WAVELET ENERGY GUIDED LEVEL SET BASED ACTIVE CONTOUR - A NOVEL METHOD TO SEGMENT HIGHLY SIMILAR INTENSITY REGIONS

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by

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LIST OF ABBREVIATIONS

CT Computed Tomography

CVS Computer Vision System

HVS Human Vision System

LSAC Level Set based Active Contour

PDE Partial Differential Equation

PDF Probability Density Function

WELSAC Wavelet Energy Guided Level Set based Active Contour

KONTUR AKTIF BERASASKAN SET PERINGKAT TERARAH OLEH DAYA WAVELET - SATU KAEDAH BARU UNTUK MENSEGMENTASI KAWASAN DENGAN KEAMATAN YANG HAMPIR SAMA

ABSTRAK

Segmentasi imej adalah salah satu peringkat permulaan yang paling penting dalam sistem pengesanan berbantukan komputer yang mempermudahkan pengesanan, pengecaman dan pengukuran objek selanjutnya. Dalam kajian-kajian yang lampau, pelbagai kaedah segmentasi imej telah digunakan berdasarkan pada domain imej, struktur yang dikehendaki dan modaliti pengimejan. Walaupun terdapat terlalu banyak kaedah segmentasi imej skala kelabu, salah satu cabaran utama yang dihadapi dalam segmentasi ialah segmentasi kawasan-kawasan yang mempunyai persamaan yang tinggi dalam nilai keamatan dengan latarbelakangnya.

Tesis ini telah dirangsangkan oleh masalah ini dan mencadangkan satu kaedah segmentasi baru yang menyatukan satu ciri baru, yang boleh menonjolkan ketidaksamaan di antara kawasan-kawasan yang mempunyai variasi keamatan yang rendah. Ciri ini disatukan untuk memformulasi satu kontur aktif berasaskan set peringkat yang baru, yang mana kaedah ini menangani segmentasi kawasan-kawasan dengan keamatan yang hampir sama, di mana tiada sempadan yang jelas di antara kawasan-kawasan ini.

Dalam fasa pertama tesis ini, kekuatan transformasi wavelet diadaptasikan untuk memformulasi ciri baru, yang dinamakan *daya wavelet*. Ujian-ujian telah dijalankan dengan imej sintetik dan juga imej nyata untuk membuktikan daya wavelet ini sememangnya dapat menangkap dan menonjolkan ketidaksamaan pada keamatan kawasan-kawasan yang perlu disegmentasikan.

Fasa kedua tesis ini didedikasikan untuk memformulasi kontur aktif berasaskan set peringkat baru yang sesuai untuk segmentasi kawasan-kawasan yang tidak mempunyai sempadan yang jelas dengan latarbelakangnya. Dalam formulasi ini, dua ciri yang mengarah kontur ini ialah ciri kawasan terkandung daya wavelet dan ciri kelicinan kontur. Menggunakan formulasi ini, persamaan-persamaan untuk mengerakkan kontur telah diterbitkan dan dilaksanakan dengan meggunakan MATLAB. Kaedah segmentasi yang dicadangkan ini dinamakan *kontur aktif berasaskan set peringkat terarah oleh daya wavelet*.

Ujian-ujian telah dijalankan dengan menggunakan tiga jenis imej perubatan untuk memastikan kaedah yang dicadangkan dan membezakannya dengan kaedah kontur aktif berasaskan set menggunakan garisan dan kontur aktif berasaskan set menggunakan kawasan yang sedia ada. Keputusan ujian menunjukkan kaedah yang dicadangkan dapat mensegmentasi kawasan yang dikehendaki dengan ketepatan yang tinggi berbanding dengan garis panduan yang disediakan oleh pakar radiologi. Kaedah yang dicadangkan juga memberi satu kadar purata kejituan iaitu 82.31% dengan kadar purata kesalahannya 1.02%.

WAVELET ENERGY GUIDED LEVEL SET BASED ACTIVE CONTOUR - A NOVEL METHOD TO SEGMENT HIGHLY SIMILAR INTENSITY REGIONS

ABSTRACT

Image segmentation is one of the most important preliminary stages in computer-aided diagnosis system that facilitates further object identification, recognition, and quantification. In past researches, various image segmentation methods have been employed depending upon the image domain, structure of interest and imaging modality. Although there are vast number of gray scale image segmentation methods available, one of the main challenges still faced in segmentation is the segmentation of regions with high similarity in intensity values with their background.

This thesis is motivated by this problem and proposes a new segmentation method that integrates a novel feature, which is able to enhance the dissimilarity between regions with low variations in intensity. This feature is integrated to formulate a new level set based active contour model, that addresses the segmentation of regions with highly similar intensities, which do not have clear boundaries between them.

In the first phase of this thesis, the strength of wavelet transform is adapted to formulate the new feature, named as *wavelet energy*. Experiments are conducted with both synthetic as well as real images to prove that the wavelet energy feature indeed captures and enhances the dissimilarities in the intensities of the regions that are to be segmented.

