INTEGRATIVE REDUCTION, CONFIRMATION, AND THE SYNTAX-SEMANTICS MAP

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ABSTRACT. Intertheoretic relations are an important topic in the philosophy of science. However, since their discussion in (Nagel, 1961), such relations have mostly been restricted to relations between pairs of theories in the natural sciences. In this paper, we present models of two new types of intertheoretic relations, called *Montague Reduction* and *Integrative Reduction*, that are assumed in Montague's (1973) theories of natural language syntax and semantics. To show the rationale behind our adoption of these two kinds of relations, we analyze them in the framework of Bayesian confirmation theory.

Keywords Intertheoretic relations, Bayesian confirmation, Bayesian networks in philosophy, Reduction, Syntax-semantics map.

1. Introduction

Methodology is one of the central concerns in philosophy of science. While scientists are occupied with the collection of data, the formulation and testing of hypotheses (or theories), and the creation of phenomena, philosophers of science are interested in the identification of methods for the theories' evaluation and justification. In the last hundred years, the majority of work on scientific methodology has focused on the natural sciences, especially on physics. However, there is also a considerable body of methodological work in other sciences like linguistics. The latter includes Chomsky's (1966) work on explanatory models in linguistics, and the recent work of Schütze, e.g. (Schütze, 1996; 2011). Yet, while these works have received due attention in their own academic field, they have been largely neglected by philosophers of science.

The present paper focuses on a particular issue in linguistic methodology, intertheoretic relations. Since the publication of (Nagel, 1961), intertheoretic relations have been an important topic in the philosophy of science. However, subsequent research has concentrated almost exclusively on relations between pairs of theories in the natural sciences. In this paper, we present models of two new types of intertheoretic relations, called *Montague Reduction* and *Integrative Reduction*, that are assumed in Montague's (1973) theory of natural language syntax and semantics. To do this, we first identify the relation of Montague Reduction and develop its associated model. The latter is then refined into (a model of) the more sophisticated relation of Integrative Reduction.

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Montague Reduction and Integrative Reduction are related to the best-studied intertheoretic relation, Nagelian reduction, and other undirected dependency relations by family resemblance. Like Nagelian reduction, Montague Reduction and Integrative Reduction aim to derive a proposition of the reduced theory (here, Montague syntax, or *Categorial Grammar*) from a proposition of the reducing theory (Montague's *Model-Theoretic Semantics*). As a consequence, our two new intertheoretic relations share the rationale of Nagelian reduction: The reduction of syntax to semantics promotes cognitive economy and simplicity, explains the success of the reduced theory in terms of the reducing theory, establishes their relative consistency, and effects a mutual flow of confirmation between the two theories, cf. (Dijzadji-Bahmani et al., 2010b).

In view of the success of the Nagelian model[1], however, the question arises whether the relation between Categorial Grammar and Model-Theoretic Semantics requires a different analysis than the (arguably similar) relation between many other pairs of scientific theories (paradigmatically, the relation between thermodynamics and statistical mechanics). Admittedly, Categorial Grammar and Model-Theoretic Semantics are mainstream theories of linguistic syntax and semantics.[2] Montague's view of their interrelation is today widely adopted. Yet, it remains unclear whether their prominence in contemporary linguistics provides sufficient reason for the introduction of a new model of Integrative (or Montague) Reduction.

Our paper answers the preceding question in the affirmative. To show the need for a new model of the relation between Categorial Grammar and Model-Theoretic Semantics, we adopt an epistemic framework. In particular, we determine that the Integrative Reduction of Categorial Grammar to Model-Theoretic Semantics is epistemically preferable to their Montague Reduction or Nagelian reduction (in the sense that it raises the prior and posterior probabilities and the degree of confirmation of the two theories), and that their Montague or Nagelian Reduction of is epistemically preferable (in the above sense) to the pre-reductive situation.

The paper is organized as follows: Sections 2 and 3 present Montague's formal framework for the analysis of natural language syntax and semantics, and review relevant concepts from Bayesian confirmation and network theory. The remaining sections focus on the simultaneous development and evaluation of our model of Integrative Reduction. Section 4 provides a Bayesian analysis of the syntax-semantics relation before and after the execution of a Montague Reduction and shows that, post-reduction, the two theories are confirmatory of each other. Section 5 identifies a problem with Montague Reduction, revises its model into a model of the relation of Integrative Reduction, and shows in which respects it is epistemically preferable to Montague Reduction. We close by indicating how our model of Integrative Reduction can be incorporated into a sophisticated variant of Schaffner's revised model of Nagelian reduction.

[1]For many years, Nagelian reduction has been considered a dead end. The present paper rejects this assumption. While Nagel's original model of reduction, cf. (Nagel, 1961), suffers from various problems, the latter are overcome in Schaffner's revised version, cf. (Nagel, 1977; Schaffner, 1974). For the present purposes, it will suffice to focus only on the Nagelian model. We outline a Schaffner-style extension of our model of Integrative Reduction in Section 6.

[2]Textbooks on linguistic syntax and semantics that are based on the two theories include (Heim and Kratzer, 1998), (Chierchia and McConnell-Ginet, 2000), and (de Swart, 1998).

2. Montague Grammar

We begin with a presentation of the two theories that we aim to relate. Section 2.1 states their objects and object-connecting mechanism. Section 2.2 compares the Montagovian account of the syntax-semantic relation with the Nagelian model of intertheoretic reduction. To enable a Bayesian analysis of our new type of intertheoretic relation, Section 2.3 identifies syntactic and semantic propositions with the objects of probabilistic evaluations.

2.1. **Montague's 'Two Theories' Theory.** Richard Montague's 'Universal Grammar' provides a formal framework for the analysis and interpretation of natural language syntax, based on (Montague, 1970a; 1973; 1970b). Montague conceives of natural languages as interpreted formal systems: The syntax of a language (hereafter 'Categorial Grammar', CG) is specified through the enumeration of grammatical categories, $\text{CAT} = \{\text{N(oun)}, \text{V(erb)}, \text{S(entence)}, \dots\}$, their associated structures ('expressions'), $\mathcal{E} = \{\mathcal{E}_\text{N}, \mathcal{E}_\text{V}, \mathcal{E}_\text{S}, \dots\}$ (with $\mathcal{E}_\text{N} = \{\mathsf{John}, \mathsf{Mary}, \mathsf{Fido}, \dots\}$), and the definition of rules, $\mathbb{G} = \{\mathbb{G}_\text{S}, \dots\}$, governing the behavior of syntactic operations like concatenation and conjunction. The latter apply to tuples of expressions to yield unique complex expressions. Montague syntax thus constitutes an algebra, $\mathcal{A}_\text{CG} = \{\{\mathcal{E}_\text{N}, \mathcal{E}_\text{V}, \dots\}, \mathbb{G}_\text{S}, \dots\}$, over the set of basic expressions. A language (e.g. English) is identified with the closure of the set $\{\mathcal{E}_\text{N}, \mathcal{E}_\text{V}, \dots\}$ under the rules of the algebra.[3]

Model-Theoretic Semantics (MS) matches the syntactic algebra on the level of natural language meaning. The interpretation function I establishes a relation between syntactic expressions and their semantic referents. For every $\mathcal{E}$-constant c (e.g. John), we assume a denotation, $[\![c]\!]$ (e.g. 🯅), such that $[\![c]\!] = I(c)$. We call the set $\{\mathcal{D}_\text{N}, \mathcal{D}_\text{V}, \mathcal{D}_\text{S}, \dots\}$ (containing the domains of individual objects, properties, truth-values, etc.) of $\mathcal{E}$-denotations '$\mathcal{D}$' and stipulate that it be non-empty. Every syntactic rule, $\mathbb{G}_{k_i}$ (with $k \in \text{CAT}$, $i \in \mathbb{N}$)[4] is associated with a semantic rule, $\mathbb{S}_{k_i}$, that governs the behavior of the syntactic operations' semantic counterparts (e.g. functional application, set intersection). The semantics of a language thus constitutes an algebra $\mathcal{A}_\text{MS} = \{\{\mathcal{D}_\text{N}, \mathcal{D}_\text{V}, \dots\}, \mathbb{S}_\text{S}, \dots\}$ over the set of denotations. Its interpretation is identified with the closure of this set under the rules in $\mathbb{S}$. Expressions and their denotations, as well as syntactic and semantic rules, are related via a map from the syntactic algebra to the (polynomial closure of the) semantic algebra.

Significantly, the above-described map[5] is not strictly one-to-one. This is due to the semantic ambiguity of nouns and verbs, and attendant impossibility of mapping every element of the syntactic algebra onto a unique element of the semantic algebra. Rather than being associated with a single class of semantic referents, nouns (e.g. John) may be interpreted either as individual objects (i.e.

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[3] For an introduction to Montague's theory of syntax and semantics, the reader is referred to (Partee, 1997) and (Gamut, 1991).

[4] Our use of category indices (i.e. i) is motivated by the fact that some categories (especially the category 'S') are associated with different rules (cf. Montague's rules S4, S9, S11, S14, S17 (Montague, 1973)). Their association with semantic rules T4, T9, T11, T14, T17 preserves the above-noted correspondence. Since the remainder of this paper will only be concerned with the sentence-formation rules S4 and T4, we hereafter suppress their number.

[5] Since Montague's syntax-semantics 'map' is multi-valued, it may be more adequate to call it a 'relation'. However, since the term *syntax-semantics map* is standard in linguistics, we hereafter adopt this term.

$= [\![\mathsf{John}]\!]'$) or as generalized quantifiers (i.e. the set, $[\![\mathsf{John}]\!]''$, of all of John's properties).[6] This is made necessary by the possibility of conjoining proper names with quantifier phrases (cf. the expression John and every woman), and the restriction of coordination to same-domain objects, cf. (Partee, 1987). To preserve function-argument structure, we similarly interpret intransitive verbs (e.g. run) either as properties of individual objects (e.g. $[\![\mathsf{run}]\!]'$) or as properties of generalized quantifiers (e.g. $[\![\mathsf{run}]\!]''$). Figure 1 illustrates the map (symbolized by dotted lines with arrows) between elements of the syntactic and semantic algebra.

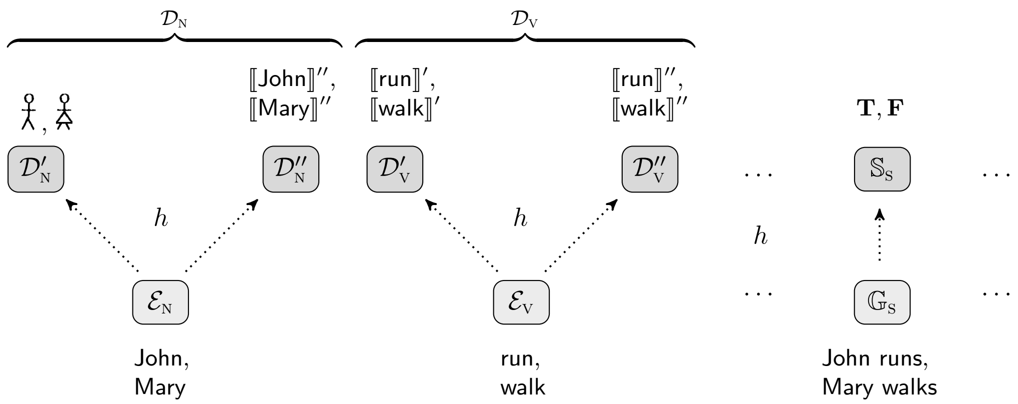

FIGURE 1. Montague's syntax-semantics relation.

Since the identification of the domains $\mathcal{D}_{\mathrm{N}}$ and $\mathcal{D}_{\mathrm{V}}$ with the set $\{\mathcal{D}'_{\mathrm{N}}, \mathcal{D}''_{\mathrm{N}}\}$, respectively $\{\mathcal{D}'_{\mathrm{V}}, \mathcal{D}''_{\mathrm{V}}\}$ preserves the structure of the syntactic algebra, we hereafter describe Montague's syntax-semantic map as the homomorphism h. To facilitate the representation of the Montagovian framework, we assume the existence of only three classes of expressions or objects. We demonstrate in Section 5.2 that our new model of reduction is easily extensible to the remaining categories (e.g. adjectives, adverbs, determiners and their semantic correspondents).

