Symplectic reduction and the problem of time in nonrelativistic mechanics

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Abstract

The deep connection between the interpretation of theories invariant under local symmetry transformations (i.e. gauge theories) and the philosophy of space and time can be illustrated nonrelativistically via the investigation of reparameterisation invariant reformulations of Newtonian mechanics, such as Jacobi's theory. Like general relativity, the canonical formulation of such theories feature Hamiltonian constraints; and like general relativity, the interpretation of these constraints along conventional Dirac lines is highly problematic in that it leads to a nonrelativistic variant of the infamous problem of time. I argue that, nonrelativistically at least, the source of the problem can be found precisely within the symplectic reduction that goes along with strict adherence to the Dirac view. Avoiding reduction, two viable alternative strategies for dealing with Hamiltonian constraints are available. Each is found to lead us to a novel and interesting re-conception of time and change within nonrelativistic mechanics. Both these strategies and the failure of reduction have important implications for the debate concerning the relational or absolute status of time within physical theory.

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1 Introduction

Certain physical systems are such that the mathematical representation describing them is degenerate – they contain what Redhead ([2003]) labels 'surplus structure'. By this we mean that the relevant equations of motion (together with the same initial data) produce multiple physically indistinguishable but mathematically distinct solutions. We can represent these formal redundancies in terms of groups of symmetry transformations on a space of possible configurations of the system and if these groups act locally we call them gauge symmetries. In order to determine a unique mathematical representation for a system displaying gauge symmetries the theory of constrained Hamiltonian mechanics was developed (Dirac [1958a],[1964]; Henneaux and Teitelboim [1992]). This theory allows us to characterise the degeneracy precisely in terms of (first class) constraint functions on phase space and regulate it by providing equations that pick out dynamics that is independent of the action of the constraints. Geometrically Dirac's procedure for eliminating degeneracy within gauge theories can be understood in terms of a process of reduction from a constraint manifold within phase space with a presymplectic geometry to a reduced phase space with a symplectic geometry (Gotay and Nester [1978]). Interpretationally, this reduction process can be endowed with the significant role of providing a passage from a dynamical arena with excess representational structure to one which provides a direct representation of the true dynamical degrees of freedom (on this point see Belot [2007] and Butterfield [2007]).

The general theory of relativity explicitly features local redundancy in the form of diffeomorphism invariance and can be cast into a constrained Hamiltonian formalism known as canonical general relativity (Dirac [1958b]; Bergmann [1961]; Arnowitt et al. [1962]; Misner et al. [1970]). Perplexingly, however, once rendered into constrained Hamiltonian form the degeneracy of the theory seems to become entangled with the dynamics since the canonical Hamiltonian is itself a first class constraint. Thus, according to Dirac's work, even though it effects the the transformations between three dimensional hypersurfaces that play the role of time in the theory, the Hamiltonian constraint should be interpreted as a gauge generator. Furthermore, if we accept that the Hamiltonian is gauge generating then it would seem that we must classify as representing observables only functions which weakly commute with it.[1] This class of observables cannot vary along entire histories of a system and are therefore unable to change with respect to any temporal variable which parameterises the history. Correspondingly, from a geometric perspective application of symplectic reduction techniques to canonical general relativity is understood as leading to a reduced phase space which, despite having a symplectic structure, can no longer be understood as representing temporal evolution of either states or observables (see Belot and Earman [2001]; Rickles [2008]; Belot [2007]). This is the essence of the the problem of time in classical gravity – it is intimately connected to various issues that beset attempts to formulate a quantum theory of gravity and are grouped together as the problem of time in quantum gravity (Kuchař [1988]; Isham [1992]; Anderson [2010]).

The chain of argument leading to the classical problem of time is controversial. In particular, Kuchař ([1992]) and Barbour ([1994]) have argued that there are characteristics peculiar to the Hamiltonian constraint which mean we should not follow the standard procedure and treat it as gauge generating. In a similar fashion, Pons and Salisbury ([2005]) argue that Dirac's analysis is incomplete (Pons [2005]) since gauge symmetry groups should be more properly thought of as acting on the space of entire solutions rather than, as Dirac assumes, at a given time. Thus, under their analysis it is simply erroneous to identify the Hamiltonian (which acts on initial data points in order to create solutions) as a gauge generator. Also in this anti-Dirac spirit, Barbour and Foster ([2009]) have explicitly considered the case of Jacobi's theory which provides a useful model for general relativity since its reparameterisation invariant action and vanishing Hamiltonian make it *timeless* in a fundamental sense. Contrary to Dirac's work they conclude that the Hamiltonian can be taken to generate genuine physical change and that observables that do not weakly commute with the Hamiltonian can be defined consistently.

[1]A function is said to weekly commute with a constraint when the Poisson bracket between the function and the constraint is zero when calculated on the sub-manifold within the phase space defined by satisfaction of the constraint.