The second phase of this thesis is dedicated to formulate a new level set based active

contour model that is suitable for segmentation of regions without a clear boundary with their background. In this formulation, the two terms that guide the contour are the wavelet energy incorporated region term and the contour smoothness term. With this formulation, the equations for evolving the contour are derived and implemented in MATLAB. The proposed segmentation method is named *wavelet energy guided level set based active contour*.

Experiments are conducted using three different types of medical images to validate the proposed method and compare it with the existing edge-based and region-based active contour. The experimental results show that the proposed method is able to segment the region of interest in close correspondence with the manual delineation by the radiologists. The proposed method also produces an encouraging average accuracy rate of 82.31% with average error rate of 1.02%.

CHAPTER 1

INTRODUCTION

1.1 Overview

Computer vision has been evolving rapidly and has enabled various computer-aided systems to be developed to assist human in decision making. It has been employed in various applications ranging from industrial inspection, robotics, biomedical and satellite surveillance systems. Computer Vision System (CVS) imitates Human Vision System (HVS) by replacing imaging modalities such as camera and x-ray in the place of human eye, and computers and image processing algorithms in the place of optic nerve and brain.

Image segmentation, being the most essential and crucial preliminary process in low-level image processing, is required to facilitate higher level image analysis such as the description, recognition, quantitative measurement and visualization of objects of interest. The main aim of image segmentation is to produce a strong correlation between the segmented object with the real world object. For example, in a computer-aided diagnosis system, the segmentation of blood vessels or tumour needs to be as accurate as a medical expert's manual delineation, considering the critical nature of such applications.

Image segmentation partitions an image into a number of isolated regions that corresponds to relevant objects in the image (Sonka et al. 1999). A segmentation of an image, I is a finite set of regions, R_1, R_2, ...,R_s, given as

$$I = \bigcup_{i=1}^{s} R_i. \tag{1.1}$$

Image segmentation continues to be an actively researched area despite its rather colourful history dating back to the 1950s. Many techniques and breakthroughs have been reported in the vast amount of literature available, however two major challenges still persists in image segmentation. The first challenge is to be able to automatically segment an object from its background in the case where distinct regions in an image are *perceived* collectively as belonging to a single entity. Such problems in image segmentation necessitates a higher level of perceptual understanding before successful segmentation is possible. Figure 1.1 are examples of such images. Which is the best segmentation algorithm that can be employed to segment the entities *humans perceive* as deers from these images?

Figure 1.1: Example of complex images that has been a challenge for segmentation. *Source: http://www.gardnergallery.com*

The second challenge is to be able to segment a region of interest that is highly similar to its background. Typical cases include medical image modalities such as ultrasound, Magnetic Resonance Imaging and Computed Tomography images. For example, the tumour within an organ has highly similar intensity with its background tissue and other adjacent structures. Figure 1.2 shows an example of medical image with high similarity in intensity values between the tumour regions and the normal liver tissue. The segmentation of the tumour regions from the surrounding tissue is the main problem faced in this type of medical images. This thesis addresses the segmentation issues related to the second challenge, that is segmenting regions of interest that are highly similar to their background.

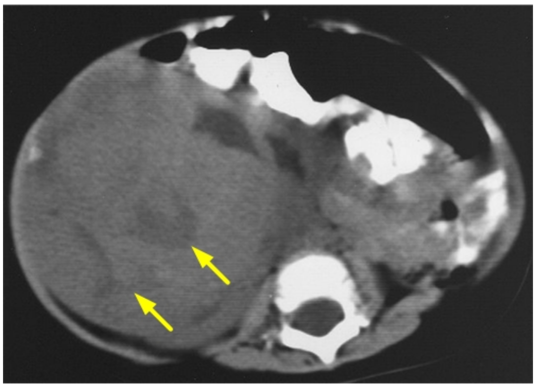

Figure 1.2: An example of medical image with liver tumour, marked by the arrows.

1.2 Approaches in Image Segmentation

Many different image segmentation algorithms have been developed in the past several decades. However, image segmentation remains acutely problem-centric. A given segmentation method may perform well on one problem domain but poorly on a different domain. Thus, it is very hard to achieve a generic segmentation method that is universally applicable for a broad range of problem domains. Thus far, image segmentation methods that have been proposed can be generally categorized into three categories, which are edge-based, region-based and hybrid approaches.

Edge-based approaches detect abrupt changes in image intensity between two dissimilar regions to partition an image. However, noise in the image, broken edges and overlapping regions (Sonka et al. 1999) fail to produce closed and completely segmented regions. Region-based segmentation constructs regions using predefined pixel homogeneity properties such as intensity, color and texture. Region growing, watershed and clustering are examples of region-based segmentation approach. The accuracy of region-based segmentation is dependent upon the homogeneity properties being applied.

