Extension of Markowitz Model for Portfolio Analysis 27207

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Abstract: - This paper focused on portfolio analysis that set-up among 10 selected stocks traded on Kuala Lumpur Stock Exchange (KLSE). Markowitz model is the main method used to build the optimal portfolio for this paper. There are two type of analysis were conducted in this paper which are daily analysis and weekly analysis. Among this 2 analysis, weekly analysis provides a higher profit level with lower risk level than daily analysis.

Key-Words: - Markowitz portfolio theory, expected return and risk, a risk-free rate.

1 Introduction

This paper will focus on stock investment through setting up an optimal portfolio based on the Markowitz model. The objective is to select an optimal portfolio which provides the most optimum return and lower risk among 10 selected stocks in KLSE.

Markowitz model assumes an investor has 2 considerations when constructing an investment portfolio: expected return and variance (risk) in return. The optimal expected return on a portfolio can be written as:

$$\overline{R}_p = \sum_{i=1}^{N} X_i \overline{R}_i \tag{1}$$

The expected risk of a portfolio can be written as:

$$\sigma_p{}^2 = \sum_{i=1}^{N} X_i{}^2 \sigma_i{}^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \tag{2}$$

This paper also constructs a portfolio from combining a risk-free asset with a risky portfolio. The fraction of funds that the investor should place in each selected stock will be determined by using Markowitz model.

2 Theory

2.1 Markowitz Portfolio Theory

The Markowitz portfolio model has had a profound effect on the investment industry. The basic model was developed Harry Markowitz, who derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under a reasonable set of assumptions, and he derived the formulas for computing the variance of a portfolio. This formula for the variance of a portfolio not only indicated the importance of diversifying the investments to reduce the total risk of a portfolio, but also showed how to effectively diversify.

2.1.1 The Expected Return and Risk of an Individual Security

The investor wished to maximize expected return from the portfolio he would place all his funds in that security with maximum expected returns. The expected return of the *i*th security can be written as:

$$R_i = \sum_{t=1}^{\infty} d_{it} r_{it} \tag{3}$$

where:

r_{it} = the anticipated return (however decided upon) at time t per ringgit invested in security i

d_{it} = the rate at which the return on the ith security at time t is discounted back to the present.

As noted, the variance or standard deviation is a measure of the variation of return, R_i from the expected of return [E (R_i)], as follows:

Variance,

$$\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}[R_i - E(R_i)]^2 \quad (4)$$

Standard deviation,

$$\sigma_i = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[R_i - E(R_i)]^2} \quad (5)$$

In portfolio analysis, the covariance of return usually is concerned rather than prices or some other variable. For two securities, i and j, the covariance of return is defined as:

$$\sigma_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\} \quad (6)$$

Covariance is affected by the variability of the two individual return series. Obviously, this covariance may be measured by taking into consideration the variability of the two individual return series as follows:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \quad (7)$$

where

ρ_{ij} = the correlation coefficient of returns

σ_i = the standard deviation of Rit

σ_j = the standard deviation of R_{jt}

2.1.2 The Expected Return and Risk for a Portfolio

Let X_i is the fraction of the investor's funds invested in the ith security, and then the return on the portfolio is

$$R_p = \sum_{t=1}^{\infty}\sum_{i=1}^{N} d_{it} r_{it} X_i = \sum_{i=1}^{N} X_i\left(\sum_{t=1}^{\infty} d_{it} r_{it}\right)$$
$$= \sum_{i=1}^{N} X_i R_i \quad (8)$$

where R_i is independent of X_i.

The expected return is also a weighted average of the expected returns on the individual securities. Taking the expected value of the expression just given for the return on a portfolio yields

$$\bar{R}_p = E(R_p) = E\left(\sum_{i=1}^{N} X_i R_i\right) = \sum_{i=1}^{N} X_i \bar{R}_i \quad (9)$$

The variance on a portfolio is a little more difficult to determine than the expected return. As noted, Harry Markowitz derived the general formula for the variance of a portfolio as follows:

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \quad (10)$$

2.1.3 Combining a Risk-free Asset with a Risky Portfolio

When one of the investments available is risk-free, then the optimal portfolio composition has a particularly simple form. Then the return on such a portfolio (Sharpe, W, 1964) is:

$$R_p = \sum_{i=1}^{N} X_i R_i + X_{N+1} R_f \quad (11)$$

Therefore the expected return of a portfolio that includes a risk-free asset is written as:

$$\bar{R}_p = E(R_p) = \sum_{i=1}^{N} X_i \bar{R}_i + X_{N+1} R_f \quad (12)$$

where

X_{N+1} =the fraction invested in the risk- free rate

R_f = the risk-free rate of return

The R_i is considered to be random variables. The X_i are not random variables, but are fixed by the investor. Since the X_i is percentages we have

$$\sum_{i=1}^{N+1} X_i = 1 \quad (13)$$

Because the returns for the risk-free asset are certain, σ_{R_f} = 0, which means $R_i - E(R_i)$ will also equal zero, and the product of this expression with any other expression will equal zero. Consequently, the covariance of the risk-free asset with any risky asset or portfolio of assets will always equal zero. Similarly, the correlation between any risky asset i, and the risk-free asset, R_f, would be zero. Therefore, the expected variance (risk) of a portfolio by combining a risk-free asset with a portfolio of risky assets such as those that exist on the Markowitz efficient frontier, is written as (Markowitz .H, 1952)

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} X_i X_j \sigma_{ij} \quad (14)$$

3 Methodology

The data used in this study were the daily closing prices of 10 selected stocks traded on KLSE. This study focuses only on 10 selected stocks that were traded from 15th October 2002 until 18th March 2003 that is 100 trading days in KLSE. Selection of 10 stocks for this study's analysis was according to the most volume traded in KLSE from 11th March 2003 until 15th March 2003. Interest rate was collected that was release by Bank Negara Malaysia on 18th March 2003 and the rate is 3% annually.

Analysis of data:

i. Stock's profit: $R_i = \dfrac{P_t - P_{t-1}}{P_{t-1}}$ and

$$\overline{R}_i = \frac{1}{N}\sum_{t=1}^{N} R_{it}$$

ii. Stock's profitable risk (variance):

$$\sigma_i^2 = \frac{1}{N-1}\sum_{t=1}^{N}\left(R_{it} - \overline{R}_i\right)^2$$

Covariance between any two stocks:

$$\sigma_{ij} = \frac{1}{N-1}\sum_{t=1}^{N}\left(R_{it} - \overline{R}_i\right)\left(R_{jt} - \overline{R}_j\right)$$

iii. Excess return to standard deviation:

$$ERSD = \frac{\overline{R}_i - R_f}{\sigma_i}$$

iv. Cut-off rate (C_i):

$$C_i = \frac{\sum_{j=1}^{i-1} \rho_{ij} \dfrac{\overline{R}_j - R_f}{\sigma_j}}{\sum_{j=1}^{i} \rho_{ij}}$$

v. Stock selection into portfolio is depend to value of *ERSD* and C_i^*. If the value of $ERSD > C_i^*$, then the stock is selected into portfolio or vice versa.

vi. The percentage invested in each selected stock in portfolio:

$$X_i = \frac{Z_i}{\sum_{j=i}^{N}\left|Z_j\right|}$$

where $Z_i = \dfrac{1}{\sigma_i}\left(\dfrac{\overline{R}_i - R_f}{\sigma_i} - C^*\right)$

vii. Coefficient variation:

$$\text{CV} = \frac{\sigma_p}{R_p}$$

4 Result and Analysis

4.1 Daily analysis

Table 1 Stock Return and Variance (Daily Analysis)

Stock	Mean Return $\overline{R}_i$ (x 10^{-3})	Excess Return $\overline{R}_i - R_f$ (x10^{-3})	Variance σ_i^2 (x 10^{-3})	*ERSD* (x10^{-2})
IOI	0.234	0.1518	0.467	0.7025
MAYBANK	0.020	-0.0622	0.296	-0.3615
AMMB	-1.620	-1.7022	0.515	-7.5008
BAT	0.510	0.4278	0.055	5.7686
SIME	0.465	0.3828	0.187	2.7994
COMMERZ	-0.200	-0.2822	0.341	-1.5282
PBB	-0.420	-0.5022	0.147	-4.1420
AFFIN	-4.290	-4.3722	0.415	-21.4622
KEMAS	0.738	0.6558	1.514	1.6854
GENTING	1.382	1.2998	0.294	7.5806

From Table 1, there are 6 stocks provide positive return with the highest value recorded by Genting which is 0.001382. On the other hand, the lowest return recorder by Affin with -0.00429. For the risk level, Kemas recorded the highest value of risk which is 0.001514 and the lowest risk recorder by BAT with 0.000055. Here it shows that profit level and risk level is positively correlated but with just a small correlation coefficient value.