A principle purpose of this paper is to examine these significant claims on a technical and interpretive level within the context of the geometric presentation of the problem of time in nonrelativistic mechanics. Within §2, we first provide a concise introduction to the relevant ideas from geometrical mechanics before presenting Dirac's argument for the classification of first class constraints as gauge generating in terms of a simplified version of the symplectic reduction procedure. In §4 we will then consider the potential application of this symplectic reduction to a class of nonrelativistic reparameterisation invariant theories (such as Jacobi's theory) within which the Hamiltonian is the only constraint. It will be argued—contra received orthodoxy—that the application of this geometric version of Dirac's work is inappropriate for this case and therefore not generally applicable. This leaves open the question of how we should define both change and observables within nonrelativistic reparameterisation invariant mechanics. Utilising the symplectic formalism that has been introduced we will then, in §5, evaluate two rival positions that offer new methodologies for defining both change and observables – these will be designated 'the emergent time strategy' and 'the correlation strategy' respectively.[2]

A further purpose is to illustrate a number of novel and important interpretative consequence which can be derived from our negative result regarding symplectic reduction together with the geometric structure of our two non-reductive schemes. In §3 we will examine the connection between gauge theory, possibility space reduction, Haecceitism, and the ontological indeterminism issue that might be seen to undermine certain interpretations. Building upon this discussion, and the results of §4-5, in §6 we will first consider the positions of relationalism and substantivalism (and their variants) as they have been discussed in the context of space and space-time. We will then introduce the corresponding notions of temporal relationalism and temporal substantivalism. The case of time in nonrelativistic reparameterisation invariant mechanics will then be demonstrated to force upon us a number of revisions to the supposedly canonical, existing analysis of gauge theory and space-time ontology. In particular: i) Within the Hamiltonian formulation of Jacobi's theory the ontological indeterminism issue is found not to threaten the Haecceitistic variant of temporal substantivalism; ii) The framework for connecting the treatment of constraints to the relationalism/substantivalism distinction will be found wanting in that the first of our timeless strategies would, under its terms, be mis-classified as temporally substantivalist; and iii) The second of our timeless strategies will be argued to lead to a position which cannot properly be understood in terms of relationalism or substantivalism with regard to time but rather is found to be timeless in a fundamentally *Parmenidian* sense.

[2]The first originates with Kuchař ([1992]), Barbour ([1994]) and Foster (Barbour and Foster [2009]). The second with Rovelli ([1990], [1991], [2002],[2004]) who was followed by Dittrich ([2006], [2008]) and Thiemann ([2007]).

2 Mechanics with a fixed parameterisation

2.1 Lagrangian mechanics

We start with the specification of the set of n independent variables, q_i where $i = 1...n$, which serve to characterise the properties of a mechanical system. These variables are elements of a manifold[3] which we call the **configuration space**, $\mathcal{C}_0$.[4] At a given point $q \in \mathcal{C}_0$ we can define a **tangent space** $T_q\mathcal{C}_0$. The disjoint union of all the tangent spaces of $\mathcal{C}_0$ is called the **tangent bundle** $T\mathcal{C}_0$. The elements of the tangent bundle are pairs $(q, \dot{q})$ of configuration variables q and vectors tangent to those variables $\dot{q}$. For formulations of mechanics with a fixed parameterisation the parameter with which the tangent vectors are defined is unique and may be interpreted as time t (this will prove not to be the case for the theories of mechanics considered in §4). Thus we have $(q, \dot{q}) \in T\mathcal{C}_0$ with $\dot{q} = \frac{\partial q}{\partial t}$.

A curve within the tangent bundle, $\gamma_0 : \mathbb{R} \to T\mathcal{C}_0$, will correspond to a history of a system – a sequence of configurations and velocities. The parameterisation of the curve will be fixed up to a choice of origin and unit by the distinguished time parameter t. This parameter can be taken to vary monotonically along each curve in configuration space. Clearly, for this picture to match up with the physics of the real world we need some restriction on which histories are nomologically possible. This is achieved by defining the **Lagrangian**, $L_0 : T\mathcal{C}_0 \to \mathbb{R}$, and the **action**, $I[\gamma_0] = \int_{\gamma_0} L_0[q_i, \dot{q}_i]dt = \int_{\gamma_0}(T - V)dt$, where T and V are kinetic energy and potential energy respectively. The extremisation of the action, $\delta I[\gamma_0] = 0$, according to the principle of least action leads to the **Euler-Lagrange equations**, $\frac{d}{dt}\left(\frac{\partial L_0}{\partial \dot{q}_i}\right) = \frac{\partial L_0}{\partial q_i}$, that specify a set of parameterised solutions, $\{\gamma_{PS}\} \subset \{\gamma_0\}$, which uniquely determine the physically possible histories of the system given an initial point in $T\mathcal{C}_0$.

