On the formal statement of the special principle of relativity

Márton Gömöri and László E. Szabó
Department of Logic, Institute of Philosophy
Eötvös University, Budapest
Emails:
Laszlo E. Szabo <leszabo@phil.elte.hu>
Marton Gomori <gomorim@gmail.com>

Preprint (6 June 2012)

Abstract

The aim of the paper is to develop a proper mathematical formalism which can help to clarify the necessary conceptual plugins to the special principle of relativity and leads to a deeper understanding of the principle in its widest generality.

1 Introduction

The principle of relativity is the assertion that "All the laws of physics take the same form in any admissible frame of reference." Since the very first formulation of the general theory of relativity, it has been a hotly debated issue whether the principle holds true if the set of admissible frames is extended from the inertial frames to arbitrary frames of reference (Norton 1993). Einstein himself was quite convinced that his requirement of general covariance, which is satisfied by the theory, expresses precisely the principle of relativity in it's extended form (Norton 1988, 1993). In his famous objection, Kretschmann (Norton 1993) drew attention that it doesn't seem to do so—any spacetime theory, including special relativity and Newtonian theory, can be given a generally covariant formulation. How covariance is related to the principle of relativity is heavily discussed in the context of general relativity up to this day, in connection with some closely related foundational issues such as the meaning of diffeomorphism symmetry and its implications for the observational basis of general relativity (Friedman 1983; Brading and Brown 2004; Rovelli 2004; Dieks 2006; Westman and Sonego 2009).

In special relativity, in contrast, these issues—the relation between the special relativity principle (RP) and Lorentz covariance, their empirical content and their applicability to physical theories—are considered completely unproblematic. As Norton (1993, p. 796) writes:

> The lesson of Einsteins's 1905 paper was simple and clear. To construct a physical theory that satisfied the principle of relativity of

inertial motion, it was sufficient to ensure that it had a particular formal property: its laws must be Lorentz covariant. Lorentz covariance became synonymous with satisfaction of the principle of relativity of inertial motion and the whole theory itself, as Einstein (1940, p. 329) later declared:

> The content of the restricted relativity theory can accordingly be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations.

Moreover, this simplification continues with the following idea. Norton (*ibid.*) writes:

> *Selecting* suitable transformation laws for the field and other quantities, Einstein was able to show that the laws of electrodynamics remained unchanged under the Lorentz transformations. That is, they were Lorentz covariant. [italics added]

There is however some tension between this simplified picture and the concrete applications and the original understanding of the RP which is supposed to reflect something from the *physical* behaviors of both the moving measuring equipments and the moving physical objects to be measured. We mention two major problems here in the introduction.

First, if it is true that the RP reduces to the requirement of covariance, and the transformation rules are "selected" such that the physical equations will be covariant against these transformations, then the RP becomes a tautology—the equations are covariant against the transformations that are derived from the presumed covariance of the equations. This contradicts to the view—shared by a number of physicists and philosophers (see Brading and Castellani 2008)—that the statement of relativity/covariance principle, like many other symmetry principles, must be considered as a contingent, empirically falsifiable, statement. As Houtappel, Van Dam, and Wigner (1963) warn us:

> The discovery of Lee, Yang, and Wu, showing, among other facts, that the laws of nature are not invariant with respect to charge conjugation, reminded us of the empirical origin of the laws of invariance in a forcible manner. Before the discoveries of Lee, Yang and Wu, one could quote Fourier's principle as an earlier example of an invariance principle which had to be abandoned because of empirical evidence.

Earman points out a more general epistemological aspect:

> [V]iewing symmetry principles as meta-laws doesn't commit one to treating them *a priori* in the sense of known to be true independently of experience. For instance, that a symmetry principle functions as a valid meta-law can be known *a posteriori* by a second level induction on the character of first-order law candidates that have passed empirical muster. (Earman 2004, p. 6)

Notice that even the transformation rules must be considered as empirically falsifiable laws of nature. For, how can we verify even a single instance of the covariance principle? One might think that the verification of the covariance of a given law of physics is only a matter of mathematical verification. But this is true only if we know the transformation laws of the physical quantities—against which the physical law in question must be covariant. Consequently, we must have an independent knowledge of the transformation rules expressible in terms of the physical behavior of the measuring equipments—in various states of motion—by means of which the physical quantities are operationally defined.[1] For, as Einstein emphasizes:

> A Priori it is quite clear that we must be able to learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

The second problem is related with the proper relationship between the RP and the requirement of covariance. In its original understanding, the RP is supposed to reflect the physical behavior of *moving* physical objects. As Harvey Brown points out, this is already true in Galileo's principle:

> The process of putting the ship into motion corresponds [...] to what today we call an active pure boost of the laboratory. A key aspect of Galileo's principle that we wish to highlight is this. For Galileo, the boost is a clearly defined operation pertaining to a certain subsystem of the universe, namely the laboratory (the cabin and equipment contained in it). The principle compares the outcome of relevant processes inside the cabin under different states of inertial motion of the cabin relative to the shore. It is simply assumed by Galileo that the same initial conditions in the cabin can always be reproduced. What gives the relativity principle empirical content is the fact that the differing states of motion of the cabin are clearly distinguishable relative to the earth's rest frame. (Brown 2005, p. 34)

This is the basis of the typical applications of the RP: the description of the behavior of a moving object can be reduced to the description of the behavior of the same object at rest, by applying the transformation laws. (A concrete example will be discussed in section 2 and in Remark 4.) However, the covariance of the physical equations in itself does not determine which solution of the equations describes the moving object. As Bell pointed out, discussing the famous two-spaceship problem:

> Lorentz invariance alone shows that for any state of a system at rest there is a corresponding 'primed' state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'primed' of the original state, rather than into the 'prime' of some *other* state of the original system. (Bell 1987, p. 75)

The concept of covariance does not even refer to the concept of a particular solution describing a particular behavior of an object. This fact seems to con-

[1] For a case study illustrating this, see Gömöri and Szabó 2011a.

tradict to the claim that the RP can be reduced to the simple requirement of covariance.

As we can see from this brief introduction, there are different, sometimes controversial, views on the actual content and status of the RP. The aim of this paper is to develop a precise language in order to provide a precise formulation of the principle. Our formalism is based on the intuitive concepts and ideas that have been already discussed in the literature. However, in view of the fact that the RP is considered as a universal meta-law, which must be valid for all physical laws in all situations, we try to keep the formalism as general as possible. The benefit of the formal reconstruction is that it makes explicit all the necessary conceptual plugins to the principle; it brings out many subtle details and the related conceptual problems. We are hopeful that our analysis helps to clarify the above mentioned controversial issues and leads to a deeper understanding of the principle of relativity in its widest generality.

2 Preliminary considerations

"All the laws of physics take the same form in any inertial frame of reference." This is usually regarded as a simple and clear statement. In trying to understand the precise meaning of this sentence one encounters however several obvious questions.

First of all, it must be clear that the laws of physics *in* a reference frame K are meant to be the laws of physics as they are ascertained by an observer living in reference frame K; less anthropomorphically, as they appear in the results of the measurements, such that the measuring equipments—and in some sense the objects to be measured, too—are *in* K. At this point we encounter the first, and, as will be discussed below, highly non-trivial conceptual problem: when can we say that a physical object is "in" an inertial frame of reference?