In recent years, hybrid approaches have gained prominence because such approaches combine the strengths of various image processing techniques to overcome limitations faced in edge-based and region-based segmentation. One such example is the active contour model, which integrates either boundary or region information with curve theory and optimization methods to locate the boundary of region of interest. The capability of active contour model in combining various image properties makes it a powerful segmentation technique. Snake model, being the pioneer to the active contour model was proposed by Kass et al. (1988). However, the snake model requires prior knowledge of the topology of region of interest. The accuracy of the segmented results is highly dependent on the exact prior knowledge of the location, shape and number of the regions to be segmented. As an improvement to this snake model, Osher & Sethian (1988) has proposed level set based active contour (LSAC). Level set based active contour model is an efficient image segmentation method that has been widely employed in various modalities of images such as medical, satellite and natural images. LSAC does not require prior knowledge of the shape or the exact location of the regions to be segmented. These advantages could be exploited to segment regions that are irregular in shape.

1.3 Motivation

As mentioned earlier, one of the major challenges in image segmentation is the segmentation of regions which are similar in intensities with their backgrounds. When examined using our naked eyes, such regions cannot be easily delineated as there is very subtle difference between region of interest and its background. In typical cases, a region's boundary may appear fuzzy and this could lead to incomplete segmentation due to the lack of edge information. Hence, simple segmentation methods that utilize intensity information such as edge detection and region growing may fail to segment this region (Sonka et al. 1999).

In edge-based segmentation methods, the high similarity that exists in the intensities of adjacent regions may not provide prominent edge information. Similarly, with region-based techniques, the high similarity in intensities between adjacent structures may cause what is known in the literature as the *under-segmentation* problem. In such cases, region-based approaches often fail to detect the very low intensity variation between adjacent regions. Hence, the use of intensity values in segmentation does not result in accurately segmented regions. Therefore, the first phase of this thesis is dedicated to investigate a set of secondary numerical representation that is better able to represent the faint dissimilarities between the two regions. The second phase of the thesis intends to formulate a level set based active contour model that incorporates the results of first phase to provide an accurate segmentation of regions that have highly similar intensities with their backgrounds.

The approach presented in this thesis intends to investigate wavelet-based image processing techniques to enhance the dissimilarities between the region of interest and its background which exhibits very low intensity variation. The coefficients of the wavelet transform will be used to infer statistical information instead of using intensity values. The obtained statistical information is then fused into a specially formulated LSAC model to address the segmentation of regions that have high similarity in intensities with their background.

1.4 Research Scope and Objectives

This research aims to propose a new segmentation method which integrates wavelet-based statistical feature with level set based active contour model to segment regions that have highly similar intensities, also referred to as *fine structures*, with the background. This thesis attempts to achieve the main objective through the following:

- Capturing and enhancing the dissimilarities between all the structures that are present in the images using wavelet-based statistical feature, called *wavelet energy*.
- Formulating a new level set based active contour by integrating wavelet energy.
- Evaluating the proposed segmentation method through chosen applications.

The proposed methodology shall be validated using real patients data comprising of Computed Tomography (CT) medical images obtained from Radiologist Department of Hospital Universiti Kubang Kerian, Malaysia.

1.5 Overview of the Proposed Method

The proposed segmentation method consists of two phases as shown in Figure 1.3. In the first phase, wavelet transform is used to compute wavelet-based statistical feature, named *wavelet energy*, to capture and enhance the dissimilarities between regions that are present in an image. The wavelet energy exhibits the region's detail of each pixel in the image. Different regions in the medical image exhibits a range of wavelet energies that are distinctive to a particular region. This phase is referred as **extraction of wavelet energy**.

In the second phase, denoted as **wavelet energy guided level set based active contour**, the wavelet energy values are incorporated with a level set based active contour model to produce an accurate delineation of fine structures.

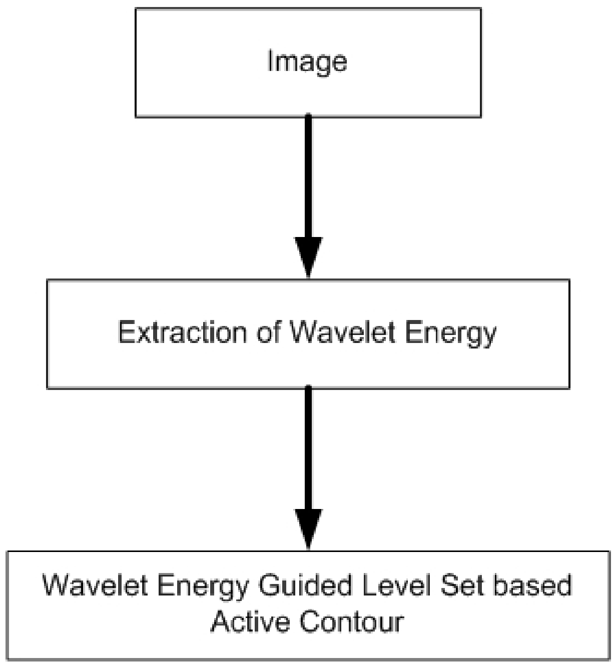

Figure 1.3: Overview of the proposed method.

