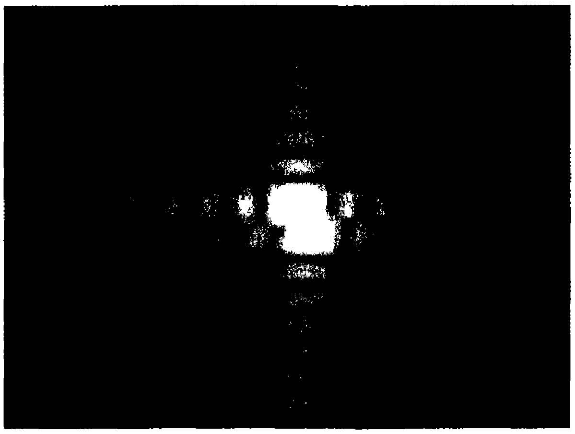
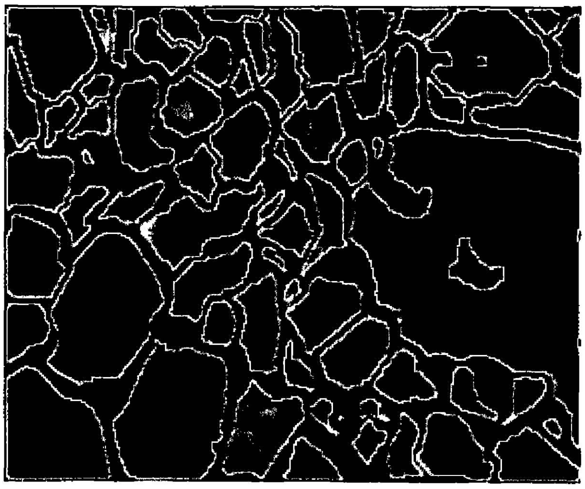
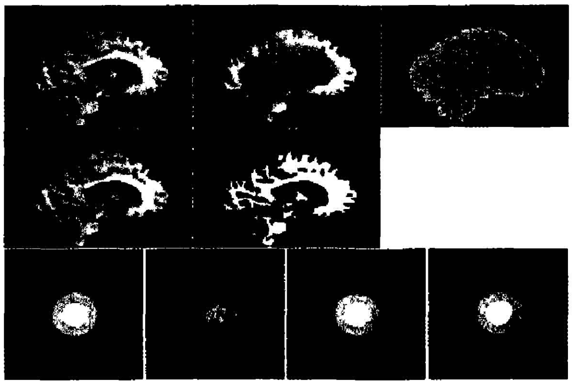
CGIV05

# Computer Graphics, Imaging and Vision

Institute of Automation, Chinese Academy of Science, Beijing, China
26 - 29 July 2005







**Editors:**
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# Proceedings

# International Conference on

# Computer Graphics, Imaging and Visualization 2004

**Institute of Automation, Chinese Academy of Science, Beijing, China**
**26 - 29 July 2005**

**Editors:**
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Bottom right

**"An Active Contour Model For Selective Segmentation",** Wang Wenyuan

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## Table of Contents

# Imaging

### Imaging & Vision

### Imaging - Recognition

### Intelligent Recognition Techniques, Applications, Systems & Tools - IRTAST

## CG - Simulation

## Animation &C. Games - DART

## Digital Entertainment: Games

## CG - Simulation

## Animation &C. Games - DART

## Digital Entertainment: Games

# Preface

During the past decades, computer science has experienced a period of unprecedented progress. This fast, near dazzling pace denotes the need and use of information that governs all walks of life and interests. Computer Graphics, Imaging and Vision (CGIV), in particular, occupies a pivotal platform in this route - it has shifted the concept of visualisation to a more mature state whereby it has become synonymous with a "Visual Culture" that gives a pleomorphic concept its skeleton and shape.

Computer Graphics, Imaging and Vision (CGIV05), is the third conference on Visualisation and Graphics that aims to explore the integration of science and art through the medium of compute technology. This conference is associated with the series of internationa conference on Information Visualisation (IV) that is held annually. Originally, it was a successfu symposium in the IV forum and has changed names from CAGD to GMAG and now it is mature to encompass range of related areas under the name of CGIV. We were overwhelmed by th success and response to this conference and by the outstanding originality and diversity of th intellectual focus brought to bear by the conference's contributors.