Of course, it is the *same* laws of physics which must take the same form in all inertial frames. It would be absurd to require that, say, the second law of thermodynamics in K must have the same form as Newton's force law in K'. But, what are the same laws of physics in different inertial frames? It is quite natural to say that the laws of physics can be identified by means of the physical phenomena they describe. If so, then one can think that the descriptions of the *same physical phenomenon* must have the same form in all frames of reference. This is however obviously not the case. For example, consider the electromagnetic field of a charged particle at rest in K. This phenomenon is described in K as it is depicted in Fig. 1A. As is well known, the description of the *same* phenomenon in K' is completely different, Fig. 1B—just take the Lorentz transformation of the situation in Fig. 1A. (For more details, see Remark 4.)

Thus, the opposite must be true: the RP is about *different* physical phenomena; different phenomena must have descriptions of the same form in the different inertial frames of reference. In our example, 'the static electromagnetic field of the rest charge' is *one* phenomenon (Fig. 1A) and 'the time-dependent stationary electromagnetic field of the same charge in motion with velocity $v = V$' is the *other* (Fig. 1C). What the RP asserts is this: the description in the co-moving inertial frame K' of the phenomenon depicted in Fig. 1C takes exactly the same form as the description of the phenomenon in Fig. 1A in inertial frame K (see Fig. 1D). But, in what general sense these two phenomena are the

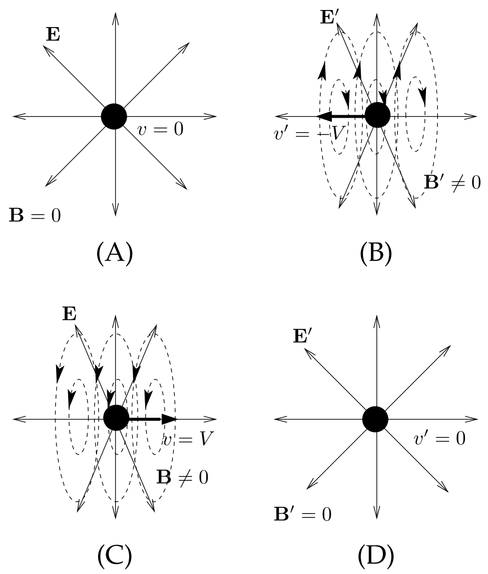

Figure 1: The descriptions of the same phenomenon in different inertial frames are different: (A) and (B). In contrast, different phenomena, (A) and (C), have descriptions of the same form in the two different (co-moving) inertial frames: (A) and (D)

counterparts of each other?

The next problem is how the phrase "same form" should be understood. For, formulas (equations, relations, functions, etc.) which are—for example logically—*equivalent* may have completely different forms/shapes in some algebraic/typographic sense. So, "same form" must be understood as "same form up to some equivalent transformations". Generally, two formulas must be regarded as equivalent if they express the same physical content, in the sense that they determine the same relations between the same physical quantities.

This immediately raises the next question: How do we identify the physical quantities defined by the different observers in different inertial frames. The obvious solution is that we identify the physical quantities which have identical empirical definitions. It is however far from obvious how these "identical empirical definitions" are actually understood. For the empirical/operational definitions require *etalon* measuring equipments. But how do the observers in different reference frames share these *etalon* measuring equipments? Do they all base their definitions on the same *etalon* measuring equipments? Is the principle of relativity really understood in this way? Is it true that the laws of physics in K and K', which ought to take the same form, are expressed in terms of physical quantities defined/measured with the same standard measuring equipments? Not exactly: if the standard measuring equipment by means of which the observer in K defines a physical quantity ξ is at rest in K and, therefore, moving in K', then the observer in K' does not define the corresponding ξ' as the physical quantity obtainable by means of the original standard equipment—being at rest in K and moving in K'—but rather as the physical quantity obtainable by means of the standard equipment *in another state of mo-*

tion; namely, at rest relative to K' and in motion relative to K.

With respect to these considerations, one can give the following preliminary formulation of the principle (Szabó 2004):

(RP) The description of a phenomenon exhibited by a physical system co-moving as a whole with an inertial frame K, expressed in terms of the results of measurements obtainable by means of measuring equipments co-moving with K, takes the same form as the description of the same phenomenon exhibited by the same physical system, except that the system is co-moving with another inertial frame K', expressed in terms of the measurements with the same equipments when they are co-moving with K'.

In what follows we translate the above considerations into a mathematical language and develop a formalism providing a more precise formulation of the RP.

3 Conceptual components of the RP

Physical quantities in K Consider an arbitrary collection of physical quantities $\xi_1, \xi_2, \ldots \xi_n$ in K, operationally defined by means of some operations with some equipments being at rest in K (see Remark 1 for a few examples).

The operational counterparts in K' Let $\xi'_1, \xi'_2, \ldots \xi'_n$ denote another collection of physical quantities that are defined by the *same operations* with the *same equipments*, but *in different state of motion*, namely, in which they are all moving with constant velocity $\mathbf{V}$ relative to K, co-moving with K'. Since, for all $i = 1, 2, \ldots n$, both ξ_i and ξ'_i are measured by the same equipment—although in different inertial states of motion—with the same pointer scale, it is plausible to assume that the possible values of ξ_i and ξ'_i range over the same $\sigma_i \subseteq \mathbb{R}$. We introduce the following notation: $\Sigma = \times_{i=1}^{n} \sigma_i \subseteq \mathbb{R}^n$.

Distinction between the quantities in K and K' It must be emphasized that quantities $\xi_1, \xi_2, \ldots \xi_n$ and $\xi'_1, \xi'_2, \ldots \xi'_n$ are, a priori, *different* physical quantities, due to the fact that the operations by which the quantities are defined are performed under *different* physical conditions; with measuring equipments of *different* states of motion. Any objective (non-conventional) relationship between them must be a contingent law of nature. Thus, the same numeric values, say, $(5, 12, \ldots 61) \in \mathbb{R}^n$ correspond to different states of affairs when $\xi_1 = 5, \xi_2 = 12, \ldots \xi_n = 61$ versus $\xi'_1 = 5, \xi'_2 = 12, \ldots \xi'_n = 61$. Consequently, $(\xi_1, \xi_2, \ldots \xi_n)$ and $(\xi'_1, \xi'_2, \ldots \xi'_n)$ are *not* elements of the *same* "space of physical quantities"; although the numeric values of the physical quantities, in both cases, can be represented in $\Sigma = \times_{i=1}^{n} \sigma_i \subseteq \mathbb{R}^n$.

