Philosophical perspectives on ad hoc-hypotheses and the Higgs mechanism

Simon Friederich,

Philosophisches Seminar, Unversität Göttingen, Humboldtallee 19, D-37073 Göttingen, Germany

email: email@simonfriederich.eu

Robert Harlander,

Theoretische Teilchenphysik, Fachbereich C, Universität Wuppertal, D- 42097 Wuppertal, Germany

email: robert.harlander@uni-wuppertal.de

Koray Karaca,

University of Wuppertal, Interdisciplinary Centre for Science and Technology Studies (IZWT),
Gaußstr. 20, D-42119 Wuppertal, Germany

email: karacak@gmail.com

Abstract: We examine physicists' charge of adhocness against the Higgs mechanism in the Standard Model of elementary particle physics. We argue that even though this charge never rested on a clear-cut and well-entrenched definition of "ad hoc", it is based on conceptual and methodological assumptions and principles which are well-founded elements of the scientific practice of high-energy particle physics. Based on our findings, we dispute the claim made by Christopher Hunt in a recent article in *Philosophy of Science* that the use of "ad hoc" by scientists reflects nothing more substantial than a judgment made on the basis of their "individual aesthetic senses". We further evaluate the implications of the recent discovery of a Higgs-like particle at the CERN Large Hadron Collider for the charge of adhocness against the Higgs mechanism.

> "Of course our model has too many arbitrary features for these predictions to be taken very seriously […]" (Weinberg 1967, p. 1265)

1. Introduction

The Higgs mechanism (HM) is a crucial part of the Standard Model (SM) of elementary particle physics. Its main purpose is to account consistently and in accordance with experimental results for the non-vanishing masses of elementary particles in the SM. Recent experimental findings at the Large Hadron Collider (LHC) in Geneva at CERN have been interpreted as a direct detection of a particle with properties as expected for the Higgs particle, which would be a spectacular confirmation of this picture.[1] In its role as part of the SM, involving a single fundamental scalar particle, we refer to the HM in what follows as the "Standard Model Higgs Mechanism" (SMHM).

Despite its predictive and explanatory success, particle physicists as well as philosophers of physics have expressed qualms about the SMHM. It has been widely regarded as problematic in several important respects. These qualms and worries have often been summarized by calling the SMHM "ad hoc".

The ad hoc-charge against the SMHM is interesting from a philosophical point of view on several grounds. First, the claim that our currently best theory of fundamental particle physics is based on an ad hoc-hypothesis sounds alarming and is certainly worthy of consideration in itself. Second, there exists a longstanding philosophical debate about the notion of adhocness, to which eminent philosophers of science such as Popper, Lakatos, Schaffner, Grünbaum, Leplin and others have made important contributions. This gives rise to the question of whether the SMHM qualifies as "ad hoc" according to any of these philosophers' accounts of adhocness, and what the possible ramifications would be. Third, it seems natural to ask what impact the recent experimental discovery of a Higgs-like particle at the LHC has on the status of the ad hoc-charge against the SMHM.

In the present work we try to assess this ad hoc-charge in a systematic manner. Section 2 gives a brief non-technical introduction to the SMHM. Section 3 reviews physicists' and philosophers' main worries, which underlie the ad hoc-charge. Section 4 recapitulates core ideas of philosophical accounts of adhocness, put forward by philosophers of science in the past. In Section 5, we show that according to a strict reading of philosophers' accounts, the SMHM does not qualify as ad hoc; however, as we argue further, the criteria which are not obeyed seem to play only a secondary role in how scientists actually classify hypotheses as ad hoc. Disregarding these criteria, Section 6 analyses in detail the SMHM in the light of Jarrett Leplin's account, which is arguably the most refined one found in the literature. We show that it captures nicely the general thrust of the ad hoc-charge against the SMHM. In the light of these observations, Section 7 critically assesses a recent claim by Christopher Hunt that "what is 'ad hoc' seems to be a judgment made by particular scientists not on the basis of any well-established definition but rather on their individual aesthetic senses" (Hunt 2012, p. 1). Finally, Section 8 discusses what implications the recent discovery of a Higgs-like scalar particle at the LHC has on the status of the ad hoc-charge against the Higgs mechanism. As we argue, this discovery discharges the SMHM from the most straightforward ad hoc-allegation, which concerns the previously lacking independent empirical support, but the main

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[1] On July 4, 2012, ATLAS and CMS, the two largest experimental collaborations at CERN, announced the observation of a new particle whose properties, as measured up to now, agree well with those of a Higgs boson with properties in the range of what is to be expected from the SMHM; see (ATLAS Collaboration 2012) and (CMS Collaboration 2012). Meanwhile, more data have been collected and analyzed, and the "observation" as been upgraded to a "discovery".

criticisms of the SMHM remain valid. On the other hand, as we note, a new tendency in the physics community towards re-evaluating or even discarding these criticisms appears to be visible.

2. The Higgs mechanism in a nutshell

The SM describes the strong and electro-weak interactions among the elementary particles in the language of quantum field theory. It is formulated in terms of a Lagrangian, which has the crucial property of being invariant under certain well-defined transformations of the fields called "local gauge transformations". Local gauge invariance is necessary for the SM Lagrangian to be renormalizable, i.e. mathematically consistent (by the standards of conventional quantum field theory) and predictive up to arbitrarily high energies.