2.2 Hamiltonian mechanics

An alternative formulation of mechanics in terms of first order equations is achieved by moving to the **cotangent bundle** of our configuration manifold, the phase space $\Gamma_0 = T^*\mathcal{C}_0$. This is the disjoint union of all the **cotangent spaces** $T^*_q\mathcal{C}_0$. A point in phase space, (q, p), consists of a point in our original configuration space, $q \in \mathcal{C}_0$, paired with a covector at q, $p \in T^*_q\mathcal{C}_0$. These covectors, which we call the conjugate momenta, are given by the Legendre transformation, $\mathcal{FL} : T\mathcal{C}_0 \to T^*\mathcal{C}_0$, which is the map between the configuration-velocity space and the phase space. It can be explicitly constructed using the definition of the **canonical momenta**, $p_i = \frac{\partial L}{\partial \dot{q}_i}$. To fix the dynamics we introduce the Hamiltonian functional, $H_0[q_i, p_i] = p^i q_i - L = T + V$, and derive Hamilton's equations, $\dot{p}_i = -\frac{\partial H_0}{\partial q_i}$ and $\dot{q}_i = \frac{\partial H_0}{\partial p_i}$. The relevant parameterised solutions $\bar{\gamma}_{PS}$ describe

[3]Those unfamiliar with the terminology of differential geometry are suggested to refer to (Baez and Muniain [1994]) or (Butterfield [2007]) for a detailed introduction

[4]The subscript 0 is used to distinguish the objects introduced here from those of the extended description of mechanics given in §4.

the system's dynamics uniquely in the phase space Γ_0 and are isomorphic to the solutions γ_{PS} in the configuration-velocity space $T\mathcal{C}_0$.

2.3 Symplectic mechanics

An elegant and powerful characterisation of mechanical systems is provided by the symplectic approach (Abraham and Marsden [1978]; Arnold [1988]; Souriau [1997]). Symplectic is a Greek word first introduced in this context by Herman Weyl ([1939]). It means roughly 'plaited together' or 'woven'. A symplectic approach to mechanics involves the generalised description of the phase space used above in terms of a natural geometric language with the canonical momenta and configuration variables explicitly represented as woven together.

Above we defined a covector as the dual of a tangent vector, similarly we can define a cotangent vector field or **one-form** as the dual of a tangent vector field. We can generalise these objects to define a k-form as a smooth section of the kth exterior power of the cotangent bundle, $\Omega^k(T^*M)$, of a manifold M. Of particular interest are **two-forms** which are functions $\Omega(x) : T_xM \times T_xM \to \mathbb{R}$ that assign to each point $x \in M$ a skew-symmetric bilinear form on the tangent space T_xM to M at x (Marsden and Ratiu [1994]). We can transform a k-form into a $k+1$-form by the action of the **exterior derivative**, $\mathbf{d} : \Omega^k(\Gamma_0) \to \Omega^{k+1}(\Gamma_0)$. It is such that $\mathbf{d}f = df$, $\mathbf{d}(\mathbf{d}\alpha) = 0$ and $\mathbf{d}(f\alpha) = df \wedge \alpha + f\mathbf{d}\alpha$ where α is a k-form and df is the differential of f.

Given a general cotangent bundle, T^*M, we can always define a corresponding **Poincaré one-form**[5], θ, in terms of a sum of products between a covector and the total differential of the vector it is paired with. Thus for our phase space, Γ_0, the Poincaré one form is $\theta = p_i dq^i$. If we then take the exterior derivative we get a two form:

$$\omega_0 = \mathbf{d}\theta = \mathbf{d}(p_i dq^i) = dp_i \wedge dq^i \tag{1}$$

This two form is called a **symplectic two form** and is both closed ($\mathbf{d}\omega_0 = 0$) and non-degenerate (if $\omega_0(X_f, X_g) = 0$ for all $X_f \in TM$ then $X_g = 0$). A manifold endowed with a symplectic two form constitutes a **symplectic geometry** (M, ω_0). Significantly, if we are given a smooth function, f, on a manifold endowed with a symplectic two form then we immediately define uniquely a smooth tangent vector field X_f through the map $f \longmapsto X_f$ given to us by $\omega_0(X_f, \cdot) = \mathbf{d}f$. The uniqueness of the vector field is guaranteed by the non-degeneracy of ω_0.

The relation between symplectic geometry and the Hamiltonian theory of mechanics outlined above can be seen immediately since Hamilton's equations can be written:

$$(\dot{q}_1, ..., \dot{q}_n, \dot{p}_1, ...\dot{p}_n) \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} = \left(\frac{\partial H_0}{\partial q_1}, ..., \frac{\partial H_0}{\partial q_n}, \frac{\partial H_0}{\partial p_1}, ..., \frac{\partial H_0}{\partial p_n} \right) \tag{2}$$

where I is the $n \times n$ identity matrix. This expression is an unknown vector multiplied by a matrix and set equal to known vector. It is equivalent to

$$\omega_0(X_{H_0}, \cdot) = \mathbf{d}H_0 \tag{3}$$

[5]See (von Westenholz [1978], pp. 392-4) for more details.

which is an unknown tangent vector field (the Hamiltonian vector field X_H) contracted with a two form and set equal to the exterior derivative of a the Hamiltonian, H. Thus we can see Hamilton's equations have an immediate connection with symplectic geometry. The dynamics of a system can be totally specified by the triple $(\Gamma_0, \omega_0, H_0)$, where Γ_0 is our phase space manifold (cotangent bundle), ω_0 is the symplectic two form, and H_0 is the Hamiltonian function on Γ_0. Together these three elements fix the value of the Hamiltonian vector field, X_{H_0}. It is the integral curves of this vector field that correspond to the parameterised phase space solutions $\bar{\gamma}_{PS}$ that we associated with the physical histories above.