To express this difference in the "physical dimensions" mathematically, we consider two different n-dimensional manifolds, Ω and Ω', each covered by one global coordinate system, ϕ and ϕ' respectively, such that $\phi : \Omega \to \Sigma$ assigns to every point of Ω one of the possible n-tuples of numerical values of physical quantities $\xi_1, \xi_2, \ldots \xi_n$ and $\phi' : \Omega' \to \Sigma$ assigns to every point

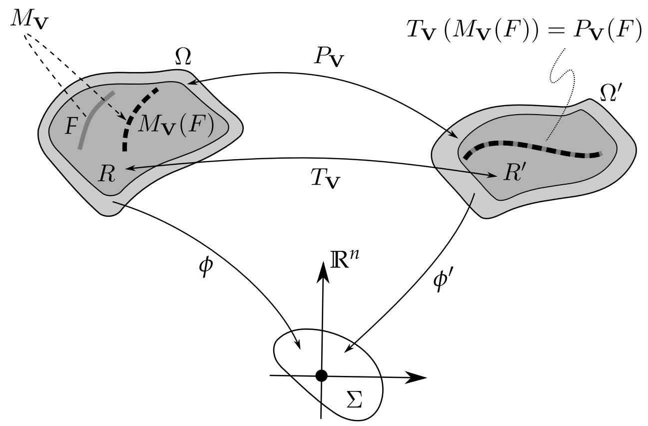

Figure 2: The relativity principle

of Ω' one of the possible n-tuples of numerical values of physical quantities $\xi'_1, \xi'_2, \ldots \xi'_n$ (Fig. 2). In this way, a point $\omega \in \Omega$ represents the *class* of physical constellations in which the quantities $\xi_1, \xi_2, \ldots \xi_n$ take the values $\xi_1 = \phi_1(\omega), \xi_2 = \phi_2(\omega), \ldots \xi_n = \phi_n(\omega)$; similarly, a point $\omega' \in \Omega'$ represents the physical constellation characterized by $\xi'_1 = \phi'_1(\omega'), \xi'_2 = \phi'_2(\omega'), \ldots \xi'_n = \phi'_n(\omega')$.[2] Again, these physical constellations are generally different, even in case of $\phi(\omega) = \phi'(\omega') \in \mathbb{R}^n$.

Admissible values In the above sense, the points of Ω and the points of Ω' range over all possible value combinations of physical quantities $\xi_1, \xi_2, \ldots \xi_n$ and $\xi'_1, \xi'_2, \ldots \xi'_n$. It might be the case however that some combinations are impossible, in the sense that they never come to existence in the physical world. Let us denote by $R \subseteq \Omega$ and $R' \subseteq \Omega'$ the physically admissible parts of Ω and Ω'. Note that $\phi(R)$ is not necessarily identical with $\phi'(R')$.[3]

Putting primes We shall use a bijection $P_{\mathbf{V}} : \Omega \to \Omega'$ ("putting primes"; Bell 1987, p. 73) defined by means of the two coordinate maps ϕ and ϕ':

$$P_{\mathbf{V}} \stackrel{def}{=} \left(\phi'\right)^{-1} \circ \phi \tag{1}$$

Transformation law In contrast with $P_{\mathbf{V}}$, we now introduce the concept of what we call the "transformation" of physical quantities. It is conceived as a bijection

$$T_{\mathbf{V}} : \Omega \supseteq R \to R' \subseteq \Omega' \tag{2}$$

determined by the contingent fact that whenever a physical constellation belongs to the class represented by some $\omega \in R$ then it also belongs to the class represented by $T_{\mathbf{V}}(\omega) \in R'$, and vice versa. Since $\xi_1, \xi_2, \ldots \xi_n$ can be various

[2] $\phi_i = \pi_i \circ \phi$, where π_i is the i-th coordinate projection in $\mathbb{R}^n$.
[3] One can show however that $\phi(R) = \phi'(R')$ if the RP, that is (7), holds.

physical quantities in the various contexts, nothing guarantees that such a bijection exists. We assume however the existence of $T_{\mathbf{V}}$.

Remark 1. It is worthwhile to consider several examples.

(a) Let (ξ_1, ξ_2) be (p, T), the pressure and the temperature of a given (equilibrium) gas; and let (ξ_1', ξ_2') be (p', T'), the pressure and the temperature of the same gas, measured by the moving observer in K'. In this case, there exists a one-to-one $T_{\mathbf{V}}$:

$$p' = p \tag{3}$$

$$T' = T\gamma^{-1} \tag{4}$$

where $\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}$ (Tolman 1949, pp. 158–159).[4] A point $\omega \in \Omega$ of coordinates, say, $p = 101325$ and $T = 300$ (in units Pa and $^\circ K$) represents the class of physical constellations—the class of possible worlds—in which the gas in question has pressure of 101325 Pa and temperature of 300 $^\circ K$. Due to (4), this class of physical constellations is different from the one represented by $P_{\mathbf{V}}(\omega) \in \Omega'$ of coordinates $p' = 101325$ and $T' = 300$; but it is identical to the class of constellations represented by $T_{\mathbf{V}}(\omega) \in \Omega'$ of coordinates $p' = 101325$ and $T' = 300\gamma^{-1}$.

(b) Let $(\xi_1, \xi_2, \ldots \xi_{10})$ be $(t, x, y, z, E_x, E_y, E_z, r_x, r_y, r_z)$, the time, the space coordinates where the electric field strength is taken, the three components of the field strength, and the space coordinates of a particle. And let $(\xi_1', \xi_2', \ldots \xi_{10}')$ be $\left(t', x', y', z', E_x', E_y', E_z', r_x', r_y', r_z'\right)$, the similar quantities obtainable by means of measuring equipments co-moving with K'. In this case, there is no suitable one-to-one $T_{\mathbf{V}}$, as the electric field strength in K does not determine the electric field strength in K', and vice versa.

(c) Let $(\xi_1, \xi_2, \ldots \xi_{13})$ be $(t, x, y, z, E_x, E_y, E_z, B_x, B_y, B_z, r_x, r_y, r_z)$ and let $(\xi_1', \xi_2', \ldots \xi_{13}')$ be $\left(t', x', y', z', E_x', E_y', E_z', B_x', B_y', B_z', r_x', r_y', r_z'\right)$, where B_x, B_y, B_z and B_x', B_y', B_z' are the magnetic field strengths in K and K'. In this case, in contrast with (b), the well known Lorentz transformations of the spatio-temporal coordinates and the electric and magnetic field strengths constitute a proper one-to-one $T_{\mathbf{V}}$. ⌟

Description of a phenomenon Next we turn to the general formulation of the concept of *description of a particular phenomenon* exhibited by a physical system, in terms of physical quantities $\xi_1, \xi_2, \ldots \xi_n$ in K. We are probably not far from the truth if we stipulate that such a description is, in its most abstract sense, a *relation* between physical quantities $\xi_1, \xi_2, \ldots \xi_n$; in other words, it can be given as a subset $F \subset R$.

Remark 2. Consider the above example (a) in Remark 1. An isochoric process of the gas can be described by the subset F that is, in ϕ-coordinates, determined

[4] There is a debate over the proper transformation rules (Georgieu 1969; Sewell 2008).

by the following single equation:

$$F \quad \{p = \kappa T \tag{5}$$

with a certain constant κ.