Table 2 *ERSD*, C_i, Z_i and Capital Investment Proportion (X_i)

Stock *i*	*ERSD*	C_i	*ERSD*><C_i	Z_i	X_i
GENTING	0.075806	0.000000	>	5.4394	0.1510
BAT	0.057686	-0.000571	>	10.1327	0.2813
SIME	0.027994	0.014695	>	3.3239	0.0923
K.EMAS	0.016854	0.012651	>	0.8819	0.0245
IOI	0.007025	0.012416	<	1.1330	0.0314
MAYBANK	-0.003615	0.015978	<	0.8047	0.0223
COMMERZ	-0.015282	0.017589	<	0.1180	0.0033
PBB	-0.041420	0.013552	<	-1.9762	-0.0548
AMMB	-0.075008	0.003056	<	-2.5359	-0.0704
AFFIN	-0.214622	-0.017460	<	-9.6783	-0.2687

In Table 2, there are 4 counters show their *ERSD* value is greater than C_i. These counter are Genting, BAT, SIME and K.EMAS. Among these 4 counters, the largest capital investment proportion will go to BAT with 28.13%, following by Genting with 15.10%, SIME with 9.23%, and the smallest proportion is 2.45% by counter K.EMAS. By using equation (2.13), the proportion invested in the risk-free rate of interest is 45.09%.

Table 3 The Expected Return and Risk on an Optimal Portfolio (Daily Analysis)

	$\overline{R}_i$	X_i	$X_i\overline{R}_i$	σ_i^2
GENTING	0.001382	0.1510	0.000209	0.000294
BAT	0.000510	0.2813	0.000143	0.000055
SIME	0.000465	0.0923	0.000043	0.000187
K.EMAS	0.000738	0.0245	0.000018	0.001514
R_f	0.000082	0.4509	0.000037	0.000000

The expected Return of an optimal portfolio is

$$R_p = 0.000209 + 0.000143 + 0.000043 + 0.000018 + 0.000037 = 0.00045 = 0.045\%$$

The variance (risk) of an optimal portfolio is

$$\sigma_p^2 = 0.00001616$$

4.2 Weekly analysis

Data on each Tuesday's closing prices is used to process the analysis of weekly data. In the event of Tuesday being a holiday, the most recently available closing price for the stock was used. There are 23 data are used for weekly analysis.

Table 4 Stock Return and Variance (Weekly Analysis)

Stock	Mean Return $\overline{R}_i$ (x 10^{-3})	Excess Return $\overline{R}_i - R_f$ (x10^{-3})	Variance σ_i^2 (x 10^{-3})	*ERSD*
IOI	1.285	0.708	2.424	0.0144
MAYBANK	0.005	-0.572	1.168	-0.0167
AMMB	-7.264	-7.841	2.398	-0.1601
BAT	2.222	1.645	0.095	0.1688
SIME	1.798	1.221	0.247	0.0777
COMMERZ	-0.966	-1.543	1.380	-0.0415
PBB	-2.066	-2.643	0.340	-0.1434
AFFIN	-18.944	-19.521	2.144	-0.4216
KEMAS	1.952	1.375	4.039	0.0216
GENTING	6.354	5.777	1.598	0.1589

In Table 4, the result is almost the same as daily analysis with 6 counters recorded positive return level with highest value is 0.006354 by Genting. There are 4 counters recorded negative return level and the lowest return level also recorded by Affin with -0.018944. On the other hand, the highest risk level recorded by KEMAS with 0.002144 and the lowest risk level is 0.000095 by BAT. If comparing relation between returns and risk level, this weekly analysis shows a negative value of correlation coefficient. This is good sign that high return obtained with a lower risk level for weekly analysis.

