth GNP X₁	Agri Growth X₂	Ind Growth X₃	Pop Growth X₄	Exp Growth X₅	Import Growth X₆
1985	29280	4.5	4.4	8.7	2.0	4.9	7.3
1986	29280	4.5	4.2	8.7	2.1	7.5	8.9
1987	31270	4.4	3.0	6.7	1.9	10.7	6.4
1988	27580	4.3	3.0	6.0	1.9	10.2	5.2
1989	31230	4.1	3.4	5.8	2.2	9.7	-0.7
1990	34680	4.0	3.7	6.1	2.2	9.4	0.4
1991	37480	4.0	3.9	6.5	2.2	9.8	3.7
1992	42400	4.0	3.8	7.1	2.3	10.3	5.6
1993	46980	2.9	3.7	7.7	2.2	10.9	7.2
1994	57568	3.2	3.6	8.0	2.0	11.3	7.9
1996	70626	5.6	2.8	9.8	2.4	17.8	15.7
1997	85311	5.7	2.6	11.0	2.4	17.8	15.7

Source: World Development Report, Oxford University Press.
Missing observations for the year 1995.

This exploration applies matrix arithmetic to obtain the parameter estimates by the *principle of least squares*. Mathematically, we seek the coefficient vector U satisfying $AU = Y$, where

$$
A = \begin{bmatrix}
1 & 4.5 & 4.4 & \cdots & 7.3 \\
1 & 4.5 & 4.2 & \cdots & 8.9 \\
1 & 4.4 & 3.0 & \cdots & 6.4 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 3.2 & 3.6 & \cdots & 7.9 \\
1 & 5.6 & 2.8 & \cdots & 15.7 \\
1 & 5.7 & 2.6 & \cdots & 15.7
\end{bmatrix}, \quad
U = \begin{pmatrix}
\hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_5 \\ \hat{\beta}_6
\end{pmatrix}, \quad
Y = \begin{pmatrix}
29280 \\ 29280 \\ 31270 \\ \vdots \\ 57568 \\ 70626 \\ 85311
\end{pmatrix}.
$$

Since A is not a square matrix, the defining matrix equation $AU = Y$ is first transformed into an equivalent form $\left(A^T A\right)U = A^T Y$. The solution U is obtained from the equation

$$
U = \left(A^T A\right)^{-1}\left(A^T Y\right).
$$

Matrix notation and arithmetic is invaluable in finding parameter estimates for multiple linear regressions. Clearly the computations involved are tedious and cumbersome, and problems of this nature are normally avoided in class. However, a graphic calculator is a very useful device to make these computations as illustrated in the Figures 8–10 below.

Figure 8. The entries of the matrix A

Figure 9. The vector Y

Figure 10. The solution of the coefficient vector U

Thus the estimated regression function is

$$\hat{\mu}_Y(x_1, x_2, x_3, x_4, x_5, x_6) = -67347.4 - 3700.31x_1 + 10274.7x_2 + 10843.6x_3$$
$$- 21622.2x_4 + 6591.04x_5 - 2531.95x_6.$$

This regression function provides a useful model for forecasting the economic performance. Further analysis may be made to arrive at the conclusions that industrial growth is positively associated $(p < 0.02)$, as well as growth in export $(p < 0.04)$ and growth in import $(p < 0.10)$. Polynomial regression models are of course special cases of multiple linear regression functions. As a further example, students were provided with a two-variable real data and were asked to find the linear and quadratic regression models from the least squares best fit method above. These models were next compared with the regression equations obtained from the calculator's *Stats Editor.* The equations

found were exactly identical. The students then had enhanced understanding of regression equations obtained from an application of statistics software.

Example 4. In this example, students retrieved from a www.census.gov website data on the population of Malaysia from 1950 – 2001. The learning objectives are to apply the concept of discrete differential data to formulate a logistic population growth model

