How to move an electromagnetic field?

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Abstract

As a first principle, it is the basic assumption of the standard relativistic formulation of classical electrodynamics (ED) that the physical laws describing the electromagnetic phenomena satisfy the relativity principle (RP). According to the standard view, this assumption is absolutely unproblematic, and its correctness is well confirmed, at least in a hypothetico-deductive sense, by means of the empirical confirmation of the consequences derived from it. In this paper, we will challenge this customary view as being somewhat simplistic. The RP is actually used in exceptional cases satisfying some special conditions. As we will see, however, it is quite problematic how the RP must be understood in the general case of a coupled particles + electromagnetic field system.

1 Introduction

As a first principle, it is the basic assumption of the standard relativistic formulation of classical electrodynamics (ED) that the physical laws describing the electromagnetic phenomena satisfy the relativity principle (RP). According to the standard view, this assumption is absolutely unproblematic, and its correctness is well confirmed, at least in a hypothetico-deductive sense, by means of the empirical confirmation of the consequences derived from it. In this paper, we will challenge this customary view as being somewhat simplistic. In the majority of cases these results are, in fact, derived merely from the covariance of the corresponding equations, by means of the transformation rules. The RP is actually used in exceptional cases satisfying some special conditions. As we will see, however, it is quite problematic how the RP must be understood in the general case of a coupled particles + electromagnetic field system.

Einstein's (1905) derivation of the relativistic Lorentz equation of motion of a charged particle, or, in the same paper, the derivation of the relativistic Doppler effect are good examples for the first group of situations in which the results are obtained merely from the covariance and the transformation rules.

According to the standard schema, the covariance of the equations of electrodynamics is taken as a straightforward consequence of the RP and the transformation rules for the basic electrodynamical quantities are derived from the required covariance. There are several problems to be raised concerning these derivations, and certain steps are questionable (Gömöri and Szabó 2011b). This is however not our main concern here. We do not question the transformation rules of the electrodynamical quantities—and, therefore, the covariance of the Maxwell–Lorentz equations—as they can be derived from the laws of electrodynamics in one single frame of reference, independently of the RP (Gömöri and Szabó 2011b). Our concern in this paper is whether the Maxwell–Lorentz electrodynamics satisfies the RP.

It is generally true that covariance is a logical consequence of the RP, at least under some plausible conditions (Gömöri–Szabó 2011a). It must be clear however that the opposite is not true: covariance of the equations is not sufficient for the RP. In Bell's words:

> Lorentz invariance alone shows that for any state of a system at rest there is a corresponding 'primed' state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'primed' of the original state, rather than into the 'prime' of some *other* state of the original system. (Bell 1987, p. 75)

While it is the very essence of the RP that it is about the connection between two situations: one is in which the system, as a whole, is at rest relative to one inertial frame, say K, the other is in which the system shows the similar behavior, but being in a collective motion relative to K, co-moving with some K'. In other words, the RP assigns to each solution F of the equations, stipulated to describe the situation in which the system is co-moving as a whole with inertial frame K, another solution $M_{\mathbf{V}}(F)$, describing the similar behavior of the same system when it is, as a whole, co-moving with inertial frame K', that is, when it is in a collective motion with velocity $\mathbf{V}$ relative to K, where $\mathbf{V}$ is the velocity of K' relative to K. And it asserts that the solution $M_{\mathbf{V}}(F)$, expressed in the primed variables of K', has exactly the same form as F in the original variables of K.[1]

To put the problem we address in this paper in perspective, let us focus on the fact that the RP is about the relationship between two states of the physical system in question: One is in which the system, as a whole, is *at rest* relative to K; the other is in which the system is *at rest* relative to K', that is, in which the system is *in motion with constant velocity* relative to K. Consequently, a minimal requirement for the RP to be a meaningful statement is the following :

Minimal Requirement for the RP (MR) *The states of the system in question must be meaningfully characterized as such in which the system as a whole is at rest or in motion with some velocity relative to an arbitrary frame of reference.*

Let us show a well-known electrodynamical example in which a particles + electromagnetic field system satisfies this condition. Consider one single charged particle moving with constant velocity $\mathbf{V} = (V, 0, 0)$ relative to K and