To give another example, consider the case (b). The relation F given by equations

$$F \quad \begin{cases} E_x(t,x,y,z) = E_0 \\ E_y(t,x,y,z) = 0 \\ E_z(t,x,y,z) = 0 \\ \qquad r_x(t) = r_0 + v_0 t \\ \qquad r_y(t) = 0 \\ \qquad r_z(t) = 0 \end{cases} \tag{6}$$

with some specific values of E_0, r_0, v_0 describes a neutral particle moving with constant velocity in a static homogeneous electric field. ⌟

Physical equations Of course, one may not assume that an arbitrary relation $F \subset R$ has physical meaning. Let $\mathcal{E} \subset 2^R$ be the set of those $F \subset R$ which describe a particular behavior of the system. We shall call $\mathcal{E}$ the *set of equations* describing the physical system in question. The term is entirely justified. In practical calculations, two systems of equations are regarded to be equivalent if and only if they have the same solutions. Therefore, a system of equations can be identified with the set of its solutions. In general, the equations can be algebraic equations, ordinary and partial integro-differential equations, linear and nonlinear, whatever. So, in its most abstract sense, a system of equations is a set of subsets of R.

Now, consider the following subsets[5] of Ω', determined by an $F \in \mathcal{E}$:

Primed solution $P_{\mathbf{V}}(F) \subset \Omega'$: the "primed F", that is a relation "of exactly the same form as F, but in the primed variables $\xi'_1, \xi'_2, \dots \xi'_n$". The quotation marks are important. Since one and the same $F \subset \Omega$ can be given in many different "forms", by means of different numbers of different equations, functions, relations, of different types. That is why we formalized the concept of a description of a phenomenon as an abstract relation between quantities $\xi_1, \xi_2, \dots \xi_n$, given in the form of a subset of Ω. Similarly, subset $P_{\mathbf{V}}(F)$ is an abstract relation between $\xi'_1, \xi'_2, \dots \xi'_n$, which can be thought of in many different equivalent "forms". So, whether F and $P_{\mathbf{V}}(F)$ are "of the same form" may or may not be manifestly apparent (cf. Friedman 1983, p. 150). Also note that relation $P_{\mathbf{V}}(F)$ does not necessarily describe a true physical situation, since it can be not realized in nature.

Same solution expressed in primed variables $T_{\mathbf{V}}(F) \subseteq R'$: which is the same description of the same physical situation as F, but *expressed* in the primed variables.

[5]We denote the map of type $\Omega \to \Omega'$ and its direct image maps of type $2^{\Omega} \to 2^{\Omega'}$ and $2^{2^{\Omega}} \to 2^{2^{\Omega'}}$ or their restrictions by the same symbol.

The same but in different state of motion We need one more concept. The RP is about the connection between two situations: one is in which the system, as a whole, is at rest relative to inertial frame K, the other is in which the system shows the similar behavior, but being in a collective motion relative to K, co-moving with K'. In other words, we assume the existence of a map $M_{\mathbf{V}} : \mathcal{E} \to \mathcal{E}$, assigning to each $F \in \mathcal{E}$, stipulated to describe a phenomenon exhibited by a system co-moving with inertial frame K, another relation $M_{\mathbf{V}}(F) \in \mathcal{E}$, that describes the same physical system exhibiting the same phenomenon as the one described by F, except that the system is *in motion* with velocity $\mathbf{V}$ relative to K, that is, co-moving with inertial frame K'.

4 The formal statement of the RP

Now, applying all these concepts, what the RP states is the following:

$$T_{\mathbf{V}}\left(M_{\mathbf{V}}(F)\right) = P_{\mathbf{V}}(F) \quad \text{for all } F \in \mathcal{E} \tag{7}$$

or equivalently,

$$P_{\mathbf{V}}(F) \subset R' \text{ and } M_{\mathbf{V}}(F) = T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(F)\right) \quad \text{for all } F \in \mathcal{E} \tag{8}$$

Remark 3. Notice that, for a given fixed F, everything on the right hand side of the equation in (8), $P_{\mathbf{V}}$ and $T_{\mathbf{V}}$, are determined *only by* the physical behaviors of *the measuring equipments* when they are in various states of motion. In contrast, the meaning of the left hand side, $M_{\mathbf{V}}(F)$, depends on the physical behavior of *the object physical system* described by F and $M_{\mathbf{V}}(F)$, when it is in various states of motion. That is to say, the two sides of the equation reflect the behaviors of *different parts* of the physical reality; and the RP expresses a law-like regularity between the behaviors of these different parts. ⌟

Remark 4. Let us illustrate these concepts with a well-known textbook example of a static versus uniformly moving charged particle. The static field of a charge q being at *rest* at point (x_0, y_0, z_0) in K is the following:

$$F \quad \begin{cases} E_x(t,x,y,z) = \dfrac{q\,(x-x_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{q\,(y-y_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{q\,(z-z_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = 0 \\ B_z(t,x,y,z) = 0 \end{cases} \tag{9}$$

The stationary field of a charge q *moving* at constant velocity $\mathbf{V} = (V, 0, 0)$ relative to K can be obtained by solving the equations of electrodynamics (in K) with the time-depending source (for example, Jackson 1999, pp. 661–665):

$$
M_{\mathbf{V}}(F) \quad \begin{cases} E_x(t,x,y,z) = \dfrac{qX_0}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = -c^{-2} V E_z(t,x,y,z) \\ B_z(t,x,y,z) = c^{-2} V E_y(t,x,y,z) \end{cases} \tag{10}
$$

where where (x_0, y_0, z_0) is the initial position of the particle at $t = 0$, $X_0 = \gamma\,(x - (x_0 + Vt))$.

Now, we form the same expressions as (9) but in the *primed* variables of the co-moving reference frame K':

$$
P_{\mathbf{V}}\,(F) \quad \begin{cases} E'_{x'}(t',x',y',z') = \dfrac{q'\,(x'-x'_0)}{\left((x'-x'_0)^2 + (y'-y'_0)^2 + (z'-z'_0)^2\right)^{3/2}} \\ E'_{y'}(t',x',y',z') = \dfrac{q'\,(y'-y'_0)}{\left((x'-x'_0)^2 + (y'-y'_0)^2 + (z'-z'_0)^2\right)^{3/2}} \\ E'_{z'}(t',x',y',z') = \dfrac{q'\,(z'-z'_0)}{\left((x'-x'_0)^2 + (y'-y'_0)^2 + (z'-z'_0)^2\right)^{3/2}} \\ B'_{x'}(t',x',y',z') = 0 \\ B'_{y'}(t',x',y',z') = 0 \\ B'_{z'}(t',x',y',z') = 0 \end{cases} \tag{11}
$$

By means of the Lorentz transformation rules of the space-time coordinates, the field strengths and the electric charge (e.g. Tolman 1949), one can express (11) in terms of the original variables of K:

$$
T_{\mathbf{V}}^{-1}\,(P_{\mathbf{V}}(F)) \quad \begin{cases} E_x(t,x,y,z) = \dfrac{qX_0}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = -c^{-2} V E_z(t,x,y,z) \\ B_z(t,x,y,z) = c^{-2} V E_y(t,x,y,z) \end{cases} \tag{12}
$$

We find that the result is indeed the same as (10) describing the field of the moving charge: $M_{\mathbf{V}}(F) = T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(F)\right)$. That is to say, the RP seems to be true in this particular case.

Reversely, *assuming* that the particle + electromagnetic field system satisfies the RP, that is, (8) holds for the equations of electrodynamics, one can *derive* the stationary field of a uniformly moving point charge (10) from the static field (9). ⌟

5 Covariance

Now we have a strict mathematical formulation of the RP for a physical system described by a system of equations $\mathcal{E}$. Remarkably, however, we still have not encountered the concept of "covariance" of equations $\mathcal{E}$. The reason is that the RP and the covariance of equations $\mathcal{E}$ are not equivalent—in contrast to what many believe. In fact, the logical relationship between the two conditions is much more complex. To see this relationship in more detail, we previously need to clarify a few things.

