Instants and Instantaneous Velocity

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Abstract

As Frank Arntzenius has shown, instantaneous velocity continues to pose philosophical puzzles. Here I recap the flaws involved in all three standard theories of velocity-the 'at-at' theory plus the calculus, impetus or dispositional theories and 'no-instants' theories. Next, I argue that, although it avoids the problems with impetus theories identified by Arntzenius, Marc Lange's dispositional theory of velocity suffers from its own variety of philosophical perplexity. I conclude by arguing for a modified 'no-instants' theory, inspired by Aristotle's responses to Zeno's paradoxes, that avoids the problems for the versions considered by Arntzenius. In addition, this theory points the way to a more philosophically perspicuous account of the internal structure of time.

1 The Problem of Instantaneous States

The concept of the instantaneous state of a physical system has been a source of philosophical puzzlement since at least the time of Zeno. Recently the coherence of the concept has come under pressure from two directions. Firstly, there has been a renewed debate over the physical and metaphysical status of instantaneous velocity, largely mirroring the classical debates over Zeno's arrow paradox.[1] Secondly, several philosophers, including Michael Dummett and Jeremy Butterfield, have put pressure directly on the "classical model" of zero duration instants or instantaneous states.[2] In this paper, I attempt to bring these two debates together in a new way. The paper defends three nested conclusions. First, that none of the current proposals for the nature of instantaneous velocity, including Marc Lange's sophisticated dispositional account, are entirely satisfactory. Second, that this problem is not likely to be solved by further "fiddling" with the definition or interpretation of velocity, but only by reconceptualizing the concept of an instant as it informs the concept of an instantaneous state. Finally, that when we carry out this "re-think," we discover another model for temporal instants as the boundaries of temporal durations, which continues to push us towards a non-classical model of time.

The paper, therefore, proceeds in three stages. First, the next two sections recapitulate the contemporary debates over the status of velocity. §2 explicates Frank Arntzenius's arguments against the standard responses to Zeno's arrow, while §3 argues that while Marc

Lange's definition, as a disposition to follow the inertial path defined by the time-derivative of the path taken from above, avoids Arntzenius's objections, it is problematic for other reasons. The primary objection to Lange's account is that it commits him to substantive and at least somewhat implausible metaphysical presuppositions. Having established that claim, I argue that there is a more plausible account of Zeno's paradox and of instantaneous velocity more generally, anchored in the recognition of an ambiguity in the application of the concept of an instant. In the final section, I attempt to connect the model of a non-Humean instantaneous state developed here with other non-classical theories of time, both contemporary and historical.

Laying out the ultimate conclusions in a bit more detail, I argue that the puzzles about instantaneous velocity, and rates of change more generally, are the result of a failure to recognize an ambiguity in the concept of an *instant*, and therefore of an instantaneous state. We will conclude that there are two distinct conceptions of a temporal instant: (i) instants conceived as fundamentally distinct zero-duration temporal atoms, *the classical model of time* and (ii) instants conceived as the boundary of, or between, temporally extended durations. If we conceive of instants as logically distinct "atoms," such that time is *merely* a set of such instants, then time lacks the the differential structure necessary in order to define an instantaneous rate. However, if we conceive of instants as gaining their identity from their position in continuously ordered time, then while instantaneous velocity can be defined, we have no grounds for believing that the states of a system at distinct times are logically independent of each other. To see how this works, we need a closer look at Zeno's Arrow Paradox and the standard responses to it.

2 Arntzenius's version of Zeno's argument against change

Zeno's arrow paradox runs basically as follows. Consider an arrow fired at a target. Assume that time consists of a collection of durationless instants. At every instant the arrow occupies some place; it possesses a particular location. But, then the arrow is not moving *at* that instant, since movement requires that one be at one location and then at another location. However, since the arrow is not moving *at* any time, the arrow is not moving and never gets to the target.