1.6 Contribution of the Thesis

This thesis proposes a new level set based active contour model that integrates the strengths of wavelet transform into level set based active contour. The proposed approach offers a viable solution to one of the main issues faced in medical image segmentation, which is the segmentation of distinct regions whose pixels are made up of highly similar intensity ranges. The main significant contributions of this thesis are summarized as follows:

- Enhancement of regions' dissimilarities using wavelet-based statistical feature, called *wavelet energy*.
- Development of a new LSAC model incorporating wavelet energy to segment fine structures in medical images.

1.7 Thesis Organisation

This thesis is organised into six chapters. The current chapter has introduced the importance of image segmentation and the major challenges faced in image segmentation. In addition, this chapter outlines the motivation and objectives of this research.

Chapter 2 presents a critical review of level set based active contour formulation. Research work in this area covering three approaches, which are edge-based, region-based and hybrid LSAC are studied and discussed in detail.

Chapter 3 lays the relevant theoretical foundation on wavelet transformation, supervised Bayes maximum likelihood classifier and level set based active contour, which may be necessary for deeper understanding of the subject matter.

Chapter 4 introduces the computation of wavelet-based feature, known as wavelet energy. The wavelet energy captures and enhance the dissimilarities between the various regions in an image. This chapter discusses the relation between wavelet energy and the objects through test images. Experiments with two test images are carried out, stressing the importance of wavelet energy in medical image analysis.

Chapter 5 presents the integration of wavelet energy into level set based active contour formulation. Experiments are carried out to validate the accuracy of integrating wavelet energy in level set based active contour. The experimental results are compared and evaluated with conventional active contour including edge-based LSAC, region-based LSAC and ground truth from medical experts to prove the accuracy of the proposed method.

Chapter 6 concludes the efficiency and limitations of the proposed wavelet energy guided level set based active contour. Recommendations for future work are discussed to improve the current proposed method.

CHAPTER 2

LITERATURE REVIEW: LEVEL SET BASED ACTIVE CONTOUR

2.1 Introduction

Active contour has become a very important and successful approach in image segmentation, that has been employed in various applications. This active contour approach can be formulated mathematically using two models: snake and level set. Early work on active contour was pioneered by Kass et al. (1988), who has introduced the snake model. A later development by Osher & Sethian (1988), introduced a new direction to active contour, which is referred to as level set based active contour (LSAC). LSAC is introduced to overcome some limitations of the snake model. A detailed explanation on the limitations faced by the snake model are covered in Section 3.4. As level set based active contour is the main focus of this work, the following literature review will cover only related work that have been employed in level set based active contour model.

Several research work have been explored in level set based active contour, which can be broadly categorized into edge-based, region-based and hybrid methods. The image force called as *term* determine which category a level set based active contour belongs to. Edge-based LSAC model uses image gradient information as edge force (edge term) to halt the contour at edges of interest. Region-based LSAC employs region statistical properties such as intensity, texture or colour as region force (region term) to evolve the contour. Hybrid LSAC approach incorporates image gradient, region properties and prior knowledge of the image such as object shape in evolving the contour. The following sections describe in detail relevant related work in each category of the LSAC model, covering its' strengths and limitations.

2.2 Edge-based LSAC

Edge-based LSAC consists of two forces that determine the contour evolution; edge term that uses image gradient to halt the contour evolution and regularizing term to minimize the length of the contour and produce a smooth contour.

Malladi et al. (1995) adapted curve evolution theory and level set method (Osher & Sethian 1988) to model *geometric active contour* for shape recovery of objects in 2D and 3D images. This model consists of a regularizing term to characterize contour velocity and an edge term computed from the image data to halt the contour. The model is defined as

$$\frac{\partial \phi(x,y)}{\partial t} = g(I(x,y))(k(\phi(x,y)) + v)|\nabla\phi(x,y)|, \tag{2.1}$$

where $g(\cdot) : \Omega \rightarrow \Re$ is the image gradient, v is a constant that controls direction of contour evolution, either inward or outward and $k(\cdot)$ is the regularizing term. The regularizing term uses geometric properties, curvature and unit normal to evolve the contour in normal direction with rate proportional to its' mean curvature. This model defines the edge term using second order image gradient, Laplacian-of-Gaussian, to stop the contour evolution. During evolution, the contour moves in normal direction to the level set curve and stops at the region boundaries where the gradient approaches values zero. However, this proposed geometric active contour only halts at edges with gradient value equal to zero, which is an ideal case of an edge. In normal case, the gradient values along the boundaries exhibit uneven gradient values. This factor has lead active contour to ignore these region boundaries and leak to adjacent region due to invalid gradient information.