Consider the following two sets: $P_{\mathbf{V}}(\mathcal{E}) = \{P_{\mathbf{V}}(F) | F \in \mathcal{E}\}$ and $T_{\mathbf{V}}(\mathcal{E}) = \{T_{\mathbf{V}}(F) | F \in \mathcal{E}\}$. Since a system of equations can be identified with its set of solutions, $P_{\mathbf{V}}(\mathcal{E}) \subset 2^{\Omega'}$ and $T_{\mathbf{V}}(\mathcal{E}) \subset 2^{R'}$ can be regarded as two systems of equations for functional relations between $\xi'_1, \xi'_2, \ldots \xi'_n$. In the primed variables, $P_{\mathbf{V}}(\mathcal{E})$ has "the same form" as $\mathcal{E}$. Nevertheless, it can be the case that $P_{\mathbf{V}}(\mathcal{E})$ does not express a true physical law, in the sense that its solutions do not necessarily describe true physical situations. In contrast, $T_{\mathbf{V}}(\mathcal{E})$ is nothing but $\mathcal{E}$ expressed in variables $\xi'_1, \xi'_2, \ldots \xi'_n$.

Now, covariance intuitively means that equations $\mathcal{E}$ "preserve their forms against the transformation $T_{\mathbf{V}}$". That is, in terms of the formalism we developed:

$$T_{\mathbf{V}}(\mathcal{E}) = P_{\mathbf{V}}(\mathcal{E}) \tag{13}$$

or, equivalently,

$$P_{\mathbf{V}}(\mathcal{E}) \subset 2^{R'} \text{ and } \mathcal{E} = T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\mathcal{E})\right) \tag{14}$$

The first thing we have to make clear is that—even if we know or presume that it holds—covariance (14) is obviously *not sufficient* for the RP (8). For, (14) only guarantees the invariance of the set of solutions, $\mathcal{E}$, against $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$, but it says nothing about which solution of $\mathcal{E}$ corresponds to which solution. While it is the very essence of the RP that it is solution $M_{\mathbf{V}}(F)$—describing the same physical system exhibiting the same phenomenon as the one described by F, except that the system is *in motion* with velocity $\mathbf{V}$ relative to K—which must be equal to solution $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}(F)$. For example, what we use in the above mentioned textbook derivation of the stationary electromagnetic field of a uniformly moving point charge (end of Remark 4) is not the covariance of the equations—that would be not enough—but statement (8), that is, what the RP claims about the solutions of the equations in detail.

It must be clear that the fact that covariance (14) does not imply the RP (8) is simply a *logical* fact. This fact is prior to the physical problem of whether the RP holds for a given physical situation or not; whether we have a physically meaningful, unambiguously defined map $M_{\mathbf{V}} : \mathcal{E} \to \mathcal{E}$; whether and under

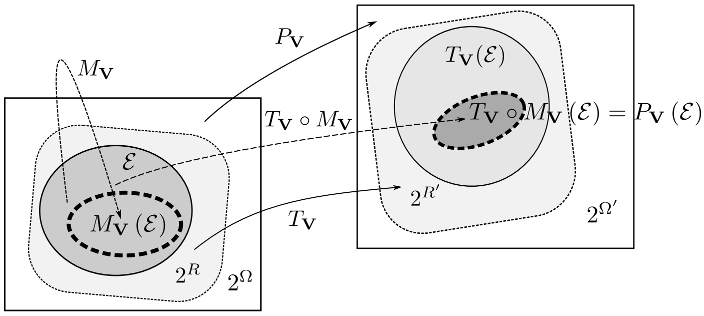

Figure 3: The RP only implies that $T_{\mathbf{V}}(\mathcal{E}) \supseteq T_{\mathbf{V}} \circ M_{\mathbf{V}}(\mathcal{E}) = P_{\mathbf{V}}(\mathcal{E})$. Covariance of $\mathcal{E}$ would require that $T_{\mathbf{V}}(\mathcal{E}) = P_{\mathbf{V}}(\mathcal{E})$, which is generally not the case

what conditions solution $M_{\mathbf{V}}(F)$ really describes "the system set anyhow in motion" in reality (cf. Jánossy 1971, pp. 207–210; Bell 1987, p. 75; Szabó 2004).

In a precise sense, covariance is not only not sufficient for the RP, but it is *not even necessary* (Fig. 3). The RP only implies that

$$T_{\mathbf{V}}(\mathcal{E}) \supseteq T_{\mathbf{V}}\left(M_{\mathbf{V}}(\mathcal{E})\right) = P_{\mathbf{V}}(\mathcal{E}) \tag{15}$$

(7) implies (13) only if we have the following *extra* condition:

$$M_{\mathbf{V}}(\mathcal{E}) = \mathcal{E} \tag{16}$$

6 Initial and boundary conditions

Let us finally consider the situation when the solutions of a system of equations $\mathcal{E}$ are specified by some extra conditions—initial and/or boundary value conditions, for example. In our general formalism, an extra condition for $\mathcal{E}$ is a system of equations $\psi \subset 2^{\Omega}$ such that there exists exactly one solution $[\psi]_{\mathcal{E}}$ satisfying both $\mathcal{E}$ and ψ. That is, $\mathcal{E} \cap \psi = \{[\psi]_{\mathcal{E}}\}$, where $\{[\psi]_{\mathcal{E}}\}$ is a singleton set. Since $\mathcal{E} \subset 2^R$, without loss of generality we may assume that $\psi \subset 2^R$.

Since $P_{\mathbf{V}}$ and $T_{\mathbf{V}}$ are injective, $P_{\mathbf{V}}(\psi)$ and $T_{\mathbf{V}}(\psi)$ are extra conditions for equations $P_{\mathbf{V}}(\mathcal{E})$ and $T_{\mathbf{V}}(\mathcal{E})$ respectively, and we have

$$P_{\mathbf{V}}\left([\psi]_{\mathcal{E}}\right) = \left[P_{\mathbf{V}}(\psi)\right]_{P_{\mathbf{V}}(\mathcal{E})} \tag{17}$$

$$T_{\mathbf{V}}\left([\psi]_{\mathcal{E}}\right) = \left[T_{\mathbf{V}}(\psi)\right]_{T_{\mathbf{V}}(\mathcal{E})} \tag{18}$$

for all extra conditions ψ for $\mathcal{E}$. Similarly, if $P_{\mathbf{V}}(\mathcal{E}), P_{\mathbf{V}}(\psi) \subset 2^{R'}$ then $T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\psi)\right)$ is an extra condition for $T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\mathcal{E})\right)$, and

$$\left[T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\psi)\right)\right]_{T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\mathcal{E})\right)} = T_{\mathbf{V}}^{-1}\left(\left[P_{\mathbf{V}}(\psi)\right]_{P_{\mathbf{V}}(\mathcal{E})}\right) \tag{19}$$

If equations $\mathcal{E}$ satisfy the covariance condition (14), we have

$$\left[T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}(\psi)\right)\right]_{\mathcal{E}} = T_{\mathbf{V}}^{-1}\left(\left[P_{\mathbf{V}}(\psi)\right]_{P_{\mathbf{V}}(\mathcal{E})}\right) \tag{20}$$

That is to say, solving the primed equation with the primed extra conditions is equivalent to first expressing the primed extra conditions in the original quantities and then solving the original equations (cf. Houtappel, Van Dam, and Wigner 1963). Notice however that it by no means follows *from the covariance* of equations $\mathcal{E}$ that the primed extra conditions determine the solution describing the moving object; that is, it can be the case that $\left[T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}\left(\psi\right)\right)\right]_{\mathcal{E}} \neq M_{\mathbf{V}}\left([\psi]_{\mathcal{E}}\right)$—this is the difference between the RP and the covariance requirement.