The SM employs the HM to account for non-zero particle masses in the Lagrangian, which would otherwise violate gauge invariance and thus spoil the renormalizability of the theory. Its basic idea is the existence of a scalar field (i.e. a field with spin 0), the so-called Higgs-field, which has an infinite number of degenerate lowest-energy field configurations. Classically, this field has a non-zero vacuum expectation value, in which respect it differs from all other fields of the SM Lagrangian. This is often expressed by saying that local gauge invariance, though not violated explicitly, is absent on the level of the ground-state of the theory in the form of a so-called "spontaneous breaking" of the local gauge symmetry.[2]

A consequence of the features just described is that particle masses are induced through the interaction between the particles and the Higgs field. The SMHM thereby makes the prediction that the strength of a particle's interaction with the Higgs boson is proportional to its mass. As a consequence, for example, the ratio of the Higgs decay rate into particles such as muons and bottom-quarks is equal to the (square of the) ratio of the muon and bottom-quark masses, up to calculable effects due to kinematics or higher order quantum fluctuations.

There is quite an amount of freedom as regards how the HM is implemented in the electro-weak theory (and thus in the SM). In particular, the behavior of the Higgs field under gauge transformations, its so-called gauge representation, is not fixed by theoretical requirements. Steven Weinberg was the first to use the HM in order to preserve gauge symmetry in electro-weak interactions. In his seminal paper (Weinberg 1967), usually considered the birth of the SM, he implemented the HM in its minimal form by including a single Higgs field in the simplest possible gauge representation. This is what we refer to as the SMHM in this paper. Other, more complicated, representations were considered later in history, but they either require a larger number of parameters (e.g. two-Higgs doublets), or are in conflict with precision measurements. We will therefore focus on the SMHM, although most of our discussion applies to other implementations as well. In particular, the aspects of the ad hoc-charge discussed in Section 3 all refer to its field-theoretic aspect, not to the particular gauge representation.

Quite often, probability (or unitarity) conservation is referred to as another benefit of the SMHM, besides renormalizability. Unitarity conservation means the following: For vector particles like the

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[2] This characterization of the SMHM in terms of spontaneous symmetry breaking and a degenerate ground state has to be taken with a grain of salt and is in some respects misleading; see (Smeenk 2006), (Healey 2007, Chapter 6.5), and (Friederich 2012) for clarifications aimed at philosophers.

Z- or the W-boson, non-zero masses are equivalent to the existence of longitudinal degrees of polarization. If gauge invariance is broken on the level of the Lagrangian, these components can contribute to scattering amplitudes that “violate unitarity”, meaning that the probability of the respective scattering reaction becomes ill-defined. If these masses are induced by the SMHM, however, the Higgs particle's contribution to the scattering amplitude exactly cancels the unitarity-violating terms. Both renormalizability and unitarity conservation are consequences of the same feature of the Lagrangian, namely, gauge invariance.

Interactions among the particles lead to observable processes involving the Higgs particles, which are detectable at particle colliders. The predicted phenomenology of effects due to Higgs particles has been studied in great detail in the physics literature (see, e.g. (LHC 2011)), and the experimental search program has been active for several decades. Indirect searches in the 1990s based on the comparison of precise measurements made by LEP at CERN and SLD at SLAC with quantum field theoretical calculations provided first clues for the value of the Higgs boson mass (see, e.g. (LEP 2003)). Direct searches were subsequently done at LEP2 with increased beam energy, leading to a lower Higgs mass limit of about 114 GeV. Further mass exclusion limits have been obtained since 2008 at the Tevatron collider at Fermilab, and later at the LHC at CERN. As mentioned already above, on July 4, 2012, a signal for a particle at around 125 GeV was observed. Its properties, as measured so far, are compatible with a SM Higgs boson.

3. Physicists’ criticisms and accusations of adhocness against the SMHM

Even though the SMHM forms part of the Standard Model as a renormalizable quantum field theory, which, to the current date, describes all phenomena observed at particle colliders to very high precision, physicists have searched for alternatives to a fundamental Higgs field already early on. Evidently, until very recently one of the main motivations for this was the lack of direct experimental evidence for the Higgs boson. However, even now after the recent discovery of a Higgs-like particle, many particle physicists would consider the confirmation of this particle being the Higgs boson with properties exactly as predicted from the SM as a disappointment. In fact, the situation that the LHC discovers a SM Higgs boson and nothing else is often called the nightmare scenario by many physicists (see Cho (2007)).

Qualitative arguments

We begin by citing an objection against the SMHM that was mostly relevant prior to the recent discovery:

1. There is not enough evidence from independent sources for the SMHM.

Before the recent detection event at the LHC, this criticism was very natural: Among all the different types of particles in the SM, the Higgs particle is the only scalar particle and it has a unique status and role among all SM-particles. At the same time, it was the only type of particle for which no direct evidence existed. In the words of Lee Smolin, this endowed the SMHM with an “ad hoc quality”:

> [While] the idea of spontaneous symmetry breaking to explain why each of the elementary particles we see in the world has different properties [...] is a beautiful idea, there is a certain ad hoc quality to how it is realized. To this date, no one has so far observed a Higgs particle and we have only a very imprecise idea of their actual properties. (Smolin 1999, p. 61.)

We will later (Section 8 in particular) discuss what impact the recent discovery has on this criticism.

2. There are no other fundamental scalar particles besides the Higgs boson in the SM, nor is there any experimental evidence for such particles.

Except for the Higgs boson, all particles of the SM – and thus all known fundamental particles – either have spin ½ (quarks and leptons) or spin 1 (gauge bosons: W, Z, gluons, photon). Lorentz invariance as required by special relativity, on the other hand, demands that the Higgs field be scalar, i.e. have spin 0. All known scalar particles are composed of more fundamental ones; examples are pions and kaons, which are composed of quarks and gluons. Concern about the SMHM on these grounds is expressed, for example, by MIT physicists Farhi and Jackiw, who criticize the introduction of the Higgs fields as a fundamental scalar field as "ad hoc":

> While the Weinberg-Salam model [i.e. the SM] is recognized to be a theoretical and experimental success, it is frequently believed that the Higgs mechanism [...] is an unsatisfactory feature of the theory. There is no experimental evidence for fundamental scalar fields, which are introduced in an ad hoc manner with ad hoc interactions solely to effect the symmetry breakdown. There is no other compelling theoretical reason for scalar fields. (Farhi and Jackiw 1982, p. 1.)