Caselles et al. (1997) proposed *geodesic active contour* to improve the drawback faced in geometric active contour, in which the geometric active contour ignores some broken edges or small gaps along the object boundaries. Caselles's model uses energy minimization formulation (Kass et al. 1988) to find a geodesic curve in the image using steepest descent method. Geodesic curve is the minimal distance length between given points in the image space. Then, the resulted geodesic curve is represented as zero level set function (Osher & Sethian 1988) to evolve the contour as in curve evolution theory. The geodesic active contour is modeled as

$$\frac{\partial \phi\left(x,y\right)}{\partial t} = g\left(I\left(x,y\right)\right)\left(k\left(\phi\left(x,y\right)\right)+v\right)\left|\nabla \phi\left(x,y\right)\right| + \nabla g\left(I\left(x,y\right)\right)\cdot \nabla \phi\left(x,y\right). \qquad (2.2)$$

Comparing the main difference between equation 2.1 and 2.2, it may be seen that a new term $\nabla g\left(I\left(x,y\right)\right)\cdot \nabla \phi\left(x,y\right)$ is added in geodesic active contour. This new term in geodesic active contour attracts the contour towards the local minima of the image, which is in the middle of two adjacent regions. Thus, real object boundaries can be detected although there are high or small variations in gradient values. Whereas in geometric active contour model, the contour only stops where the gradient of the edge becomes zero. Hence, geometric active contour model fails to capture broken edges or small gaps in the object boundary. Kichenssamy et al. (1995) proposed another similar model based on geodesic active contour and suggested an extension of this approach to 3D segmentation.

Edge-based LSAC approaches depend on a stopping function, which is the image gradient to stop the contour evolution. Thus, this approach does not capture very weak edges and big gaps because the evolving contour may ignore these weak edges. Edge-based approach also faces problem in noisy images because the local minima of the noise may interfere with the object gradient. This will cause the active contour to be attracted to the wrong local maxima of the image that may exhibit similar gradient attribute as the object gradient. In addition, the use of Gaussian filters to obtain the gradient image may smooth or remove very weak edges, thus will cause leakage of the contour to adjacent regions at the weak edge points.

2.3 Region-based LSAC

Region-based LSAC basically consists of two forces to evolve the contour, which are region term and regularizing term. Region term uses homogeneity properties of each region in the image such as intensity and texture to minimize the energy force of the contour. Regularizing term is used to produce a smooth contour.

Chan & Vese (2001) introduced *active contour without edges* that integrate Mumford-Shah segmentation functional (Mumford & Shah 1989) with level set using energy minimization approach. The region term used in this model is based on Mumford-Shah segmentation technique. This technique approximates the regions inside and outside the contour with a distinct intensity value, which is known as *piecewise-constant intensities*. These constant values are computed by taking the average intensity value of each region. Least mean square error method is used to minimize the region term by minimizing the difference between actual image intensity at every pixel and the piecewise-constant of the corresponding region. Each region in the final partitioned image is represented with the best approximate of piecewise-constant. A regularizing term that minimizes the length of the contour is used to produce a smooth contour. This active contour without edges model is formulated using zero level set function, which is given as

$$\frac{\partial \phi(x,y)}{\partial t} = \delta_\epsilon \phi(x,y) \left[\mu div \left(\frac{\nabla \phi(x,y)}{|\nabla \phi(x,y)|} \right) - \nu - \lambda_1 (\mu_0(x,y) - c_1)^2 + \lambda_2 (\mu_0(x,y) - c_2)^2 \right]. \tag{2.3}$$

where c_1 and c_2 are the means of the image intensities inside and outside the region, parameters μ, ν, λ_1 and $\lambda 2$ are fixed parameters that controls the strength of each term. This model has produced encouraging results using synthetic and real images. Brieva et al. (2005) have adopted this method to segment vessel structures in coronary angiography.

An extension of Chan-Vese model (Chan & Vese 2001) to vector valued such as RGB and multispectral images is proposed by Chan et al. (2000) (The earlier work by (Chan & Vese 2001) was only published in 2001 in IEEE Transactions on Image Processing). Vector valued images can be represented as multichannels, and each channel carries some information about the object of interest. For example, a colour image can be divided into three channels: Red, Green and Blue channel. This proposed method extends Chan-Vese model into multichannel images by taking the mean of energy minimization of each channel. Experiments were computed on images with various channels and images with added noise to prove the robustness of this method.

Vese & Chan (2002) proposed a multiphase level set framework to segment images with more than two regions. This work is an extension to the earlier work by Chan and Vese (Chan & Vese 2001) to solve the problem of multiple regions overlapping each other and multiphase edges such as T-junctions in the image. This framework considers *m*-phase level set functions as the edges of the segmented image with 2^m disjoint subsets of the entire image. An example of multiphase active contour model for two-phase level sets (ϕ_1, ϕ_2) is defined as

$$\frac{\partial \phi}{\partial t} = \delta \phi_1 \left[\nu div \left(\frac{\nabla \phi_1}{|\nabla \phi_1|} \right) - \left((\mu_0 - c_{11})^2 - (\mu_0 - c_{01})^2 \right) H(\phi_2) - \left((\mu_0 - c_{10})^2 - (\mu_0 - c_{00})^2 \right) 1 - H(\phi_2) \right] \tag{2.4}$$

$$\frac{\partial \phi}{\partial t} = \delta \phi_2 \left[\nu div \left(\frac{\nabla \phi_2}{|\nabla \phi_2|} \right) - \left((\mu_0 - c_{11})^2 - (\mu_0 - c_{01})^2 \right) H(\phi_1) - \left((\mu_0 - c_{10})^2 - (\mu_0 - c_{00})^2 \right) 1 - H(\phi_1) \right] \tag{2.5}$$

where $\{c_{00}, c_{01}, c_{10}, c_{11})$ denote the mean intensity of the corresponding subsets in the image. Samson et al. (2000) have also proposed a similar multiphase level sets framework (Vese & Chan 2002) for classifying pixels into corresponding regions in the image.