The standard response to this argument invokes the so-called 'at-at' theory of motion. On such a theory of motion speed, rate of change of position, is not a distinct element of the instantaneous state of the arrow. Rather, it is merely a defined or constructed quantity that encodes facts about the path of the arrow at other times. As Russell puts it in the classic modern statement of the at-at theory, in §447 of *The Principles of Mathematics*,

> It is to be observed that in consequence of the denial of the infinitesimal and in consequence of the allied purely technical view of the derivative of a function we must entirely reject the notion of a state of motion. Motion consists merely in the occupation of different places at different times subject to continuity as

> explained in Part V. There is no transition from place to place, no consecutive moment or consecutive position, no such thing as velocity except in the sense of a real number which is the limit of a certain set of quotients. The rejection of velocity and acceleration as physical facts (*i.e.* as properties belonging *at each instant* to a moving point, and not merely real numbers expressing limits of certain ratios) involves as we shall see some difficulties in the statement of the laws of motion but the reform introduced by Weierstrass in the infinitesimal calculus has rendered this rejection imperative. (Russell, 1938)

Thus, on the "at-at" theory of motion, the ball moves by occupying different locations at different times and the rate of change is determined by the relations between those various locations-at-a-time. On this theory objects move, but they do so without possessing an instantaneous velocity.

However, as Frank Arntzenius points out this seems to have extremely troubling consequences for our interpretation of classical mechanics. First, it seems to create a severe problem for our ability to formulate any concept of determinism in classical physics. If the instantaneous velocity of a classical particle is not part of its instantaneous state, then nothing about that state serves to determine much of anything about it's future path. More precisely, the motion of a classical particle depends on two kinds of facts about the particle, kinematic and dynamic.[3] Clearly, the future motion of a particle depends on the current forces to which the particle is subject. However, it also depends on the previous motion of the particle, normally encoded in the instantaneous velocity. If instantaneous velocity is not an element of the instantaneous state of the particle, then that state, by itself, cannot determine the particle's future motion. Secondly, as Arntzenius also points out, classical physics does treat instantaneous velocity as a component of the instantaneous state.[4] If instantaneous velocity is *not* an element of the state, we seem to be required to claim that two particles occupying the same spatial location, but moving in different directions, have the same state.

The most straightforward way to visualize this problem is to consider a modified version of the arrow paradox. Instead of a single archer, consider the situation with two archers each firing at a target standing next to the other. For simplicity, consider two baseballs instead of two arrows, since the balls have an axis of symmetry in the direction of motion. Consider the precise instant when the two balls pass each other in flight. At that instant, the at-at theory tells us that, absent considerations of other times, there is nothing to distinguish the states of motions of the two balls. Therefore, nothing which tells us which ball is moving in which direction. More picturesquely, if we were to examine a snapshot of the two balls in flight, nothing in the content of the snapshot would tell us where either ball would be next.[5]

What then might the options be, other than merely "biting the bullet?" Frank Arntzenius has discussed three basic options. The first two assume that the classically defined instantaneous velocity, standardly defined as the time-derivative of the position at each time, picks out an element of the instantaneous state, while the third, the "no-instants" view is more radical and will be discussed below. Before continuing, here is a bit of terminological clarification. In what follows, "instantaneous velocity" always refers to the standardly defined *quantity* given by the time-derivative of position at each time. The interpretative question then becomes whether that quantity "picks out" an element of the state at each time, and if it does so what that element of the state *is*.

The first option is what Arntzenius calls the "at-at, plus neighborhood properties" interpretation of the instantaneous state. On this interpretation, one simply accepts that instantaneous velocity is another element of the kinematic state, on a par with position. At first glance this might seem to solve the problems above. Unfortunately, the instantaneous velocity, so defined, does not seem to be properly instantaneous, in two senses. First, the relevant limits are defined only over neighborhoods surrounding the instant, not at the instants separately. This seems, minimally, at least in tension with the reasons for adopting the "at-at" theory in the first place. Roughly, the basic principle here is the Humean principle that the state at *each* time is logically independent from the state at *all* other times. Again following Arntzenius, while the *value* of the velocity does not imply anything about the value of the position or velocity at a particular other time, it does imply that the set of states around the time at issue has a particular structure. Second, it is not clear that the limit or neighborhood velocity actually solves the determinism problem above. The limit property is really a disguised relation between the current state and earlier, or earlier and later, states of the particle. However, we normally look for future states to be determined by the intrinsic state. Or to follow Lange, on the standard understanding of relations, they can only be causally efficacious if at least some of the relata are independently causally efficacious. However, on the limit account of velocity, the relation plays a causal role that it does not inherit from any of the relata(cf. Lange, 2005, 439).