In line with the constructive requirements of syntax and semantics, our presentation of the Montagovian framework has only stipulated rules for the formation of complex expressions and entities (e.g. sentences and truth-values). For future use, we also assume rules for the identity of basic objects. Thus, the rules $\mathbb{G}_{\mathrm{N}}, \mathbb{G}_{\mathrm{V}}$ are simple rewrite rules, taking words in the lexicon to expressions in the syntax. The rules $\mathbb{S}_{\mathrm{N}}, \mathbb{S}_{\mathrm{V}}$ constitute their semantic counterparts. Note that, by the ambiguity noun- and verb-interpretations, the rules $\mathbb{S}_{\mathrm{N}}, \mathbb{S}_{\mathrm{V}}$ (but not $\mathbb{G}_{\mathrm{N}}, \mathbb{G}_{\mathrm{V}}$) will include two different cases, covering $\mathcal{D}'_{\mathrm{N}}$ and $\mathcal{D}''_{\mathrm{N}}$, and $\mathcal{D}'_{\mathrm{V}}$ and $\mathcal{D}''_{\mathrm{V}}$, respectively.

The formation of minimally complex sentences (e.g. John runs) is governed by the rule $\mathbb{G}_{\mathrm{S}}$, below:

Let $[AB]$ represent the concatenation of the expressions A and B. The rule $\mathbb{G}_{\mathrm{S}}$ is then defined as follows, cf. (Montague, 1973, p. 251):

[6]Intuitively, $[\![\mathsf{John}]\!]''$ abbreviates Montague's $[\![\lambda P.P(\mathsf{John})]\!]$, where P ranges over first-order properties, with John an individual constant. Properties of generalized quantifiers (e.g. the property $[\![\mathsf{run}]\!]''$) constitute similar abbreviations.

$\mathbb{G}_{\mathrm{S}}$. If $R \in \mathcal{E}_{\mathrm{V}}$ and $j \in \mathcal{E}_{\mathrm{N}}$, then $[jR'] \in \mathcal{E}_{\mathrm{S}}$,

where R' is the result of replacing the main verb in R (e.g. run) by its third person singular present form (runs). The concatenation of a noun (e.g. John) with an inflected verb (runs) thus yields a sentence (John runs).

Semantic rules follow their syntactic counterparts: Given the replacement of syntactic categories, $\mathcal{E}_k$, by referential domains, $\mathcal{D}_k$, and interpretation of concatenation and agreement as functional application, nothing changes. Clause $\mathbb{S}_{\mathrm{S}}$ (below) defines the semantic correspondent of rule $\mathbb{G}_{\mathrm{S}}$, cf. (Montague, 1973, p. 254).

$\mathbb{S}_{\mathrm{S}}$. If $[\![R]\!] \in \mathcal{D}_{\mathrm{V}}$ and $[\![j]\!] \in \mathcal{D}_{\mathrm{N}}$, then $[\![R]\!]([\![j]\!]) \in \mathcal{D}_{\mathrm{S}}$.

Note that, by the set-like character of $\mathcal{D}_{\mathrm{N}} = \{\mathcal{D}'_{\mathrm{N}}, \mathcal{D}''_{\mathrm{N}}\}$ and $\mathcal{D}_{\mathrm{V}} = \{\mathcal{D}'_{\mathrm{V}}, \mathcal{D}''_{\mathrm{V}}\}$, the rule $\mathbb{S}_{\mathrm{S}}$ is understood as the conjunction of rules $\mathbb{S}'_{\mathrm{S}}$ and $\mathbb{S}''_{\mathrm{S}}$, below:

$\mathbb{S}'_{\mathrm{S}}$. If $[\![R]\!]' \in \mathcal{D}'_{\mathrm{V}}$ and $[\![j]\!]' \in \mathcal{D}'_{\mathrm{N}}$, then $[\![R]\!]'([\![j]\!]') \in \mathcal{D}_{\mathrm{S}}$.

$\mathbb{S}''_{\mathrm{S}}$. If $[\![R]\!]'' \in \mathcal{D}''_{\mathrm{V}}$ and $[\![j]\!]'' \in \mathcal{D}''_{\mathrm{N}}$, then $[\![R]\!]''([\![j]\!]'') \in \mathcal{D}_{\mathrm{S}}$.

According to $\mathbb{S}'_{\mathrm{S}}$, the application of the first-order property $[\![\text{runs}]\!]'$ to the denotation, 🯅, of John yields either truth (i.e. $[\![\text{runs}]\!]'($🯅$) = \mathbf{T}$) or falsity ($\mathbf{F}$). According to $\mathbb{S}''_{\mathrm{S}}$, the application of the higher-order property $[\![\text{runs}]\!]''$ to the generalized quantifier, $[\![\text{John}]\!]''$, denoted by John yields either truth (i.e. $[\![\text{runs}]\!]''\big([\![\text{John}]\!]''\big) = \mathbf{T}$) or falsity ($\mathbf{F}$).

2.2. **Montague's Theory and Intertheoretic Reduction.** In the context of our introduction (cf. Sect. 1), we have already described the syntax-semantics pair as the instantiation of a specific type of intertheoretic relation. To emphasize its similarities and differences with the Nagelian account of reduction, we next describe the reduction of syntax to semantics on the Nagelian model. Given the description of Montague's syntax-semantics relation from the previous subsection, a comparison between accounts of the two types of relations is easily carried out.

In the following, we assume a *reduced* (or *phenomenological*) theory, $\mathcal{T}_2$, and a *reducing* (or *fundamental*) theory, $\mathcal{T}_1$.[7] Renownedly, Nagelian reduction is a three-step process, involving the establishment of connections (via bridge laws) between terms in the non-logical vocabulary of the theories $\mathcal{T}_1$ and $\mathcal{T}_2$ (i), the substitution of terms from $\mathcal{T}_1$ by their bridge-law correspondents from $\mathcal{T}_2$ (ii), and the derivation of every proposition in $\mathcal{T}_2$ from a proposition in $\mathcal{T}_1$ (iii). Accordingly, the use of the Nagelian model of reduction for the reduction of the syntax-semantics pair requires a formulation of the bridge laws $\mathcal{D}_{\mathrm{N}} \simeq \mathrm{N}$, $\mathcal{D}_{\mathrm{V}} \simeq \mathrm{V}$, $\mathcal{D}_{\mathrm{S}} \simeq \mathrm{S}$, etc. (i), the substitution of the domain-denoting expressions '$\mathcal{D}_{\mathrm{N}}$', '$\mathcal{D}_{\mathrm{V}}$', and '$\mathcal{D}_{\mathrm{S}}$' by the category-denoting expressions 'N', 'V', resp. 'S' (ii), and the derivation of every proposition in $\mathcal{A}_{\mathrm{CG}}$ (e.g. the proposition $\mathbb{G}_{\mathrm{S}}$) from the corresponding proposition (here, $\mathbb{S}_{\mathrm{S}}$) in $\mathcal{A}_{\mathrm{MS}}$ (iii).

Clearly, the Nagelian model of reduction (hereafter *Nagel Reduction*, or NR) and the Montagovian model of the syntax-semantics relation (hereafter *Montague Reduction*, or MR) share the final step (iii) of the above-described process.

[7] The use of the adjectives *phenomenological* and *fundamental*, which suggests the directionality of the reduction relation, is in accordance with the treatment of theories in physics.

The two models further agree with respect to the connectability of (the denotations of) terms in the two theories' non-logical vocabularies. However, while Nagel Reduction satisfies the requirement of intertheoretical connectability (i) through the formulation of (syntactic) bridge laws, Montague reduction satisfies this requirement through the assumption of a (semantic) homomorphism between the objects of the syntactic and the semantic algebra. Since the latter also establishes connections between propositions of the two theories, Montague Reduction obviates Nagel's substitution step (ii).

But the semantic characterization of connectability and the absence of the substitution step are not the only salient properties of Montague Reduction. Importantly, the latter is also defined by the homomorphism's *non-symmetry*. Arguably, the Nagelian and the 'Montagovian' model both characterize reduction as a directed dependency relation (This is reflected in the identification of one of the two theories with the phenomenological theory and of the other with the fundamental theory). However, while the semantic ambiguity of some syntactic categories in the Montagovian model makes the directedness of the syntax-semantics relation explicit, Nagel's formalization of bridge laws as biconditional statements conceals this property. As a consequence, the Nagelian – but not the Montagovian – model of reduction represents the reduction relation as a *symmetric* relation. To emphasize the symmetric representation of Nagel Reduction, we will sometimes describe the latter as an *undirected* relation.[8]

The directed dependency between Categorial Grammar and Model-Theoretic Semantics (described above) motivates our identification of Categorial Grammar with the phenomenological theory and of Model-Theoretic Semantics with the fundamental theory. Figure 2 compares Montague's description of the syntax-semantics relation (right) with the Nagelian account of reduction (left):

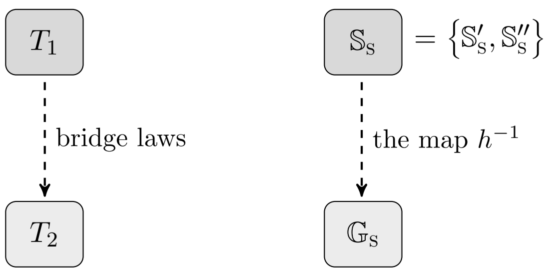

FIGURE 2. The Nagelian model (left) and the Montagovian model of reduction (right).

Note our use of dashed (rather than dotted) in the above figure. This change in notation is required by the directedness of the syntax-semantics relation, and an attendant a change in the arrows' denotation. While the 'dotted' arrows in Figure 1 (cf. Sect. 2.1) represent Montague's map $h : \mathcal{A}_{\text{CG}} \to \mathcal{A}_{\text{MS}}$, the 'dashed' arrows in Figure 2 represent its inverse $h^{-1} : \mathcal{A}_{\text{MS}} \to \mathcal{A}_{\text{CG}}$.

For future reference, we provide the following definitions of Nagel Reduction and of Montague Reduction in terms of their salient properties:

[8]For a discussion of this issue and a Nagelian solution, the reader is referred to (Kuipers, 1982) and (Dijzadji-Bahmani et al., 2010a).

Definition 1 (Nagel Reduction (NR))**.** A type of *undirected* (i.e. *symmetrically represented*) dependency relation, described by Nagel (1961), that is defined by the existence of intertheoretical connections between *terms* in the related theories' non-logical vocabularies, and by the derivability of every proposition in the phenomenological theory from a corresponding proposition in the fundamental theory.

Definition 2 (Montague Reduction (MR))**.** A type of *directed* (or *non-symmetric*) dependency relation, implicit in (Montague, 1973), that is defined by the existence of intertheoretical connections between *objects* of the two related theories, and by the derivability of every proposition in the phenomenological theory from a corresponding proposition in the fundamental theory.

The commonalities and differences between Nagel Reduction and Montague Reduction are captured in Figure 3.

$$\text{Derivability}\begin{cases} \text{NR} & \left.\begin{matrix}\textit{Syntactic}\text{ interth. connectability (via bridge laws),}\\ \textit{Undirected}\text{ dependency}\end{matrix}\right. \\ \text{MR} & \left.\begin{matrix}\textit{Semantic}\text{ interth. connectability (via the map } h^{-1}\text{),}\\ \textit{Directed}\text{ dependency}\end{matrix}\right. \end{cases}$$

FIGURE 3. Nagel Reduction vs. Montague Reduction.

As is clear from the above, the Montagovian model of the syntax-semantic relation instantiates only one particular type of intertheoretic relation. There are many others, ranging from 'strict' Nagel Reduction (cf. Def. 1) via the 'weaker' Nagel-Schaffner reduction (Schaffner, 1967; 1974), cf. (Nagel, 1977), to undirected dependency relations, cf. (Darden and Maull, 1977; Hartmann, 1999; Mitchell, 2003). We expect that the relation between Categorial Grammar and Model-Theoretic Semantics be found in the mid-range of this spectrum.

Significantly, the previously discussed Montague Reduction (cf. Def. 2, and Sect. 4) may not be identified with the relation of Integrative Reduction, which will be introduced in Section 5. Admittedly, both models of the syntax-semantics relation share salient properties (e.g. semantic intertheoretic connectability and directedness). Notably, however, Montague Reduction lacks the latter's defining property: *intra*theoretic connectability. In this sense, our model of Montague Reduction constitutes only an intermediate step towards the development of our model of Integrative Reduction. This is not to deny that Montague Reduction be taken as an intertheoretic relation in its own right. We will see, however, that Integrative Reduction constitutes a considerable generalization and, in certain respects, improvement of Montague Reduction.