In a similar vein, Andrei A. Slavnov notes that "[t]his mechanism is based on the ad hoc introduction of scalar fields which appear as fundamental elementary particles together with leptons, quarks and Yang-Mills mesons [i.e. the particles mediating the interactions between quarks and leptons]", and he adds that this introduction of fundamental scalars is not an "attractive possibility."(Slavnov 1979, 289) Further below, we point out two especially worrisome features of the introduction of fundamental scalars, namely, the so-called naturalness and triviality problems.

3. The conception of the vacuum as pervaded by a non-vanishing Higgs field is conceptually problematic.

As explained in the previous section, the SMHM is often characterized by saying that it involves a ground state degeneracy and leads to a non-zero expectation value for the Higgs field. This aspect of the SMHM is sometimes criticized as conceptually suspect (though not explicitly "ad hoc", as far as we know). For example, in an introductory textbook on gauge theories physicist K. Moriyasu compares the "Higgs field [...] to an old-fashioned 'aether' which pervades all space-time [and] acts like a continuous background medium even at very short distances." (Moriyasu 1983, p. 120)

In a similar vein, philosopher of science Margaret Morrison contends that, in the case of the SMHM, "we are dealing with fields whose average value is non-zero, where the vacuum is said to have a non-zero expectation value" (Morrison 2003, p. 359) and characterizes the vacuum of the SM as a "plenum". (Morrison 2003, p. 357) Alluding to this aspect of the SMHM she argues that "the various vacuum hypotheses which provide the necessary theoretical foundations [of the

SMHM] are essentially problematic, for both physical and philosophical reasons." (Morrison 2003, p. 361)

However, unless this criticism is understood metaphorically as a mere rephrasing of the other criticisms mentioned, it is based on conceptual misunderstandings. We do not want to enter the details of a discussion about gauge symmetry breaking and the claimed vacuum degeneracy, but to indicate that matters are more subtle than suggested by Moriyasu and Morrison, it suffices to note that the Higgs field itself is not gauge invariant, which means that its vacuum expectation value is gauge-dependent and thus not a physical quantity. Consequently, whether or not that vacuum expectation value is non-zero under certain conditions does not by itself have any physical import. This already indicates that the SMHM does not in fact rely on a questionable "theoretical story about the vacuum" (Morrison 2003, p. 361), contrary to what Morrison – and similarly Moriyasu – seem to suggest.[3]

4. All other known cases of symmetry breaking in nature are dynamical, that is, due to composite, rather than fundamental fields. In the SMHM, in contrast, a dynamical account is lacking and the symmetry breaking is implemented by fiat through the shape of the Higgs potential.

The conceptual framework of the HM applies not only in particle physics, but also in condensed matter physics, where, for example, the HM can be used as a conceptual tool to account for phenomenological features of superconductors. In that case, the field that plays the role of the Higgs field is not fundamental, but associated with pairs of electrons (so-called Cooper pairs), which are held together by the complex interplay of electro-magnetic interactions. Within particle physics, a similar situation is found in chiral symmetry breaking in QCD due to what is called a "quark condensate", which is likewise a composite system. Symmetry breaking which occurs due to a field associated with composite particles is said to be *dynamical*. In the SMHM, in contrast, where the Higgs field is fundamental rather than composite, the symmetry breakdown due to the Higgs field is non-dynamical. According to the following passage by Jackiw, this feature of the SMHM makes it ad hoc:

> Spontaneous symmetry breaking is adopted from many-body, condensed matter physics, where it is well understood: the dynamical basis for the instability of symmetric configurations can be derived from first principles. In the particle physics application, we have not found the dynamical reason for the instability. Rather, we have postulated that additional fields exist, which are destabilizing and accomplish the symmetry breaking. But this ad hoc extension introduces additional, a priori unknown parameters and yet-unseen particles, the Higgs mesons [i.e. Higgs bosons]. (Jackiw 1998, 12777)

As emphasized by Jackiw, unlike in condensed matter physics, in the SMHM symmetry breaking is implemented non-dynamically through the choice of parameters in the so-called "Higgs potential" (the part of the Lagrangian which accounts for the mass of the Higgs boson and its self-interaction). For this reason, physicist Bruce Schumm characterizes this potential as an "arcane and ad hoc notion" in itself (Schumm 2004, p. 329). According to theoretical physicist Michael Peskin, since the parameters of this potential are put in by hand, "we should be ashamed of ourselves if we are satisfied" (Peskin 2012, p. 12) with the understanding of mass generation provided by the SMHM.

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[3] See (Friederich 2012, Sects. 5 and 6) for more detailed considerations and arguments.

5. The SMHM leads to a large number of independent parameters for the SM, and it does not explain their values or reduce their number.

Not only the parameters of the Higgs potential but also the couplings between the Higgs particle and the other particles of the SM (which are directly related to their masses, as explained in Section 2) cannot be derived from more fundamental principles. They must all be introduced as independent parameters on the basis of experimental data without further explanation for their values. According to CERN theorist Gian Francesco Giudice, this is highly unsatisfactory (where the last sentence in the quote reiterates the previously discussed criticism concerning the non-dynamical character of spontaneous symmetry breaking in the SMHM):

> Unlike the rest of the theory, the Higgs sector is rather arbitrary, and its form is not dictated by any deep fundamental principle. For this reason its structure looks frightfully ad hoc. [...]
>
> The Higgs sector explains the structure of quark and lepton masses that we observe, but only at the price of introducing 13 adjustable input parameters determined by experimental measurements. Quark and lepton masses can certainly be accounted for by the Higgs sector, but unfortunately the theory is unable to predict their values. Moreover, although the Higgs sector can generate the spontaneous breaking of electroweak symmetry, it provides no deep explanation about the force that is ultimately responsible for the phenomenon. (Guidice 2009, p. 174)

Even though the values of parameters used in the SMHM are theoretically unconstrained and therefore arbitrary, they appear by no means random. For example, the so-called CKM matrix which governs transitions among the three quark generations has a curious structure, with rapidly decreasing elements as one moves away from the diagonal ones, which, in turn, are close to unity. Other oddities are the enormous spread of the charged-fermion masses, spanning six orders of magnitude, with the largest one (the top-quark mass) being suspiciously close to the Higgs field's vacuum expectation value.