Given these issues, we can consider the second class of options, what Arntzenius calls impetus theories. These include both the original late Medieval impetus theories and any modern theory that postulates a distinct property that causes a system to follow a particular path, but is not "logically supervenient on facts about position over time."(Tooley, 1988, 225) The fundamental problem with such theories is that whatever the impetus *is*, it does not seem to be a velocity nor is it properly connected to velocity in general. There are two ways to explicate the problem with the impetus theories. First, if the impetus is merely causally connected to the path, then it must be *possible* for two systems to follow exactly the same path and to possess different states of motion, different values of impetus, at every moment along that path. But, that seems, if not contradictory, at least bizarre in two distinct ways. First, the whole point of postulating impetus is to explain the role of instantaneous velocity in generating and/or explaining the path of classical systems. It doesn't seem able to do that if we can have two systems with identical paths, and therefore identical instantaneous velocity, but with different values of impetus. Second, consider one of the possible systems absent the natural law connecting impetus and velocity. Such a system would remain otherwise subject to classical mechanics and thus the instantaneous velocity would continue to figure critically in the prediction or explanation of the path of the system. What element of the state does it "pick out" in that world, since it is not impetus? Both of these issues point to the fact that whatever it is that instantaneous velocity picks out, it must be logically supervenient on positions over time. That is, it must be a velocity.

Here's another reason to think that impetus theories do not establish the correct relationship between the state of motion and the path. Given Galilean invariance, we know that under a variety of transformations of the path of system, the physics of the system remains unchanged. Part of the reason for that is that transformations of the path *impose* transformations of the velocities, including the limit or instantaneous velocities at each time. On the one hand, since impetus is not logically supervenient on the path, we have no grounds to expect it to transform properly. On the other, even if it does, by fiat, transform properly it seems to once again turn into a metaphysical dangler.[6]

The final alternative, the 'no-instants' view, treats time as composed of a collection of finite duration non-atomic regions connected *via* an appropriate Borel algebra.Arntzenius's no instants view seems to be a more formally developed version of the position Michael Dummett has labeled 'fuzzy realism.'(cf. Dummett, 2000, 505 ff.) Arntzenius points out two basic problems with such a "no-instants" view. First, it seems to leave our original worries with determinism (Arntzenius) or causal role(Lange) untouched. It still will be the case that the state of a system at a time fails to determine its future trajectory; if only because the system does not have an instantaneous state. Secondly, our actual ability to do mechanics seems to be entirely parasitic on the more traditional formulations. One formulates and solves problems in the more usual point-set and point function formalism, and then uses the homomorphisms between the point algebra and the Borel algebra to translate into the "no-instants" view. To put it slightly differently, this version of a no instants view does not so much offer an interpretation of classical physics as an alternative, and a not very attractive one.

In sections 4 and 5 below, I will propose another model on which time is also not composed of instants. However, the version to be defended below more closely resembles Michael Dummett's constructive model, than his fuzzy realist model. Before introducing that model, we need to examine one more attempt to salvage the classical model.

3 Instantaneous velocity as a pure disposition

In "How Can Instantaneous Velocity Fulfill Its Causal Role?"(2005) Marc Lange proposes that we interpret the instantaneous velocity in classical physics as a disposition to follow a particular future trajectory, with that path specified by the time derivative from above of its trajectory at each point. For clarity, let us call this property the "Lange velocity." In terms of Arntzenius's classification above, Lange's proposal is a sophisticated impetus theory of motion; at each time, the Lange velocity is merely causally supervenient on the path up to and including that point. However, it avoids both of the criticisms of impetus theories discussed above.