[1] For a more precise formulation of the relativity principle and covariance, see Gömöri–Szabó 2011a.

the coupled stationary electromagnetic field (Jackson 1999, pp. 661):

$$M_{\mathbf{V}}(F) \quad \begin{cases} E_x = \dfrac{qX_0}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_y = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_z = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ B_x = 0 \\ B_y = -c^{-2}VE_z \\ B_z = c^{-2}VE_y \end{cases} \tag{1}$$

where where (x_0, y_0, z_0) is the initial position of the particle at $t = 0$, $X_0 = \gamma\,(x - (x_0 + Vt))$ and $\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}$. In this case, it is no problem to characterize the particle + electromagnetic field system as such which is, as a whole, in motion with velocity $\mathbf{V}$ relative to K; as the electromagnetic field is in collective motion with the point charge of velocity $\mathbf{V}$ (Fig. 1) in the following sense:[2]

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) \tag{2}$$
$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) \tag{3}$$

that is,

$$-\partial_t \mathbf{E}(\mathbf{r}, t) = \mathrm{D}\mathbf{E}(\mathbf{r}, t)\mathbf{V} \tag{4}$$
$$-\partial_t \mathbf{B}(\mathbf{r}, t) = \mathrm{D}\mathbf{B}(\mathbf{r}, t)\mathbf{V} \tag{5}$$

where $\mathrm{D}\mathbf{E}(\mathbf{r}, t)$ and $\mathrm{D}\mathbf{B}(\mathbf{r}, t)$ denote the spatial derivative operators (Jacobians for variables x, y and z); that is, in components:

$$-\partial_t E_x(\mathbf{r}, t) = V_x \partial_x E_x(\mathbf{r}, t) + V_y \partial_y E_x(\mathbf{r}, t) + V_z \partial_z E_x(\mathbf{r}, t) \tag{6}$$
$$-\partial_t E_y(\mathbf{r}, t) = V_x \partial_x E_y(\mathbf{r}, t) + V_y \partial_y E_y(\mathbf{r}, t) + V_z \partial_z E_y(\mathbf{r}, t) \tag{7}$$
$$\vdots$$
$$-\partial_t B_z(\mathbf{r}, t) = V_x \partial_x B_z(\mathbf{r}, t) + V_y \partial_y B_z(\mathbf{r}, t) + V_z \partial_z B_z(\mathbf{r}, t) \tag{8}$$

[2]It must be pointed out that velocity $\mathbf{V}$ conceptually differs from the speed of light c. Basically, c is a constant of nature in the Maxwell–Lorentz equations, which can emerge in the solutions of the equations; and, in some cases, it can be interpreted as the velocity of propagation of changes in the electromagnetic field. For example, in our case, the stationary field of a uniformly moving point charge, in collective motion with velocity $\mathbf{V}$, can be constructed from the superposition of retarded potentials, in which the retardation is calculated with velocity c; nevertheless, the two velocities are different concepts. To illustrate the difference, consider the fields of a charge at rest (9), and in motion (1). The speed of light c plays the same role in both cases. Both fields can be constructed from the superposition of retarded potentials in which the retardation is calculated with velocity c. Also, in both cases, a small local perturbation in the field configuration would propagate with velocity c. But still, there is a consensus to say that the system described by (9) is at rest while the one described by (1) is moving with velocity $\mathbf{V}$ (together with K', relative to K.) A good analogy would be a Lorentz contracted moving rod: $\mathbf{V}$ is the velocity of the rod, which differs from the speed of sound in the rod.

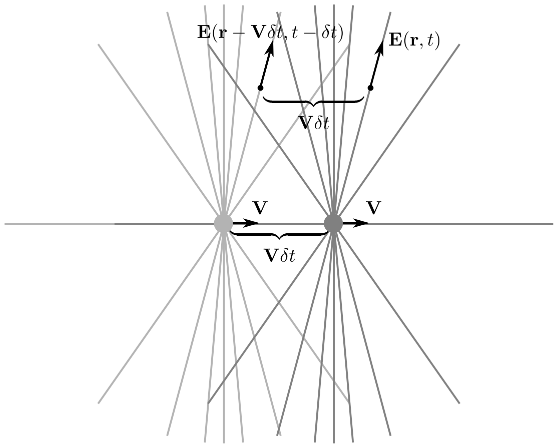

Figure 1: The stationary field of a uniformly moving point charge is in collective motion together with the point charge