The Hamiltonian vector field that we have just defined is unique. This implies that it will generate a unique $\mathbb{R}$-action on phase space.[6] This $\mathbb{R}$-action, and the associated flow, are what we conventionally identify as temporal evolution since they take us from a point in phase space (instantaneous state of a physical system) to a second point (state) that is t units along a solution (physical history). Thus, we see that there is a intimate connection between the Hamiltonian and time.

This connection is made even more explicit by the introduction of the **Poisson bracket**, which is a special case of the Lie bracket, that can be defined via the symplectic two form for any pair of functions, $f, g \in C^\infty(\Gamma_0)$, as $\{f, g\} := \omega_0(X_f, X_g)$. The Poisson bracket can be related to the action of a vector field on a smooth function $\{f, g\} = X_g(f) \equiv df(X_g) \equiv \mathcal{L}_{X_g}(f)$. This means that if we take the Poisson bracket of the Hamiltonian with an arbitrary smooth function we will get the change of this function along the flow defined by the Hamiltonian vector field. This is equal to the variation of the function with respect to the flow parameter of X_{H_0} which is, of course, how change with respect to time is represented:

$$\{f, H_0\} = X_{H_0}(f) = \frac{df}{dt} = \dot{f} \tag{4}$$

Conversely, the commutation condition $\{f, H_0\} = 0$ indicates that a function is conserved – it does not change with respect to time.

2.4 Presymplectic geometry and symplectic reduction

A physical system within which a Lie group, G, acts on the tangent bundle, $T\mathcal{C}_0$, such that the Lagrangian, L, is invariant and the group is local (i.e. it can be parameterised in a natural way by a family of arbitrary functions on space-time) is said to display a gauge symmetry. In such systems the assumption that the Legendre transformation is an isomorphism which was implicit in our construction of mechanics above no longer holds. This is because the bijectivity of the map $\mathcal{FL} : T\mathcal{C}_0 \to T^*\mathcal{C}_0$ is dependent on the Lagrangian being such that it determines tangent vectors $\dot{q}$ uniquely through the definition of the canonical momenta. Gauge symmetries $g \in G$ manifestly subvert this since we have that $L(q', \dot{q}') = L(gq, g\dot{q}) = L(q, \dot{q})$ for $\forall g \in G$. In phase space terms the existence of a gauge symmetry group corresponds to the p_i's and q_i's not all being independent

[6] The additive real group, $\mathbb{R}$, allows us to define an **$\mathbb{R}$-action** as $\Phi : \mathbb{R} \times M \to M$. We associate it with a one parameter group of diffeomorphisms from M to M called a **flow** $\{\alpha^t\}$ through the relation $\alpha^t(x) = \Phi(t, x)$ for $x \in M$.

- there exists some functional relationship between them of the form $\varphi(p,q) = 0$. We call such functions **constraints**.[7]

Geometrically we can understand the collection of all the constraints, φ_j where $j = 1, ...m$, as defining an m-dimensional sub-manifold, $\Sigma = \{(p,q) \in \Gamma_0 | \forall_j : \varphi_j(p,q) = 0\}$, within phase space, Γ_0, that we call the constraint surface. The phase space itself will, as in the unconstrained case, have a symplectic geometry characterised by the pair (Γ_0, ω) – where ω is again a closed and non-degenerate two form constructed by taking the total differential of the Poincaré one form $\theta = p_i dq^i$. However, points in this space which do not lie on the constraint surface will not correspond to physically possible states since they constitute solutions which violate the gauge symmetry (they are *inaccessible* or *merely unphysical* in the language of Rickles [2004] p.177). It is the geometry particular to the class of points lying on the constraint surface that is nomologically significant.

We can characterise the geometry of the constraint surface explicitly by first restricting θ to Σ to get a new characteristic one form, $\tilde{\theta} = \theta_{|_\Sigma}$. The total derivative of $\tilde{\theta}$ will then give us a two form $\tilde{\omega} = \mathbf{d}\tilde{\theta}$ which endows the constraint manifold with the geometry $(\Sigma, \tilde{\omega})$. This new two form will be closed but whether it is degenerate or not depends on the particular properties of the constraint surface itself. In cases where it is non-degenerate we again have a symplectic geometry and the dynamics is as described above only now with the triple $(\Sigma, \tilde{\omega}, \tilde{H}_0)$ defining the system (where $\tilde{H}_0 : \Sigma \rightarrow \mathbb{R}$ is the restriction of H_0 to Σ).