First, Lange's instantaneous velocity is a purely positional property of the system. We do not need any knowledge about the dynamics of the system beyond its trajectory to determine its velocity. That is, although merely causally supervenient on the prior path, it is logically supervenient on the path as whole, in particular on the path *later* than any time at which we specify the Lange velocity. Second, as long as the trajectory is smooth, the Lange velocity, the derivative from above, will match the ordinary velocity as given by the prior trajectory. Thus, while we require a natural law to link Lange velocity to ordinary velocity, it is one that we already possess. This follows from the fact that classical physics implies that all trajectories are continuous and smooth.(cf. Lange, 2005, § 2)

However, Lange's purely subjunctive definition of instantaneous velocity remains problematic in at least two ways. First, these subjunctive properties are not only never realized in the actual world, it is nomologically impossible that they could be actualized in any world governed by classical physics and containing at least two massive particles. Second, it seems odd that while I can predict the present instantaneous velocity of a particle, I cannot *measure* it.

Let's consider these in reverse order. Given Lange's definition, one can at best calculate or predict the present Lange velocity. Since the present Lange velocity is merely causally dependent on the prior trajectory, information about that trajectory provides merely causal information about the current state. On the one hand, this might seem trivial. All measurements depend on the causal connection between the state of measured system and the "pointer state" of the relevant measuring instrument. Unfortunately, the problem is worse than that. Consider a hypothetical measuring instrument that reports the velocity of a system at time t. The only possible way it could function is to report the results of an interaction with the system beginning at some time before t and concluding, in the ideal case, precisely at t. But then, at best, it is reporting an approximation to the ordinary instantaneous velocity. In this case, we need not only the natural law connecting the measured state to the pointer state; we need an additional natural law connecting the measured state to the state that we intended to measure, the Lange velocity. We are sometimes practically unable to determine aspects of the current state of the system except by inferring it from other causally relevant information; Lange, however, postulates a distinct component of the *present* state which is *in principle* subject only to prediction rather than measurement. But, once again, this normally involves detecting directly causal consequences of the state of interest and inferring back to it. In this case, we inferring the state not from its effect but from one of its causes. Technically the Lange velocity and any velocity measurement are both joint effects of a common cause. But, then it seems reasonable to ask what substantive oomph the Lange velocity adds, other than a kind of metaphysical simplicity–a velocity that does not depend on prior position. If not a dangler, the Lange velocity seems at least somewhat *ad hoc*. An impression that the next concern merely reinforces.

It is a well known problem for operationalizing classical physics that, for all possible worlds governed by classical physics and containing anything more than a single point particle, Newtonian universal gravitation implies that no object follows an inertial trajectory.[7] Therefore, the Lange velocity is an unrealized disposition in particularly strong sense. Not only is it not realized in the actual world, it seems to be impossible to actualize. At first glance, this may not seem like a particularly serious problem; there is an extensive literature on vacuous natural laws.[8] However, things are not quite that simple.

Lange velocity is vacuous in a particularly strong sense, *nomologically vacuous.* Consider one of the standard examples of a vacuous law–a possible world containing less than a critical mass of U^{235}. It seems plausible to claim that the U^{235} nuclei that do exist in such a possible world possess the same disposition to participate in critical nuclear reactions as do their counter-parts in this world. That plausibility, however, seems to flow from an intuition that the absence of sufficient U^{235} in that world is fundamentally an accident. What if there were a law of nature in the world *prohibiting* the formation of additional U^{235}. Does it still seem plausible to claim that U^{235} in that world can participate in self-sustaining nuclear reactions? One's first intuition should probably be no; it seems odd that anything could be physically disposed to do something whose occurrence is prohibited by physical law.

However, there are other options. Perhaps, U^{235} in the actual world, simply doesn't possess counterparts in such a world. Perhaps this is a world where laws of nature and dispositions "come apart" in some strange way. Perhaps, ... The right answer seems to be that we simply don't have even halfway decent intuitions about these kinds of situations.

Unfortunately, Lange velocity seems to be a similar disposition. The realization of the path that Lange velocity disposes objects to follow is prohibited in worlds where classical physics applies by the existence of gravity in those worlds. The exception might be single point particle worlds. However, such worlds are sufficiently far from the actual world that it is not clear whether we have any reason to expect that single particle to follow a smooth path or any path in that world. It is far from clear that either instantaneous velocity or Lange velocity have any application in such a world.