The uniformly moving point charge + electromagnetic field system not only satisfies condition MR, but it satisfies the RP: Formula (1) with $\mathbf{V} = 0$ describes the static field of the particle when they are at rest in K :

$$F \quad \left\{ \begin{array}{l} E_x = \dfrac{q\,(x - x_0)}{\left((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right)^{3/2}} \\ E_y = \dfrac{q\,(y - y_0)}{\left((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right)^{3/2}} \\ E_z = \dfrac{q\,(z - z_0)}{\left((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right)^{3/2}} \\ B_x = 0 \\ B_y = 0 \\ B_z = 0 \end{array} \right. \tag{9}$$

By means of the Lorentz transformation rules one can express (1) in terms of the 'primed' variables of the co-moving reference frame K':

$$\begin{aligned}
E'_x &= \frac{q'\left(x'-x'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}} \\
E'_y &= \frac{q'\left(y'-y'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}} \\
E'_z &= \frac{q'\left(z'-z'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}} \\
B'_x &= 0 \\
B'_y &= 0 \\
B'_z &= 0
\end{aligned} \tag{10}$$

and we find that the result is indeed of the same form as (9).

So, in this well-known particular textbook example the RP is meaningful and satisfied. In accordance with the standard realistic interpretation of electromagnetic field (Frisch 2005, p. 41), the states F and $M_{\mathbf{V}}(F)$ can be meaningfully characterized as such in which both parts of the physical system, the particle and the electromagnetic field, are at rest or in motion with some velocity relative to an arbitrary frame of reference. We will show, however, that this is not the case in general.

2 How to understand the RP for a general electrodynamical system?

What meaning can be attached to the words "a coupled particles + electromagnetic field system is *in collective motion* with velocity $\mathbf{V}$" ($\mathbf{V}=0$ included) relative to a reference frame K, in general? One might think, we can read off the answer to this question from the above example. However, focusing on the electromagnetic field, the partial differential equations (4)–(5) imply that

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}-\mathbf{V}t) \tag{11}$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r}-\mathbf{V}t) \tag{12}$$

with some time-independent $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{B}_0(\mathbf{r})$. In other words, the field must be a stationary one, that is, a translation of a static field with velocity $\mathbf{V}$. But, (11)–(12) is certainly not the case for a general solution of the equations of ED; the field is not necessarily translating with a collective velocity. The behavior of the field can be much more complex. Whatever this complex behavior is, we believe that the following general metaphysical principle holds:

Humean Supervenience of Motion (HSM) *If an extended object as a whole is at rest or is in motion with some velocity relative to an arbitrary reference frame K, then all local parts of it are in motion with some local instantaneous velocity* $\mathbf{v}(\mathbf{r},t)$ *relative to K.*

Combining HSM with MR, we obtain the following:

Local Minimal Requirement for the RP (LMR) *The states of the extended physical system in question must be meaningfully characterized as such in which all local parts of the system are at rest or in motion with some local instantaneous velocity relative to an arbitrary frame of reference.*

Consequently, in case of electrodynamics, a straightforward minimal requirement for the RP to be a meaningful statement is that (2)–(3) must be satisfied at least *locally* with some local and instantaneous velocity $\mathbf{v}(\mathbf{r},t)$: it is quite natural to say that the electromagnetic field at point $\mathbf{r}$ and time t is moving with *local* and *instantaneous* velocity $\mathbf{v}(\mathbf{r},t)$ if and only if

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \mathbf{E}\left(\mathbf{r}-\mathbf{v}(\mathbf{r},t)\delta t, t-\delta t\right) && (13)\\ \mathbf{B}(\mathbf{r},t) &= \mathbf{B}\left(\mathbf{r}-\mathbf{v}(\mathbf{r},t)\delta t, t-\delta t\right) && (14) \end{aligned}$$

are satisfied *locally*, in an *infinitesimally* small space and time region at $(\mathbf{r},t)$, for infinitesimally small δt. In other words, the equations (4)–(5) must be satisfied *locally* at point $(\mathbf{r},t)$ with a local and instantaneous velocity $\mathbf{v}(\mathbf{r},t)$:

$$\begin{aligned} -\partial_t\mathbf{E}(\mathbf{r},t) &= \mathsf{D}\mathbf{E}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) && (15)\\ -\partial_t\mathbf{B}(\mathbf{r},t) &= \mathsf{D}\mathbf{B}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) && (16) \end{aligned}$$