We close the present subsection with a number of caveats about the syntax-semantics relation. Our previous considerations have identified Montague Reduction as a weak, i.e. directed, variant of Nagel Reduction. Significantly, however, the presented intertheoretic relation is even weaker than has been previously established. This is due to the greater structural richness of Categorial Grammar, and attendant impossibility of providing a semantic account of all syntactic properties. Word order and agreement are a case in point: To obtain the 'right' complex expressions, Montague's syntactic rules specify the order of their constituent basic expressions, and the conditions for their agreement. Without this specification, we would concatenate the expressions John and run into the complex expression Run John rather than John runs. Other problems regard the denotation of the same semantic object by differently-formed expressions and the existence of purely syntactic well-formedness constraints. All of the above motivate our description of Montague (and also of Integrative) Reduction as a distinct type of intertheoretic *relation*, rather than strong Nagelian *reduction*.

Our characterization of the syntax-semantics relation as a (very weak) reductive relation requires a further clarification: All popular accounts of reduction (including (Nagel, 1961)) assume the existence of the relevant theories' common domain of application. On this account, the reduced and the reducing theory both make (more-or-less) the same claims (e.g. about the behavior of a given physical system). This is not the case for our syntax-semantics pair. While Categorial Grammar accounts for the well-formedness of syntactic structures, Model-Theoretic Semantics accounts for their denotations' constructive properties. Admittedly, the denotation relation (formalized by the interpretation function I (cf. Sect. 2.1)) establishes a firm connection between the objects of the two theories. This does not change the fact, however, that their 'reductive achievement' will be comparatively weaker.

All of the above admonitions characterize our new type of intertheoretic relation. While they will be ignored in the rest of this paper, their neglect would distort our representation of the syntax-semantics relation. To enable the Bayesian analysis of its associated model, the following subsection discusses the use of probabilities in this part of linguistics. Section 3 gives a primer on Bayesian confirmation and network theory.

2.3. **Montagovian Rules and Probabilities.** Our presentation of Montague's theory has presupposed the existence of two sets of rules, $\mathbb{G}$, $\mathbb{S}$, for the formation of complex expressions and entities. Like hypotheses of any scientific theory, the latter are obtained by the scientific method (discussed, here, for the formulation of $\mathbb{G}_S$): Following the isolation of simple sentences in a given data-set (typically, an electronic text collection like the *British National Corpus*), linguists abstract information about the sentences' structural properties and propose a hypothesis (e.g. $\mathbb{G}_S$) about their formation. Hypotheses are tested through the analysis of other (new) corpora: A given string of expressions (e.g. the sentence John runs) is taken to support the hypothesis if its structure does, questions it if it does not reflect the assumed formation process (i.e. if it 'positively/negatively instantiates' $\mathbb{G}_S$).

To enable a Bayesian analysis of our model(s) of the syntax-semantics relation, we assign a probability to every syntactic or semantic rule. A rule's probability is informed by the frequentist data available at the time. Thus, the probability of the truth of the hypothesized rule $\mathbb{G}_S$ will be very high (low) if a very large (small) percentage of expressions of the described form instantiates $\mathbb{G}_S$. Intuitively, a rule's frequentist probability will influence a linguist's psychological confidence in its descriptive adequacy. Consequently, if a very large (small) percentage of expressions of a given form instantiates the rule $\mathbb{G}_S$, we expect the linguist's belief in the truth of $\mathbb{G}_S$ to be similarly high (or low).

Our previous considerations have defined the probability of a given rule via their positive instantiations' frequency in a given sample. Notably, however, only syntactic rules are directly instantiated. This difference, which motivates our reductive endeavor, will recur in the two theories' pre-reductive confirmation (cf. Thm. 1, Sect. 4.1). The semantic rule $\mathbb{S}_{\mathrm{S}}$ derives its support from the linguistic support of $\mathbb{G}_{\mathrm{S}}$ via the assumption of the homomorphism h. While rules for the construction of more complex objects (e.g. the denotation of the sentence Mary walked for an hour or John runs fast) are directly supported by the established entailment relations and speakers' validity judgements, cf. (Dowty, 1979), the restriction of entailment to sentences prevents the direct semantic support of the simple rules $\mathbb{S}_{\mathrm{N}}$, $\mathbb{S}_{\mathrm{V}}$ and (by extension) $\mathbb{S}_{\mathrm{S}}$. Their probabilities are defined by the probabilities of their syntactic counterparts.

This concludes our discussion of the reductive and probabilistic aspects of Montague's theory. Before we move to our introduction to Bayesianism, however, one final caveat is in order: Importantly, our attribution of probabilities to Montagovian rules does not constitute a probabilistic extension of Montague Grammar. The central aim of our paper is methodological, not substantive. Consequently, we do not intend any revisions or additions to (our fragment of) Montague's theory. The attribution of probabilities is only a means to an end, i.e. the possibility of providing a Bayesian analysis of its associated model. For the latter, it will suffice to restrict ourselves to the use of probabilistic variables. While nothing prevents us from plugging in actual values, this is not necessary for the success of our analysis.

3. A Primer on Bayesianism

We analyze a rule's evidential support via Bayesian confirmation theory: Its central idea constitutes the interpretation of confirmation as probability-raising, and associated distinction between two notions of probability, relative to the receipt of a new piece of evidence: The initial, or prior, probability of a proposition H (for 'hypothesis') is the probability of H before, its final, or posterior, probability the probability after the evidence E is considered.

Bayesian conditionalization on E requires an update of the prior probability, $\boldsymbol{P}(\mathrm{H})$, to the posterior probability, $\boldsymbol{P}'(\mathrm{H})$, of H, where $\boldsymbol{P}'(\mathrm{H})$ is typically expressed in terms of the original probability measure, i.e. $\boldsymbol{P}'(\mathrm{H}) = \boldsymbol{P}(\mathrm{H}|\mathrm{E})$, provided that $\boldsymbol{P}(\mathrm{E}) > 0$. Our use of Bayes' Theorem, a result from probability theory, yields the following expression for the posterior probability of H:

$$\begin{aligned}\boldsymbol{P}(\mathrm{H}|\mathrm{E}) &= \frac{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{E})} \\ &= \frac{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H}) + \boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})\boldsymbol{P}(\neg\mathrm{H})} \\ &= \frac{\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{H}) + \boldsymbol{P}(\neg\mathrm{H})\,x}\,.\end{aligned} \tag{1}$$

In the above, the expression $x := \boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})/\boldsymbol{P}(\mathrm{E}|\mathrm{H})$ is the likelihood ratio.

According to Bayesian confirmation theory, a piece of evidence E confirms a hypothesis H if the posterior probability of H (given E) is greater than the prior probability of H, i.e. if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) > \boldsymbol{P}(\mathrm{H})$. The piece of evidence, E, disconfirms H if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) < \boldsymbol{P}(\mathrm{H})$, and is irrelevant for H if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) = \boldsymbol{P}(\mathrm{H})$.[9]

While the case of two propositions is easy to compute, the confirmatory situation is often much more complicated. This is due to the fact that the respective hypothesis may have a fine structure, and that there may be different pieces of evidence that stand in certain probabilistic relations to each other. As we will see is due course, the relation between syntax and semantics, upon which we focus in this paper, exhibits a similarly high degree of complexity.

Bayesian networks prove to be a highly efficient tool for the computation of the above-described scenarios.[10] A Bayesian network is a directed acyclical graph whose nodes represent propositional variables and whose arrows encode the conditional independence relations that hold between the variables. The rest of this paragraph introduces some useful vocabulary: *Parent nodes* are nodes with outgoing arrows; *child nodes* nodes with incoming arrows. *Root nodes* (marked in grey) are unparented nodes; *descendant nodes* are child nodes, or the child of a child node, etc.

By the special choice of graph, paths of arrows may not lead back to themselves (thus motivating the graph's acyclicity). Variables at each node can take different numerical values, assigned by the probability function $\boldsymbol{P}$. Thus, Bayesian networks do not only provide a direct visualization of the probabilistic dependency relations between variables, but come along with a set of efficient algorithms for the computation of whichever conditional or unconditional probability over a (sub-)set of the variables involved we are interested in.

We illustrate the use of Bayesian networks by framing the confirmatory relation between H and E. To do so, we first introduce two binary propositional variables, H and E (printed in italic script). Each of them has two values (printed in roman script): H or ¬H (i.e. 'the hypothesized rule is true/false'), and E or ¬E ('the evidence obtains/does not obtain'), respectively. The relation between E and H can then be represented in the graph in Figure 4.

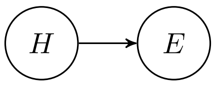

FIGURE 4. Bayesian network representation of the dependence between E and H.

The arrow from H to E denotes a direct influence of the variable in the parent to the variable in the child node. The truth or falsity of the hypothesis affects the probability of the obtaining of E.

To turn the graph from Figure 4 into a Bayesian network, we further require the marginal probability distribution for each variable in a root node (i.e. the

[9] Bayesianism is presented and critically discussed in (Howson and Urbach, 2005) and in (Earman, 1992). These texts also discuss Jeffrey conditionalization, which is an alternative updating rule. For an introduction to Bayesian epistemology, see (Hájek and Hartmann, 2010) and (Hartmann and Sprenger, 2010).

[10] For an introduction to Bayesian networks, see (Neapolitan, 2003; Pearl, 1988). (Bovens and Hartmann, 2003) discusses applications from epistemology and the philosophy of science, and provides a short introduction to the theory of Bayesian networks.

prior probability, $\boldsymbol{P}(\mathrm{H})$, of H), and the conditional probability distribution for every variable in a child node, given its parents. In the present case, the latter involves fixing the likelihoods $\boldsymbol{P}(\mathrm{E}|\mathrm{H})$ and $\boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})$. Using the machinery of Bayesian networks, we can then obtain all other probabilities. As will be relevant below, the graph's probability distribution respects the Parental Markov Condition (PMC): A variable represented by a node in a Bayesian network is independent of all variables represented by its non-descendant nodes in the Bayesian network, conditional on all variables represented by its parent nodes.

4. Reduction and Confirmation I: Nagelian Pairwise Reduction

Our previous considerations of Montague Reduction have been restricted to the presentation of its associated model. To provide a rationale for the introduction of this model as a 'new' model of intertheoretic relations (in addition to the established Nagelian model), we next provide a Bayesian analysis of Montague Reduction. The present section investigates the result of performing a Montague Reduction of Categorial Grammar to Model-Theoretic Semantics. Section 5 investigates the probabilistic and confirmation-theoretic advantages of performing a refined variant of Montague Reduction, called *Integrative Reduction*.

To enable an analysis of Montague Reduction in the Bayesian framework, we hereafter focus on propositions in $\mathbb{G}$ and $\mathbb{S}$. The ostensible exclusion of expression- and entity classes (cf. the two leftmost triples of nodes in Fig. 1) from our probabilistic considerations does not hamper the success of our proposed model of reduction. This is due to the strong correspondence between basic-type objects and their corresponding (syntactic or semantic) rules (cf. Sect. 2.1).

The reductive relation between Categorial Grammar and Model-Theoretic Semantics can be represented via the Bayesian network in Figure 5:

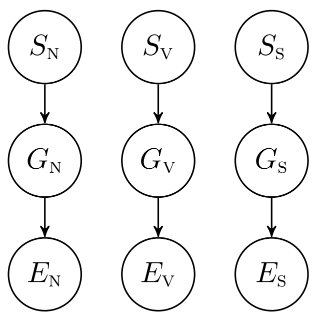

Figure 5. Post-reductive relations between pairs of propositions $\langle S_k, G_k\rangle$.

For simplicity, we assume that every rule in $\mathbb{G}$ is supported by exactly one (relevant set of) piece(s) of evidence. As specified in Section 2.3, we take evidence for a given syntactic rule to be an intuitively well-formed expression whose structure reflects the rule's assumed formation process. The replacement of dotted or dashed arrows (cf. Fig. 1 resp. 2) by 'continuous' arrows $\downarrow$ (cf. Fig. 4) is motivated by their denotation of probabilistic dependence relations between propositional variables (rather than of the map h or h^{-1}), and the attendant dependence of the probability of the truth of syntactic rules on the probability of the truth of semantic rules. Moreover, the conditional dependency of syntactic on semantic rules enables us to obtain an aligned chain of arrows, and, thus, to represent a flow of evidence from the syntactic to the semantic theory.

The independence of proposition pairs $\langle S_k, G_k \rangle$ (cf. the lack of horizontal arrows between nodes) in Figure 5 justifies our preliminary restriction to the single-proposition case. Correspondingly, we abbreviate S_{S} as S, G_{S} as G, and E_{S} as E. Figures 6 and 7 display the graphs associated with the dependence relations between S, G and E before and after the establishment of the relation of Montague Reduction:

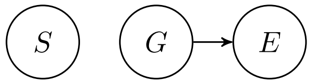

FIGURE 6. Pre-reductive dependence relations between S, G, and E.

