R_{FACTOR} TO DETERMINE THE REYNOLDS NUMBER OF SATURATED WATER FLOWING IN TUBES

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ABSTRACT

Fluid flowing in a tube is commonly used in practice for heating and cooling applications. The flow in a tube can be laminar or turbulent, entirely depending on the flow conditions. It is therefore, useful to know the nature of the flow, especially in evaluating the Nusselt number. Osborn Reynolds, in the 1880's, discovered that the flow depends mainly on the mean flow velocity V, inner tube diameter D_i and the kinematic viscosity ν and this led to the formulation of the Reynolds number, a dimensionless physical quantity. The parameters defining Reynolds number are dependent on temperature, whereby an accurate representation of this number will be critical in the determination of the flow regimes. This problem was solved by obtaining a temperature dependent model, which represented these parameters. The temperature dependent model obtained by using the reciprocal quadratic fit is called the R_{FACTOR} , determined in the temperature range of 0°C to 360°C, and for saturated water.

INTRODUCTION

The evaluation of the Reynolds number is important to determine the flow regime in a tube. The flow regimes can be classified as laminar, transition and turbulent. Laminar regime is characterised by smooth streamlines and highly ordered motion. Turbulent regime is usually associated with highly disordered motion caused by velocity fluctuations. Transition is the intermediate regime, because the change from laminar to turbulent flow does not occur suddenly, but goes through a transition region where here is a hesitation between the laminar and turbulent flows, before the flow becomes a fully developed turbulent flow.

Reynolds number is usually evaluated by using the mean fluid velocity, kinematic viscosity and the tube's inner diameter. Problems are usually encountered in this evaluation, especially if it involves simulation process. The determination of this number needs the information related to the temperature of the fluid that is flowing in the tube. Although there is no parameter representing the temperature directly, but the effect depends on the proper selection of the kinematic viscosity value from the handbook. Normally the handbooks provide the kinematic viscosity values at fixed interval of temperatures. Odd temperature values are obtained through linear interpolation. If one is carrying out a simulation process, whereby the fluid temperature is obtained through iteration, the difficulty lies in the evaluation of the Reynolds number, as the

  Editor: A.A.M. Sayigh

simulation process will be halted until the corresponding value of kinematic viscosity is entered or the value is read from a large database system created just to cater for this. The processing time will also be effected. Therefore, the need to obtain a mathematical model to represent the temperature dependent parameters is inevitable [1].

THEORY

Osborn Reynolds, in the 1880's discovered that the flow depends mainly on the mean flow velocity V, inner tube diameter $\boldsymbol{D_i}$ and the kinematic viscosity ν [2]. He formulated the Reynolds number, which is given as equation (1) below,

$$\mathrm{Re} = \frac{VD_i}{\nu} \tag{1}$$

Fluid flowing in a tube is considered as an internal flow, where the fluid is completely confined by the inner surfaces of the tube. This sets the limit on how much the boundary layers can grow. The fluid velocity changes from zero at the surface to a maximum at the centre of the tube. So, it is usually convenient to work with the mean velocity, as the velocity gradient is not that great. The mean velocity is assumed to remain constant in an incompressible flow with the cross-sectional area being constant. Laminar flow is characterised by smooth streamlines and highly ordered motion while velocity fluctuations and highly disordered motion describe turbulent flow. There is a transition region, where the change over from laminar to turbulent flow takes place. In a fully developed flow, the Reynolds number showing the onset of turbulence is approximately 2300. So therefore, the critical Reynolds number for flow in a tube is generally accepted to be 2300. Reynolds number below this critical value indicates a laminar flow, while a transition regime occurs in a range of between 2300 and 4000. The fully developed turbulent flow happens when the Reynolds number exceeds 4000 [3].

METHODOLOGY

The kinematic viscosity of a fluid can be replaced by its dynamic viscosity, by using the fact that the ratio of the dynamic viscosity to kinematic viscosity is the density of the fluid as shown in equation (2):

$$\rho = \frac{\mu}{\nu} \tag{2}$$

The mean velocity too can be introduced as the ratio of the mass flow rate J to the product of density, ρ and the cross-sectional area A of the tube as shown below:

$$J = VA\rho \tag{3}$$

The tubes used are of circular cross-section and the cross-sectional area is as given by equation (4):

$$A = \frac{\pi D_i^2}{4} \tag{4}$$

By using equations (3) and (4), the mean velocity can be represented as equation (5).