None of these issues are immediately fatal for Lange's proposal. What they do seem to show is that Lange velocity is more problematic than it might initially seem to be. And, it seems to me, all of them point to something odd going on with the concept of an instantaneous rate, or even with the concept of an instantaneous state more generally. That is, it seems odd that one needs to twist oneself in metaphysical pretzels to deal with such a physically straightforward concept as instantaneous velocity. In the next section, we consider whether the underlying problem with instantaneous rates rests not fundamentally with the concept of a rate, but with the concept of an instant. In particular, I will argue that for systems with well-defined instantaneous rates of change in fundamental quantities, the instants, and thus the instantaneous states, are *not* fundamentally distinct.

4 Instantaneous Velocity and the Humean Presupposition

Given that both no-instant and impetus views have their own difficulties, perhaps we should take another look at the standard view? On this view, the instantaneous velocity, the derivative of position with respect to time, is a component of the state of the system. As was discussed above, the principle problem with this claim is that the velocity at a time is not logically independent of the state at other times. In what follows, I argue that at least some of the reasons for including instantaneous velocity in the state of the system, also provide reasons for denying that the states at distinct times are logically distinct.

David Hume famously argued that the states of the world at distinct times are metaphysically and logically independent of each other. At most these distinct states can be the causal product of earlier states. Hume, equally famously, reduced these causal relations to spatiotemporal contiguity and resemblance. However, the fundamental underlying presumption is precisely that the states of the universe, or of any other system, at different times are distinct from each other. We will call this the *Humean* presumption and the instants at which a system possesses each of these distinct states, *Humean* instants.

As an analogy, we can think of the Humean presumption as the belief that we can capture the sequential states of a system in a sequence of photographs of the system, or perhaps, a film strip.[9] We then imagine the "frame-rate" of our camera increasing without limit. The higher the frame rate the more accurately our movie approximates the actual continuous sequence of distinct states. However, even the actual instantaneous state of the system shares with the approximations their fundamentally distinct, although of course not discrete, nature.

However, this is precisely the problem; characterizing distinct states in a way that does not imply discreteness is quite difficult.[10] We simply cannot get from a discrete set of distinct states to a truly continuous set of states having a differentiable structure through a sequence of interpolations or approximations. As a matter of mathematics, no matter how many elements we add to a discrete set it remains discrete. Let us assume that we can provide such a characterization of distinct but not discrete sets. Intuitively, the basic idea seems to be that for any sequence of times, and states at those times, we can "shuffle" the order of those times and get a logically, although not causally, possible sequence of states. Notice two features of this situation. First, this picture of fundamentally distinct states leads naturally to the "at-at" theory of motion. Being in motion is a relation between the various states, the various distinct pictures of the system. Intuitively, if the state at a time consists only of position, then, as long as we do not "worry about" how or why the system changed locations, any sequence of positions is permitted.

However, it is also the case that if this set theoretic structure is all the structure we have then *instantaneous rates of change are not even definable.* Consider the set of instants in the previous paragraph. The arbitrary "shuffles" are permitted only because the set does not have an order relation or even a topology that such transformations must respect. But, absent such a structure the derivative of functions over that set are simply not well-defined.

Derivatives are defined for *spaces* not merely for sets. That is, the instantaneous velocity of a system moving in one-dimension is

$$\lim_{\delta t \to 0} \frac{x(t + \delta t) - x(t)}{\delta t}$$

But, in determining the value of this limit, or even whether it exists, one must know not merely the value of t; one must also know the set of neighborhoods around t. When one carries out an arbitrary re-ordering, a "shuffle," we have no guarantee that the limits will have the same values, or even will exist. If time is *merely* a *set* of instants, we simply don't have enough structure to define or work with concepts of instantaneous rates of change. That is, if one insists that the evolution of a system is given by a sequence of states at a sequence of Humean instants, then instantaneous rate of change is not defined and one can apply the "at-at" theory of motion.