Region-based LSAC does not restrict the placement of initial contour compared to edge-based active contour, which requires the contour to be either inside or outside object of interest. This approach is also insensitive to noise and does not require preprocessing step of noise removal. However, the use of intensity values to represent each region is only applicable for regions with uniform intensity. Thus, the region-based LSAC may fail on highly textured images and encounter oversegmentation or undersegmentation problems.

2.4 Hybrid LSAC

Hybrid LSAC integrates edge and region information to incorporate the strength of edge-based and region-based LSAC. This method basically consists of edge term, region term and other image properties' term such as shape to evolve the contour and regularizing term as contour's smoothing factor. This term are then, integrated and formulated in zero of the level set function.

Paragios (2000) proposed *geodesic active region*, which is an extension to geodesic active contour (Caselles et al. 1997) unifies region information and edge information in the LSAC model. The edge term is modeled to be of minimal length (geodesic curve) and attracted towards the real boundaries of objects of interest. In a related work by Paragios & Deriche (2002), the boundary information is determined using probabilistic edge detector. Probabilistic edge detector estimates the likelihood of a pixel being on the object boundary rather than being in the neighbourhood regions. Paragios (Paragios 2002) proposed another variation to this boundary information by using gradient vector flow (GVF). Region term of this model is defined using Bayes rule, which maximizes the probability of each pixel belonging to a given region based on training sample that is assumed to be Gaussian. The geodesic active region framework has been applied successfully in various computer vision applications, such as medical image segmentation (Paragios 2002), texture segmentation (Paragios & Deriche 2002) and motion estimation and tracking (Paragios & Deriche 1999).

A framework for cardiac image segmentation is proposed by Charnchai Pluempitiwiriyawej & Ho (2005) that models the contour evolution based on edge, region and prior knowledge of the heart and its' shape. The gradient magnitude of the image is used to model the edge term. The regions in the image are modeled using statistical method based on maximum likelihood function that maximizes the probability of the given pixels belonging to each corresponding region. The region probabilities are estimated assuming the training regions are Gaussian models. The shape of ellipse described parametrically is used as the shape of object of interest to force the contour to be ellipse in shape. The parameters that control the strength of each term are defined using simulated annealing approach. The experiments conducted have produced encouraging results compared to GVF Snake by Xu & Prince (1997), which are based only on edge information.

Xie & Mirmehdi (2004) presented *region-aided geometric snake* that integrates GVF Snake (Xu & Prince 1997) and region information to detect weak and noisy edges in the LSAC model. Magnitude of gradient using Gaussian filter produces the edge map of the image that models the edge term. Region information is used to overcome leakage at weak edges and to ignore noise. This method computes the region term by applying mean shift segmentation algorithm to the image to obtain preliminary segmented regions. Region boundary map are computed by applying gradient filter to the preliminary segmented results. Then, the gradient map vectors of region boundary map are extended using gradient vector diffusion to produce a larger capture range from the edge points. This method is applied on natural images and medical images covering heart and optic disk images. The experimental results using gray level and colour images proved that, this model is able to detect weak edges. However, the accuracy of this model is dependent upon the accuracy of the mean shift algorithm. Poor preliminary segmented region boundary maps may lead to false edge detection.

The work by Sandberg et al. (2002) and Aujol et al. (2003) has introduced filters such as Gabor and wavelet to be integrated in the LSAC model for segmenting textured images. The LSAC model proposed in (Chan et al. 2000) is extended to texture images using Gabor filter that describes textures in scales, orientation and local frequencies. Gabor filter transforms the original image into multichannel feature images obtained through different parameters of scale, orientation and local frequency. Then these multichannel images are used in the vector-valued active contour without edges model formulation, that has been proposed by Chan et al. (2000).

Wavelet-based classification (Aujol et al. 2003) is implemented by minimizing the level set function in order to obtain optimal classification of texture images using wavelet packet transform. The minimizing functional consists of partition, region and regularizing terms. The partition term ensures no pixels are left unclassified or a pixel been assigned to two classes simultaneously in order to obtain a complete partition of the image. In region term, each pixel position is assigned with a feature vector computed from decomposed wavelet packet subbands' coefficients. The feature, named energy is obtained by squaring the wavelet coefficients at the particular pixel position. The feature vector at each pixel position comprises of the energy values of all the subbands at the corresponding pixel position. Then maximum likelihood criterion is employed to maximize the probability of a pixel's feature vector belonging to a class. Regularizing term is added to discard irregular contour and produces smooth contour. This model has been experimented with texture images that consists of more than two texture classes. However, the main drawback of this model is its' computational burden due to the use of wavelet packet approach to decompose the image, in which every subband is decomposed further to the next level, which may lead to redundancy.