$$\frac{dP}{dt} = kP(C - P), \quad P(0) = 6,433,799,$$

where C is the carrying capacity, and to make predictions. Data can be transmitted from one calculator to the others by using a TI-92Plus cable, thus not everyone needs to input the data. Figure 11 gives a scatter plot of the population data.
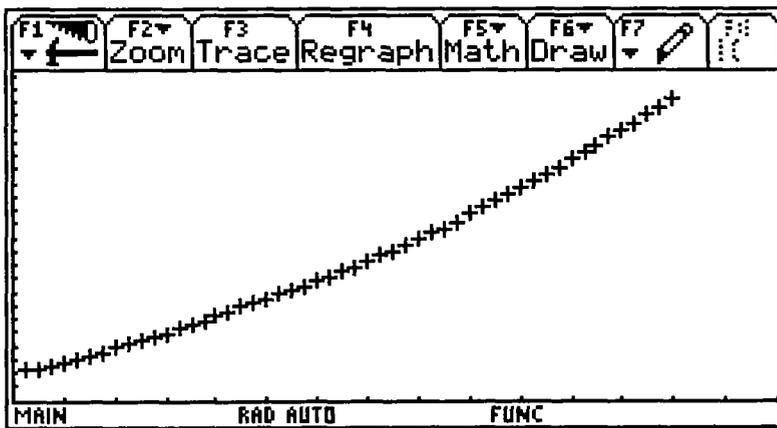
Figure 11. A scatter plot of the Malaysian population from 1950 – 2001
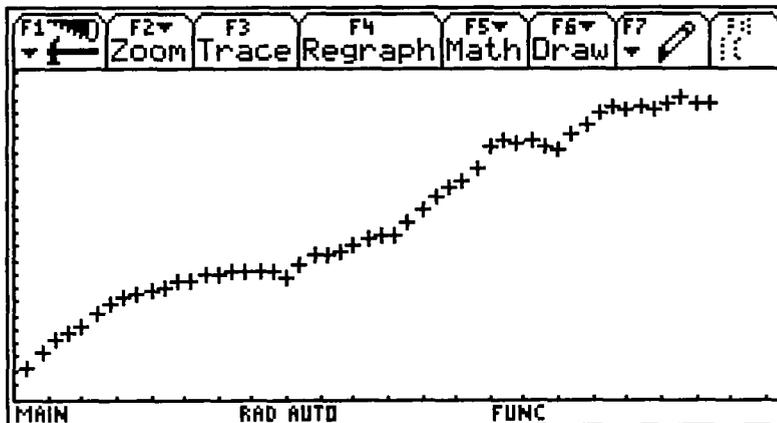
Figure 12. A scatter plot of the discrete derivative

The equation above can be expressed in the form

(1)
$$\frac{1}{P}\frac{dP}{dt} = -kP + kC := ax + b$$

with $x = P$. The discrete derivatives

$$P'(t_{n-1}) = \frac{P(t_n) - P(t_{n-1})}{t_n - t_{n-1}},$$

are computed (Figure 12). Equation (1) requires us to compute the points $\left(P_n, \frac{P'_n}{P_n}\right)$ with $P_n = P(t_n)$, and the best linear regression equation to this data is next obtained from the *Stats Editor* (Figures 13 and 14).
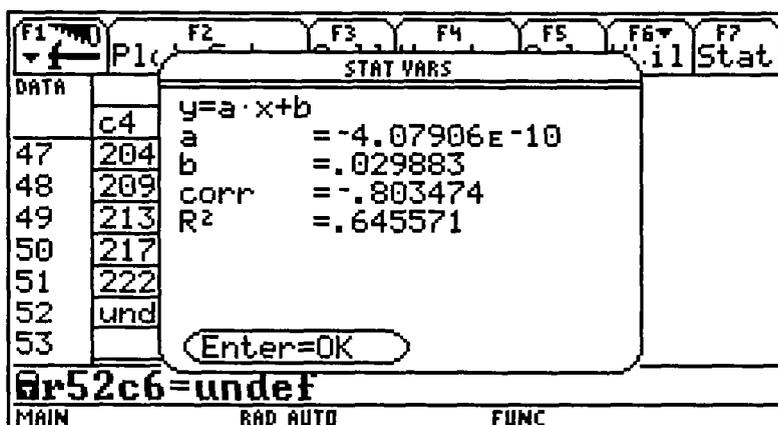
Figure 13. The linear regression equation
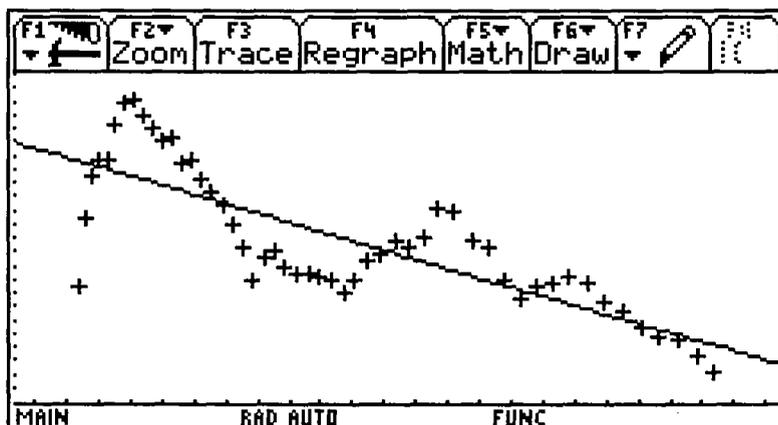
Figure 14. A scatter plot of population growth and the regression line