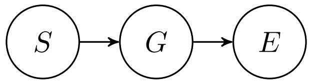

FIGURE 7. Post-reductive dependence relations between S, G, and E.

We determine the confirmation of S and G via their relevant probabilities, beginning with the pre-reductive situation (cf. Fig. 6). Its comparison with the post-reductive situation (Sect. 4.2, 4.3, cf. Fig. 7) will show that a Montague Reduction of syntax to semantics effects a boost of the joint (prior and posterior) probabilities of and a flow of confirmation between the two theories.

4.1. **Pre-Reductive Confirmation.** Let $\boldsymbol{P}_1(\mathrm{S})$ and $\boldsymbol{P}_1(\mathrm{G})$ be the marginal probabilities of the root nodes S, G of the Bayesian network in Figure 6, with $\boldsymbol{P}_1$ the corresponding probability measure. Let $\boldsymbol{P}_1(\mathrm{E}|\mathrm{G})$ and $\boldsymbol{P}_1(\mathrm{E}|\neg\mathrm{G})$ be the conditional probabilities of the child node E. For convenience, we use the following abbreviation scheme:[11]

$$\begin{aligned} \boldsymbol{P}_1(\mathrm{S}) = \sigma \quad &, \quad \boldsymbol{P}_1(\mathrm{G}) = \gamma\,, \\ \boldsymbol{P}_1(\mathrm{E}|\mathrm{G}) = \pi \quad &, \quad \boldsymbol{P}_1(\mathrm{E}|\neg\mathrm{G}) = \rho\,. \end{aligned} \tag{2}$$

We assume a positive confirmatory relation between E and G, such that $\pi > \rho$.

From the network structure in Figure 6, we can read off the conditional and unconditional independences $E \perp\!\!\!\perp S|G$ resp. $S \perp\!\!\!\perp G$ such that $\boldsymbol{P}_1(\mathrm{S}|\mathrm{E}) = \boldsymbol{P}_1(\mathrm{S})$. Evidence E does not confirm (or disconfirm) S. Hence, there is no flow of confirmation from the syntactic to the semantic theory. In the absence of the map $h : \mathbb{G} \to \mathbb{S}$ (or related bridge laws), the variables G and S are probabilistically independent before the reduction:

$$\boldsymbol{P}_1(\mathrm{S}, \mathrm{G}) = \boldsymbol{P}_1(\mathrm{S})\, \boldsymbol{P}_1(\mathrm{G}) = \gamma\, \sigma\,. \tag{3}$$

By equation (3), the prior probability of the conjunction of S and G equals the product of the marginal probabilities of the positive instantiations of their root

[11]To prevent the equivocation of probabilistic and (subsequently introduced) type variables, we denote numerical values by lowercase Greek letters.

nodes. Using the methodology of (Bovens and Hartmann, 2003), we obtain their posterior probability as follows:

$$\begin{aligned} \boldsymbol{P}_1^* &:= \frac{\boldsymbol{P}_1(\mathrm{S},\mathrm{G},\mathrm{E})}{\boldsymbol{P}_1(\mathrm{E})} = \frac{\boldsymbol{P}_1(\mathrm{S},\mathrm{G},\mathrm{E})}{\sum_{S,G}(S,G,\mathrm{E})} \\ &= \frac{\gamma\,\pi\,\sigma}{\gamma\,\pi+\bar{\gamma}\,\rho}\,, \end{aligned} \tag{4}$$

where the denominator of the fraction in the final line is a convex combination of π and ρ weighed by γ, and where $\bar{\gamma} := 1-\gamma$.[12]

We close the present subsection by assessing the degree of confirmation of the conjunction of S and G. To this aim, we use the difference measure d, cf. (Carnap, 1950), defined for our case as follows:[13]

$$d_1 := \boldsymbol{P}_1(\mathrm{S},\mathrm{G}|\mathrm{E}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G})\,. \tag{5}$$

Thus, E confirms G if its consideration raises the probability of the conjunction of S and G. By calculating d_1, we show that the latter is indeed the case:

$$d_1 = \frac{\gamma\,\bar{\gamma}\,\sigma\,(\pi-\rho)}{\gamma\,\pi+\bar{\gamma}\,\rho}\,. \tag{6}$$

Assuming that γ, π, ρ, and σ lie in the open interval $(0, 1)$ with $\pi > \rho$, the above fraction is always strictly positive. We summarize our observation in the following theorem:

Theorem 1. *E confirms S and G iff E confirms G.*

We next investigate the joint probabilities of S, G in the post-reductive situation.

4.2. **Post-Reductive Confirmation.** The consideration of Montague's inter-theory mapping (cf. the arrow from S to G in Fig. 5, 7) requires a restatement of the above probabilities. Since G is no longer a root node in Figure 7 (and is, thus, not assigned a prior probability), we replace the second equation in (2) by (7), below, where $\boldsymbol{P}_2$ is the new probability measure:

$$\boldsymbol{P}_2(\mathrm{G}|\mathrm{S}) = 1 \quad , \quad \boldsymbol{P}_2(\mathrm{G}|\neg\mathrm{S}) = 0\,. \tag{7}$$

Equation (7) is warranted by Montague's homomorphism h. All other assignments are as for $\boldsymbol{P}_1$. Our introduction of the new measure $\boldsymbol{P}_2$ is motivated by the move to a different probabilistic situation, and attendant need to assign the received Montagovian propositions possibly distinct probabilistic values. Equality statements of the form $\boldsymbol{P}_2(\mathrm{S}) = \boldsymbol{P}_1(\mathrm{S})$ ensure the possibility of comparing the respective propositions' confirmation in different situations.

As encoded by the arrow from S to G, Montague's mapping effects a flow of evidence from syntax to semantics. The confirmation of the relevant semantic rule is defined simply as follows:

Theorem 2. *E confirms S iff* $\pi > \rho$.

[12]We will hereafter always abbreviate $1-x$ as $\bar{x}$.

[13]As discussed in (Fitelson, 1999; Eells and Fitelson, 2000), results may depend on our choice of confirmation measure. Whether (and to what extent) they do, will be a question for future research.

According to the above theorem, our piece of evidence confirms the semantic proposition if, as assumed in Section 4.1, E supports G. Equations (7) ensure a positive flow of confirmation from G to S.

The prior and posterior probabilities of the conjunction of S and G are as follows (all calculations are in the Appendix):

$$\boldsymbol{P}_2(\mathrm{S},\mathrm{G}) = \sigma\,, \tag{8}$$

$$\boldsymbol{P}_2^* := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) = \frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{9}$$

The degree of confirmation of the conjunction of S and G under $\boldsymbol{P}_2$ is recorded below:

$$d_2 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) - \boldsymbol{P}_2(\mathrm{S},\mathrm{G}) = \frac{\sigma\,\bar{\sigma}\,(\pi-\rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{10}$$

To show the epistemic value of Montagovian reduction, we next compare the conjunction's probabilities and degree of confirmation in the two scenarios. We accept a reduction if it raises the conjunction's probabilities or evidential support, and reject (or ignore) it otherwise.

4.3. **Comparing Situations.** We begin by comparing the conjunction's prior probabilities, $\boldsymbol{P}_1(\mathrm{S},\mathrm{G})$ and $\boldsymbol{P}_2(\mathrm{S},\mathrm{G})$. While the propositional variables S, G are independent before the reduction, they have become dependent after the reduction. This is due to the fact that G is no longer a root node in Figure 7. In order to compare the joint probabilities of S and G, we assume the identity of $\boldsymbol{P}_2(\mathrm{G})$ and $\boldsymbol{P}_1(\mathrm{G})$, and $\boldsymbol{P}_2(\mathrm{E}|\mathrm{G})$ and $\boldsymbol{P}_1(\mathrm{E}|\mathrm{G})$, respectively. By the first equality in (7), we further assume the equality in (11)

$$\boldsymbol{P}_2(\mathrm{G}) = \boldsymbol{P}_2(\mathrm{G}|\mathrm{S})\,\boldsymbol{P}_2(\mathrm{S}) = \sigma\,, \tag{11}$$

such that $\gamma = \sigma$.

Using the above, we calculate the difference, Δ_0, between the conjunction's pre- and post-reductive prior probabilities and obtain

$$\Delta_0 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G}) = \sigma\,\bar{\sigma}\,. \tag{12}$$

Intuitively, the Montague Reduction of Categorial Grammar to Model-Theoretic Semantics is epistemically valuable if the conjunction's prior probability is higher post- than pre-reduction, i.e. if $\Delta_0 > 0$. Since we assume all non-h-based probabilities to be non-extreme, we know that the latter is indeed the case.

The difference between the conjunction's posterior probabilities is also strictly positive:

$$\Delta_1 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G}|\mathrm{E}) = \frac{\pi\,\sigma\,\bar{\sigma}}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{13}$$

We show this via the above assumptions, together with the fact that $\pi > \rho$.

Our propositions' confirmation witnesses a similar increase. To establish this, we calculate the difference between their conjunction's pre- and post-reductive degree of confirmation under the difference measure and obtain

$$\Delta_2 := d_2 - d_1 = \frac{\sigma\,\bar{\sigma}^2\,(\pi-\rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{14}$$

As can be read off from the expression in (14), the positivity of Δ_2 – and the attendant positive confirmatory impact of Montague Reduction – is conditional on the requirement that $\sigma \in (0,1)$ and that $\pi > \rho$.

We have seen that Montague Reduction increases the joint probabilities and evidential support of the relevant conjunction. Given the similarity between the models of Montague Reduction and of Nagel Reduction (cf. Sect. 2.2), however, the probabilies and degree of confirmation of the truth of syntactic and semantic propositions are not higher after the Montague Reduction than they are after the Nagel Reduction of syntax to semantics.[14] Admittedly, our model of Montague Reduction captures the syntax-semantics relation more accurately than the Nagelian model. This does not change the fact, however, that the epistemic rationales for their adoption are equally strong.

But the similarities between Nagel Reduction and Montague Reduction give rise to another, more serious, problem: The latter lies in the fact that both types of intertheoretic relations are established pairwise (such that $\mathbb{G}$ is reduced to $\mathbb{S}$ by reducing $\mathbb{G}_{\mathrm{S}}$ to $\mathbb{G}_{\mathrm{S}}$, $\mathbb{G}_{\mathrm{N}}$ to $\mathbb{G}_{\mathrm{N}}$, etc.). While Montague's syntax-semantics relation thus renders the evidence variable E probabilistically relevant for S, the stipulation of independent morphisms between all pairs $\langle S_k, G_k\rangle$ does not assign the reduction an optimal epistemic value. This is due to the fact that the probability of syntax reduced to semantics will correspond to the product of the probabilities of all proposition pairs:[15]

$$\begin{aligned} \boldsymbol{P}_2(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k\rangle) &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{G}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}, \mathrm{G}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}) && (15)\\ &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}), && \text{(by (7))} \end{aligned}$$

respectively

$$\begin{aligned} \boldsymbol{P}_2(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k|\mathrm{E}_k\rangle) &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{G}_{\mathrm{N}}|\mathrm{E}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}, \mathrm{G}_{\mathrm{V}}|\mathrm{E}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}|\mathrm{E}_{\mathrm{S}})\\ &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}|\mathrm{E}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}|\mathrm{E}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}|\mathrm{E}_{\mathrm{S}})\,, && (16) \end{aligned}$$

such that the probability of the conjunction decreases in inverse proportion to the number of its conjuncts. Contrary to what is claimed for Nagel Reduction, cf. (Dijzadji-Bahmani et al., 2010b), the optimal generalization of Bayesian network representations of Montague Reduction to theories with multiple propositional elements may not be conceptually straightforward, but requires insight into the mutual dependencies between same-theory propositions or proposition-reducing principles.

5. Reduction and Confirmation II: Integrative Reduction

Integrative Reduction (abbreviated IR) accounts for such intratheoretical connections. Its model (presented in Sect. 5.2) is developed in abstraction from a sophisticated version of Montague's 'two theories' theory (cf. Sect. 2.1). To increase the perspicuity of the rule-connecting mechanism, Montague (1973) stipulates a third level of *types*, i.e., logico-semantic rôles which mediate between syntactic categories and their referential domains, cf. (Russell, 1908; Church, 1940). Every syntactic category k is thus correlated with a semantic type α, whose associated domain, $\mathcal{D}_\alpha$, constitutes the familiar denotation set of all expressions in $\mathcal{E}_\alpha$. Figure 8 schematizes the use of types on the level of objects:

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[14]For a Bayesian analysis of (Schaffner's revised version of) Nagel Reduction, the reader is referred to (Dijzadji-Bahmani et al., 2010b).