It should be mentioned, however, that the main alternatives to the SMHM enhance the number of independent parameters even further, and they do not explain any of the curious structures just mentioned. The minimal supersymmetric extension of the SM, for example, which is probably the most studied model beyond the SM, requires more than 100 new parameters. So, the SMHM is in fact less problematic than its most-discussed alternatives as far as the criticism raised by Guidice is concerned. Most of these alternatives are conceived as responses to the criticism of the SMHM to be discussed next, the so-called “naturalness problem”.

Formal arguments

1. Fine Tuning and Naturalness

The argument against the fundamental scalar character of the Higgs field that is widely regarded as the most severe is the so-called Naturalness or Fine Tuning Problem (FTP). Its precise formulation is within the renormalization formalism and thus beyond the scope of this paper (see (Susskind 1979) for a classic reference). We will nevertheless try to give a qualitative description: According to quantum field theory, any particle mass measured in experiment (the so-called physical mass) can be seen as the sum of the bare mass and a contribution due to the interaction of the particle with so-called vacuum fluctuations, called its interaction mass. Usually, one assumes the theory to be valid only up to a (large but finite) energy scale, the "cut-off", beyond which effects accounted for by a more fundamental (yet presently unknown) theory are supposed to set in. One can calculate the interaction mass as a function of that cut-off. The bare mass then follows as the difference between the physical and the interaction mass.

For spin ½ and spin 1 particles, the dependence on the cut-off is logarithmic, leading to an interaction mass that is at most of the same order of magnitude as the bare mass. Bare and physical mass are thus typically also of the same order of magnitude. For scalar particles, on the other hand, the dependence of the interaction mass on the cut-off is quadratic. Assuming the SM to be valid up to the Planck scale, the bare mass and the effect of the vacuum fluctuations would have to balance each other by about 34 orders of magnitude in order to result in a physical Higgs mass of 125 GeV. The disturbing fact about such a large cancellation is that the bare mass of the Higgs is an arbitrary parameter; the reason why Nature would have fine-tuned it to almost exactly the interaction mass is unexplained in the SMHM. As a consequence, the SM appears "unnatural" in this respect, which is why the problem is called the "naturalness problem".

The FTP arises from the scalar nature of the Higgs field and is therefore directly linked to the ad hoc-charge against the introduction of a scalar Higgs field. It is unsurprising that the most popular models devoid of the FTP solve at least one of the issues outlined before as well: they assume either a non-fundamental, composite Higgs boson and provide a dynamic account of gauge symmetry breaking, or they try to explain the shape of the Higgs potential (e.g. supersymmetry; the term which is quadratic in the Higgs field is driven negative by its renormalization group evolution). However, none of them seems to be presently privileged with respect to the SM by available experimental data, and many of them are ruled out.

2. Triviality

Finally, let us mention a further issue related to the Higgs boson, which is also considered unsatisfactory by many physicists. Again, its precise formulation requires the renormalization group formalism, but even then the argument requires approximations and extrapolations (see (Callaway 1988)). The worry is that the self-coupling of the Higgs field may diverge at large but finite momentum transfers, which would mean that the SM cannot be consistently extrapolated to arbitrary high energy scales and breaks down at or below the corresponding energy. The precise position of this so-called "Landau pole" depends on the Higgs mass. For the experimentally hinted at mass M_H=125 GeV, however, the Landau pole seems to be beyond the Planck scale, which, reassuringly, would mean no serious limitation for the SM. The name "triviality" reflects the fact that the Landau pole can only be made to disappear completely if the self-coupling of the Higgs boson is assumed to be vanishing, i.e. trivial, while this self-coupling must be non-vanishing for the HM to generate non-vanishing particle masses.

4. Philosophical accounts of "ad hoc-hypothesis"

The notion of an ad hoc-hypothesis[4] owes its widespread use Karl Popper[5], who employed it to characterize hypotheses which are added to scientific theories in response to nonconforming empirical data. According to him, "a conjecture [is] "ad hoc" if it is introduced […] to explain a particular difficulty, but if […] it cannot be tested independently" (Popper 1974, p. 986), thus reducing the degree of falsifiability of the theory.

Kenneth Schaffner departs from Popper in that he calls not only those hypotheses "ad hoc" which *cannot* be tested independently, but also those for which, relative to a given scientific context, there is no "independent theoretical or experimental support." (Schaffner 1974, p.68) Thus, for him, "ad hocness [is] a property of specific hypotheses, as embedded [...] in a constellation of other hypotheses constituting the theory" (Schaffner 1974, p. 67), not a feature of hypotheses in themselves. Adolf Grünbaum (1976) also conceives of adhocness as context-relative by taking the ad hoc-character of ad hoc-hypotheses to be time-dependent. On his account, an ad hoc-hypothesis at a time t may no longer qualify as ad hoc at a later time t* – provided the theory which incorporates it makes the right kind of empirical progress in between. We shall later consider in which sense this may hold for the SMHM.

The account of ad hoc-hypotheses which we choose as the basis of our discussion of the SMHM is due to Jarrett Leplin (1975). In what follows we review the criteria which he proposes as separately necessary and jointly sufficient for auxiliary hypotheses to qualify as ad hoc.