Now consider a set of extra conditions $\mathcal{C} \subset 2^{2^R}$ and assume that $\mathcal{C}$ is a *parametrizing set of extra conditions* for $\mathcal{E}$; by which we mean that for all $F \in \mathcal{E}$ there exists exactly one $\psi \in \mathcal{C}$ such that $F = [\psi]_{\mathcal{E}}$; in other words,

$$\mathcal{C} \ni \psi \mapsto [\psi]_{\mathcal{E}} \in \mathcal{E} \tag{21}$$

is a bijection.

$M_{\mathbf{V}} : \mathcal{E} \to \mathcal{E}$ was introduced as a map between solutions of $\mathcal{E}$. Now, as there is a one-to-one correspondence between the elements of $\mathcal{C}$ and $\mathcal{E}$, it generates a map $M_{\mathbf{V}} : \mathcal{C} \to \mathcal{C}$, such that

$$[M_{\mathbf{V}}(\psi)]_{\mathcal{E}} = M_{\mathbf{V}}\left([\psi]_{\mathcal{E}}\right) \tag{22}$$

Thus, from (17) and (22), the RP, that is (7), has the following form:

$$T_{\mathbf{V}}\left([M_{\mathbf{V}}(\psi)]_{\mathcal{E}}\right) = [P_{\mathbf{V}}(\psi)]_{P_{\mathbf{V}}(\mathcal{E})} \qquad \text{for all } \psi \in \mathcal{C} \tag{23}$$

or, equivalently, (8) reads

$$[P_{\mathbf{V}}(\psi)]_{P_{\mathbf{V}}(\mathcal{E})} \subset R' \text{ and } [M_{\mathbf{V}}(\psi)]_{\mathcal{E}} = T_{\mathbf{V}}^{-1}\left([P_{\mathbf{V}}(\psi)]_{P_{\mathbf{V}}(\mathcal{E})}\right) \tag{24}$$

One might make use of the following theorem:

Theorem 1. *Assume that the system of equations $\mathcal{E} \subset 2^R$ is covariant, that is, (13) is satisfied. Then,*

(i) for all $\psi \in \mathcal{C}$, $T_{\mathbf{V}}\left(M_{\mathbf{V}}\left(\psi\right)\right)$ is an extra condition for the system of equations $P_{\mathbf{V}}\left(\mathcal{E}\right)$, and, (23) is equivalent to the following condition:

$$\left[T_{\mathbf{V}}\left(M_{\mathbf{V}}(\psi)\right)\right]_{P_{\mathbf{V}}(\mathcal{E})} = [P_{\mathbf{V}}(\psi)]_{P_{\mathbf{V}}(\mathcal{E})} \tag{25}$$

(ii) for all $\psi \in \mathcal{C}, P_{\mathbf{V}}\left(\psi\right) \subset 2^{R'}$, $T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}\left(\psi\right)\right)$ is an extra condition for the system of equations $\mathcal{E}$ and (24) is equivalent to the following condition:

$$[M_{\mathbf{V}}(\psi)]_{\mathcal{E}} = \left[T_{\mathbf{V}}^{-1}\left(P_{\mathbf{V}}\left(\psi\right)\right)\right]_{\mathcal{E}} \tag{26}$$

Proof. (i) Obviously, $T_{\mathbf{V}}\left(\mathcal{E}\right) \cap T_{\mathbf{V}}\left(M_{\mathbf{V}}\left(\psi\right)\right)$ exists and is a singleton; and, due to (13), it is equal to $P_{\mathbf{V}}\left(\mathcal{E}\right) \cap T_{\mathbf{V}}\left(M_{\mathbf{V}}\left(\psi\right)\right)$; therefore this latter is a singleton, too. Applying (18) and (13), we have

$$T_{\mathbf{V}}\left([M_{\mathbf{V}}(\psi)]_{\mathcal{E}}\right) = \left[T_{\mathbf{V}}\left(M_{\mathbf{V}}\left(\psi\right)\right)\right]_{T_{\mathbf{V}}(\mathcal{E})} = \left[T_{\mathbf{V}}\left(M_{\mathbf{V}}\left(\psi\right)\right)\right]_{P_{\mathbf{V}}(\mathcal{E})} \tag{27}$$

therefore, (25) implies (24).

(ii) Similarly, due to $P_{\mathbf{V}}(\psi) \subset 2^{R'}$ and (14), $\mathcal{E} \cap T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(\psi))$ exists and is a singleton. Applying (19) and (14), we have

$$T_{\mathbf{V}}^{-1}\left([P_{\mathbf{V}}(\psi)]_{P_{\mathbf{V}}(\mathcal{E})}\right) = \left[T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(\psi))\right]_{T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(\mathcal{E}))} = \left[T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(\psi))\right]_{\mathcal{E}} \tag{28}$$

that is, (26) implies (24). □

Remark 5. Let us note a few important—but often overlooked—facts which can easily be seen in the formalism we developed:

(a) The covariance of a set of equations $\mathcal{E}$ does *not* imply the covariance of a subset of equations separately. It is because a smaller set of equations corresponds to an $\mathcal{E}^* \subset 2^R$ such that $\mathcal{E} \subset \mathcal{E}^*$; and it does not follow from (13) that $T_{\mathbf{V}}(\mathcal{E}^*) = P_{\mathbf{V}}(\mathcal{E}^*)$.

(b) Similarly, the covariance of a set of equations $\mathcal{E}$ does *not* guarantee the covariance of an arbitrary set of equations which is only satisfactory to $\mathcal{E}$; for example, when the solutions of $\mathcal{E}$ are restricted by some extra conditions. Because from (13) it does not follow that $T_{\mathbf{V}}(\mathcal{E}^*) = P_{\mathbf{V}}(\mathcal{E}^*)$ for an arbitrary $\mathcal{E}^* \subset \mathcal{E}$.

(c) The same holds, of course, for the combination of cases (a) and (b); for example, when we have a smaller set of equations $\mathcal{E}^* \supset \mathcal{E}$ together with some extra conditions $\psi \subset 2^R$. For, (13) does not imply that $T_{\mathbf{V}}(\mathcal{E}^* \cap \psi) = P_{\mathbf{V}}(\mathcal{E}^* \cap \psi)$.