What this amounts to is that the role of instantaneous velocity in classical physics is deeply connected to the fact that classical physics represents time as isomorphic to the *real line,* not merely the set of real numbers. In fact, it seems misleading to think of the real numbers themselves as a set, even if we grant that there is a set of all real numbers. The reals are the totally-ordered, Dedekind complete field, and to claim that time is represented by the reals is to claim that it can be parametrized by such a field. But, this claim is much stronger than that time is a set of instants, even a set of instants with cardinality equivalent to that of the set of all real numbers. Even ignoring constraints on the structure of time imposed by the field operations or the total-ordering, Dedekind completeness imposes close connections between "distinct" elements of the field. Dedekind completeness, or the least upper bound property, requires that every subset of the reals with an upper bound has a least upper bound. Or, equivalently, that every Cauchy sequence of the reals converges to a limit in the reals.

Given this definition, it seems misleading to think of the reals as a simple set of distinct mathematical "entities" sitting out there in some Platonic realm waiting to be arranged according some additional structure. We can see this in, at least, two ways. First, consider a non-algebraic real number, e.g. π. π *is* the least upper bound of certain sets of rational numbers. We can of course *define* a real number as the least upper bound of a set of rational numbers, many of which will be rational as well. In this sense, it seems that each real number "carries" along the whole structure of $\mathbb{R}$. Alternatively, when claiming that π is a real number, one is not merely stating that π is an element of a particular set, one is claiming as well that the results of various operations involving π and *all* of the other elements of the set are well-defined and are also elements of that set and claiming that the set of subsets of that set has a certain structure, that there is at least one such subset of which π is the least upper bound. The various elements of $\mathbb{R}$ simply are not logically independent of each other.

What then does this tell us about time and instantaneous velocity? First, that there is something philosophically odd about the complaint that velocity violates the Humean presupposition. The Humean presupposition of logical distinctness is part of a package which includes spatio-temporal discreteness. And, as part of that package it makes perfectly good sense, since the instantaneous rate of change of position is simply not well defined when space and time are discrete.[11] Second, it points us towards an interpretation of the structure of time that takes temporal intervals as basic, without denying that there is a set of instants. On this interpretation, each instant is the boundary of the appropriate temporal interval(s). In this sense, it has the advantage that it keeps the problem of interpreting instantaneous velocity as a problem of interpretation, not as a problem in physics [Smith]. A classical point particle following a smooth trajectory has an instantaneous velocity, just as classical physics specifies, at every instant. The problem only arises when one then insists that the classical state obey the Humean presupposition, a presupposition that one has no *physical* reason to expect the system to obey.

Finally, it allows us to provide the clearest diagnosis of the problem with which we began–Zeno's arrow. Zeno's arrow paradox arises, on this interpretation, from an ambiguity in the specification of the time through which the arrow moves–either as a mere set of instants with the cardinality of the reals or as having the full structure of the reals. On the first reading, the set of instants are each logically independent of the others, and thus the specification of the distinct places that the arrow occupies at each of those instants are also distinct. Here one begins with the "at-at" theory and rigorously holds to it all the way through the description. On this reading, it's basically an accident that the path happens to be smooth and that the instantaneous velocity happens to be a handy logical construction; the sequence of states of the arrow remains merely the sequence of positions that it occupies. It is, on this account, literally false to include the instantaneous velocity in the state at each time. As the defenders of the "at-at" theory have argued, this provides a perfectly adequate account of the Arrow Paradox; one denies the claim that one must have an instantaneous state of motion in order to be moving. Unfortunately, it does not seem to provide an adequate interpretation of the classical physics of motion, where instantaneous velocity is part of the instantaneous state.

However, now consider the classical physics account of the arrow, using instantaneous velocity. The state at each time consists both of the present location and of the way in which the object is presently *pointed at* certain future locations. At first glance, this seems odd; shouldn't the present state be independent of what the object is going to do later? However, the oddity here is not with instantaneous velocity in particular. Rather the fact that instantaneous velocity can have such a significant causal role in physics, results from the fact that the present instant is, in its essential nature, pointed at the future. That is, physical time, as parametrized by the real numbers, is essentially non-Humean and that non-Humean nature reveals itself particularly clearly in the fact that instantaneous rates of change play an unavoidable role in physical explanations. Here, one resolves the arrow paradox by denying that at each time the object is merely *at* a particular place. Instead it is at one place while also being "pointed at" other places.