Leplin's first condition is familiar from Popper, Schaffner, and Grünbaum:

> *Condition of experimental anomaly*: If a hypothesis H is introduced into a theory T in response to an experimental result E, then if H is ad hoc, E is anomalous for T but not for T as supplemented by H. (Leplin 1975 p. 317)

In Leplin's sense, the phrase "in response to" is to be understood as implying that T was known before E. This excludes situations where the suggested theory T is in conflict with existing experimental data E and gets "rescued" from this conflict by an auxiliary hypothesis H at some point. We will see that already this restriction (henceforth referred to as "T before E") prevents the SMHM from being ad hoc according to Leplin.

Note also that the condition of experimental anomaly does not require E to be the only aspect where T is in disagreement with experiment. This will be used explicitly in the condition of non-fundamentality further below.

Let us remark on a certain lack of definition in the expression "an experimental result": it is unclear whether this includes only a single number (say, the maximal brightness of a certain type of supernovae), a distribution of numbers (e.g., the time evolution of the brightness of these supernovae), or maybe even more general data samples (the time evolution of all types of supernovae). The fuzziness of this condition affects its application to the SMHM, as we will see later.

Also the second of Leplin's conditions is in agreement with the previously stated accounts of ac hoc-hypotheses. It concerns their context of justification, exemplified in various historical cases of theory modification, including, e.g., the Lorentz-Fitzgerald contraction hypothesis, Pauli's neutrino

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[4] See (Karaca 2010), Chap. 6., for a more detailed overview of philosophical accounts of ad hoc-hypotheses.

[5] (Popper 1959), Sects. 5, 19, and 46.

hypothesis and the hypothesis of a trans-Uranian planet by Adams and Leverrier. In Leplin's words, an "ad hoc hypothesis is one introduced in response to an experiment that provides its only support" (Leplin 1975, p. 319).[6] We consider this one the central condition of adhocnes, carrying the actual meaning of the Latin expression "ad hoc" (="for this"):

> *Condition of justification*: If a hypothesis H is introduced into a theory T in response to an experimental result E, then if H is ad hoc, E is evidence for H but:
>
> 1. No available experimental results other than E are evidence for H.
>
> 2. H has no application to the domain of T apart from E.
>
> 3. H has no independent theoretical support. (Ibid. p. 320)

Here, what Leplin means by "the domain of a theory" is the totality of all "experimental results, statements of problems, hypotheses of previous theories, descriptions of postulated entities" (Ibid. p. 318).

In Leplin's view, an ad hoc hypothesis is added into a scientific theory without destroying its internal coherence. This results in the following

> *Consistency condition*: If a hypothesis H is introduced into a theory T is ad hoc, then H is consistent with accepted theory and with the essential propositions of T. (Ibid. p. 327)

Here, Leplin defines an "essential proposition" of a theory to be a proposition whose rejection would count, in the judgment of the scientific community, as a rejection of T irrespective of the retention of other hypotheses (Ibid., p. 327).

Leplin does not think that the features of ad hoc-hypotheses presented in the previous condition are by themselves sufficient for endorsing or rejecting those hypotheses. What he calls "condition of tentativeness" states that the allegation of adhocness against a hypothesis should not be construed as a commitment to either its future confirmation or disconfirmation:

> *Condition of tentativeness*: If a hypothesis H is ad hoc, then there are no sufficient grounds for holding that H is true and no sufficient grounds for holding that H is false. (Ibid. p. 321)

This condition is the hardest to apply to hypotheses in the wild, since it is usually very difficult to decide in which cases there actually are sufficient grounds for holding that a hypothesis is true. However, we will see that for the SMHM the condition of tentativeness can be construed as met in a rather well-defined and broadly accepted way.

Unlike the previous conditions, the last condition Leplin sets forth concerns the way in which an experimental anomaly is understood prior to its treatment by an ad hoc-auxiliary hypothesis:

> *Condition of non-fundamentality*: If a hypothesis H introduced into a theory T in response to an experimental result E is ad hoc, then there are problems other than E confronting T which there is good reason to believe are connected with E in the following respects:
>
> 1. These problems together with E indicate that T is non-fundamental.

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[6] The fact that the neutrino and the trans-Uranian planet hypotheses were *independently* "confirmed" in the past and, as a result of this, no charge of "ad hocness" is raised against them any more seems to vindicate the general consensus on ad hoc hypotheses indicated by Leplin's *condition of justification*.

> 2. None of these problems including E can be satisfactorily solved unless this non-fundamentality is removed.
> 3. A satisfactory solution to any one of these problems will contribute to the solution of the others. (Ibid. p. 331)

Again, this condition carries a significant amount of arbitrariness, manifest in the expressions “good reason”, “believe”, and “satisfactory”. Also, it is left unspecified how severe the “problems other than E” need to be.

Lastly, we stress that Leplin restricts the use of “ad hoc hypothesis” to cases in which a hypothesis criticized as “ad hoc” is introduced as a response to a disconfirming experimental result. Leplin is aware that this requirement limits his analysis to cases of “experimental” rather than “theoretical anomaly”. However, he does not think that this limitation takes away from the generality of his analysis. Leplin’s consideration here is that even though there is in principle the possibility that there could be cases in which “adhocness” has application outside the cases of experimental anomaly, no such case has yet arisen in scientific practice. However, as we shall argue later, the SMHM constitutes precisely such a case.

5. The SMHM is not strictly speaking ad hoc according to Leplin

It is clear that the SMHM cannot be ad hoc according to accounts such as Popper' according to which ad hoc hypotheses are in principle unable to make independently testable predictions. The SMHM made at least one such prediction (which turned out to be *actually* testable), namely, that a neutral scalar particle with spin 0 should exist. Therefore, any account which rules out that ad hoc-hypotheses may make independently testable predictions fails to apply to the SMHM. However, it is far less obvious (and much more rewarding to investigate) how the SMHM fares according to more liberal accounts of adhocness such as Leplin's.