Equation (1) shows that the linear regression line models the differential equation

$$\frac{dP}{dt} = aP^2 + bP.$$

Subject to an initial condition $P(0)$, the differential equation solver *deSolve* (Figure 15) easily determines the population model

$$P(t) = \frac{7.32584E7 \cdot (1.03033)^t}{(1.03033)^t + 10.3865}.$$
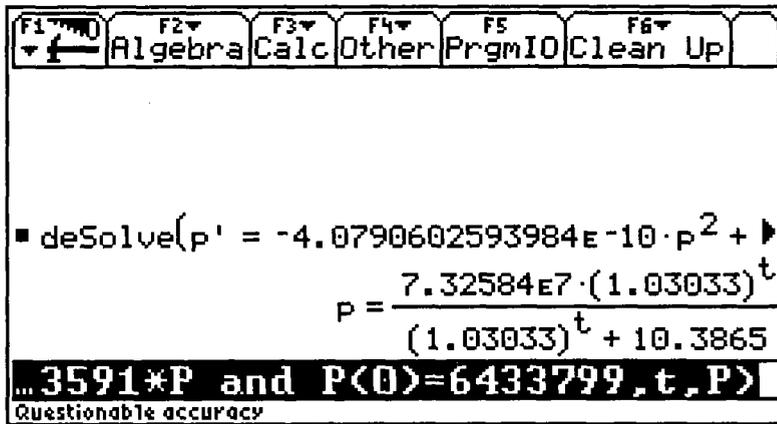
Figure 15. A symbolic solution of the population model
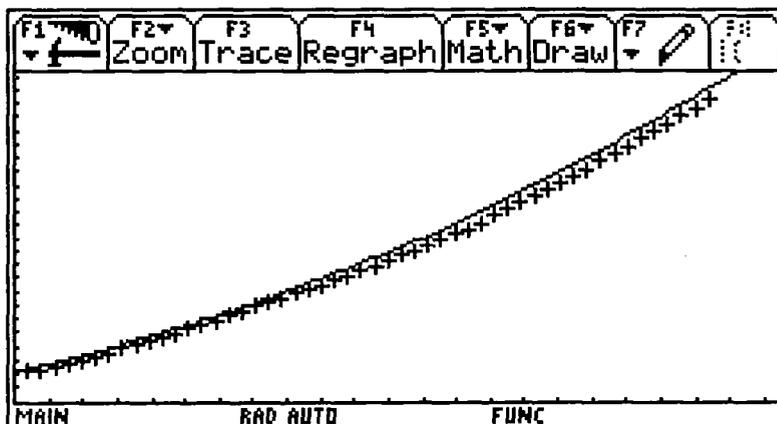
Figure 16. Graphs of the population scatter plot and the population model

The population model $P(t)$ and the population scatter plot are graphed on the same graph in Figure 16. The model is a very good fit to the population data.

The graph of the population model is next plotted over a period of 400 years in Figure

17. Predictions can be made; for instance, the population of Malaysia in 2020 will be $P(70) = 32.09$ *million*. In addition, the carrying capacity is seen to be about 73 million.
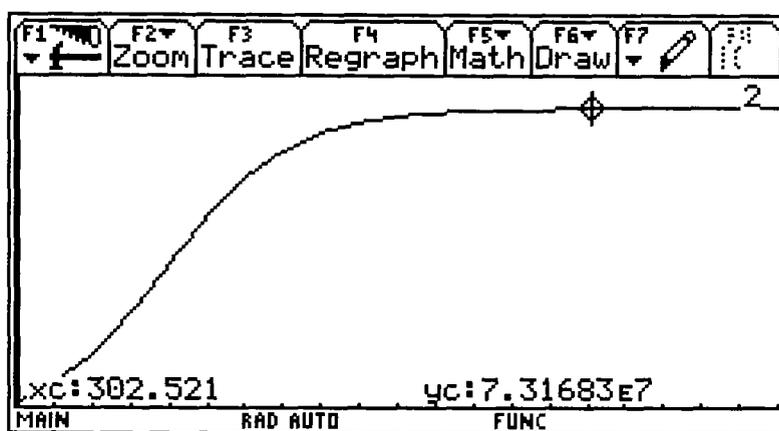
Figure 17. Graph of the population model