Hybrid LSAC allows the integration of prior knowledge into the level set based active contour model with edge and region information. The adaptation of prior knowledge in this approach proved to produce a more efficient and robust results, especially for medical images. This approach also allows the conventional LSAC model to be extended by incorporating other filtering or segmentation techniques.

2.5 Summary

In this chapter, the related work that has been researched so far in level set based active contour is reviewed in detail. The review covers on edge-based, region-based and hybrid level set based active contour. This review shows that each category of LSAC has it's own strengths and weaknesses. The LSAC model being employed is highly dependent on the problem domain to be solved. It may be concluded that region-based LSAC is the most suitable method to solve the segmentation problem in images with very weak edge points.

CHAPTER 3

THEORETICAL BACKGROUND

3.1 Introduction

This chapter presents the theoretical background on topics that are relevant to this research. Section 3.2 introduces wavelet theory and wavelet multi resolution representation. Section 3.3 presents the theory of supervised Bayes maximum likelihood classifier. Section 3.4 concludes this chapter with level set theory and overview of level set based active contour formulation.

3.2 Wavelet Theory

Wavelet is a mathematical function that allows representation of signal at various frequencies (Mallat 1989). The frequencies are also referred to as scales. Hence, wavelets are used to represent images in multi resolution (multi scale) in order to analyze and capture the image structures at various scales. The focus of this section is to introduce wavelet representation in images.

Wavelet transform analyzes a signal at various time-frequency domain, which represents the signal in multi resolution or multi scale (Graps 1995). As multi resolution and multi scale gives the same definition, these two terms will be used interchangeably throughout this chapter. A signal contains both coarse or fine structures at any given resolution. High resolution exhibits fine signal structure, whereas low resolution exhibits coarse signal structure. The original signal is analyzed using a scalable window, referred to as the *mother wavelet*, which is shifted continuously along the signal. This shifting process is iterated to the desired number of levels by varying the scale (window) size of the mother wavelet, either by dilating or compressing the original mother wavelet at each level. The process of varying the scale or window size is called as the scaling process. The level number corresponds to the iteration number of the shifting and scaling process. This whole process of shifting and scaling of the mother wavelet is called *continuous wavelet transform*. During the shifting of the mother wavelet at each location of the signal, the correlation between the wavelet and the signal is computed. This correlation process is referred to as convolution operation. The correlation value or the coefficients is a measure of the similarity between the signal and the mother wavelet at the corresponding location of the signal being analyzed.

Multi scale wavelet transformation is generated by scaling the mother wavelet and correlating it with the signal. This may be expressed as in the following equation 3.1.

$$\psi_{s,\tau}(x) = \frac{1}{\sqrt{s}}\psi\left(\frac{x-\tau}{s}\right) \tag{3.1}$$

where $\psi_{s,\tau}(x)$ is the scaled and translated function of the mother wavelet $\psi(x)$, s is the scaling factor and τ is the shifting factor.

The continuous wavelet transform generates infinite amount of data, which leads to data redundancy besides being time consuming. As a solution to these problems, the *discrete wavelet transform* has been introduced. The discrete wavelet transform decomposes the signal by discretizing the scaling and shifting factors by powers of two. These discretized scaling and shifting are known as *dyadic* scales and positions. The discrete wavelet transform implementation for multi resolution analysis was proposed by Mallat (1989). In this thesis, Mallat's multi resolution wavelet representation has been adopted and the description herein will cover Mallat's wavelet representation for images.

According to Mallat's multi resolution analysis, given a sequence of increasing resolutions $(r_j)_{j \in Z}$, the structure of an image at resolution r_j are defined as the intensity variation between its approximation at resolution r_j and its approximation at the lower resolution r_{j-1}. This structure of the image at a corresponding resolution provides the coarse to fine intensity variation of the image. The following description explains the concepts of multi resolution representation using wavelet to extract all of the image's intensity variation using the term *signal* referring to the image. A rigorous introduction to the wavelet multi resolution theory is available in (Mallat 1989).

With **Z** and **R** denoting the set of integers and real numbers respectively, let the two-dimensional image signal as a finite energy function $f(x, y) \in \mathbf{L}^2(\mathbf{R}^2)$. $\mathbf{L}^2$ is the Hilbert vector space of square-integrable one dimension functions $f(x, y) \in \mathbf{R}$. A multi resolution approximation of $\mathbf{L}^2(\mathbf{R}^2)$ is a sequence of subspaces $\mathbf{L}^2(\mathbf{R}^2)$, which follows multi resolution properties as stated by Mallat (1989). The multi resolution properties are restated here for explanation purposes:

1. Let A_{2^j} be the approximation operator of signal $f(x, y) \in \mathbf{L}^2(\mathbf{R}^2)$ at resolution 2^j. Operator A_{2^j} is equal to orthogonal projection operator on a vector space $\mathbf{V}_{2^j} \subset \mathbf{L}^2(\mathbf{R}^2)$.