In the case that $\tilde{\omega}$ is degenerate, however, we have a **presymplectic** geometry and our regular description of dynamics is no longer available to us. This is because presymplectic geometries have a degenerate structure that does not allow us to associate a unique vector field with every smooth function. This means that we are not provided with a straightforward characterisation of time evolution either via a unique $\mathbb{R}$-action or by the usual Poisson bracket with the Hamiltonian. Even more worryingly, the existence of local symmetry groups allows for indeterministic (or more properly underdetermined) evolution since at a given point the degeneracy of the Hamiltonian vector field allows for multiple mathematically distinct but dynamically equivalent solutions irrespective of the path leading up to that point. Thus, it would seem that the degeneracy inherent in presymplectic geometries is of a pernicious variety such that we can no longer establish a direct representational relationship between the relevant mathematical and ontological objects – there is no longer a one-to-one correspondence between the phase space solutions and the physical histories which are distinguished by unique values of the action and so our theory is underdetermined.

To get a better hold on the nature of this degeneracy we can define the null tangent vector space $N_x \subset T_x\Sigma$ as the collection of vectors that satisfy the equation $\tilde{\omega}(X, \cdot) = 0$. This is equivalent to the null space or kernel, $Ker(\tilde{\omega})$, of the presymplectic form. A kernel of dimension greater than zero is characteristic of the non-trivial structure of the presymplectic form just as a kernel of dimension equal to zero is characteristic of the trivial structure of the symplectic form. An equivalence relation between two points $x, y \in \Sigma$ can be defined based upon the

[7] These particular constraints are known as *primary* constraints. See footnote 11 for details.

condition of being joined by a curve, $\bar{\gamma} : \mathbb{R} \to \Sigma$, with null tangent vectors. Sets of points for which this equivalence relation holds are sub-manifolds called gauge orbits, $[x]$, and we say that the action of our presymplectic form is to partition phase space into these orbits. Equivalently we can say that the orbits are defined by the integral curves of the null vector field of $\tilde{\omega}$. The non-uniqueness that we understood in terms of the existence of gauge orbits is, therefore, also characterised by $Ker(\tilde{\omega})$.

Critically for our purposes the quotient $\Pi_R = \Sigma / Ker(\tilde{\omega})$ will necessarily be both symplectic and a manifold. The first is assured since the quotient is with respect to a *sectional foliation*.[8] The second is assured because the quotient is of a presymplectic manifold with respect to the kernel of its own presymplectic form and it can be shown that this implies that the resulting quotient manifold will be endowed with a closed two form with a kernel of zero dimension – i.e. it will have a symplectic geometry.[9] We can now represent evolution in terms of a unique $\mathbb{R}$-action defined in Π_R. We call Π_R the reduced phase space and using the projection map $\pi : \Sigma \to \Pi_R$ can define the symplectic geometry (Π_R, ω_R, H_R) where ω_R is the two form whose pullback to Σ by π is $\tilde{\omega}$ (i.e. $\tilde{\omega} = \pi^* \omega_R$ where $\pi^* : \Pi_R \to \Sigma$). An equation of the form $\omega_R(X_{H_R}, \cdot) = \mathbf{d}H_R$ then gives us a unique Hamiltonian vector field along with the associated Poisson bracket and $\mathbb{R}$-action that allows us to uniquely represent both time and the physical histories uniquely within our formalism.

The pullback by π also allows us to consider the properties that smooth functions on the reduced phase space will have with respect to the constraint manifold. Given such a function, $f_R \in C^\infty(\Pi_R)$, we can define $f_\Sigma \in C^\infty(\Sigma)$, by $f_\Sigma = \pi^* f_R$. Since points connected by a gauge orbit on Σ will be represented by a single point on Π_R we have that f_Σ will be constant along such gauge orbits. We can also talk about functions on the full phase space as being constant along gauge orbits. Since the constraints are by definition functions of the form $\varphi_j : \Gamma_0 \to 0$ the symplectic form on phase space will associate them each with a vector field X_{φ_j}. If we then take the Poisson bracket between them and an arbitrary function, $f \in C^\infty(\Gamma_0)$, we will have $\{f, \varphi_j\} = \omega(X_f, X_{\varphi_j})$. On the constraint surface it must be the case that the X_{φ_j} coincide with the null vector fields N – the integral curves of which are the gauge orbits. So, given that on the constraint surface f must be a function which is unchanging along the gauge orbits, the definition of the Poisson bracket implies that the expression $\{f, \varphi_j\}$ must vanish on the constraint surface – i.e. we have that $\{f, \varphi_j\} \approx 0$, where the weak equality is understood to mean vanishing upon the constraint surface.

We can therefore distinguish a class of functions on phase space, *Dirac-Bergmann observables*, by the satisfaction of three equivalent conditions:

1. Constancy along gauge orbits on the constraint manifold

2. Weakly commuting with all the constraints

[8]See (Souriau [1997], p.42 and pp. 82-3). It is a sectional foliation because the orbits which partition Σ constitute manifolds which are suitably transverse.

[9]See (Souriau [1997], theorem 9.10).

3. Equivalence to a function on the reduced phase space

The name observable seems sensible since it is only these functions that are specified uniquely for every value of the flow parameter defined by the vector field generated by the reduced Hamiltonian, H_R. Thus, given our reliance on an underlying symplectic structure to define time, precise restrictions are placed upon the mathematical objects with which we would want to associate physical quantities.