This year, CGIV05 once again aims to reflect the intellectual breadth, research, and the empiric scope of the evolving spectra of Computer Graphics, Imaging and Vision in the light of the visu culture. Joining us in this endeavour are some 130 researchers who have chosen to share chapter of their efforts with other fellow researchers and all of those who are in pursuit of the jo of knowledge.

With the hope that this conference would open even wider awareness of research, we welcom you to CGIV05.

Muhammad Sarfraz
Yangsheng Wang
Ebad Banissi

## Acknowledgments

The $2^{nd}$ Conference on Computer Graphics, Imaging, and Vision was organized by Chinese Academy of Science, Institute of Automation - CASIA -, GraphicsLink and Visualisation and Graphics Research Unit - VGRU - London South Bank University in London UK.

Through out the preparing stages of this conference, we have received invaluable help from our colleagues at Chinese Academy of Science, Institute of Automation - CASIA - who hosted the conference this year.

We are particularly in debt to the colleagues of CASIA for their day to day support and help to facilitate the local operation particularly Peng Lu and Ge Li. We also received encouragement and support from team members of VGRU-Visualisation & Graphics Research Unit of London South Bank University, professor Anna Ursyn of University of Northern Colorado, professor Gordon Clapworthy, University of Luton, UK and GraphicsLink for their financial support.

We are in debt to all the authors for their contribution and reviewers for their infinite patience and helpful suggestions and ideas, who reviewed the papers and helped us with their expertise and thoughtful advice and consequently made this event a truly joint project.

The conference event and the publications are the work of many members of the Programme committee, the organising committee, all the reviewers, technical members and administrative team and most importantly, the contributors to whom we remain beholden in every sense of the word.

Finally, we would offer our sincere thanks to Anita D'Pour of GraphicsLink for her continuous effort in preparing, organizing and handling of the conference administration. Our appreciation also goes to Peng Lu, Ge Li and other colleagues at CASIA, for producing high standard editorial production of the Proceedings of conference papers.

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# Second Order Partial Derivatives Estimation Using a Modification of the Lagrange Quadratic Interpolant

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## Abstract

*Partial derivative values are required to construct a smooth interpolated surface which passes through given three dimensional scattered data points These partial derivative values are not available in practice for raw three dimensional scattered data and the estimation of these values is thus desirable This research concentrates on estimating the partial derivative values up to the second order Many current methods concentrate on first order partial derivatives estimation as most applications require only up to the first order derivatives Higher order derivatives are required for higher continuity conditions in the interpolating surfaces In this paper, we use a modification of the Lagrange quadratic interpolant to estimate second order partial derivatives This method does not require results from the first order derivatives, hence, we are able to estimate the second order partial derivatives directly from the three dimensional scattered data points Standard test functions are used for evaluating our estimated results*

*Keywords*--- **Second order partial derivatives estimation.**

## 1. Introduction

The purpose of this work is to develop a method to estimate the second order partial derivatives at three dimensional scattered data points The only data available are just the coordinate values of these data points and not the derivative values To facilitate the smooth joining of interpolating piecewise curves or surfaces at common vertices or boundaries, partial derivative values are required The order of the continuity of the adjoining piecewise curves or surfaces depends on the partial derivative values available With first order partial derivative values, we can generate surfaces with tangential continuity across the boundaries. With first and second order partial derivative values, we can generate patchwise surfaces that join with curvature continuity

There are ways to estimate the derivative values at the data points. Global estimation implies the estimation based on the entire set of data points available in the domain where all the data points influence the estimation at each and every data point. Such global influence might cause inaccuracies of the estimated partial derivatives Peaks or dips at certain places in the domain influence the derivative estimation even if they may be quite a distance away. Local estimation methods are preferred as they require less computation and the accuracy is better

In this paper, we introduce a new method that can be used to estimate second order partial derivatives This method involves the computation of the planar gradients and the projected distances parallel to the $x$ and $y$ axes on the $x$-$y$ plane It requires less computation compared to other currently known methods and yet provides good estimated results The given data points are first of all triangulated and then base distances parallel to the $x$ and $y$ axes on the $x$-$y$ plane are calculated from the obtained triangles to be used in our method

This paper concludes with a comparison of the results obtained by using this method with results from other methods available in the literature, carried out using the same set of data points and test functions.