Table 5 *ERSD*, C_i, Z_i and Capital Investment Proportion (X_i)

Stock *i*	*ERSD*	C_i	*ERSD*><C_i	Z_i	X_i
GENTING	0.168848	0.000000	>	20.0657	0.3918
BAT	0.158927	-0.067277	>	4.6417	0.0906
SIME	0.077663	0.003111	>	6.6346	0.1295
K.EMAS	0.021637	0.038184	<	0.7599	0.0149
IOI	0.014374	0.044116	<	0.8333	0.0163
MAYBANK	-0.016737	0.030130	<	0.2902	0.0057
COMMERZ	-0.041528	0.041248	<	-0.4004	-0.0078
PBB	-0.143427	0.016571	<	-6.3371	-0.1237
AMMB	-0.160124	-0.021782	<	-2.7257	-0.0532
AFFIN	-0.421582	-0.026652	<	-8.5290	-0.1665

In Table 5, the C* when all stocks are included is C* = C_{10} = -0.026652. Recall that stocks have an *ERSD* above C* are held long, but stocks with an *ERSD* below C* are sold short. Thus, the value of C* = -0.026652 implies that the first 6 counters are held long and counters 7 to 10 are sold short. There are 3 counters with a ratio of *ERSD* above C_i. So these 3 counters are going to form the optimal portfolio for weekly analysis. These 3 counters are BAT, Genting and SIME. The largest capital investment proportion will go to BAT with 39.18%, SIME with 12.95% and Genting with 9.06%. Therefore, the proportion invested in the risk-free rate of interest is 38.81%.

Table 6 The Expected Return and Risk for optimal Portfolio

Stock i	$\overline{R}_i$	X_i	$X_i\overline{R}_i$	σ_i^2
BAT	0.002222	0.3918	0.000871	0.000095
GENTING	0.006354	0.0906	0.000576	0.001598
SIME	0.001798	0.1295	0.000233	0.000247
R_f	0.000577	0.3881	0.000224	0.000000

From Table 6, the expected return for this weekly optimal portfolio is

$$R_p = 0.000871{+}0.000516{+}0.000233{+}\ 0.00022 = 0.001903\ \ = 0.19\%$$

The expected variance (risk) for this optimal portfolio is

$$\sigma_p = 0.0000243$$

4.3 Coefficient of Variation

Therefore, comparison have made between daily and weekly analysis to choose the analysis that provide high return and low risk through coefficient variation value.

Daily analysis portfolio:

$$\mathrm{CV} = \frac{\sigma_p}{R_p} = \frac{\sqrt{0.00001616}}{0.00045} = 8.9332$$

Weekly analysis portfolio:

$$\mathrm{CV} = \frac{\sigma_p}{R_p} = \frac{\sqrt{0.00002430}}{0.001903} = 2.5904$$

Based on the coefficient variation, weekly analysis showed a lower CV value that is 2.5904 while daily analysis is 8.9332. This result indicate that portfolio of weekly analysis will be chosen and this portfolio will provide a higher return with certain risk level if compared to daily analysis.

5 Conclusions

This study focused on the expected return for an optimal portfolio of securities and an expected risk measure by using the Markowitz portfolio model. The criteria of choosing stock into optimal portfolio involve the comparison of excess return to standard deviation (*ERSD*) with cut-off rate (C_i). Besides this, we also have to consider the value of expected return of each stock.

For daily analysis, there are 5 counters forming the optimal portfolio with expected return of this portfolio is 0.045% at 0.00001616 risk level. These 5 counters are BAT, Genting, SIME, K.EMAS and a risk-free rate of interest with capital investment proportion 28.13%, 15.10%, 9.23%, 2.45% and 45.09% respectively.

On the other hand, there are just 4 counters selected into optimal portfolio of weekly analysis with expected return of 0.19% with risk level of 0.0000243. These 4 counters are BAT, Genting, SIME, and a risk-free rate of interest with capital investment proportion 39.18%, 9.06%, 12.95% and 38.81% respectively.

Coefficient variation value for daily analysis is 8.9332 and if compared to weekly analysis which is 2.5904, it shows that the optimal portfolio of weekly analysis provides higher return level with lower risk level. This mean the best portfolio is from weekly analysis that is combination of counter BAT, Genting, SIME and a risk-free rate of interest.

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