This idea of passing from a presymplectic to a symplectic manifold by quotienting with respect to the kernel of the presymplectic form is what we will call symplectic reduction and has an important connection[10] with Dirac's theory of constraints. In particular, in cases (such as those considered in the next section) where there is only one *primary* constraint and no *secondary* constraints[11] the application of symplectic reduction is identical to following the Dirac procedure in that it leads to the same conditions on observable functions we have just outlined. A theory in which all *first class primary* constraints are gauge generating is said to obey Dirac's theorem (Barbour and Foster [2009]) and we can therefore say that the applicability of symplectic reduction is equivalent to satisfaction of Dirac's theorem in all theories with a single primary constraint.

3 Reductionism, Haecceitism and gauge symmetry

The identification between gauge theories treated according to Dirac's constraint procedure and the re-construction of such theories in terms of reduced phase spaces arrived at via symplectic reduction has important interpretational consequences. As we have seen above conventional Hamiltonian mechanics can be characterised in terms of a phase space which has a symplectic geometry and within which solutions (the integral curves of the Hamiltonian vector field) are in one-to-one correspondence with physical histories. In these circumstances it seems natural to identify the phase space as a *possibility space* since each point can be considered to represent a distinct possible instantaneous physical state and each curve a distinct possible physical history. On the other hand, when we have a constrained Hamiltonian system the relevant phase space is clearly not a suitable candidate for a possibility space it contains *inaccessible* points (i.e. those not on the constraint surface) which can not be thought of as representing physically possible states. Furthermore, even if we exclude such points and focus on the *physical* section of phase space (i.e. consider only points on the constraint surface) then we again do not have a natural candidate for a possibility space since the weaker presymplectic geometry only equips us with an equivalence class of solutions corresponding to each physical history. This leaves the theory open

[10] See (Gotay et al. [1978]) and (Pons et al. [1999]) for explicit examination of this connection.

[11] Primary constraints are those that arise directly from the fact that the conjugate momenta are not independent functions of the velocities. Secondary constraints arise from the application of consistency conditions that ensure the primary constraints are conserved.

to pernicious underdetermination such that if points are identified as representing distinct instantaneous states then specifying an initial sequences of states fails to uniquely determine future states. Since the class of classical constrained Hamiltonian theories features theories, such as electromagnetism, which are manifestly deterministic in the sense of giving unique predictions for all measurable quantities the appearance of indeterminism should be seen to be interpreted as a sign of inadequacy in our standard representative formalism[12] – we cannot identify the constraint surface as a possibility space in a conventional sense.

Rather, we could treat it as an unconventional possibility space by weakening the representational connection between points on the constraint surface and instantaneous states. The classic philosophical strategy to enable such a weakening would be to adopt a position that disavows Haecceitism. Following Lewis ([1983]), we can designate as *Haecceitists* those who admit 'nonqualitative determinants of cross-identification' (p.19) between entities or objects in distinct worlds or structures. To adopt such a position is to allow for real differences which are only with respect to which objects play which role within the structure; since one is allowed to cross-identify each of a pair of qualitatively identical objects whose roles are permuted between two structures, we may ground a non-qualitative differentiation *of the structures* in terms of the cross-identification *of the objects*. The standard *literal* way of interpreting a possibility space—i.e. each point represents a distinct instantaneous state—can be understood in terms of Haecceitism. We can seen this since: i) The literal interpretation licences us to consider as distinct two histories represented by sequences of points which differ solely with respect to a gauge transformation; ii) Such a difference is only with regard to which instantaneous states (represented by points) play which roles; iii) This means that if we take a history to be the relevant structure and instantaneous states (labelled by the points to which they correspond) to be the relevant objects, then the ontological difference between gauge related histories in the literal interpretation can be naturally *cashed out* in Haecceitistic terms.[13]

When applied to the constraint manifold of a gauge theory (such as electromagnetism) such an approach becomes problematic because its combination with the presence of pernicious underdetermination forces us into interpreting an empirically deterministic theory as ontologically indeterministic – two sequences of instantaneous states can initially coincide but then differ in a real but non-qualitative manner as determined by a purely haecceitistic differentiation.[14] An anti-Haecceitist, on the other hand, denies the possibility of non-qualitative determinants of cross-identification and so will disavow exactly the haecceitistic

[12]As pointed out by Belot and Earman ([2001] p.8) the only other alternative in such circumstances would be to accept that there exist physically real quantities that are not measurable. Although potentially consistent, this would seem like a very unnatural approach and would require us to construct a highly unorthodox account of the concept of measurement.

[13]This is not to imply that there many not be other methodologies to ground such differences. For example Butterfield's ([1988]) response to the hole argument in general relativity makes use of counterpart theory rather than Haecceitism to establish a non-qualitative yet ontologically significant difference between gauge related histories.

[14]See (Belot [2003], §7) and (Belot and Earman ([2001] §2.3) for explicit treatment of the electromagnetic case.

differentiation that allows for two gauge related sequences of points in a possibility space to represent distinct structures.[15] Thus, by adopting anti-Haecceitism we can relieve ourselves of the burden of having to endorse ontological indeterminism by instituting a many-to-one relationship between gauge related sequences of points on the constraint surface and the unique sequences of instantaneous states they represent.