5 Conclusion: Time and Times

If the above considerations are correct, than it might seem that we really haven't made any progress at all. We are still left with a choice between three accounts of motion:

1. the standard account of instantaneous velocity as limit, interpreted against a backdrop of non-Humean time
2. Lange's sophisticated version of an impetus theory
3. a "no-instants" [Arntzenius] or "fuzzy realist" [Dummett] theory of time denying the existence of instantaneous states altogether.

However, there are three classes of reasons to prefer the first option. First, only the non-Humean option seems to get the order of explanation between physics and metaphysics right. Both the second and third options are attempts to salvage a metaphysical intuition at the expense of a plain reading of the physics. One might expect puzzling metaphysics out of puzzling physics, but instantaneous velocity seems to be perfectly well-understood by high school students in the first week of basic physics. An interpretation that locates the problem in the proverbially troubling metaphysics of time rather than in the physically straightforward application to basic kinematics seems to have a key advantage.

Second, there are various other pressures on the "classical" theory of time as composed of a continuum of instants. The first is psychological, or perhaps epistemological. It seems clear that our "time-sense" is constructed out of the reports of various cognitive and sensory systems regarding the contents of intervals of various lengths. Modern work on the sensory psychology shows that this is more than merely the truism that the specious, or psychological, present is temporally extended. Rather the brain/mind somehow synchronizes the reports of various peripheral systems operating on very different time-scales into a single "consciousness." It seems at least plausible that the commitment to the possibility of arbitrarily close approximations to a perfect synchrony serves as an essential regulatory principle in organizing the contents of these various neural systems. Just as above, then, the instants are ideal abstractions from our awareness of temporal intervals. Among other puzzles, this allows us to make sense of how we get the concept of an pure instant from the temporally distributed behavior of brains.[12]

Second, both Michael Dummett and Jeremy Butterfield have brought pressure to bear on the Humean conception of instants or instantaneous states from directions related to but distinct from those directly related to velocity and Zeno's paradox. Michael Dummett has argued against the classical model of time on the grounds that it distinguishes certain possibilities that we have no ground to distinguish, related both to "jump discontinuities" and "removable discontinuities" in physical quantities. Dummett suggests that we replace the "classical model" of time as a composed of a classical continuum of instants with a "constructive model" on analogy with the intuitionistic theory of the continuum. In this sense, Dummett's conclusions resemble those defended here. Unfortunately, they are unnecessarily burdened by his verificationist anti-realism. Dummett argues that we should replace the classical continuum with a constructive one according to which the instants are merely "notional" constituents of the continuum. Now there may or may not be good reasons for thinking that the classical mathematical theory of the continuum needs to be replaced by constructive mathematics; however, the arguments of the previous section show that even on a classical theory of the continuum there is something "off" in conceiving of the temporal continuum as "composed of its points, the real numbers, each of which exists independently of the others: each represents a determinate position on the rational line, a point on that line, if it is rational, a dimensionless gap in that line, if it is irrational." Picturesquely, even on a classical conception, the set of instants conceived of as isomorphic to the real line is not merely a *set* of locations, it is a set of *locations*.

Jeremy Butterfield's campaign against "pointillisme" takes a somewhat different tack. Although I find Butterfield's *positive* claims difficult to identify, he seems to be claiming that violations of pointillisme arise in classical mechanics at the level of state descriptions, especially in the use of velocity and other vectorial quantities. He seems to be allowing that the state at a point of space, time or space-time is entirely *at* that point, even if it has implications about other points. Butterfield seems to be accepting a "pointillistic" theory of space and/or time, and arguing that the states at those various points violate pointillisme. I have argued that the states at the points violate pointillisme *because* there is no pointillistic theory of points.