It is clear from the examples above that the graphic calculator allows student learning to occur at a higher cognitive level and serves to facilitate inquiries, explorations, and problem-solving activities. Our course put emphasis on its use as

- a tool for the symbolic manipulation or graphical display of mathematical functions and equations,
- a facility for the collection, examination and analysis of data,
- a tool to foster collaborative learning and teach students to work as a team,
- a tool to aid in solving realistic problems that enables the student to concentrate on problem aspects and interpretation rather than computational aspects, and
- a tool to discover, visualize, or investigate mathematical theories.

In this student-centered learning environment, the graphic calculator encourages students to reflect on and elaborate not just their own ideas, but those of their peers as well.

4. SURVEY SUMMARY

To monitor the impact of graphic calculators (GC) in the course, a semi-structured survey was prepared and implemented upon completion of the course. This survey requested information on students' perception of their understanding and impression of the course taught with GC, and also sought their views on the educational value of

integrating the GC into mathematics in general. The four main aspects observed in this study are the *cognitive* domain including *tool competency*, the *affective* domain, *behavioral* domain and the *value* domain.

Quantitative data

The instrument used in this study was a survey questionnaire made up of 53 items. Item 1 to item 11 (11 items) measure the confidence in the learning of mathematics with GC (MatGC) and item 12 to item 23 (12 items) measure the confidence in using GC (ConfGC). These items were adapted from an instrument called the Attitudes to Technology in Mathematics Learning Questionnaire (Mtech) developed and validated by Fogarty et al. (2001). Fogarty et al. reported an internal consistency with Cronbach alpha value of .90 for both the confidence in learning mathematics with GC and confidence in using GC respectively.

The overall breakdown of the items in the instrument of this study is grouped into five categories or aspects. There are 22 items in the cognitive category measuring students' intellectual knowledge applied to the learning of the lab course, 5 items in the tool competency gathered to measure the required skills in using GC in the lab course, 11 items in the affect category concentrating on the students' positive and negative feelings concerning the application of GC technology in the lab course, 12 items on value measuring the use, relevance and worth of GC in personal and professional career. Finally there are 3 items in the behavioural category measuring the nature of activity (group or activity-oriented) in the process of learning. All items are 5-point Likert scale ranging from 1 (strongly disagree) through 3 (neutral) to 5 (strongly agree).

A mean score of more than 3.0000 indicates a favorable response towards the usage of GC in learning the lab course. The questionnaires were administered to the entire population ($N = 39$) at the end of the lab course.

Table 2. A summary of the items according to the categories in the survey questionnaire

Category/aspect	Related Items
Cognitive	1,7,10,11,24,25,26,27,28,29,30,33,34,37,38,39,40,43,47,49,51,53
Tool competency	12,13,14,15
Affect	8,9,16,17,18,19,20,21,22,23,45
Value	2,3,4,5,6,31,35,36,46,48,50,52
Behaviour	41,42,44

Table 3. Comparing means from the different aspects observed in the study

Aspects	N	Mean	Std. Deviation
Cognitive	39	3.7155	.57296
Tool competency	39	2.4718	1.05978
Affect	39	3.5000	.77040
Behaviour	39	3.1197	1.03304
Value	39	3.4190	.80116
MatGC	39	3.6981	.60589
ConfGC	39	3.2168	.89477

Table 4. Comparing gender mean between the different aspects observed in the study

Gender		Cognitive	Tool	Affect	Behaviour	Value	MatGC	ConfGC
Male	Mean	3.6167	2.4000	3.4697	3.4444	3.0000	3.4848	3.1970
	N	6	6	6	6	6	6	6
	Std.dev.	.50563	1.10272	.82204	1.12875	1.17851	.75551	.80989
Female	Mean	3.7335	2.4848	3.5055	3.0606	3.4952	3.7369	3.2204
	N	33	33	33	33	33	33	33
	Std.dev.	.58968	1.06891	.77396	1.02217	.71154	.58023	.92025
Total	Mean	3.7155	2.4718	3.5000	3.1197	3.4190	3.6867	3.2015
	N	39	39	39	39	39	39	39
	Std.dev	.57296	1.05978	.77040	1.03304	.80116	.58221	.87673