[15]To simplify notation, we write $\boldsymbol{P}_2(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k\rangle)$ for $\boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{S}_{\mathrm{V}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{N}}, \mathrm{G}_{\mathrm{V}}, \mathrm{G}_{\mathrm{S}})$, etc.

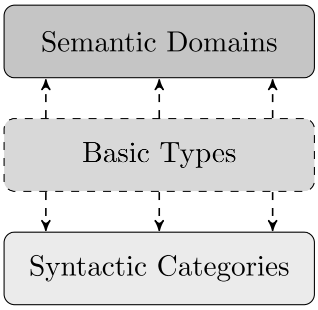

FIGURE 8. The type basis of syntactic and semantic categories.

To demonstrate the requirements on the use of an intermediate type-level, we present three different cases, stipulating the existence of multiple, two, or a single basic type for the reduction of syntactic to semantic propositions. We begin with a discussion of the multi-type case.

5.1. **Case 1: Separate Types.** Notably, the assumption of a separate type for each category pair does not improve upon the above-observed independence (cf. Sect. 4.3). Let the types e, p, and t be associated with individual objects ('entities'), properties, and truth-values, respectively. Assume that the type correspondent, $\mathbb{T}_{\mathrm{S}}$, of the semantic rule $\mathbb{S}_{\mathrm{S}}$ is defined as follows:

$\mathbb{T}_{\mathrm{S}}$. If $\boldsymbol{R} \in X_{\mathrm{V}}$ and $\boldsymbol{j} \in X_{\mathrm{N}}$, then $\boldsymbol{R} * \boldsymbol{j} \in X_{\mathrm{S}}$,

where X is neutral between the notation for expression sets $\mathcal{E}$ and referential domains $\mathcal{D}$, and where $*$ is neutral between the concatenation/agreement operator and the designation of functional application. For reasons of simplicity, we restrict ourselves to the presentation of the type-correspondent of the semantic rule $\mathbb{S}'_{\mathrm{S}}$. On the assumption of further types (e.g. the types q and r, associated with the categories of generalized quantifiers and properties of generalized quantifiers, respectively), the correspondent of the rule $\mathbb{S}''_{\mathrm{S}}$ is analogously defined. Note that we will hereafter use the name '$\mathbb{T}_{\mathrm{S}}$' as a hyperonym for the rule $\mathbb{T}'_{\mathrm{S}}$, such that the type-correspondent of the rule $\mathbb{S}_{\mathrm{S}}$ will be ambiguous.

Following the notational convention from the end of Section 3, we denote the values of variables $T_{\mathrm{S}}, T_{\mathrm{N}}$, and T_{V} by $\mathrm{T_S}, \neg\mathrm{T_S}, \mathrm{T_N}, \neg\mathrm{T_N}$, and $\mathrm{T_V}, \neg\mathrm{T_V}$, respectively. The graph in Figure 9 encodes the dependencies of propositional variables after the reduction via the newly introduced type level:
As in the untyped case (cf. Sect. 4.3), the independence of triples $\langle S_k, T_k, G_k\rangle$ warrants the derivation of their joint probabilities via the product of their individual probabilities. For this reason, we initially restrict ourselves to the prior and posterior probabilities of the conjunction of $\mathrm{T_S}, \mathrm{S_S}$, and $\mathrm{G_S}$. To emphasize our model's connection with the network from Figure 7, we use a similar abbreviation scheme, with $\boldsymbol{P}_3$ the new probability measure:

$$\begin{aligned}
\boldsymbol{P}_3(\mathrm{T_S}) &= \tau \quad , \\
\boldsymbol{P}_3(\mathrm{S_S}|\mathrm{T_S}) &= 1 \quad , \quad \boldsymbol{P}_3(\mathrm{S_S}|\neg\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_3(\mathrm{G_S}|\mathrm{T_S}) &= 1 \quad , \quad \boldsymbol{P}_3(\mathrm{G_S}|\neg\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_3(\mathrm{E_S}|\mathrm{G_S}) &= \pi \quad , \quad \boldsymbol{P}_3(\mathrm{E_S}|\neg\mathrm{G_S}) = \rho\,.
\end{aligned} \tag{17}$$

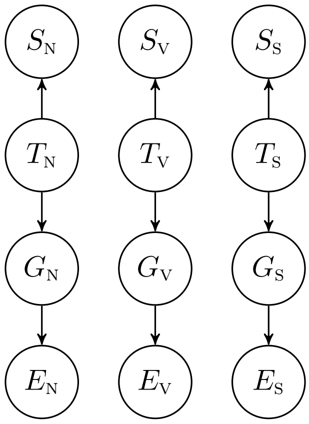

FIGURE 9. Case 1: Separate types.

The equations in the last line are as for Figures 6, 7, above. The identities in lines two and three are necessitated by the replacement of S_{S} and G_{S} by T_{S} as root nodes. Since, thus,

$$\boldsymbol{P}_3(\text{S}_\text{S}) = \boldsymbol{P}_3(\text{T}_\text{S}) = \boldsymbol{P}_3(\text{G}_\text{S}), \tag{18}$$

Theorem 2 also holds in the new model. This is due to the strong dependence of syntactic and semantic on type rules, and the assumption of a positive confirmatory relation between E and G.

Admittedly, the attribution of probabilities to our type rules seems less intuitive than the assignment of probabilities to their syntactic or semantic counterparts. This is amended by the equalities in the second and third line of (17). Thus, the probabilities of T_S, T_N, and T_V can be obtained via the probabilities of their semantic (or syntactic) correlates S_S, S_N, and S_V (resp. G_S, G_N, and G_V).

The close association of semantic and type rules prompts a general remark: Our introduction to this paper (cf. Sect. 1) has announced the development of a new type of intertheoretic relation on the model of Montague's characterization of the syntax-semantics relation. As we will show at the end of Section 5.2.2, the introduction of a separate level of types only serves to elucidate the relation between same-theory objects and propositions. Given the establishment of their constructive relations, and attendant identification of propositional interdependencies, the set of types (and associated type-propositions) is dispensable.

Let us proceed to the confirmation of the conjunction of the positive instantiations of the variables $T_{\text{S}}, S_{\text{S}}$, and G_{S}. The conjunction's prior and posterior probability are as follows:

$$\boldsymbol{P}_3(\text{T}_\text{S}, \text{S}_\text{S}, \text{G}_\text{S}) \;=\; \tau \tag{19}$$

$$\boldsymbol{P}_3^* := \boldsymbol{P}_3(\text{T}_\text{S}, \text{S}_\text{S}, \text{G}_\text{S}|\text{E}_\text{S}) \;=\; \frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\,. \tag{20}$$

The probabilistic equivalence of the separately typed and the untyped model is obvious: Given the equalities $\boldsymbol{P}_3(\text{S}_\text{S}) = \boldsymbol{P}_2(\text{S})$, $\boldsymbol{P}_3(\text{G}_\text{S}) = \boldsymbol{P}_2(\text{G})$, and $\boldsymbol{P}_3(\text{E}_\text{S}|\text{G}_\text{S}) = \boldsymbol{P}_2(\text{E}|\text{G})$ such that $\tau = \sigma$, it is easy to see the identity between the prior and posterior probabilities of the conjunction of $\text{T}_\text{S}, \text{S}_\text{S}$, and G_S, respectively of S and G. Like the joint probability of the latter, the joint probability of former form converges to 0 as their number increases.

The difference, Δ_3, of their associated degrees of confirmation witnesses confirmation stasis. Consequently, the conjunction of $\mathrm{S_S}$ and $\mathrm{G_S}$ is not better confirmed than its untyped competitor in the post-reductive situation. We summarize our findings in the following theorem, where $|\mathrm{CAT}|$ and $|\mathrm{TY}|$ denote the number of basic (syntactic or semantic) categories and types, respectively:

Theorem 3. *If* $|\mathrm{TY}| = |\mathrm{CAT}|$*, then* $\bigcap_k \langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k \rangle$ *has the same prior and posterior probability and is confirmed to the same degree as* $\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k \rangle$ *under the difference measure.*

5.2. **Case 2: Two Types.** The desired increase in confirmation requires the identification of connections between same-theory propositions. Montague's framework provides this link: Rather than taking different semantic or syntactic categories and rules to be structurally independent, Montague observes a strong connection in their constructive properties. Thus, he distinguishes two different classes of types, i.e. *simple* (or *basic*) types and *complex* (or *derived*) types. The types for entities e and truth-values t are identified as simple types; the type for properties p is identified as a complex type. The simple/complex distinction is preserved at the level of syntactic categories and semantic domains: As the category of type-e expressions, the category N (of nouns) is characterized as a *basic* category; the category V (of verbs) of type-p expressions as a *derived* category. Similarly, the domain of type-e expressions $\mathcal{D}_\mathrm{N}$ is described as a basic (or *primitive*) domain; the domain of type-p expressions $\mathcal{D}_\mathrm{V}$ is described as a complex (*constructed*, or *functional*) domain.

A formal account of the construction of complex-type domains is easily provided: According to Montague's type logic IL and its predecessor, Church's Simple Theory of Types (Church, 1940), a domain is constructed iff it is the function space over primitive (or less complex) domains. Arguably, such function spaces (e.g. the space $\mathcal{D}_\mathrm{N} \to \mathcal{D}_\mathrm{S}$) provide a suitable representation of other domains (here, the property domain $\mathcal{D}_p$). This is due to the fact that characteristic functions $\mathcal{D}_\mathrm{N} \to \{\mathbf{T}, \mathbf{F}\}$ represent the set of entities of which a given property is true (false). In a world w, that is inhabited by John, Mary, and Fido, the property $[\![\textsf{is a dog}]\!]$ is, thus, identified with the set $\{x \in \mathcal{D}_\mathrm{N} \mid [\![\textsf{is a dog}]\!](x) = \mathbf{T}\} = \big\{[\![\textsf{Fido}]\!]\big\}$. In line with the above, we call members of the former (i.e. entities) *primitive* objects; members of the latter (i.e. properties) are called *constructed* objects (or *functions*). We denote types for function spaces $\mathcal{D}_\alpha \to \mathcal{D}_\beta$ (with α, β types in IL) by $\alpha \to \beta$.

Let us return to our discussion of the two-typed syntax-semantics reduction. While the basic types e and t (or their associated rules) are directly involved in the reduction of the rules $\mathbb{S}_\mathrm{S}$, $\mathbb{S}_\mathrm{N}$ and $\mathbb{G}_\mathrm{S}$, $\mathbb{G}_\mathrm{N}$, they only serve as 'building blocks' in the formulation of the rules $\mathbb{S}_\mathrm{V}, \mathbb{G}_\mathrm{V}$. The above-described mechanism leaves open two possibilities for the construction of derived-type rules: While rules for the behavior of complex expressions or objects can be directly formulated in terms of derived types (such that $\mathcal{D}_\mathrm{V}$ and V (or their associated rules) are defined in terms of the derived type $e \to t$ (or its rule), cf. the graph in Fig. 10), their statement can, alternatively, involve rules for the obtaining of basic expressions and objects (such that $\mathcal{D}_\mathrm{V}$ and V (or their rules) are only defined in terms of the domains $\mathcal{D}_\mathrm{S}, \mathcal{D}_\mathrm{N}$ resp. the categories S, N (or their rules), cf. the graph in Fig. 11). As we will see in due course, both formulations yield the same probabilities.

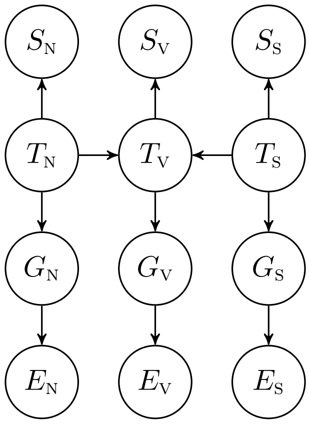

FIGURE 10. Case 2.i: Two types direct.

5.2.1. *Case 2.i: Two Types Direct.* We begin by determining the joint probabilities and confirmation of the conjunction of directly typed propositions (cf. Fig. 10).