## 2. Lagrange quadratic interpolant

Let us introduce the Lagrange quadratic interpolant In general, we consider a two-dimensional case where a unique quadratic interpolant, also known as a parabolic function, in the Lagrange form interpolates three consecutive univariate data points $P_{i-1}=(x_{i-1},y_{i-1})$, $P_i=(x_i,y_i)$ and $P_{i+1}=(x_{i+1},y_{i+1})$ as shown in Figure 1,

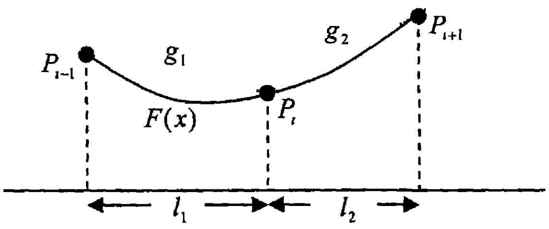

**Figure 1 Three consecutive data points, $P_{i-1}$, $P_i$ and $P_{i+1}$**

where $l_1=x_i-x_{i-1}$, $l_2=x_{i+1}-x_i$, $g_1=\frac{y_i-y_{i-1}}{l_1}$ and $g_2=\frac{y_{i+1}-y_i}{l_2}$.

The quadratic interpolant can be written as follows.

$$\begin{aligned}F(x)=&\frac{(x-x_i)(x-x_{i+1})}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})}y_{i-1}\\&+\frac{(x-x_{i-1})(x-x_{i+1})}{(x_i-x_{i-1})(x_i-x_{i+1})}y_i\\&+\frac{(x-x_{i-1})(x-x_i)}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}y_{i+1}\end{aligned}\qquad(1)$$

By differentiating the parabolic function (1) with respect to x, the first order derivative can be obtained as

$$\begin{aligned}F'(x)=&\frac{2x-x_i-x_{i+1}}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})}y_{i-1}\\&+\frac{2x-x_{i-1}-x_{i+1}}{(x_i-x_{i-1})(x_i-x_{i+1})}y_i\\&+\frac{2x-x_{i-1}-x_i}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}y_{i+1}\end{aligned}\qquad(2)$$

Differentiating for the second time with respect to x, we obtain the second order derivative as follows:

$$\begin{aligned}F''(x)=&\frac{2}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})}y_{i-1}\\&+\frac{2}{(x_i-x_{i-1})(x_i-x_{i+1})}y_i\\&+\frac{2}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}y_{i+1}\end{aligned}\qquad(3)$$

## 3. Second order derivatives estimation

Let us suppose $P_{i-1}$ is an end-point and $P_i$ is an inner point We can substitute $x=x_{i-1}$ or $x=x_i$ into equation (3) and simplify it to obtain

$$F''(x_{i-1})=\frac{2(g_2-g_1)}{l_1+l_2}\quad\text{and}\quad F''(x_i)=\frac{2(g_2-g_1)}{l_1+l_2}\qquad(4)$$

the second order derivatives at the end-point and inner point of a set of univariate data points.

Analogously, we are now able to extend the second order derivatives estimation for the two dimensional case as in (4) to that for the three dimensional case. We want to estimate the partial derivatives at three dimensional scattered data points. For this method we are proposing the initial following steps have to be carried out:

1. Take the projection of all the data points ont the $x$-$y$ plane.
2. Triangulate these projected points using, fo example, the Delaunay triangulation metho (Brassel & Reif [2], Low et al. [8]). We wi have triangles on the x-y plane with th projected points as the vertices
3. For each point, draw two lines passing throug it, one parallel to the $x$-axis and another parall to the $y$-axis. This allows us to calculate th base distances, i.e. distances on the $x$-$y$ pla from each point to the opposite sides triangles that intersect the lines parallel to the axis and the $y$-axis This is illustrated in Figu 2.

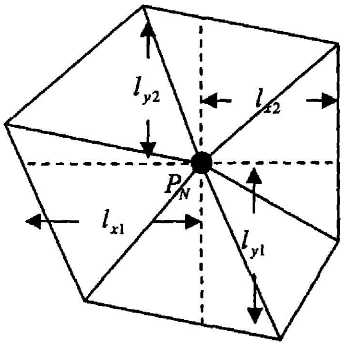

**Figure 2 Inner point $P_N$ with base distance parallel to the $x$-axis and the $y$-axis**

4. If the point in question is a boundary p based on Step No. 3, we can also calculate distance along the line parallel to the $x$-axi the $y$-axis, if applicable) from the opposite of the triangle to the intersecting side of adjoining triangle. This is illustrated in Fig for the case of a line parallel to the $x$-axis. distances for the line parallel to the $y$ follow analogously.