$$V = \frac{J}{\rho\left(\frac{\pi D_i^2}{4}\right)} \tag{5}$$

The Reynolds number can then be transformed from its original appearance as given in equation (6) to the one shown in equation (7).

$$\mathrm{Re} = \frac{VD_i}{v} \tag{6}$$

$$\mathrm{Re} = \frac{14J}{11D_i}\left(\frac{1}{\mu}\right) \tag{7}$$

After refining the equation to determine the Reynolds number, the next step is to perform a curve fit on the dynamic viscosity, μ against temperature. The curve fit that suits the data will hence be the R_{FACTOR}, as indicated in equation (8).

$$\mathrm{Re} = \frac{14J}{11D_i}\left(\frac{1}{R_{FACTOR}}\right) \tag{8}$$

RESULTS

The method used to find the R_{FACTOR} is by using the linear curve fitting method [4]. The curve fitting method with the best correlation factor and one that gives the lowest standard error is selected. R_{FACTOR} is an equation which varies with temperature and is obtained by doing a curve fitting on the plot of dynamic viscosity of saturated water against temperature, in the range of 0°C to 360°C. Two different curve fitting methods were employed, namely the polynomial fit and reciprocal quadratic fit as shown in Figure 1. The reciprocal quadratic fit was selected as it gave a very low standard error that is 9.2 x 10^{-4}% as compared to the polynomial fit with a percentage error of 3.2 x 10^{-3} %. In Figure 2, it can be seen that the better fit is the reciprocal quadratic fit. The reciprocal fit equation is selected to be the R_{FACTOR} and is given as equation (9).

$$R_{Factor} = \frac{1}{(552.748 + 21.206T + 0.0768T^2)} \tag{9}$$

A further analysis on the R_{FACTOR} shows that the R_{FACTOR} and the dynamic viscosity is related by correlation factor of 0.99 and Figure 3 shows the almost perfect straight line that verifies the correlation.

CONCLUSION

The use of R_{FACTOR} , instead of the dynamic viscosity has a significant edge, especially if the Reynolds number is used in a computer simulation. Normally the fluid temperature in the tube is determined by using iterative method. Therefore once the temperature is determined, by using the R_{FACTOR} correlation found, the Reynolds number can be computed rather easily. A sample of MATLAB program on how the R_{FACTOR} is used in determining the Reynolds number is shown in Table 1.

The program was used to analyse the effect of varying the tube's diameter on the Reynolds number, at various different temperatures, but at a fixed mass flow rate. The results of this simulation program are as shown in Figure 4. The alternative way of analysing will involve using the values of dynamic viscosity from the handbook of constants, at intervals that are normally increasing with fixed even increments. But it clearly showed by using the above program, that R_{FACTOR} capacity is not limited in this way, except for the temperature range in which the R_{FACTOR} is determined and the type of fluid used. The R_{FACTOR} can obviously be evaluated at different temperature ranges and for different type of fluids as well.

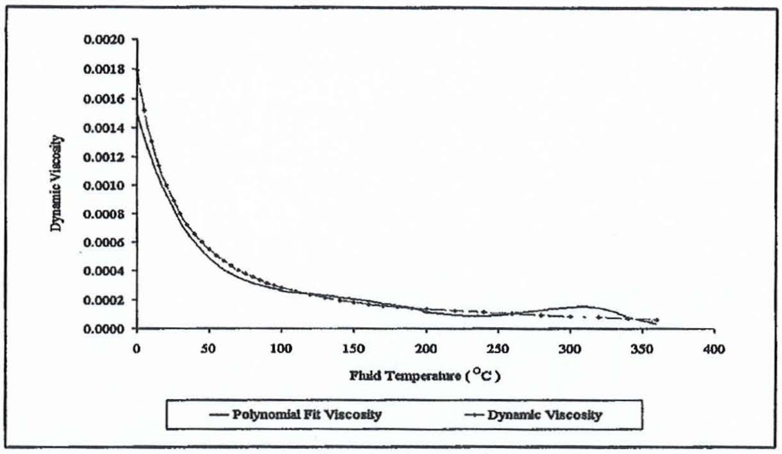

Figure 1: Graph of actual dynamic viscosity and polynomial fit viscosity versus fluid temperature in degrees celsius