Let us introduce the notation “SM0” for the SM without the Higgs sector, implying the symbolic equation SM=SM0+SMHM. The SM0 is a well-defined theory no less than the SM itself, fulfilling all the properties of a consistent quantum field theory, such as renormalizability, absence of anomalies, and unitarity. However, the kind of physical world that it describes is radically different from our own: In particular, all the particles it accounts for are massless. In addition, a direct consequence of the SM0 is that the photon and the weak gauge bosons are physically indistinguishable (and therefore not recognizable as distinct particles), just as the electron and the electron-neutrino.[7]

Certainly, the SM0 has never been proposed as a candidate theory of elementary particle physics for our world. This is the first reason why, strictly speaking, the SMHM does not qualify as an ad hoc-hypothesis according to Leplin. Contrary to the criteria he proposes, physicists do not seem to regard it as obligatory for the hypothesis H to qualify as ad hoc that the “pre-ad hoc” theory T has ever been held as a serious candidate theory in its own right; it seems fully sufficient that T be, in some hardly specifiable sense, a viable theory.

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[7] In Section 5, “A world without the Higgs mechanism”, of (Quigg 2007), Quigg offers more detailed considerations as to what a world described by the SM0 would be like.

On the other hand, in a slightly looser sense, we can in fact find an earlier theory that may be considered as the "pre-ad hoc" version of the (electroweak part of the) SM, namely Glashow's model[8] (Glashow 1961), which is based on the same gauge group SU(2) x U(1). Disregarding the mass terms which Glashow introduced at the cost of breaking gauge invariance explicitly (rather than spontaneously), it results in something very close to the SM0. In that sense, the SM0 "existed" before the SM, whose origin is usually taken to be Weinberg's paper of 1967.

Nevertheless, again taking Leplin's conditions at face value, the SMHM does not qualify as an ad hoc-hypothesis added to the SM0 for a second reason. As discussed above, Leplin's conditions for an ad hoc-hypothesis involve the restriction "T before E": an existing theory runs into conflict with "new" experimental data and is subsequently "rescued" by an auxiliary hypothesis. However, it was clearly known long before Glashow's paper that the electron mass is non-zero, and also that the electromagnetic and the weak interactions are distinct (both of which could be taken as the "experimental result E"). After all, this is why Glashow did not actually suggest the SM0, but introduced explicit mass terms. Therefore, considering the SMHM, also the requirement made by Leplin that H be introduced into an existing theory T in order to count as (possibly) ad hoc does not conform to how physicists actually characterize hypotheses as "ad hoc".[9]

In a final attempt to apply Leplin's criteria to the SMHM one may argue that Weinberg's theory was an ad hoc-modification not of the SM0, but of Glashow's actual theory, i.e. a theory which includes explicitly gauge symmetry-breaking mass terms.[10] Eliminating these mass terms and instead accounting for particle masses by the SMHM renders the theory renormalizable (as later shown by 't Hooft (1979)) and, as such, mathematically consistent and predictive up to arbitrarily high energies.[11] However, Leplin's conditions do not include such a situation where an auxiliary hypothesis cures an internal, or mathematical, deficit of a theory; the hypothesis is always assumed to remove a disagreement with experimental data. This is the third reason why the SMHM does not qualify as ad hoc on Leplin's account.

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[8] In 1979 S. Glashow shared the Nobel prize with S.Weinberg and A.Salam "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles" (http://www.nobelprize.org/nobel_prizes/physics/laureates/1979/)

[9] See (Karaca 2010), Chapter 7, for the original statement of the claim that the SMHM is not strictly speaking ad hoc on any account according to which adhocness requires that a conflict be removed between an established theory and incoming empirical data. Rather, Karaca argues that the SMHM is an ad hoc hypothesis that was proposed to solve a conceptual problem, namely, the problem of how to account for the non-zero masses of vector bosons while preserving the gauge invariance of the theory.

[10] Whether Weinberg himself considered his theory a modification of Glashow's seems difficult to decide. His only reference to Glashow's model is in a footnote, where he characterizes it as "similar to ours [...]; the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions." (Weinberg 1967)

[11] The failure of renormalizability in Glashow's theory may have been the chief reason why it was not recognized earlier as an important contribution. According to the Science Citation Index (Glashow 1961) was cited only once per year between 1961 and 1967. In fact, even Weinberg's paper was cited less than five times in the three years between its publication and 't Hooft's proof of renormalizability ('t Hooft 1971). For a historical comparison of Glashow's and Weinberg's theories, see (Karaca 2013).

To summarize the discussion up to this point: No matter whether we consider the SM as a modification of (a) the SM0 or of (b) Glashow's theory, the constraint “T before E” prevents the SMHM from being straightforwardly ad hoc by Leplin's criteria. Additional objections against the ad hoc-charge arise for option (a) from the requirement that H be introduced into a previously established theory, and for option (b) from the fact that the SMHM solves a conceptual, rather than an experimental, difficulty.

Apparently, from the perspective of many physicists, all of these reasons one might have for not regarding the SMHM as ad hoc are only of circumstantial importance. However, instead of modifying Leplin's criteria in response, we will simply assume that, say, the condition of experimental anomaly can be formulated in a slightly more permissive way. The refined version should somehow capture the basic idea that, in order for a hypothesis H to qualify as ad hoc, there has to be some (conceptually) viable theory T which suffers from a certain shortcoming E (not necessarily arising from experimental data), while T supplemented by H does not. This idea evidently applies to the SMHM (T=SM0 or T=Glashow's theory, H=SMHM). In the next section, we will see the relation of the SMHM with respect to what we regard as the more essential among Leplin's criteria.