2. The approximation of a signal at resolution 2^{j+1} contains all the information to compute the same signal at finer resolution 2^j. Thus, it is equivalent to

$$\mathbf{V}_{2^j} \subset \mathbf{V}_{2^{j+1}}, \quad \forall\, j \in \mathbf{Z} \tag{3.2}$$

with

$$\bigcup_{j=-\infty}^{+\infty} \mathbf{V}_{2^j} \ is\ dense\ in\ \mathbf{L}^2(\mathbf{R}), \tag{3.3}$$

and

$$\bigcap_{j=-\infty}^{+\infty} \mathbf{V}_{2^j} \ = \ \{0\}\,. \tag{3.4}$$

3. In the dyadic scale, function $f(x, y)$ that belongs to one of the vector spaces can be derived as

   - Dilation

$$f(x) \ \in \ \mathbf{V}_{2^j} \ \Leftrightarrow \ f(2x) \ \in \mathbf{V}_{2^{j+1}}, \quad \forall\, j \ \in \ \mathbf{Z} \tag{3.5}$$

   - Translation

$$f(x) \ \in \ \mathbf{V}_0 \ \Leftrightarrow \ f(x-n) \ \in \mathbf{V}_0, \quad \forall\, n \ \in \ \mathbf{Z} \tag{3.6}$$

Following the multi resolution properties from properties 3.2 to 3.6, let $(\mathbf{V}_{2^j})_{j \in Z}$ be the multi resolution approximation of $\mathbf{L}^2(\mathbf{R}^2)$. The approximation of a signal $f(x, y)$ at a resolution 2^j is equal to its orthogonal projection on the vector space $\mathbf{V}_{2^j}$. Then, a *scaling function*, $\Phi(x, y)$ is introduced, which its' dilation and translation is an orthonormal basis of each space $\mathbf{V}_{2^j}$. Let $\Phi_{2^j}(x, y) = 2^{2^j}\Phi(2^j x, 2^j y)$, hence the family of functions

$$\left(2^{-j}\Phi_{2^j}\left(x - 2^{-j}n,\ y - 2^{-j}m\right)\right)^2_{(n,m)\ \in\ Z} \tag{3.7}$$

forms an orthonormal basis of $\mathbf{V}_{2^j}$. For separable multi resolution approximations of $\mathbf{L}^2(\mathbf{R}^2)$, each vector space $\mathbf{V}_{2^j}$ is decomposed as a tensor product of two identical multi resolution approximation subspaces of $\mathbf{L}^2(\mathbf{R})$

$$\mathbf{V}_{2^j} \ = \ \mathbf{V}^1_{2^j} \ \otimes \ \mathbf{V}^1_{2^j}. \tag{3.8}$$

From equation 3.8, the scaling function $\Phi(x, y)$ can be written as

$$\Phi(x, y) \;=\; \phi(x)\phi(y) \tag{3.9}$$

where $\phi(x)$ is the one-dimensional scaling function of multi resolution approximation $(\mathbf{V}^1_{2^j})_{j\in Z}$. In images, the importance is given to the horizontal and vertical directions with the separable multi resolution approximation. The orthogonal basis of $\mathbf{V}_{2^j}$ is given by

$$\left(2^{-j}\Phi_{2^j}\left(x-2^{-j}n,\; y-2^{-j}m\right)\right)^2_{(n,m)\in Z} \;=\; \left(2^{-j}\phi_{2^j}\left(x-2^{-j}n\right)\;\phi_{2^j}\left(y-2^{-j}m\right)\right)^2_{(n,m)\in Z}. \tag{3.10}$$

Therefore, the *discrete approximation* of signal $f(x, y)$ at a resolution 2^j is characterized by the set of inner products

$$A^d_{2^j}f \;=\; \left(\left\langle f(x,y),\; \phi_{2^j}\left(x-2^{-j}n\right)\;\phi_{2^j}\left(y-2^{-j}m\right)\right\rangle\right)^2_{(n,m)\in Z}. \tag{3.11}$$

The intensity variation between the approximation of a function $f(x, y)$ at the resolutions 2^{j+1} and 2^j is called the *detail signal*. The *detail signal* at the resolution 2^j is equal to the orthogonal projection of the signal on the orthogonal complement of $\mathbf{V}_{2^j}$ in $\mathbf{V}_{2^{j+1}}$. Given $\mathbf{O}_{2^j}$ as the orthogonal complement,

$$\mathbf{O}_{2^j}\; is\; orthogonal\; to\; \mathbf{V}_{2^j},\; \mathbf{O}_{2^j} \;\oplus\; \mathbf{V}_{2^j} \;=\; \mathbf{V}_{2^{j+1}}. \tag{3.12}$$

The orthonormal basis of $\mathbf{O}_{2^j}$ is build by scaling and translating three wavelet functions, $\Psi^1(x, y)$, $\Psi^2(x, y)$ and $\Psi^3(x, y)$. Let $\Phi(x, y) = \phi(x)\phi(y)$ be the associated two-dimensional scaling function and $\psi(x)$ be the one-dimensional wavelet associated with the scaling