Let $\boldsymbol{P}_4(\mathrm{T_S}) = \tau$, $\boldsymbol{P}_4(\mathrm{T_N}) = \tau'$, and $\boldsymbol{P}_4(\mathrm{T_V}) = \tau''$ be the marginal probabilities of $\mathrm{T_S}, \mathrm{T_N}$, and $\mathrm{T_V}$, respectively. We specify conditional probabilities via the following scheme:

$$\begin{aligned} \boldsymbol{P}_4(\mathrm{T_V}|\mathrm{T_N},\mathrm{T_S}) = 1 \quad &, \quad \boldsymbol{P}_4(\mathrm{T_V}|\neg\mathrm{T_N},\mathrm{T_S}) = 0\,, \\ \boldsymbol{P}_4(\mathrm{T_V}|\mathrm{T_N},\neg\mathrm{T_S}) = 0 \quad &, \quad \boldsymbol{P}_4(\mathrm{T_V}|\neg\mathrm{T_N},\neg\mathrm{T_S}) = 0\,, \\ \boldsymbol{P}_4(\mathrm{S_V}|\mathrm{T_V}) = 1 \quad &, \quad \boldsymbol{P}_4(\mathrm{S_V}|\neg\mathrm{T_V}) = 0\,, \\ \boldsymbol{P}_4(\mathrm{G_V}|\mathrm{T_V}) = 1 \quad &, \quad \boldsymbol{P}_4(\mathrm{G_V}|\neg\mathrm{T_V}) = 0\,, \\ \boldsymbol{P}_4(\mathrm{E_V}|\mathrm{G_V}) = \pi'' \quad &, \quad \boldsymbol{P}_4(\mathrm{E_V}|\neg\mathrm{G_V}) = \rho''\,. \end{aligned} \tag{21}$$

The probabilities of $\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}, \mathrm{E_S}$ resp. $\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N}, \mathrm{E_N}$ are as in (17). The equations in lines 1, 2, 4 and 1, 2, 3 ensure a positive flow of confirmation from $\mathrm{G_V}$ to $\mathrm{T_V}$ and from $\mathrm{T_V}$ to $\mathrm{S_V}$, respectively.

Note that, by the equations in the first two lines, the positive instantiation of the propositional variable T_V is not only conditionally dependent on the obtaining of the evidence E_V (given the truth of G_V and S_V), but also on the obtaining of the evidence E_N and E_S (given the truth of G_N, G_S and S_N, S_S). It is this conditional dependence, and the attendant flow of confirmation not only from $\mathrm{G_V}$ and $\mathrm{S_V}$, but also from $\mathrm{G_N}, \mathrm{G_S}$ and $\mathrm{S_N}, \mathrm{S_S}$ via $\mathrm{T_N}$ and $\mathrm{T_S}$ to $\mathrm{T_v}$ that effects the higher posterior probabilities and the (possibly) higher degree of confirmation of the conjunction of the positive instantiation of the above variables.

The prior and posterior probabilities of the conjunction of the variables' positive instantiations are as follows:

$$\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) = \tau\,\tau'\,, \tag{22}$$

$$\boldsymbol{P}_4^* \;:=\; \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}|\mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})\,. \tag{23}$$

To show the epistemic value of the two- over the three-typed case (cf. Sect. 5.1), we must first specify the probabilities of the (initially neglected) rules for entity- and property-types: The values of $\boldsymbol{P}_3(\mathrm{S_N}), \boldsymbol{P}_3(\mathrm{G_N})$, $\boldsymbol{P}_3(\mathrm{E_N})$ are as for $\boldsymbol{P}_4$. The conditional probabilities of S_V, G_V and E_V are analogous to their probabilities from (21), lines 3–5.

By the above-observed independence (Sect. 5.1, cf. Sect. 4.3), the prior and posterior probability of the conjunction $(T_S, T_N, T_V, S_S, S_N, S_V, G_S, G_N, G_V)$, cf. (24) and (25), below, amount to the product of their respective probabilities.

$$\boldsymbol{P}_3(T_S, T_N, T_V, S_S, S_N, S_V, G_S, G_N, G_V) \quad = \quad \tau\,\tau'\,\tau'', \tag{24}$$

$$\begin{aligned} (\boldsymbol{P}_3^*)' &:= \boldsymbol{P}_3(T_S, T_N, T_V, S_S, S_N, S_V, G_S, G_N, G_V | E_S, E_N, E_V) \\ &= \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right). \end{aligned} \tag{25}$$

For comparability, we assume equalities between $\boldsymbol{P}_4(S_k|T_k)$ and $\boldsymbol{P}_3(S_k|T_k)$, $\boldsymbol{P}_4(G_k|\ T_k)$ and $\boldsymbol{P}_3(G_k|T_k)$, and $\boldsymbol{P}_4(E_k|G_k)$ and $\boldsymbol{P}_3(E_k|G_k)$ (with $(e \to t) \equiv p \in \mathrm{TY}$). We begin by comparing the conjunction's prior probabilities, reflected in the difference Δ_4:

$$\begin{aligned} \Delta_4 &:= \boldsymbol{P}_4(T_S, T_N, S_S, S_N, S_V, G_S, G_N, G_V) - \boldsymbol{P}_3\Big(\bigcap_k \big\langle T_k, S_k, G_k\big\rangle\Big) \\ &= \tau\,\tau'\,\bar{\tau}'' \end{aligned} \tag{26}$$

As is clear from the relevant term in (26), the conjunction's prior probability is higher after the performance of a two-typed Montague Reduction than it is after a separately typed Montague Reduction. In particular, the positivity of Δ_4 depends on the non-certainty of T_V such that $\tau'' \neq 1$. Under the usual assumptions (in particular, the assumption that $\pi'' > \rho''$), the difference of the conjunction's posterior probabilities is also positive.

The conditions for a higher degree of confirmation are motivated by our previous observations: Given the difference $\Delta_6 := d_4 - d_5$ (with d_4 and d_5 the degree of confirmation of the conjunction of the positively instantiated variables from Figures 10 resp. 9), the replacement of three- by two-typed propositions of the syntactic and the semantic theory will increase the flow of confirmation between the two theories only if $\Delta_6 > 0$. Simple manipulations show that the latter is the case if and only if $\Delta_5 > \tau\,\tau'\,\bar{\tau}''$

As can be seen from the above, it is hence 'easier' to raise the posterior probability of Montague's theories by establishing a relation between different syntactic and semantic objects than it is to increase their degree of confirmation. Especially if $(\boldsymbol{P}_3^*)'$ is (comparatively) high, the confirmation may not be greater after the reduction.

Our findings are captured in the following theorem, where TY_2 and TY_n are the basic-type sets associated with theories of two- and n-typed syntax/ semantics, with $\mathrm{TY}_2 \subseteq \mathrm{TY}_n$.

Theorem 4. *If $|\mathrm{TY}_2| < |\mathrm{TY}_n|$, then the conjunction of two-typed propositions has a higher prior and posterior probability and is better confirmed under the difference measure than the conjunction of their n-typed correspondents if the following holds:*

- *i. The marginal probability of the truth of the propositions for members of TY_n is non-extreme.*
- *ii. For every proposition $\mathbb{T}_i$ associated with a member, i, of $\mathrm{TY}_n \backslash \mathrm{TY}_2$, the likelihood of T_i on G_i is higher than the likelihood of $\neg T_i$ on G_i.*

iii. The difference between the posterior probability of the conjunction of two- and n-typed propositions is greater than the product of the marginal probability of the truth of the propositions for members of TY_2 *and the probability of the falsity of the propositions for members of* $\mathrm{TY}_n \backslash \mathrm{TY}_2$.

Since conditions (i) and (ii) are satisfied by our standard assumptions (cf. Sect. 4), we regard Theorem 4 as a rationale for the introduction of a (non-Nagelian) model of the two-typed version of Montague Reduction (cf. Def. 3, below).

5.2.2. *Case 2.ii: Two Types Indirect.* To compare the probabilities of the directly with those of the indirectly typed propositions, we next consider the probabilities of the network in Figure 11.

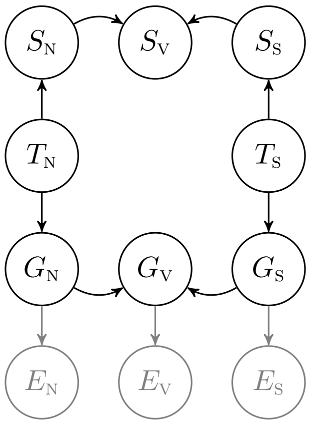

FIGURE 11. Case 2.ii: Two types indirect.

The mediated formulation of derived-type rules (cf. the chains of arrows from T_N and T_S via S_N and S_S to S_V) requires the replacement of the first three lines in (21) by the equalities in (27), below (with $\boldsymbol{P}_5$ the new probability measure):

$$\boldsymbol{P}_5(\mathrm{S_V}|\mathrm{S_N}, \mathrm{S_S}) = 1 \quad , \quad \boldsymbol{P}_5(\mathrm{S_V}|\neg\mathrm{S_N}, \mathrm{S_S}) = 0\,, \qquad (27)$$
$$\boldsymbol{P}_5(\mathrm{S_V}|\mathrm{S_N}, \neg\mathrm{S_S}) = 0 \quad , \quad \boldsymbol{P}_5(\mathrm{S_V}|\neg\mathrm{S_N}, \neg\mathrm{S_S}) = 0\,.$$

The conditional probability of $\mathrm{G_V}$ is similarly defined: Rather than depending only on the probability of $\mathrm{T_V}$, the probability of the truth of S_V is now dependent on the probabilities of $\mathrm{S_N}, \mathrm{S_S}$. This is not to claim a fundamental difference between the presently and previously introduced models: Notably, our choice of different type-rule formulations does not impact the theories' probabilities and confirmation. This is due to the probabilistic equivalence of chains of arrows $(T_S \to S_S) \circ (S_S \to S_V)$, $(T_S, T_N \to T_V) \circ (T_V \to S_V)$, and the corresponding identities $\boldsymbol{P}_5(\mathrm{S_V}) = \boldsymbol{P}_4(\mathrm{S_V})$, $\boldsymbol{P}_5(\mathrm{G_V}) = \boldsymbol{P}_4(\mathrm{G_V})$. The prior and posterior probabilities and the degree of confirmation of indirectly typed propositions are thus the same as those of directly typed propositions.

We have motivated our presentation of the model of a refined version of Montague Reduction by the need to identify dependencies between same-theory objects and propositions. Our investigation into the probabilistic impact of different rule-formulations yields further insight into the latter requirement: While the use of types (as a surrogate for term-connecting bridge laws) increases the perspicuity of the effected reduction, the improvement of our theories' probabilities and degree of confirmation is not conditional on the introduction of an intermediate type level. This is warranted by the identity of $\boldsymbol{P}_5(\mathrm{S_S}), \boldsymbol{P}_5(\mathrm{T_S}), \boldsymbol{P}_5(\mathrm{G_S})$ and $\boldsymbol{P}_5(\mathrm{S_N}), \boldsymbol{P}_5(\mathrm{T_N}), \boldsymbol{P}_4(\mathrm{G_N})$, respectively.[16] The only requirement lies in the establishment of definitional connections between same-theory objects (such that $\mathcal{D}_\mathrm{V} := (\mathcal{D}_\mathrm{N} \rightarrow \mathcal{D}_\mathrm{S})$).

The latter constitute the core feature of Integrative Reduction. We define Integrative Reduction as a sophisticated version of Montague Reduction, that differs from the latter with respect to the establishment of constructive relations between same-theory objects and propositions. Since the introduction of a different basic type for every category/domain pair prevents the establishment of this type of *intra*theoretic relation, a separately-typed variant of Montague Reduction (along the lines of Sect. 5.1) does not qualify as a *proper* case of Integrative Reduction.[17] This is due to the probabilistic equivalence of their associated propositions, and the attendant collapse of the separately-typed model of Integrative Reduction into a variant of Montague Reduction.

Our definition of Integrative Reduction runs as follows:

Definition 3 (Integrative Reduction (IR))**.** A type of *directed* (or *non-symmetric*) dependency relation, implicit in (Montague, 1973), that is defined by the existence of intertheoretical and *intra*theoretical connections (i.e. constructibility relations) between objects of the two related theories, and by the derivability of every proposition in the phenomenological theory from a corresponding proposition of the fundamental theory.

Figure 12 (below) captures the commonalities and differences between Nagel Reduction, Montague Reduction, and Integrative Reduction.

Derivability {
- NR } *Syn.* interth. connect'y, *Undirected* dependency
- MR, IR } *Sem.* interth. connect'y, *Directed* dependency
  - IR } *Intra*th. connectability (constructibility rel's)

FIGURE 12. Nagel Reduction vs. Montague Reduction vs. Integrative Reduction.