Let me conclude with a two points not directly related to contemporary worries about physics or the calculus. The contemporary tendency is to help ourselves to "times" or "instants" understood as parts of time is quite contemporary. The best metaphysicians of the past of long been worried about a time or an instant might be, especially relative to TIME more generally. Here, briefly, are two important examples, largely selected because they otherwise have so little in common. Aristotle, pretty clearly, sees that the continuity of time implies that the instants, his *nows*, cannot be conceived as parts of time, out of which it is constructed. Rather, they are the boundaries of temporal intervals. The present is essentially the boundary of the past, not merely a temporal location that I happen to occupy and that happens to be later than other such temporal locations. Consider, for example,

> Hence time is not number in the sense in which there is 'number' of the same point because it is beginning and end, but rather as the extremities of a line form a number, and not as the parts of the line do so, . . . because obviously the 'now' is no part of time nor the section any part of the movement, any more than the points are parts of the line-for it is two lines that are parts of one line. [220a15][13]

In a, somewhat, similar vein, Immanuel Kant claims, *The Critique of Pure Reason*: in *The Critique of Pure Reason*,

> The infinitude of time signifies nothing more than that every determinate magnitude of time is possible only through limitations of one single time that

> underlies it. The original representation, *time*, must therefore be given as unlimited.[A32/B47-48].[14]

All of these considerations serve to support the belief that, absent a prior grasp of TIME, whether we think of that as obtained via a pure Kantian intuition or some other way, we have an extremely difficult time making good sense of temporal instants and of the physics of instantaneous states and rates of change. However, once we abandon the Humean conception of instants, and the accompanying distinctness of instantaneous states, the physics of instantaneous velocity takes care of itself.

Notes

[1]The contemporary debate regarding the status of instantaneous velocity has several tributaries. Particularly relevant to the role of Zeno's arrow, see Arntzenius (2000) and Arntzenius's exchange with Sheldon Smith(Arntzenius, 2003; Smith, 2003a,b).For related discussions, not directly focused on Zeno's Arrow, see Carroll (2002); Jackson and Pargetter (1988); Meyer (2003); Sherry (1986); Tooley (1988).

[2]Dummett's original paper is Dummett (2000). Also see his exchange with Ulrich Meyer(Dummett, 2005; Meyer, 2005). For Butterfield see Butterfield (2006a,b)

[3]Given that we are restricting attention to classical physics in this essay, I will normally drop the 'classical' modifier.

[4]This seems particularly worth pointing out as Sheldon Smith(2003a; 2003b) seems convinced that Arntzenius objects to the use of classical mechanics. Instead, it seems clear that Arntzenius is raising issues with *the interpretation* of classical mechanics.

[5]Of course, in a real snapshot, it's likely that "blur" would allow us to distinguish the direction of motion of the balls. However, that is entirely an artifact of the non-instantaneous nature of actual photographs.

[6]Michael Tooley, at least, recognizes this problem with his theory of "states of motion." Unfortunately, he seems to believe that the problem with absolute velocity only arises in special relativity. It is, of course, well known that Galilean invariant theories, such as Newtonian mechanics, have no more basis for absolute velocity than Lorentz invariant ones. (cf. Tooley, 1988, §6.2)

[7]For a detailed discussion see (Torretti, 1996, especially Chapter 1)

[8]The classic discussion is in Armstrong (1983), especially Chapter 8. See also the papers in Carroll (2004), especially Chapters 10 and 11.

[9]We can postulate a "metaphysical" lens on our camera if you like, for example one that can take pictures of dispositions as well as occurrent properties.

[10]Note that Hume himself solves this problem by arguing that instants are both distinct and discrete.While Hume, obviously, doesn't make use of the modern terminology, this seems pretty clearly the "upshot" of Part 2 of his *Treatise*. On the overall unity of his system see especially §§2.4 and 2.5.

[11]Once again note, that Hume himself was perfectly clear on this. You can only have the whole package, not bits and pieces.

[12]For a discussion of this problem from a philosophical perspective, sometimes called the problem of temporal integration, see Callender (2008) and references therein.

[13]For more detailed historical discussions of Aristotle's response to the Arrow and his accompanying denial of the compositional conception of instants see, for example, Lear (1981); Vlastos (1966).

[14]In this context, one might also note the close connection between the pure intuition of time and the interpretation of calculus in Michael Friedman's interpretation of Kant.(Friedman, 1992)

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