(d) However, covariance is guaranteed if a covariant set of equations is restricted with a *covariant* set of extra conditions; because $T_{\mathbf{V}}(\mathcal{E}) = P_{\mathbf{V}}(\mathcal{E})$ and $T_{\mathbf{V}}(\psi) = P_{\mathbf{V}}(\psi)$ trivially imply that $T_{\mathbf{V}}(\mathcal{E} \cap \psi) = P_{\mathbf{V}}(\mathcal{E} \cap \psi)$. ⌟

7 Concluding discussions and open problems

As we have seen, the RP does not reduce to the covariance of the physical equations, and the precise formulation of the RP is a much more difficult matter. It requires several conceptual plugins, without which the RP would be simply meaningless. In section 3 we gave the explicit formulation of these concepts in our formalism. The concrete meanings of these generally formalized conceptual plugins are to be specified in the concrete physical contexts.

It must be mentioned that one of these concepts, $M_{\mathbf{V}} : \mathcal{E} \to \mathcal{E}$, which carries an essential part of the physical content of the RP, seems to be especially problematic. For, what does it generally mean to say that a solution, $M_{\mathbf{V}}(F)$, describes the same physical system exhibiting the same phenomenon as the one described by F, except that the system is in motion with velocity $\mathbf{V}$ relative to K? As it was pointed out in (Szabó 2004), there is no clear and unambiguous answer to this question, even in very simple situations.

In fact the same question can be asked with respect to the definitions of quantities $\zeta'_1, \zeta'_2, \ldots \zeta'_n$—and, therefore, with respect to the meanings of $T_{\mathbf{V}}$ and $P_{\mathbf{V}}$. For, $\zeta'_1, \zeta'_2, \ldots \zeta'_n$ are not simply arbitrary variables assigned to reference

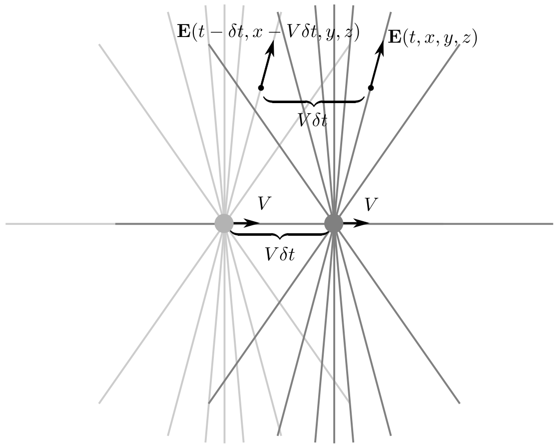

Figure 4: The stationary field of a uniformly moving point charge is in collective motion together with the point charge

frame K', in one-to-one relations with $\xi_1, \xi_2, \ldots \xi_n$, but the physical quantities obtainable by means of the same operations with the same measuring equipments as in the operational definitions of $\xi_1, \xi_2, \ldots \xi_n$, except that everything is in a collective motion with velocity $\mathbf{V}$. Therefore, we should know what we mean by "the same measuring equipment but in collective motion". From this point of view, it does not matter whether the system in question is the object to be observed or a measuring equipment involved in the observation.

At this level of generality we only want to point out two things. First, whatever is the definition of $M_{\mathbf{V}} : \mathcal{E} \to \mathcal{E}$ in the given context, the following is a *minimum* requirement for the RP to have the assumed physical meaning:

(M) Every relation $F \in \mathcal{E}$ must describe a phenomenon which can be meaningfully characterized as such that the physical system exhibiting this phenomenon is co-moving with some inertial frame of reference.

Recall that this minimum requirement is, tacitly, already there in Galileo's principle. As Brown points out in the passage we quoted in the Introduction: "The principle compares the outcome of relevant processes inside the cabin under different states of inertial motion of the cabin relative to the shore."

A simple example for a system satisfying condition (M) is the one discussed in Remark **4**: solutions (9) and (10) both describe a system of charged particle + electromagnetic field which are in collective rest and motion respectively. The electromagnetic field is in collective motion with the point charge of velocity V (Fig. 4) in the following sense:

$$\mathbf{E}(t,x,y,z) = \mathbf{E}(t-\delta t, x - V\delta t, y, z) \tag{29}$$

$$\mathbf{B}(t,x,y,z) = \mathbf{B}(t-\delta t, x - V\delta t, y, z) \tag{30}$$

But, generally, condition (M) by no means requires that the system be in a simple stationary state and all parts move with the same collective velocity—the objects contained in Galileo's cabin may exhibit very complex time-dependent behavior; the fishes may swim with their fins, the butterflies may move their wings, the particles of the smoke may follow a very complex dynamics.

Notice that requirement (M) does not even say anything about whether and how the fact that the system in question is co-moving with a reference frame is reflected in a solution $F \in \mathcal{E}$. It does not even require that this fact can be expressed in terms of quantities $\xi_1, \xi_2, \ldots \xi_n$. It only requires that each $F \in \mathcal{E}$ belong to a physical situation in which it is meaningful to say—perhaps in terms of quantities different from $\xi_1, \xi_2, \ldots \xi_n$—that the system is at rest or in motion relative to an inertial reference frame. How a concrete physical situation can be so characterized is a separate problem, which can be discussed in the particular contexts.[6]

The second thing to be said about $M_{\mathbf{V}}(F)$ is that it is a notion determined by the concrete physical context; but it is *not* equal to the "Lorentz boosted solution" $T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(F))$ *by definition* —as a little reflection shows:

(a) In this case, (8) would read

$$T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(F)) = T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(F)) \tag{31}$$

That is, the RP would become a tautology; a statement which is always true, independently of any contingent fact of nature; independently of the actual behavior of moving physical objects; and independently of the actual empirical meanings of physical quantities $\xi'_1, \xi'_2, \ldots \xi'_n$. But, the RP is supposed to be a fundamental *law of nature*. Note that a tautology is entirely different from a fundamental principle, even if the principle is used as a fundamental hypothesis or fundamental premise of a theory, from which one derives further physical statements. For, a fundamental premise, as expressing a contingent fact of nature, is potentially falsifiable by testing its consequences; a tautology is not.

(b) Even if accepted, $M_{\mathbf{V}}(F) \stackrel{def}{=} T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(F))$ can provide physical meaning to $M_{\mathbf{V}}(F)$ only if we know the meanings of $T_{\mathbf{V}}$ and $P_{\mathbf{V}}$, that is, if we know the empirical meanings of the quantities denoted by $\xi'_1, \xi'_2, \ldots \xi'_n$. But, the physical meaning of $\xi'_1, \xi'_2, \ldots \xi'_n$ are obtained from the operational definitions: they are the quantities obtained by "the same measurements with the same equipments when they are co-moving with K' with velocity $\mathbf{V}$ relative to K". Symbolically, we need, priory, the concepts of $M_{\mathbf{V}}(\xi_i\textit{-equipment at rest})$. And this is a conceptual circularity: in order to have the concept of what it is to be an $M_{\mathbf{V}}(\textit{brick at rest})$ the (size)′ of which we would like to ascertain, we need to have the concept of what it is to be an $M_{\mathbf{V}}(\textit{measuring rod at rest})$—which is exactly the same conceptual problem.

(c) One might claim that we do not need to specify the concepts of $M_{\mathbf{V}}(\xi_i\textit{-equipment at rest})$ in order to know the *values* of quantities

[6] For example, even this minimum requirement can raise non-trivial questions in electrodynamics (Gömöri and Szabó 2011b).