We conclude the present section with considerations about the optimal number of basic types (or primitive semantic domains).

5.3. **Case 3: One Type.** Our previous findings suggest an inverse proportionality between the theories' probabilities, or degree of confirmation, and the number of basic types: As the latter decreases, the former rises. To check this

[16]This constitutes the probabilistic basis for linguists' choice between 'direct' and 'indirect' interpretations of natural language into set-theoretic models, cf. (Partee, 1997).

[17]Note, however, the possibility of treating Integrative Reductions of any kind (including non-proper reductions) as a generalization of Montague Reduction.

hypothesis, and identify possible constraints, we now turn to the last case. Figure 13 displays a graph associated with the assumption of a single type, e, for the formulation of syntactic and semantic rules. By the results from Section 5.2, our type choice does not influence the confirmation of Montagovian propositions.

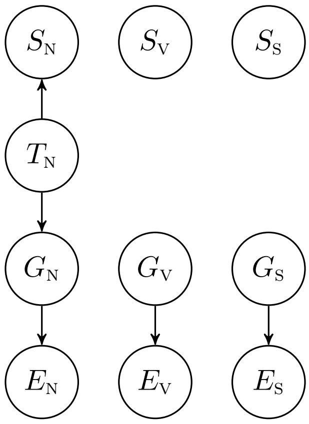

FIGURE 13. Case 3: One type.

A glance at the graph in Figure 13 reveals the large number of root nodes and conditional independencies. This is due to the impossibility of constructing the remaining types (e.g. p (or $e \to t$) and t) from a single base type (here, e), and the related need to separately introduce their associated syntactic and semantic rules.

The abbreviation scheme, below, contains the marginal and conditional probabilities of all nodes in the Bayesian network in Figure 13:

$$\begin{array}{rclcrclcrcl} \boldsymbol{P}_6(\mathrm{T_N}) = \tau' & , & \boldsymbol{P}_6(\mathrm{S_S}) = \sigma & , & \boldsymbol{P}_6(\mathrm{S_V}) = \sigma'' \\ \boldsymbol{P}_6(\mathrm{G_S}) = \gamma & , & \boldsymbol{P}_6(\mathrm{G_V}) = \gamma'' & , & \boldsymbol{P}_6(\mathrm{S_N}|\mathrm{T_N}) = 1 \\ \boldsymbol{P}_6(\mathrm{S_N}|\neg\mathrm{T_N}) = 0 & , & \boldsymbol{P}_6(\mathrm{G_N}|\mathrm{T_N}) = 1 & , & \boldsymbol{P}_6(\mathrm{G_N}|\neg\mathrm{T_N}) = 0 \\ \boldsymbol{P}_6(\mathrm{E_N}|\mathrm{G_N}) = \pi' & , & \boldsymbol{P}_6(\mathrm{E_N}|\neg\mathrm{G_N}) = \rho' & , & \boldsymbol{P}_6(\mathrm{E_S}|\mathrm{G_S}) = \pi \\ \boldsymbol{P}_6(\mathrm{E_S}|\neg\mathrm{G_S}) = \rho & , & \boldsymbol{P}_6(\mathrm{E_V}|\mathrm{G_V}) = \pi'' & , & \boldsymbol{P}_6(\mathrm{E_V}|\neg\mathrm{G_V}) = \rho'' \end{array} \tag{28}$$

The probabilities of $\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N}$, and $\mathrm{E_N}$ are as in (17). The other values in the first, second, ultimate and penultimate lines correspond to those from (2). By the absence of property- or truth-value types, the positive flow of confirmation between $\mathrm{G_N}, \mathrm{T_N}$, and $\mathrm{S_N}$ (cf. ll. 4, 5) is disabled at the verbal and sentential level.

The independence of tuples $\langle T_N, S_N, G_N\rangle$, $\langle S_S, G_S\rangle$, $\langle S_V, G_V\rangle$ facilitates the comparative assessment of our theories' probabilities and confirmation. While the prior and posterior probabilities of the conjunction of $\mathrm{T_N}, \mathrm{S_N}$, and $\mathrm{E_N}$ correspond to the probabilities of the separately typed case in Section 5.1 (granted the usual comparability conditions), the probabilities of $(\mathrm{S_S}, \mathrm{G_S})$ and $(\mathrm{S_V}, \mathrm{G_V})$ are parallel to those of the pre-reductive untyped case (cf. Sect. 4.1; Fig. 6). Their multiplication yields the following prior probability:

$$\boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) = \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N})\boldsymbol{P}_6(\mathrm{S_S}, \mathrm{G_S})\boldsymbol{P}_6(\mathrm{S_V}, \mathrm{G_V}) \tag{29}$$

By the above argument, (29) is greater than the prior probability of the conjunction of untyped propositions in the pre-reductive Montagovian, but smaller than the conjunction of separately typed propositions in the post-reductive Montagovian situation. The same holds, by an argument from $\boldsymbol{P}_2^*$ and $(\boldsymbol{P}_3^*)'$, of the conjunction's posterior probability:

$$\boldsymbol{P}_6^* \quad := \quad \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}|\mathrm{E_N}, \mathrm{E_S}, \mathrm{E_V})\,. \tag{30}$$

To compare our theories' degree of confirmation, d_6, with the support of the separately typed model, we calculate $\Delta_7 := d_6 - d_5$. Notably, Δ_7 is negative under the familar conditions (i.e. a positive confirmatory relation between every E and G, and non-extreme marginal probabilities of the truth of syntactic, semantic, or type propositions). Thus, the conjunction of the positive instantiation of single-typed propositional variables is confirmed to a lower degree than the conjunction of the positive instantiation of separately typed variables.

The concession of a Montagovian map between the elements in $\langle S_{\mathrm{S}}, G_{\mathrm{S}}\rangle$ and $\langle S_{\mathrm{V}}, G_{\mathrm{V}}\rangle$ (cf. Sect. 4.2) hardly improves this situation: While the homomorphism h cancels some of the above-observed independencies – requiring a restatement of the relevant probabilities in (31) (below) – the theories' probabilities and degree of confirmation under the difference measure do not exceed that of the separately typed model. To mark the move to a different probabilistic situation, we introduce the new probability measure $\boldsymbol{P}_7$. Significantly,

$$\begin{aligned} \boldsymbol{P}_7(\mathrm{G_S}|\mathrm{S_S}) = 1 \quad &, \quad \boldsymbol{P}_7(\mathrm{G_S}|\neg\mathrm{S_S}) = 0\,, \\ \boldsymbol{P}_7(\mathrm{G_V}|\mathrm{S_V}) = 1 \quad &, \quad \boldsymbol{P}_7(\mathrm{G_V}|\neg\mathrm{S_V}) = 0\,, \end{aligned} \tag{31}$$

(cf. Section 4.2, (7)). All other assignments are as above.

Since tuples $\big\langle (\mathrm{T}_k,)\,\mathrm{S}_k, \mathrm{G}_k\big\rangle$ remain independent, we calculate their joint probabilities via the mechanism, above. The prior and posterior probability of the conjunction are

$$\boldsymbol{P}_7(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) = \sigma\, \sigma''\, \tau' \tag{32}$$

$$\boldsymbol{P}_7^* = \left(\frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\right)\left(\frac{\pi''\,\sigma''}{\pi''\,\sigma'' + \rho''\,\bar{\sigma}''}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right). \tag{33}$$

It is easy to see that, granted the above requirements, the conjunction's prior and posterior probabilities under $\boldsymbol{P}_7$ are exactly the probabilities of (24) and (25).

We summarize the results of our investigation of the separate-, the two-, and the single-type case in the following theorem, where TY_m and TY_n are different basic-type sets such that $\mathrm{TY}^m \subseteq \mathrm{TY}^n$:

Theorem 5 (Fewness)**.** *If* TY_m *enables the construction of all linguistically relevant types, then, granted the conditions from Theorem 4,* $\bigcap_m \langle \mathrm{S}_m, \mathrm{G}_m\rangle$ *has a higher prior and posterior probability and is better confirmed under the difference measure than* $\bigcap_n \langle \mathrm{S}_n, \mathrm{G}_n\rangle$*.*

Given the derivability of all syntactic or semantic propositions, the minimal number of basic types yields the highest probabilities and (given certain conditions) effects a maximal flow of confirmation between the two theories that are related by an Integrative Reduction. As a result, the adoption of our model of Integrative Reduction (in addition to the Nagelian Model) is epistemically warranted.

6. Conclusion

In this paper, we have identified two new types of intertheoretic relations, *Montague Reduction* and *Integrative Reduction*, that are inspired by Montague's (1973) presentation of the relation between Categorial Grammar and Model-Theoretic Semantics. We have shown their commonalities with Nagel Reduction and established their salient differences. To provide an epistemic rationale for the introduction of our model of Integrative Reduction, we have given its analysis in the framework of Bayesian confirmation theory. We have shown that the Integrative Reduction of syntax to semantics yields higher prior and posterior probabilities and (given certain conditions) a higher degree of confirmation than their Montague Reduction or Nagel Reduction.

We close our paper by indicating how our model of Integrative Reduction can be incorporated into a variant of Schaffner's revised model of Nagel Reduction. As is well known, Schaffner's variant of Nagel Reduction accommodates the directed dependency of intertheoretic reduction relations through the introduction of a dedicated level of 'corrected' propositions, cf. (Schaffner, 1967; 1974). Thus, every proposition T_1 resp. T_2 of the fundamental or phenomenological theory has a corrected variant T_1^*, resp. T_2^*, such that the deduction of T_2 from T_1 proceeds via T_1^* and T_2^*. To transform our model of Integrative Reduction into a variant of Schaffner's revised model, we thus only need to introduce a corrected version, $\mathbb{S}_k^*$ resp. $\mathbb{G}_k^*$, of every proposition, $\mathbb{S}_k$ resp. $\mathbb{G}_k$, in $\mathbb{S}$ or $\mathbb{G}$. We leave the elaboration of this 'Schaffner-style' variant of our model of Integrative Reduction for another occasion.

Appendix A: Proofs and Calculations for Section 4

We have calculated the pre-reductive probabilities of the conjunction of the positive instantiations of S and G in Section 4.1. The joint distribution, $\boldsymbol{P}_2(S,G,E)$, of the (post-reductive) graph in Figure 7 is given by the expression

$$\boldsymbol{P}_2(S)\,\boldsymbol{P}_2(G)\,\boldsymbol{P}_2(E|G).$$

Using the methodology employed in (Bovens and Hartmann, 2003), the prior probability of the conjunction of S and G is obtained as follows:

$$\boldsymbol{P}_2(\mathrm{S},\mathrm{G}) = \sum_E \boldsymbol{P}_2(\mathrm{S},\mathrm{G},E) = \pi\,\sigma + \bar{\pi}\,\sigma = \sigma\,. \tag{34}$$

We yield the posterior probability, $\boldsymbol{P}_2^* := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E})$, thus:

$$\boldsymbol{P}_2^* = \frac{\boldsymbol{P}_2(\mathrm{S},\mathrm{G},\mathrm{E})}{\boldsymbol{P}_2(\mathrm{E})} = \frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{35}$$

To obtain the difference Δ_0, we calculate

$$\boldsymbol{P}_2(\mathrm{S},\mathrm{G}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G}) = \sigma - \sigma^2 = \sigma\,\bar{\sigma}\,.$$

This proves the following proposition:

Proposition 1. $\Delta_0 = 0$ *iff* $\sigma = 0$ *or* 1*;* $\Delta_0 > 0$ *iff* $\sigma \in (0,1)$*.*

The difference Δ_1 between the conjunction's pre- and post-reductive posterior probabilities is obtained as follows:

$$\Delta_1 := \boldsymbol{P}_2^* - \boldsymbol{P}_1^* = \frac{\pi\,\sigma - \pi\,\sigma^2}{\pi\,\sigma + \rho\,\bar{\sigma}} = \frac{\pi\,\sigma\,\bar{\sigma}}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{36}$$

From the difference measure

$$d_2 := \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}|\mathrm{E}) - \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}) = \frac{\sigma\,\bar{\sigma}\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,, \tag{37}$$

we calculate the difference Δ_2 between the conjunction's degree of confirmation before and after the Montague Reduction as follows:

$$\Delta_2 := d_2 - d_1 = \frac{\sigma\,\bar{\sigma}\,(\pi - \rho) - \sigma^2\,\bar{\sigma}\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}} = \frac{\sigma\,\bar{\sigma}^2\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{38}$$

This completes our proofs and calculations for Section 4.