6. In which sense the Higgs mechanism is ad hoc

In this section, we evaluate the SMHM with respect to each of Leplin's criteria of adhocness, except for the condition of experimental anomaly, which we have already discussed in the previous section. It may be helpful for the reader to explicitly recall the individual conditions as we go through them one-by-one in what follows.

Condition of consistency

Obviously Leplin’s condition of consistency applies to the SM. The crucial significance of the SMHM can be seen in the fact that it allows to formulate a renormalizable gauge theory that accounts for non-vanishing particle masses. Its most important achievement is to make the SM mathematically consistent and predictive up to arbitrarily high energies.

Condition of tentativeness

The condition of tentativeness is perfectly met by the SMHM. In fact, the SMHM is very widely regarded as tentative among physicists, namely as “something like an effective description of a much more complicated situation” (Ross and Veltman 1975, p. 136). A theory that accounts for the supposed “more complicated situation” could be one with dynamical symmetry breaking, or a supersymmetric theory. The tentativeness charge is thus reflected in the criticism number 4 in Section 3.

At this stage, it is interesting to note that “effective theories” are familiar and frequently used in particle physics; they have proven extremely useful in analyzing experimental data for signals of physics “beyond the SM”, for example. Typically, however, such effective theories involve non-renormalizable field operators, and in this case there are “sufficient grounds” for holding that they are “false” (in the sense of being mere approximations to a more consistent, fully-fledged theory). In contrast, the SMHM does not involve non-renormalizable operators, so this argument does not apply. However, this does not provide sufficient grounds for holding that it is true, either; for all we know, the SM, as based on the SMHM, might still be “only” an effective theory.

Note that most physicists would agree that the tentative character of the SMHM cannot be decoupled from that of the rest of the SM; if it turns out that the SMHM is in fact only an effective description, a more fundamental mechanism would probably affect all of the SM. In the case of the SMHM, the tentativeness condition is therefore tightly connected to the condition of non-fundamentality, which we discuss next.

Condition of non-fundamentality

Argument number 5 in Section 3 clearly falls into this category. The SM suffers from a number of explanatory "problems" related to the number and distribution of its parameters, in particular those associated with the particle masses. Unless one contents oneself with accepting that these parameters are completely random and accidental, they indicate that the SM is non-fundamental. Criterion number 1 of the condition of non-fundamentality is therefore met. By construction, the SM has no explanation for these "problems", which corresponds to criterion number 2. And since many of these problems are directly related to the Higgs sector of the SM, also criterion number 3 is met.

Arguably, also the naturalness (and potentially the triviality) problem suggests that the SM is non-fundamental. Both are linked to the scalar nature of the Higgs particle. So, if one could solve the mass problem for the SM without invoking scalar particles, one would most likely avoid all the criticisms of the SMHM listed in Section 2. This is exactly the spirit of criterion number 3 of Leplin's condition of non-fundamentality.

Condition of justification

As already mentioned in Section 4, the condition of justification is a key condition for classifying auxiliary hypotheses as ad hoc. Its wording, however, leaves a lot of room for interpretation. Like the condition of non-fundamentality, it involves three criteria. Let us start with the second and third criteria, before we discuss the first one, which gives rise to the most interesting considerations in our case.

Criterion number 2 (that H should have "no application to the domain of T apart from E") is perfectly reflected in our criticism number 4 of the SMHM: Non-dynamical spontaneous symmetry breaking is used in particle physics only to account for the non-vanishing particle masses through the SMHM, it has "no application" apart from that. One may argue that an additional application of the SMHM is to save the SM from unitarity violation at high energies. However, this problem is so tightly connected to gauge invariance that it cannot be seen as independent; the SMHM preserves gauge invariance, and, therefore, unitarity is guaranteed.

Criterion number 3 (that H "has no independent theoretical support") matches criticism number 2 of Section 3: The Higgs particle has a peculiar status as the only fundamental scalar particle among all the particles of the SM. This can be naturally paraphrased by saying that the hypothesis that there are fundamental scalars has no "independent theoretical support" other than being able to account for non-vanishing particle masses.

Let us now turn to criterion number 1 (that "[n]o available experimental results other than E are evidence for H"). It is here where the recent discovery has crucial implications for the ad hoc-charge against the SMHM. We will first take a pre-discovery point of view and discuss the impact of the discovery afterwards.

Criterion number 1 refers to some “experimental result E” which in the case of the SMHM can be taken as the observation of non-zero particle (in particular, weak gauge boson) masses (recall that we decided to ignore the fact that this observation had been made long before the SM was formulated). This choice is not unique, however: one could also argue for the distinguishability of the electro-magnetic and the weak interactions, or other phenomena related to the Higgs sector of the SM. The question is now whether the fact that there actually are several phenomena connected to the Higgs sector is in contradiction with the first criterion of the condition of justification.

The most crucial of these phenomena is the one referred to by Weinberg in his above-mentioned footnote on Glashow's paper. As noted by Ross and Veltman, “for experimental purposes the difference is that in the Weinberg model the mass of the neutral vector boson is fixed relative to the charged masses if the mixing angle is known.” (See Ross and Veltman 1975, p. 136). In other words, Weinberg's model predicts a relation between the “mixing angle”, which accounts for the relative strengths of the neutral and charged weak interactions and the masses of the Z- and W-bosons. This relation can be determined experimentally and agrees perfectly with the prediction. With the increase in experimental precision due to large particle colliders like LEP, its importance has even advanced from a test of Weinberg's theory to a probe of quantum effects. For example, the precise measurement of the mixing angle allowed the prediction (assuming that the SM is correct) of the top quark mass before its discovery at the Tevatron. Until recently, it also provided the most stringent constraints on the Higgs boson mass.