In other words, if the RP, as it is believed, applies to all situations in electrodynamics, there must exist a local instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$ satisfying (15)–(16) for all possible solutions of the following system of Maxwell–Lorentz equations:

$$\begin{aligned} \nabla\cdot\mathbf{E}(\mathbf{r},t) &= \sum_{i=1}^{n} q^i\delta\left(\mathbf{r}-\mathbf{r}^i(t)\right) && (17)\\ c^2\nabla\times\mathbf{B}(\mathbf{r},t)-\partial_t\mathbf{E}(\mathbf{r},t) &= \sum_{i=1}^{n} q^i\delta\left(\mathbf{r}-\mathbf{r}^i(t)\right)\mathbf{v}^i(t) && (18)\\ \nabla\cdot\mathbf{B}(\mathbf{r},t) &= 0 && (19)\\ \nabla\times\mathbf{E}(\mathbf{r},t)+\partial_t\mathbf{B}(\mathbf{r},t) &= 0 && (20)\\ m^i\gamma\left(\mathbf{v}^i(t)\right)\mathbf{a}^i(t) &= q^i\left\{\mathbf{E}\left(\mathbf{r}^i(t),t\right)+\mathbf{v}^i(t)\times\mathbf{B}\left(\mathbf{r}^i(t),t\right)\right.\\ &\quad \left.-c^{-2}\mathbf{v}^i(t)\left(\mathbf{v}^i(t)\cdot\mathbf{E}\left(\mathbf{r}^i(t),t\right)\right)\right\} && (21)\\ &\quad (i=1,2,\ldots n) \end{aligned}$$

where, $\gamma(\ldots)=\left(1-\frac{(\ldots)^2}{c^2}\right)^{-\frac{1}{2}}$, q^i is the electric charge and m^i is the rest mass of the i-th particle. That is, substituting an arbitrary solution[3] of (17)–(21) into

[3] Without entering into the details, it must be noted that the Maxwell–Lorentz equations (17)–(21), exactly in this form, have *no* solution. The reason is that the field is singular at precisely the points where the coupling happens: on the trajectories of the particles. The generally accepted answer to this problem is that the real source densities are some "smoothed out" Dirac deltas, determined by the physical laws of the internal worlds of the particles—which are, supposedly, outside of the scope of ED. With this explanation, for the sake of simplicity we leave the Dirac deltas in the equations. Since our considerations here focuses on the electromagnetic field, satisfying the four Maxwell equations, we must only assume that there is a coupled dynamics—approximately described by equations (17)–(21)—and that it constitutes an initial value problem. In fact, Theorem 1 could be stated in a weaker form, by leaving the concrete form and dynamics of the source densities unspecified.

(15)–(16), the overdetermined system of equations must have a solution for $\mathbf{v}(\mathbf{r}, t)$.

However, one encounters the following difficulty:

Theorem 1. *There is a dense subset of solutions* $\left(\mathbf{r}^1(t), \ldots \mathbf{r}^n(t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\right)$ *of the coupled Maxwell–Lorentz equations (17)–(21) for which there cannot exist a local instantaneous velocity field* $\mathbf{v}(\mathbf{r}, t)$ *satisfying (15)–(16).*

Proof. The proof is almost trivial for a locus $(\mathbf{r}, t)$ where there is a charged point particle. However, in order to avoid the eventual difficulties concerning the physical interpretation, we are providing a proof for a point $(\mathbf{r}_*, t_*)$ where there is assumed no source at all.