$\xi_1', \xi_2', \ldots \xi_n'$ we obtain by the measurements with the moving equipments, given that we can know the transformation rule $T_{\mathbf{V}}$ independently of knowing the operational definitions of $\xi_1', \xi_2', \ldots \xi_n'$. Typically, $T_{\mathbf{V}}$ is thought to be derived from the assumption that the RP (8) holds. If however $M_{\mathbf{V}}$ is, by definition, equal to $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$, then in place of (8) we have the tautology (31), which does not determine $T_{\mathbf{V}}$.

(d) Therefore, unsurprisingly, it is not the RP from which the transformation rules are routinely deduced, but the covariance (14). As we have seen, however, covariance is, in general, neither sufficient nor necessary for the RP. Whether (8) implies (14) hinges on the physical fact whether (16) is satisfied. But, if $M_{\mathbf{V}}$ is taken to be $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$ by definition, the RP becomes true—in the form of tautology (31)—but does not imply covariance $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}(\mathcal{E}) = \mathcal{E}$.

(e) Even if we assume that a "transformation rule" function $\phi' \circ T_{\mathbf{V}} \circ \phi^{-1}$ were derived from some independent premises—from the independent assumption of covariance, for example—how do we know that the $T_{\mathbf{V}}$ we obtained and the quantities of values $\phi' \circ T_{\mathbf{V}} \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ are correct plugins for the RP? How could we verify that $\phi' \circ T_{\mathbf{V}} \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ are indeed the values measured by a moving observer applying the same operations with the same measuring equipments, etc.?—without having an independent concept of $M_{\mathbf{V}}$, at least for the measuring equipments?

(f) One could argue that we do not need such a verification; $\phi' \circ T_{\mathbf{V}} \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ can be regarded *as the empirical definition* of the primed quantities:

$$\left(\xi_1', \xi_2', \ldots \xi_n'\right) \stackrel{def}{=} \phi' \circ T_{\mathbf{V}} \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n) \tag{32}$$

This is of course logically possible. The operational definition of the primed quantities would say: ask the observer at rest in K to measure $\xi_1, \xi_2, \ldots \xi_n$ with the measuring equipments at rest in K, and then perform the mathematical operation (32). In this way, however, even the transformation rules would become tautologies; they would be true, no matter how the things are in the physical world.

(g) Someone might claim that the identity of $M_{\mathbf{V}}$ with $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$ is not a simple stipulation but rather an analytic truth which follows from the identity of the two *concepts*. Still, if that were the case, RP would be a statement which is true in all possible worlds; independently of any contingent fact of nature; independently of the actual behavior of moving physical objects.

(h) On the contrary, as we have already pointed out in Remark 3, $M_{\mathbf{V}}(F)$ and $T_{\mathbf{V}}^{-1}(P_{\mathbf{V}}(F))$ are *different concepts*, referring to different features of different parts of the physical reality. Any connection between the two things must be a contingent fact of the world.

(i) $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$ is a $2^R \to 2^R$ map which is completely determined by the physical behaviors of the *measuring* equipments. On the other hand, whether the elements of $\mathcal{E} \subset 2^R$ satisfy condition (M) and whether $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}(\mathcal{E}) \subseteq \mathcal{E}$ depend on the actual physical properties of the *object* physical system.

(j) Let us note that in the standard textbook applications of the RP $M_{\mathbf{V}}$ is used as an independent concept, without any prior reference to the Lorentz boost $T_{\mathbf{V}}^{-1} \circ P_{\mathbf{V}}$. For example, we do not need to refer to the Lorentz transformations in order to understand the concept of 'the stationary electromagnetic field of a uniformly moving point charge'; as we are capable to solve the electrodynamical equations for such a situation, within one single frame of reference, without even knowing of the Lorentz transformation rules. ⌟

Acknowledgment

The research was partly supported by the OTKA Foundation, No. K100715.

References

Bell, J.S. (1987): How to teach special relativity, in *Speakable and unspeakable in quantum mechanics*. Cambridge, Cambridge University Press.

Brading, K. and Brown, H.R. (2004): Are Gauge Symmetry Transformations Observable? *British Journal for the Philosophy of Science* **55**, 645.

Brading, K. and Castellani, E. (2008): Symmetry and Symmetry Breaking, *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, E. N. Zalta (ed.), http://plato.stanford.edu/archives/fall2008/entries/symmetry-breaking.

Brown, H.R. (2005): Physical relativity – space-time structure from a dynamical perspective. Oxford, New York, Oxford University Press.

Dieks, D. (2006): Another Look at General Covariance and the Equivalence of Reference Frames, *Studies In History and Philosophy of Modern Physics* **37**, 174.

Einstein, A (1905): Zur Elektrodynamik bewegter Körper, *Annalen der Physik* **17**, 891. (On the Electrodynamics of Moving Bodies, in H. A. Lorentz et al., *The principle of relativity: a collection of original memoirs on the special and general theory of relativity.* London, Methuen and Company 1923)

Einstein, A (1940): The Fundaments of Theoretical Physics, *Ideas and Opinions*, New York, Bonanza, pp. 33–35.

Earman, J. (2004): Laws, Symmetry, and Symmetry Breaking: Invariance, Conservation Principles, and Objectivity, *Philosophy of Science* **71**, 1227.

Friedman, M. (1983): *Foundations of space-time theories*. Princeton NJ, Princeton University Press.

Georgiou, A. (1969): Special relativity and thermodynamics, *Proc. Comb. Phil. Soc.* **66**, 423.

Gömöri, M. and Szabó, L. E. (2011a): Operational understanding of the covariance of classical electrodynamics, (http://philsci-archive.pitt.edu/id/eprint/8783)

Gömöri, M. and Szabó, L. E. (2011b): How to move an electromagnetic field?, (http://philsci-archive.pitt.edu/id/eprint/8784).

Houtappel, R. M. F, Van Dam, H. and Wigner, E. P. (1965): The Conceptual Basis and Use of the Geometric Invariance Principles, *Reviews of Modern Physics* **37**, 595.

Jackson, J.D. (1999): *Classical Electrodynamics (Third edition).* Hoboken (NJ), John Wiley & Sons.

Jánossy, L. (1971): *Theory of relativity based on physical reality.* Budapest, Akadémiai Kiadó.

Norton, J.D. (1988): Coordinates and Covariance: Einstein's view of spacetime and the modern view, *Foundations of Physics* **19**, 1215.

Norton, J.D. (1993): General Covariance and the Foundations of General Relativity: Eight Decades of Dispute, *Reports on Progress in Physics* **56**, 791.

Rovelli, C. (2004): *Quantum Gravity*, Cambridge, Cambridge University Press, Sections 2.2–2.3.

Sewell, G. L. (2008): On the question of temperature transformations under Lorentz and Galilei boosts, *Journal of Physics A: Mathematical and Theoretical* **41**, 382003.

Szabó, L.E. (2004): On the meaning of Lorentz covariance, *Foundations of Physics Letters* **17**, 479.

Tolman, R.C. (1949): *Relativity, Thermodynamics and Cosmology.* Oxford, Clarendon Press.

Westman, H. and Sonego, S. (2009): Coordinates, observables and symmetry in relativity, *Ann. Phys.* **324**, 1585.