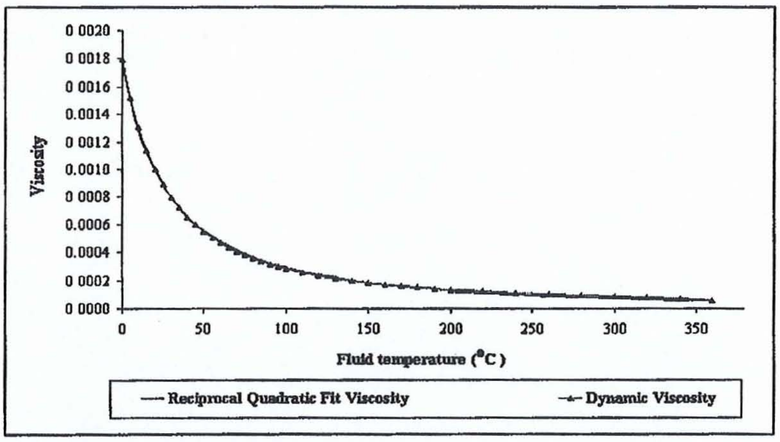

Figure 2: Graph of actual dynamic viscosity and reciprocal quadratic fit viscosity versus fluid temperature in degrees celsius

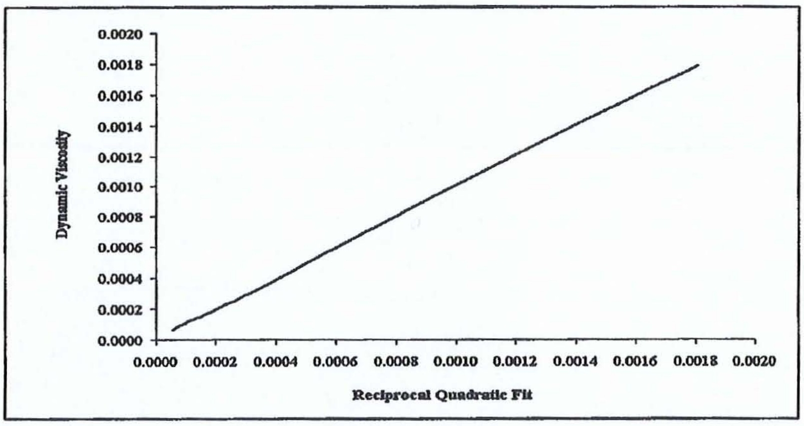

Figure 3: Graph of actual dynamic viscosity versus reciprocal quadratic fit viscosity

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TABLE 1
A SAMPLE OF MATLAB PROGRAM CODES SHOWING HOW THE REYNOLDS NUMBER CAN BE CALCULATED BY USING R_{FACTOR}

```
Di=0.09:0.01:1;
Tc=350;
J=0.003;
% To calculate Reynolds Number by using RFACTOR
% RFACTOR : based on the reciprocal quadratic linear regression
% curve fit method : for saturated water only.
% Can be expanded for other working fluids
R1=(552.74814+(21.206246*Tc)+(0.076808066*(Tc.^2)))
Rfactor=1./R1;
% Reynolds Number
Re=((4*J)./(pi*Di))*(Rfactor.^-1);
Results=[Di',Re']
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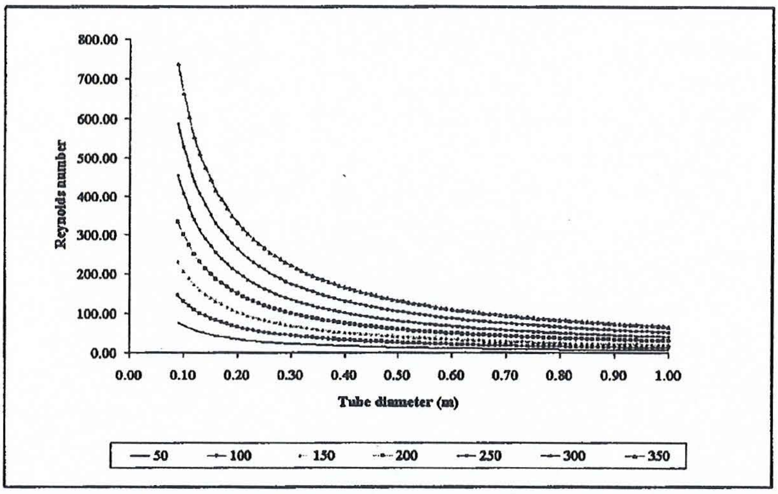

Figure 4: Graph of Reynolds number versus tube diameter, evaluated at 7 different temperatures (°C)

ACKNOWLEDGEMENTS

The authors acknowledge the research grant provided by Universiti Sains Malaysia, Penang that has made this research possible.

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