Although providing space for a viable interpretation of the possibility space structure found in gauge theory the anti-Haecceitist approach does nothing about removing what would seem like superfluous mathematical structure – to dispense with this surplus structure we need to move to the reduced phase space. Now, this space has obvious interpretational benefits since, as seen above, if all goes well the reduced space will be a symplectic manifold with the integral curves of the reduced Hamiltonian vector field naturally identified as representing physical histories and points as representing physically distinct instantaneous states. Thus the reduced space will, by definition, not feature any underdetermination associated with gauge symmetry and if we endow it with the privileged status as our fundamental possibility arena we reap the reward of recovering the ability to use our conventional representational scheme for theories which display gauge symmetry. Since we have regained a one-to-one correspondence between possibility space points and physically distinguishable instantaneous states the Haecceitism/anti-Haecceitism distinction discussed above becomes less significant. The superiority of, when possible, reduction as an interpretational stance has been advocated principally by Gordon Belot and John Earman (Belot [1996], [2000], [2003], [2003]; Earman [2002]; Belot and Earman [1999],[2001]). We will call it the *reductionism* with regard to constrained Hamiltonian theory and a close association can be made between it and Dirac's theorem as defined above – in fact, it would seem fair to say that the reductive philosophical stance is the natural interpretational consequence of a strict reading of Dirac's theorem. Arguments towards the non-applicability of symplectic reduction (corresponding to Dirac's theorem) for the specific case of nonrelativistic reparameterisation invariant mechanics will be the major preoccupation of the next section. §5 will then focus on techniques for representing time and change within the unreduced phase space before we will return, in §6, to interpretational issues connected with both reductionism/anti-reductionism and Haecceitism/anti-Haecceitism.

4 Reparameterisation invariant mechanics

4.1 Extended Lagrangian mechanics

The description of mechanics and gauge symmetry given thus far has made use of a distinguished background parameter; time t. Within the Lagrangian scheme this parameter was associated with both the tangent vectors or velocities, $\dot{q} = \frac{\partial q}{\partial t}$

[15] They need not, however, also deny primitive identity of the objects concerned (i.e. instantaneous states) since such primitive identity may be conceived of contextually. See (Ladyman [2007]) on this point.

$\in T\mathcal{C}_0$, and with the preferred parameterisation of the solutions, $\gamma_{PS} : \mathbb{R} \to T\mathcal{C}_0$. An alternative methodology for constructing a mechanical theory is to instead treat time as an additional coordinate, $q_0 = t$, in a $n+1$ dimensional **extended configuration space**, $\mathcal{C} = \mathbb{R} \times \mathcal{C}_0$. Velocities in this space are then defined for all of the $q_\mu \in \mathcal{C}$ by differentiation with respect to an arbitrary parameter τ so we have that $q'_\mu = \frac{dq_\mu}{d\tau}, (q_\mu, q'_\mu) \in T\mathcal{C}$. This arbitrary parameter is also taken to vary monotonically along curves in extended configuration space, $\gamma : \mathbb{R} \to T\mathcal{C}$. Following Lanzcos ([1966], §5)[16] we can use an extended Lagrangian, $L_{ex}[q_\mu, q'_\mu] : T\mathcal{C} \to \mathbb{R}$ to define an action of the form:

$$I = \int_\gamma d\tau L_{ex}[q_\mu, q'_\mu] = \int_\gamma d\tau(\frac{\bar{T}}{q'_0} - q'_0 V) \tag{5}$$

where $\bar{T} = q_0'^2 T$ and all masses are set to unity.

An important property of the extended Lagrangian is that it is homogenous of degree one in the extended set of velocities q'_μ: for some positive number k the transformation $q'_\mu \to kq'_\mu$ implies $L_{ex}[q_\mu, q'_\mu] \to kL_{ex}[q_\mu, q'_\mu]$. This means that the action of our theory will be invariant under re-scalings of the parameter τ. Theories which display such a dynamic insensitivity to parameterisation are said to be **reparameterisation invariant**. The interpretation of this theory will be non-standard since reparameterisation is a symmetry of the action which maps between distinct solutions in the extended configuration space – this is because the velocities are parameterisation dependent. Thus these solutions cannot be used to provide a straightforward characterisation of physical histories as in §3.1.

4.2 Extended Hamiltonian mechanics

In correspondence with §3.2 we can define an extended phase space as the cotangent bundle to our extended configuration manifold, $(q_\mu, p_\mu) \in \Gamma = T^*\mathcal{C} = T^*(\mathbb{R} \times \mathcal{C}_0)$, with $p_\mu = \frac{\partial L_{ex}}{\partial q'_\mu}$. The relevant Hamiltonian functional, $H_{ex}[q_\mu, p_\mu] : \Gamma \to \mathbb{R}$ takes the form:

$$H_{ex}[q_\mu, p_\mu] = p^\mu q'_\mu - L_{ex}[q_\mu, q'_\mu] \tag{6}$$

which is homogenous of degree one in the set of extended velocities and defines a reparameterisation invariant action

$$I = \int_\gamma d\tau(p^\mu q'_\mu - H_{ex}[q_\mu, p_\mu]) \tag{7}$$

By definition we have that the momentum conjugate to time is:

$$p_0 = \frac{\partial L_{ex}}{\partial q'_0} = L_0 - \frac{\partial L_0}{\partial \dot{q}_i}\frac{q'_i}{t'} = -H_0 \tag{8}$$

which means the extended Hamiltonian is equivalent to:

$$\begin{aligned} H_{ex}[q_\mu, p_\mu] &= t'(p_0 + H_0) \\ &= 0 \end{aligned} \tag{9}$$

[16] Also see (Johns [2005], §11-12) and (Rovelli [2005], §3.1).