Table 5. Comparing program mean between the different aspects observed in the study

Program		Cognitive	Tool	Affective	Behavioural	Value	MatGC	ConfGC
B.Sc.	Mean	3.7680	2.6000	3.5692	2.9167	3.4943	3.7461	3.3182
	N	28	28	28	28	28	28	28
	Std.dev.	.56812	.98583	.76809	1.10601	.76661	.62020	.93351
B.Sc.Ed.	Mean	3.5444	2.0000	3.2343	3.5926	3.1111	3.5020	2.8384
	N	9	9	9	9	9	9	9
	Std.dev.	.60386	1.16619	.68487	.59577	.90906	.42010	.56732
Total	Mean	3.7137	2.4541	3.4877	3.0811	3.4011	3.6867	3.2015
	N	37	37	37	37	37	37	37
	Std.dev	.57668	1.04844	.75360	1.04055	.80759	.58221	.87673

Discussion

Using SPSS, the quantitative data were analyzed. The reliability coefficient alpha was 0.6032 for MatGC and 0.7460 for ConfGC. Nonetheless, the reliability coefficient alpha obtained from the pilot study was 0.7484 for MatGC and 0.7740 for ConfGC showing

that the items have a good internal consistency that is reliable. Tables 3–5 show the results of the data analyses.

All means recorded for the five major aspects (Table 3) including the confidence in using GC to learn mathematics (MatGC) and confidence in using GC (ConfGC) except for tool competency scored higher than 3.0000 indicating that students favoured the introduction of GC in the lab course. Tool competency managed only a mean score of 2.4718 showing that students still lack some of the required skills in using GC to learn the lab course efficiently and effectively.

The gender mean in Table 4 shows that the female students scored generally higher than the male students in all the aspects observed. The overall score for the cognitive domain (mean = 3.7155) is the highest followed by the MatGC (mean = 3.6867).

The program mean in Table 5 displays a favorable response on the usage of GC except for the *tool* competency which scored only a mean of 2.4541. The Bachelor of Science (B.Sc.) program scored low in the behaviour domain (mean = 2.9167) showing that the students in this program preferred to study individually as compared to those in the Bachelor of Science with Education (B.Sc. Ed., mean = 3.5926). In the aspect of confidence in using GC (ConfGC), the Bachelor of Science with Education students scored only a mean of 2.8384 which was below the average of 3.0000 indicating that these students were not confident in handling the GC in general.

The overall response shown by the students towards the graphing technology in teaching the lab course is positive and favorable. The tool competency scored the lowest among all means and an interview with the instructor of the lab course confirmed that this is true because students were consistently forced to learn and use new GC commands when switching from topic to topic during the whole course. As a result, many students might be confused or not be able to recall the specific GC commands as fast as they should in the process of solving a problem without any reference. Such inadequacy eventually slowed down the whole process of problem solving and is likely the main cause of frustration among the students.

Qualitative data

The qualitative data were obtained from the four open-ended questions accompanying the survey questionnaires. The four questions asked were,

- What do you think are the benefits of GCs in learning mathematics?
- Do you think GC is a useful tool to learn mathematics? Why?
- What is your opinion of mathematics now after you are exposed to the use of GCs to learn mathematics?
- Would you prefer to be a person with the knowledge of GC? Why?

The results of a content analysis found that majority of the students have positive opinions towards learning mathematics with GC. These opinions can be classified into three major aspects such as *cognitive competence, affect,* and *value.* The comments commonly given by the students to the above questions were that GC helps to save time in solving a problem, allows more exploration than the traditional method, tests many concepts in a shorter time, arouses interest in learning mathematics in general, reduces careless mistakes, makes mathematics easier to learn, gives more accurate answers and is important to the future generation and careers.

Below are some of the interesting comments grouped according to the three major aspects made by the students:

Cognitive competence

Cognitive competence is associated with the students' ability to learn mathematics with GC (*cognitive* domain) and also their skills in handling the GC (*tool* competence). Their opinions on the advantages and disadvantages of using GC in the learning of mathematics were analyzed. The following are examples of the comments:

Student 2: "... GC helps to solve difficult questions where normal people cannot do so in a short period."