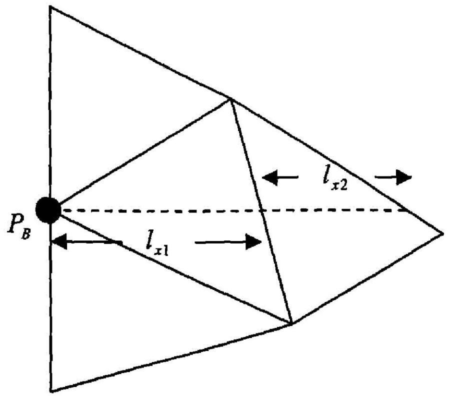

**Figure 3 Boundary point $P_B$ with base distances parallel to the $x$-axis**

5. We let $g_{x1}$, $g_{x2}$, $g_{y1}$ and $g_{y2}$ be the gradients of the pre-projected triangular planes with base distances $l_{x1}$, $l_{x2}$, $l_{y1}$ and $l_{y2}$ respectively.

After we have carried out the initial steps Nos 1 to 5, we adapt the two equations in (4) to estimate the second order partial derivatives at each scattered data point as follow:

$$\frac{\partial^2 F}{\partial x^2}=\frac{2(g_{x2}-g_{x1})}{l_{x1}+l_{x2}} \quad \text{and} \quad \frac{\partial^2 F}{\partial y^2}=\frac{2(g_{y2}-g_{y1})}{l_{y1}+l_{y2}} \qquad (5)$$

By rearranging the estimation of the second order partial derivatives with respect to $x$ and $y$ as shown in Equations (5), we can obtain the second order partial derivative with respect to $x$ followed by $y$ and the derivative with respect to $y$ followed by $x$ as

$$\frac{\partial^2 G}{\partial y\partial x}=\frac{2(g_{x2}-g_{y1})}{l_{x2}+l_{y1}} \quad \text{and} \quad \frac{\partial^2 G}{\partial x\partial y}=\frac{2(g_{y2}-g_{x1})}{l_{y2}+l_{x1}} \qquad (6)$$

Theoretically, the second order differentiation with respect to $x$ followed by $y$ should be identical to the differentiation with respect to $y$ followed by $x$. We now use convex combination to merge both estimated second order derivatives shown in Equations (6) using the base lengths as weights as follow

$$\frac{\partial^2 F}{\partial x\partial y}=\frac{\partial^2 F}{\partial y\partial x}=\frac{(l_{x2}+l_{y1})\frac{\partial^2 G}{\partial x\partial y}+(l_{y2}+l_{x1})\frac{\partial^2 G}{\partial y\partial x}}{l_{x1}+l_{x2}+l_{y1}+l_{y2}} \qquad (7)$$

$$=\frac{\frac{l_{x2}+l_{y1}}{l_{y2}+l_{x1}}(g_{y2}-g_{x1})+\frac{l_{y2}+l_{x1}}{l_{x2}+l_{y1}}(g_{x2}-g_{y1})}{l_{x1}+l_{x2}+l_{y1}+l_{y2}}$$

In the case of boundary points where we will only obtain either one of the two equations in (6), we cannot use the convex combination shown in (7). In this case, we take $\frac{\partial^2 F}{\partial x\partial y}$ and $\frac{\partial^2 F}{\partial y\partial x}$ as either one of the two obtainable equations in (6) without the convex combination (7)

## 4. Tests and Results

In this section, we test our suggested method and compare it with the Simple Quadratic Method (Chang [3]) The test domain is a set of 36 data points taken from Whelan [11] (see Table 1) and triangulated using Delaunay's method as shown in Figure 4