Appendix B: Proofs and Calculations for Section 5

Let us consider the confirmation of the conjunction of the positive instantiations of propositional variables for the separate-type case (case 1). The joint distribution $\boldsymbol{P}_3(T_{\mathrm{S}}, S_{\mathrm{S}}, G_{\mathrm{S}}, E_{\mathrm{S}})$ is given by the expression

$$\boldsymbol{P}_3(T_{\mathrm{S}})\boldsymbol{P}_3(S_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_3(G_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_3(E_{\mathrm{S}}|G_{\mathrm{S}}).$$

To obtain the prior probability of the conjunction of $\mathrm{T_S}, \mathrm{S_S}$, and $\mathrm{G_S}$, we calculate

$$\boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}) = \sum_E \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}, E_{\mathrm{S}}) = \tau\,. \tag{39}$$

The posterior probability, $\boldsymbol{P}_3^* := \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}|\mathrm{E_S})$, is obtained as follows:

$$\boldsymbol{P}_3^* = \frac{\boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}, \mathrm{E_S})}{\boldsymbol{P}_3(\mathrm{E_S})} = \frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\,. \tag{40}$$

The difference, Δ_3, between the degree of confirmation of separately typed and untyped propositions witnesses confirmation stasis:

$$\Delta_3 := d_3 - d_2 = 0\,, \tag{41}$$

where d_2 is as above, and $d_3 = d_2$.

We next discuss the probabilities and degree of confirmation of the two-type case (case 2). The joint distribution $\boldsymbol{P}_4(T_{\mathrm{S}}, T_{\mathrm{N}}, T_{\mathrm{V}}, S_{\mathrm{S}}, S_{\mathrm{N}}, S_{\mathrm{V}}, G_{\mathrm{S}}, G_{\mathrm{N}}, G_{\mathrm{V}}, E_{\mathrm{S}}, E_{\mathrm{N}}, E_{\mathrm{V}})$ is given by the expression

$$\begin{aligned}\boldsymbol{P}_4(T_{\mathrm{S}})\boldsymbol{P}_4(T_{\mathrm{N}})\boldsymbol{P}_4(T_{\mathrm{V}}|T_{\mathrm{S}}, T_{\mathrm{N}})\boldsymbol{P}_4(S_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_4(S_{\mathrm{N}}|T_{\mathrm{N}})\boldsymbol{P}_4(S_{\mathrm{V}}|T_{\mathrm{N}}, T_{\mathrm{S}})\boldsymbol{P}_4(G_{\mathrm{S}}|T_{\mathrm{S}})\\ \boldsymbol{P}_4(G_{\mathrm{N}}|T_{\mathrm{N}})\boldsymbol{P}_4(G_{\mathrm{V}}|T_{\mathrm{N}}, T_{\mathrm{S}})\boldsymbol{P}_4(E_{\mathrm{S}}|G_{\mathrm{S}})\boldsymbol{P}_4(E_{\mathrm{N}}|G_{\mathrm{N}})\boldsymbol{P}_4(E_{\mathrm{V}}|G_{\mathrm{V}})\,.\end{aligned} \tag{42}$$

The prior probability of the conjunction of positive instantiations of the above variables is as follows:

$$\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) = \tau\,\tau'\,. \tag{43}$$

Their posterior probability, $\boldsymbol{P}_4^* := \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}|\mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})$, is obtained thus:

$$\begin{aligned}\boldsymbol{P}_4^* &= \frac{\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}, \mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})}{\boldsymbol{P}_4(\mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})}\\ &= \frac{\pi\,\pi'\,\pi''\,\tau\,\tau'}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}'}\,.\end{aligned} \tag{44}$$

Rather than calculating all 2^8 possibilities, we use the equalities in (17)–(18) and (22) to isolate the significant, i.e. non-zero, cases. Since the non-uniform (i.e. positive or negative) instantiation of T_k, S_k, and G_k renders the product in (42) zero, we restrict our attention to the following two cases:

i. $\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}$;
ii. $\neg\mathrm{T_S}, \neg\mathrm{T_N}, \neg\mathrm{T_V}, \neg\mathrm{S_S}, \neg\mathrm{S_N}, \neg\mathrm{S_V}, \neg\mathrm{G_S}, \neg\mathrm{G_N}, \neg\mathrm{G_V}$.

The degree of confirmation of the conjunction of the truth of the propositions under the difference measure is as follows:

$$\begin{aligned} d_4 &:= \boldsymbol{P}_4^* - \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) \\ &= \frac{\tau\,\tau'\,\bar\tau\,\bar\tau'\,(\pi\,\pi'\,\pi'' - \rho\,\rho'\,\rho'')}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau'}\,. \end{aligned} \tag{45}$$

To obtain the difference Δ_4 between the prior probabilities of the conjunction of three- and two-typed propositions, we calculate

$$\begin{aligned} \Delta_4 &:= \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) - \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) \\ &= \tau\,\tau' - \tau\,\tau'\,\tau'' = \tau\,\tau'\,\bar\tau''\,. \end{aligned} \tag{46}$$

Proposition 2, below, summarizes the positivity conditions for Δ_4:

Proposition 2. $\Delta_4 = 0$ *iff either (i)* $\tau = 0$*, (ii)* $\tau' = 0$*, or (iii)* $\tau'' = 1$*.* $\Delta_4 > 0$ *iff* τ, τ'*, and* $\tau'' \in (0, 1)$*.*

The difference Δ_5 between the conjunction's posterior probabilities is obtained as follows:

$$\Delta_5 := \boldsymbol{P}_4^* - (\boldsymbol{P}_3^*)', \tag{47}$$

with $\boldsymbol{P}_4^*$ as above and

$$(\boldsymbol{P}_3^*)' = \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar\tau}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar\tau'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar\tau''}\right). \tag{48}$$

To show that $\Delta_5 > 0$, we first observe that the function

$$f(\tau'') := \frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar\tau''} \tag{49}$$

is strictly monotonically increasing in τ''. Consequently,

$$f(\tau'') \leq f(1) = 1\,. \tag{50}$$

By the assumption that $\pi, \pi', \rho, \rho', \tau, \tau' \in (0, 1)$, it thus holds that

$$(\boldsymbol{P}_3^*)' \leq \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar\tau}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar\tau'}\right).$$

Then,

$$\begin{aligned} \Delta_5 &\geq X \cdot \left(\frac{1}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau'} - \frac{1}{(\pi\,\tau + \rho\,\bar\tau)(\pi'\,\tau' + \rho'\,\bar\tau')\,\pi''}\right) \\ &= X' \cdot \left((\pi\,\tau + \rho\,\bar\tau)(\pi'\,\tau' + \rho'\,\bar\tau')\,\pi'' - (\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau')\right) \\ &= X' \cdot \left(\pi\,\pi''\,\rho'\,\tau\,\bar\tau' + \pi'\,\pi''\,\rho\,\bar\tau\,\tau' + \rho\,\rho'\,\bar\tau\,\bar\tau' \cdot (\pi'' - \rho'')\right). \end{aligned} \tag{51}$$

with

$$\begin{aligned} X &:= \pi\,\pi'\,\pi''\,\tau\,\tau' \\ X' &:= \frac{\pi\,\pi'\,\pi''\,\tau\,\tau'}{(\pi\,\tau + \rho\,\bar\tau)(\pi'\,\tau' + \rho'\,\bar\tau')\pi'' \cdot (\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau')}\,. \end{aligned}$$

The expression in the final line of (51) is greater than 0 if $\pi'' > \rho''$. Note that this is a sufficient, not a necessary condition. However, since our rules' confirmation by the relevant piece(s) of evidence constitutes one of our permanent assumptions (cf. Sect. 4.1), we content ourselves with this criterion.

To assess the confirmatory status of the direct two-typed case, we first identify the measures d_4, d_5, with

$$\begin{aligned} d_4 &:= \boldsymbol{P}_4^* - \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) \\ &= \frac{\tau\,\tau'\,\bar{\tau}\,\bar{\tau}'\,(\pi\,\pi'\,\pi'' - \rho\,\rho'\,\rho'')}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}'} \end{aligned} \tag{52}$$

and

$$\begin{aligned} d_5 &:= (\boldsymbol{P}_3^*)' - \boldsymbol{P}_3(\mathrm{T_N}, \mathrm{T_S}, \mathrm{T_V}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) \\ &= \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) - \tau\,\tau'\,\tau''\,. \end{aligned} \tag{53}$$

Their difference, $\Delta_6 := d_4 - d_5$, is easily obtained though the use of the fact that $d_i - d_k = (\boldsymbol{P}_i^* - \boldsymbol{P}_i(\ldots)) - (\boldsymbol{P}_k^* - \boldsymbol{P}_k(\ldots)) = (\boldsymbol{P}_i^* - \boldsymbol{P}_k^*) - (\boldsymbol{P}_i(\ldots) - \boldsymbol{P}_k(\ldots))$ such that

$$\begin{aligned} \Delta_6 &= \Big(\boldsymbol{P}_4^* - \boldsymbol{P}_4\big(\bigcap_k \langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k\rangle\big)\Big) - \Big((\boldsymbol{P}_3^*)' - \boldsymbol{P}_3\big(\bigcap_k \langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k\rangle\big)\Big) \\ &= \Delta_5 - \tau\,\tau'\,\bar{\tau}''. \end{aligned} \tag{54}$$

We close by considering the confirmation of the conjunction in the single-type case (case 3). The joint distribution $\boldsymbol{P}_6(T_\mathrm{N}, S_\mathrm{S}, S_\mathrm{N}, S_\mathrm{V}, G_\mathrm{S}, G_\mathrm{N}, G_\mathrm{V}, E_\mathrm{S}, E_\mathrm{N}, E_\mathrm{V})$ is given by the expression

$$\begin{aligned} &\boldsymbol{P}_6(T_\mathrm{N})\boldsymbol{P}_6(S_\mathrm{S})\boldsymbol{P}_6(S_\mathrm{V})\boldsymbol{P}_6(G_\mathrm{S})\boldsymbol{P}_6(G_\mathrm{V}) \\ &\boldsymbol{P}_6(S_\mathrm{N}|T_\mathrm{N})\boldsymbol{P}_6(G_\mathrm{N}|T_\mathrm{N})\boldsymbol{P}_6(E_\mathrm{N}|G_\mathrm{N})\boldsymbol{P}_6(E_\mathrm{S}|G_\mathrm{S})\boldsymbol{P}_6(E_\mathrm{V}|G_\mathrm{V})\,. \end{aligned} \tag{55}$$

Our calculation of the conjunction's prior and posterior probabilities exploit the independence of pairs, $\langle S_k, G_k\rangle$, together with the results from Sections 4.1, 4.2, 5.1 such that

$$\begin{aligned} \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) &= \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N})\boldsymbol{P}_6(\mathrm{S_S}, \mathrm{G_S})\boldsymbol{P}_6(\mathrm{S_V}, \mathrm{G_V}) \\ &= \gamma\gamma''\,\sigma\sigma''\,\tau'\,, \end{aligned} \tag{56}$$

and

$$\begin{aligned} \boldsymbol{P}_6^* &:= \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}|\mathrm{E_N}, \mathrm{E_S}, \mathrm{E_V}) \\ &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi + \bar{\gamma}\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi'' + \bar{\gamma}''\,\rho''}\right). \end{aligned} \tag{57}$$

We assess the conjunction's evidential support via the measure d_6 and observe that, under the positivity conditions from d_1 and d_5, the difference d_6 is also positive:

$$\begin{aligned} d_6 &:= \boldsymbol{P}_6^* - \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) \\ &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi + \bar{\gamma}\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi'' + \bar{\gamma}''\,\rho''}\right) - \gamma\,\gamma''\,\sigma\,\sigma''\,\tau'\,. \end{aligned} \tag{58}$$

From the measures d_5 and d_6, above, we obtain Δ_7 as follows:

$$\begin{aligned}\Delta_7 &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi + \bar{\gamma}\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi'' + \bar{\gamma}''\,\rho''}\right) - \gamma\,\gamma''\,\sigma\,\sigma''\,\tau' \qquad (59)\\ &\quad - \left(\left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) - \tau\,\tau'\,\tau''\right)\\ &= -\,\sigma\,\sigma''\,\tau'\,(1-\gamma\gamma'')\left(\left(\frac{\pi}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) - 1\right).\end{aligned}$$

Since expressions of the form $\pi/(\pi\,\tau + \rho\,\bar{\tau})$ are greater than 1 for every π, ρ, τ of the same type if $\pi > \rho$ and $\tau \in (0,1)$, the difference Δ_7 is always negative. This completes our calculations.

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