Consider a solution $\left(\mathbf{r}^1(t), \ldots \mathbf{r}^n(t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\right)$ of the coupled Maxwell–Lorentz equations (17)–(21), which satisfies (15)–(16). At point $(\mathbf{r}_*, t_*)$, the following equations hold:

$$
\begin{aligned}
-\partial_t \mathbf{E}(\mathbf{r}_*, t_*) &= \mathrm{D}\mathbf{E}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) && (22)\\
-\partial_t \mathbf{B}(\mathbf{r}_*, t_*) &= \mathrm{D}\mathbf{B}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) && (23)\\
\partial_t \mathbf{E}(\mathbf{r}_*, t_*) &= c^2 \nabla \times \mathbf{B}(\mathbf{r}_*, t_*) && (24)\\
-\partial_t \mathbf{B}(\mathbf{r}_*, t_*) &= \nabla \times \mathbf{E}(\mathbf{r}_*, t_*) && (25)\\
\nabla \cdot \mathbf{E}(\mathbf{r}_*, t_*) &= 0 && (26)\\
\nabla \cdot \mathbf{B}(\mathbf{r}_*, t_*) &= 0 && (27)
\end{aligned}
$$

Without loss of generality we can assume—at point $\mathbf{r}_*$ and time t_*—that operators $\mathrm{D}\mathbf{E}(\mathbf{r}_*, t_*)$ and $\mathrm{D}\mathbf{B}(\mathbf{r}_*, t_*)$ are invertible and $v_z(\mathbf{r}_*, t_*) \neq 0$.

Now, consider a 3×3 matrix J such that

$$
J = \begin{pmatrix} \partial_x E_x(\mathbf{r}_*, t_*) & J_{xy} & J_{xz} \\ \partial_x E_y(\mathbf{r}_*, t_*) & \partial_y E_y(\mathbf{r}_*, t_*) & \partial_z E_y(\mathbf{r}_*, t_*) \\ \partial_x E_z(\mathbf{r}_*, t_*) & \partial_y E_z(\mathbf{r}_*, t_*) & \partial_z E_z(\mathbf{r}_*, t_*) \end{pmatrix} \qquad (28)
$$

with

$$
\begin{aligned}
J_{xy} &= \partial_y E_x(\mathbf{r}_*, t_*) + \lambda && (29)\\
J_{xz} &= \partial_z E_x(\mathbf{r}_*, t_*) - \lambda \frac{v_y(\mathbf{r}_*, t_*)}{v_z(\mathbf{r}_*, t_*)} && (30)
\end{aligned}
$$

by virtue of which

$$
\begin{aligned}
J_{xy} v_y(\mathbf{r}_*, t_*) + J_{xz} v_z(\mathbf{r}_*, t_*) &= v_y(\mathbf{r}_*, t_*)\partial_y E_x(\mathbf{r}_*, t_*) \\
&\quad + v_z(\mathbf{r}_*, t_*)\partial_z E_x(\mathbf{r}_*, t_*) && (31)
\end{aligned}
$$

Therefore, $J\mathbf{v}(\mathbf{r}_*, t_*) = \mathrm{D}\mathbf{E}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*)$. There always exists a vector field $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ such that its Jacobian matrix at point $\mathbf{r}_*$ is equal to J. Obviously, from (26) and (28), $\nabla \cdot \mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) = 0$. Therefore, there exists a solution of the Maxwell–Lorentz equations, such that the electric and magnetic fields $\mathbf{E}_{\lambda}(\mathbf{r}, t)$ and $\mathbf{B}_{\lambda}(\mathbf{r}, t)$ satisfy the following conditions:[4]

$$
\begin{aligned}
\mathbf{E}_{\lambda}(\mathbf{r}, t_*) &= \mathbf{E}^{\#}_{\lambda}(\mathbf{r}) && (32)\\
\mathbf{B}_{\lambda}(\mathbf{r}, t_*) &= \mathbf{B}(\mathbf{r}, t_*) && (33)
\end{aligned}
$$

[4] $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ and $\mathbf{B}_{\lambda}(\mathbf{r}, t_*)$ can be regarded as the initial configurations at time t_*; we do not need to specify a particular choice of initial values for the sources.