Nevertheless, in spite of this highly predictive character, it is questionable whether the relation between the mixing angle and the Z- and W-boson masses pointed out by Ross and Veltman counts as independent support for the SMHM. What underlies this relation is only the gauge structure of the Higgs-field, not its more specific features as a fundamental scalar which give rise to the worries discussed in Section 3. This prediction of Weinberg's model would be the same for a non-fundamental Higgs-field which transforms under the same gauge representation. The fact that Weinberg chose the minimal representation (i.e. the one that leads to the minimum number of physical Higgs bosons) is hardly ever seriously criticized by physicists (even though models with alternative gauge structures were considered later).

Other aspects that might be proposed as independent experimental support for the SMHM prior to the discovery of the Higgs boson are rejected more easily: for example, the fact that the SMHM provides mass to more than one particle obviously cannot be taken as independent evidence, as each mass requires an additional, arbitrary parameter in the SMHM. CP-violation in the quark sector, as another example, is tied to the existence of non-vanishing quark masses, not to the mechanism that provides these masses. In conclusion, until the recent discovery at the LHC of what could be the Higgs boson, criterion number 1 of the condition of justification was met.

7. Rejecting Hunt’s dismissal of adhocness

To summarize the preceding section, although the ad hoc-charge against the SMHM never rested on a “well-established definition”, to use Hunt's phrase (2012, p. 1), it does obey the key characterizations of what philosophers consider an ad hoc-hypothesis. As our considerations show, the physicists’ charges of adhocness against the SMHM rest on both qualitative and formal arguments, which are based on widely used, rationally and empirically justifiable, methodological principles of particle physics: for example, that methodologically acceptable instances of symmetry breaking are dynamical rather than fundamental, that the values of fundamental parameters should not involve excessive fine-tunings, etc. It is therefore unjustified to claim that the grounds on which the ad hoc-charge rests are “merely aesthetic”, safe perhaps in an extremely wide and stretched sense of “aesthetic”. So, based on our considerations concerning the SMHM, we dispute the claim made by Hunt that scientists’ allegations of adhocness reflect nothing more substantial than their “individual aesthetic sense.”

Arguably, not only the criticisms of the SMHM themselves, but also the “ad hoc”-label to frame them go beyond the scientists' “individual aesthetic sense”. As we have argued, even though in the light of these criticisms the SMHM is not literally ad hoc according to Leplin's definition, it is certainly ad hoc according to its spirit. More importantly, some of Leplin's criteria of adhocness capture precisely the point of these criticisms in more general terms. It remains to be seen whether the end of the ad hoc-status of the SMHM due to the recent discovery will motivate physicists to reconsider all their concerns, not just those concerning the lack of independent empirical evidence.

8. The SMHM after the discovery of a Higgs-like particle

As a consequence of the recent discovery of a Higgs-like particle, there is now independent experimental evidence for the SMHM: the signal of a particle which the SMHM says should exist. Therefore, the most crucial characteristic of an ad hoc-hypothesis, formulated in the first criterion of Leplin’s condition of justification, is no longer obeyed. From the point of view of most physicists, however, the discovery has not laid to rest their main worries concerning the SMHM. All the other conceptually sound criticisms of Section 3 still apply, and, in particular, the naturalness problem stands unresolved. Its future fate depends strongly on whether or not there is “new physics” (i.e. hitherto unknown particles with mass) around the TeV-scale currently probed at the LHC. As long as the search for such particles turns out negative, any experimental result which confirms the existence of a Higgs boson with properties exactly as predicted by the SMHM rather than confirming any of the proposed alternatives (such as supersymmetry or dynamical symmetry breaking) seems to indicate that the naturalness problem is here to stay. Thus, the consequence of the most recent experimental data – confirming the SMHM and ruling out theoretical alternatives – may be that physicists' worries with respect to the SMHM are actually intensified. In that case, paradoxically, independent experimental evidence in favor of an ad hoc-hypothesis, when it finally arrives, would not alleviate, but in fact aggravate, the concerns which motivate calling it “ad hoc”.

However, contrary to this possibility, there seems to be a tendency among physicists to critically reconsider the validity (or applicability) of the unnaturalness criticism. For example, according to theoretical physicist Jonathan Feng:

> For decades, the unnaturalness of the weak scale has been the dominant problem motivating new particle physics […]. This paradigm is now being challenged by a wealth of experimental data. (Feng 2013)

In a similar vein, theoretical physicist Michael Krämer writes in a blog entry:

> Naturalness arguments did provide useful insight into particle physics in the past [...] but they do not work all the time. Will the naturalness problem of the Higgs mechanism lead to profound new insights or is it just a red herring? It is still too early to conclude, but the lack

> of any signal of new physics at the LHC, and the landscape of solutions of string theory[12], have led us to reconsider the role of naturalness as a particle physics paradigm. (Krämer 2013)

An alternative route to a more relaxed stance towards the naturalness problem would be to re-think its conceptual presuppositions. For example, Wetterich (2012) argues that the need for fine-tuning indicates nothing more than “a shortcoming of the perturbative expansion series” (p. 573) and interprets the measured value of the Higgs mass as an indication that the SM may remain valid up to the Planck scale. On his view, the SMHM is neither more tentative nor less fundamental than the other parts of the SM.

To conclude, it may well happen that the alleviation (or even the end) of the ad hoc-character of the SMHM due to the recent discovery leads to a shift in perspective as regards the naturalness problem in particular and the conceptual foundations of the SMHM in general. Of course the criticisms mentioned in Section 3 are still regarded as serious among physicists, but their relevance and force may now be seen in a different light, since the SMHM has lost its most conspicuous characteristic of adhocness.

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[12] The idea alluded to here is that considerations involving the string theory landscape may help dispelling the naturalness problem by integrating the SM in a multiverse scenario, where anthropic arguments may be used to explain away the perceived fine tuning of fundamental parameters as an unproblematic observation-selection effect. See (Donoghue 2007) for details.

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