Student12: " GC gives simulation...Something that you have never imagine before...No difficult programming (*e.g.* MATLAB)...GC enhanced my learning of linear algebra, calculus, differential equation and statistics...I used to have difficulties in imagining statistical inference, normal distribution etc. With GC, everything can now be seen right in front of my eyes."

Student 22: "With GC, we (students) can achieve a higher level of learning in math at an earlier stage rather than just memorizing formula like what we usually do."

Student 25: "GC provides answers graphically and numerically. Given an equation, the graph is plotted itself with the right commands and scales. This has made me realized that basic calculus could be easily done with the help of GC since I do have a shaky basis... GC helps students to explore a question from different angles. We can play with the data and can reuse them again and again..."

Student 26:"GC can handle big data that may be confusing in manual calculation."

Student 30:"... the ideas/concepts of math used to be very abstract and caused me to lost my interest in learning math further... But now, with GC I am able to visualize the concepts better..."

Affect

The term *affect* looks into how students felt about the lab course taught with GC, enjoyed the course, or were stressed by and daunted by the introduction of the new tool in the mathematics curriculum. For example:

Student 12:"GC is the best device... With GC I am now confident to tackle any math problem on my own... I am lucky to know and have a chance to learn the GC technology."

Student 29: "GC helps to alleviate my drudgery towards math... math is more exciting now compare to before the introduction of GC."

Student 30: "Math is easier now as learning concepts and ideas are clearer with GC. Before this I was afraid of math because it was difficult."

Student 37: "... I am happy because I know and can use more methods to solve a mathematics problem with GC..."

Value

Value is associated with the importance of graphing technology in relation to students' daily life, future career, and professional job. Examples of students' comments are:

Student 1:"GC is an improvement in math gadget and I want to be one of those who know about GC..."

Student 7: "... GC makes me think analytically... teaching math with GC emphasizes more on interpretation skills in solving problems... these skills are useful to my future career."

Student 22: "We can appreciate math better with the use of GC. In the traditional methods of learning math, sometimes the beauty of math is not enhanced... with GC, we look forward to explore the subject and integrate it with other fields."

Student 23: "With the knowledge of GC, I can appreciate Linear Algebra more after attending the Lab Course."

Student 25: "... since we are moving towards the technology era, I would not want to miss my chance of getting my hands on this sophisticated tool. It could be of great use in the future."

Student 29: "... I prefer to use technology when performing my daily tasks..."

Student 31: "... with much guidance, students will be able to learn how to interpret mathematical results in real life."

Student 39: "If I have enough money, I will buy a GC and use it to enhance my understanding of mathematical concepts. I can now understand better the

concepts in algebra, statistics, calculus, and differential equations... I can become a professional teacher with the knowledge of GC."

While some of the neutral comments are:

Student 16:"I don't think GC is a useful tool because student doesn't know the traditional way of calculation by just referring to GC... but it will be useful if the traditional method of solving the problem proves to be too difficult to do by hand."

Student 36: "GC is only useful if we have a trainer to teach the teacher to use it."

Discussion

The qualitative results showed an overall favorable attitude towards the use of GC technology in the lab course. The students' positive response on visualization and exploration coincides with Scariano and Calzada's (1994) assertion that GC "enhances visualization and invites self-discovery (p. 61)." The expressions illustrated in the affect domain showed that students were highly motivated and excited about the idea of introducing GC technology in the lab course. This result is supported by Stick (1997) who observed that university students were captivated with the visual displays and were highly motivated throughout the semester. Similarly the feedback from Milou's (1999) study on the teachers also concluded that calculators help to motivate student's learning. He reported further that using calculators can increase the level of achievement and understanding in algebra and precalculus. Other comment such as GC allows students to focus on concepts and brings real world data into the classroom were also reported by Martinez-Cruz and Ratliff (1998).

5. CONCLUDING REMARKS

We are very encouraged with the survey findings, which showed that the majority of students responded positively and favorably towards undergraduate mathematics enhanced with graphing technology. Our efforts in making changes to a curriculum impacted by graphing technology and the adoption of the new pedagogical practice have been rewarding. During the course, students were seldom seen working alone and but tutoring each other became a common sight. Evidently, technology promotes student's motivation, inspires critical thinking and improves problem-solving skills. Our next initiative is to incorporate graphing technology into other courses to enable students the continued familiarity in using the most advanced features of the graphing technology. We also need to face the priority challenge of developing appropriate and valid test items for

the purpose of classroom assessment in this hand-held technology enabled classroom-learning environment.

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