At $(\mathbf{r}_*, t_*)$, such a solution obviously satisfies the following equations:

$$\begin{aligned} \partial_t \mathbf{E}_\lambda(\mathbf{r}_*, t_*) &= c^2 \nabla \times \mathbf{B}(\mathbf{r}_*, t_*) && (34) \\ -\partial_t \mathbf{B}_\lambda(\mathbf{r}_*, t_*) &= \nabla \times \mathbf{E}_\lambda^{\#}(\mathbf{r}_*) && (35) \end{aligned}$$

therefore

$$\partial_t \mathbf{E}_\lambda(\mathbf{r}_*, t_*) = \partial_t \mathbf{E}(\mathbf{r}_*, t_*) \quad (36)$$

As a little reflection shows, if $\mathsf{D}\mathbf{E}_\lambda^{\#}(\mathbf{r}_*)$, that is J, happened to be not invertible, then one can choose a *smaller* λ such that $\mathsf{D}\mathbf{E}_\lambda^{\#}(\mathbf{r}_*)$ becomes invertible (due to the fact that $\mathsf{D}\mathbf{E}(\mathbf{r}_*, t_*)$ is invertible), and, at the same time,

$$\nabla \times \mathbf{E}_\lambda^{\#}(\mathbf{r}_*) \neq \nabla \times \mathbf{E}(\mathbf{r}_*, t_*) \quad (37)$$

Consequently, from (36) , (30) and (22) we have

$$-\partial_t \mathbf{E}_\lambda(\mathbf{r}_*, t_*) = \mathsf{D}\mathbf{E}_\lambda(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) = \mathsf{D}\mathbf{E}_\lambda^{\#}(\mathbf{r}_*)\mathbf{v}(\mathbf{r}_*, t_*) \quad (38)$$

and $\mathbf{v}(\mathbf{r}_*, t_*)$ is uniquely determined by this equation. On the other hand, from (35) and (37) we have

$$-\partial_t \mathbf{B}_\lambda(\mathbf{r}_*, t_*) \neq \mathsf{D}\mathbf{B}_\lambda(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) = \mathsf{D}\mathbf{B}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) \quad (39)$$

because $\mathsf{D}\mathbf{B}(\mathbf{r}_*, t_*)$ is invertible, too. That is, for $\mathbf{E}_\lambda(\mathbf{r}, t)$ and $\mathbf{B}_\lambda(\mathbf{r}, t)$ there is no local and instantaneous velocity at point $\mathbf{r}_*$ and time t_*.

At the same time, λ can be arbitrary small, and

$$\begin{aligned} \lim_{\lambda \to 0} \mathbf{E}_\lambda(\mathbf{r}, t) &= \mathbf{E}(\mathbf{r}, t) && (40) \\ \lim_{\lambda \to 0} \mathbf{B}_\lambda(\mathbf{r}, t) &= \mathbf{B}(\mathbf{r}, t) && (41) \end{aligned}$$

Therefore solution $\left(\mathbf{r}_\lambda^1(t), \ldots \mathbf{r}_\lambda^n(t), \mathbf{E}_\lambda(\mathbf{r}, t), \mathbf{B}_\lambda(\mathbf{r}, t)\right)$ can fall into an arbitrary small neighborhood of $\left(\mathbf{r}^1(t), \ldots \mathbf{r}^n(t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\right)$. □

Thus, the meaning of the concept of "electromagnetic field moving with a local instantaneous velocity $\mathbf{v}(\mathbf{r}, t)$ at point $\mathbf{r}$ and time t", that we obtained by a straightforward generalization of the example of the stationary field of a uniformly moving charge, is untenable. We do not see other available rational meaning of this concept; which would be, however, a necessary conceptual plugin to the RP. In any event, lacking a better suggestion, we must conclude that the RP is a statement which is meaningless for a general electrodynamical situation. So it is hard to understand how the RP can be the first principle of relativistic electrodynamics.

Finally, notice that our investigation has been concerned with the general laws of Maxwell–Lorentz electrodynamics of a coupled particles + electromagnetic field system. The proof of the theorem was essentially based on the presumption that all solutions of the Maxwell–Lorentz equations, determined by *any* initial state of the particles + electromagnetic field system, corresponded to physically possible configurations of the electromagnetic field. It is sometimes claimed, however, that the solutions must be restricted by the so called retardation condition, according to which all physically admissible field configurations must be generated from the retarded potentials belonging to some pre-histories of the charged particles (Jánossy 1971, p. 171; Frisch 2005, p. 145). There is no obvious answer to the question of how Theorem 1 is altered under such additional condition.

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