**Table 1 The 36 test data points**

| Point No. | Coordinates $X$ | $Y$ | Point No. | Coordinates $x$ | $y$ |
|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 19 | 0.80 | 0.85 |
| 2 | 0.50 | 0.00 | 20 | 0.85 | 0.65 |
| 3 | 1.00 | 0.00 | 21 | 1.00 | 0.50 |
| 4 | 0.15 | 0.15 | 22 | 1.00 | 1.00 |
| 5 | 0.70 | 0.15 | 23 | 0.50 | 1.00 |
| 6 | 0.50 | 0.20 | 24 | 0.10 | 0.85 |
| 7 | 0.25 | 0.30 | 25 | 0.00 | 1.00 |
| 8 | 0.40 | 0.30 | 26 | 0.25 | 0.00 |
| 9 | 0.75 | 0.40 | 27 | 0.75 | 0.00 |
| 10 | 0.85 | 0.25 | 28 | 0.25 | 1.00 |
| 11 | 0.55 | 0.45 | 29 | 0.00 | 0.25 |
| 12 | 0.00 | 0.50 | 30 | 0.75 | 1.00 |
| 13 | 0.20 | 0.45 | 31 | 0.00 | 0.75 |
| 14 | 0.45 | 0.55 | 32 | 1.00 | 0.25 |
| 15 | 0.60 | 0.65 | 33 | 1.00 | 0.75 |
| 16 | 0.25 | 0.70 | 34 | 0.19 | 0.19 |
| 17 | 0.40 | 0.80 | 35 | 0.32 | 0.75 |
| 18 | 0.65 | 0.75 | 36 | 0.79 | 0.46 |

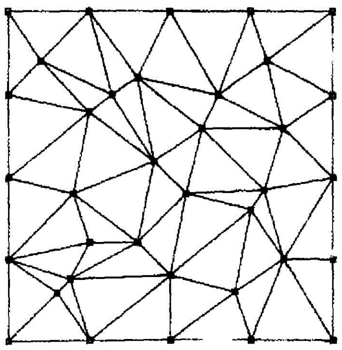

**Figure 4 Delaunay triangulation of the 36 test data points**

For the test functions, we use two well-known functions, i e

1. Franke's exponential function

$$F1(x,y)=0.75e^{-\left(\frac{(9x-2)^2+(9y-2)^2}{4}\right)}+0.75e^{-\left(\frac{(9x+1)^2}{49}+\frac{9y+1}{10}\right)}$$
$$+0.50e^{-\left(\frac{(9x-7)^2+(9y-3)^2}{4}\right)}-0.20e^{-\left((9x-4)^2+(9y-7)^2\right)}$$

2. Saddle function

$$F2(x,y)=\frac{1.25+\cos(5.4y)}{6+6(3x-1)^2}$$

We table in Table 2 the mean absolute errors from our suggested method and from the Simple Quadratic Method (SQM)

**Table 2 Mean absolute errors from our method and the SQM**

| | | Franke | | Saddle | |
|---|---|---|---|---|---|
| | | Our method | SQM | Our Method | SQM |
| $\frac{\partial^2 F}{\partial x^2}$ | Non-boundary | 4 188605 | - | 0 637816 | - |
| | Boundary | 1 066708 | - | 0 694207 | - |
| | Overall | 2 801096 | 10 78880 | 0 662879 | 1 11517 |
| $\frac{\partial^2 F}{\partial y^2}$ | Non-boundary | 4 899358 | - | 0 729190 | - |
| | Boundary | 1 390451 | - | 0 736823 | - |
| | Overall | 3 339844 | 9 73624 | 0 732583 | 0 77222 |
| $\frac{\partial^2 F}{\partial x \partial y}$ | Non-boundary | 5 404631 | - | 1 108742 | - |
| | Boundary | 3 677457 | - | 1 129274 | - |
| | Overall | 4 693442 | 5 78621 | 1 117196 | 0 73083 |

## 5. Discussions and Conclusions

In this paper, we introduce a new method for the estimation of second order partial derivatives at 3-dimensional scattered data points The scheme is simple to understand and implement For the estimation of second order derivatives at non-boundary points, the shape of the domain may be irregular However, we admit there are drawbacks to our suggested method when estimating second order derivatives at boundary points, i.e

1 For the estimation at boundary points, the domain must be rectangular
2 The estimation of mixed derivatives, $\frac{\partial^2 F}{\partial x \partial y}$ or $\frac{\partial^2 F}{\partial y \partial x}$, cannot be done at the top right corner point and also cannot be done at the bottom left corner point

Whether we can overcome these drawbacks or not, can perhaps be the topics for future research.

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