The Hamiltonian is therefore a constraint and the dynamics of our theory will be defined upon a surface within extended phase space, $\Sigma = \{x \in \Gamma : H_{ex}(x) = 0\}$. The geometry of the constraint surface is given (as above) by taking the restriction of the relevant Poincaré one form, $\theta = p_\mu dq^\mu$, to Σ:

$$\theta_{|\Sigma} = p_i dq^i - H_0 dt \tag{10}$$

and taking the total differential to get a two form $\tilde{\omega} = \mathbf{d}(\theta_{|\Sigma})$ with highly non-trivial structure.[17]

Significantly, this two form is closed and degenerate. Thus the dynamics of extended mechanics is framed within a presymplectic geometry, $(\Sigma, \tilde{\omega})$. That this should be the case can be seen quite simply since our definition of a degenerate two form is equivalent to Hamilton's equations of motion with a zero Hamiltonian:

$$\begin{aligned} \tilde{\omega}(X, \cdot) &= dH_{ex} && (11) \\ &= 0 && (12) \end{aligned}$$

The immediate consequence of the degeneracy is that no unique Hamiltonian vector field is defined within the constraint surface and thus that we cannot define a unique temporal $\mathbb{R}$-action or flow. Correspondingly, our equation of motion (12) is only solvable up to an arbitrary factor[18] meaning that the dynamical solutions can only be unparameterised curves in the tangent bundle $\bar{\gamma}_{UPS}$.

The question is then; can we now simply follow a symplectic reduction procedure and then avail ourselves of the standard description of time, change and observable functions? Or does reparameterisation have some unusual feature that necessitates a different approach? To tackle these issues we need to take a closer look at the physical interpretation of both time and its conjugate momentum and in doing so construct a more elegant and general version of reparameterisation invariant mechanics.

4.3 Jacobi's principle and timeless theory

We can associate the time coordinate t (q_0) in extended mechanics with the value taken by a clock external to our mechanical system. In the case of an open system such an interpretation would seem appropriate; but what about if the system is a closed subsystem of the universe? – or even the universe as a whole? In this case there is clearly no physical basis for an external clock and as such we would look to eliminate t as an independent variable. We can do this by the process of *Routhian reduction*[19] which serves to eliminate a cyclic independent variable (i.e. one which only appears in the Lagrangian as a velocity) by using the equations of motion to set its conjugate momentum equal to a constant. Since

[17] This should come as no surprise as this two form must encode the full structure of the constraint and, since this constraint is the Hamiltonian, therefore the dynamics.

[18] This is because it can be thought of as a linear homogenous equation which only determines the velocities up to a scaling factor applied everywhere along a solution.

[19] A fuller discussion of Routhian reduction in general, and in this case in particular, is given in (Lanzcos [1966], §5) and (Arnold [1988], §3.s2).

we have seen above that the conjugate momentum to time is equal to minus the un-extended Hamiltonian of the system we will give the physical interpretation of the constant involved as minus the total energy, E, of the system. Setting the energy as equal to a constant is of course justified for a closed system. Explicitly, following Lanzcos ([1966], §5), the Jacobi action is given by

$$I = \int_{\gamma_0} d\tau 2\sqrt{(E-V)T} \tag{13}$$

This action can be understood as defining geodesics in the un-extended space, $T\mathcal{C}_0$, without making any reference to time or parameterisation – as such it is reparameterisation invariant. We can define the *lapse* as:

$$N = \sqrt{\frac{T}{(E-V)}} \tag{14}$$

The Jacobi Hamiltonian (Barbour and Foster [2009], p.7), $H_J : T^*\mathcal{C}_0 \to \mathbb{R}$ can then be expressed as:

$$H_J = \sum_i p_i.q_i' - L_J = Nh \tag{15}$$

where

$$h = \frac{1}{2}\sum_i p_i.p_i + V - E = 0 \tag{16}$$

This is again a first class primary constraint. In fact it is the same constraint as was encountered in extended mechanics merely with p_0 replaced by $-E$ and the multiplier t' replaced by N. Thus, reparameterisation invariant theories of mechanics have a Hamiltonian of the form

$$H = Nh \tag{17}$$

where N is a arbitrary multiplier, the choice of which determines the parameterisation, and h is some function of the conjugate variables that is equal to zero. Such *timeless* theories will inevitably be constrained Hamiltonian theories with the Hamiltonian itself playing the role of the constraint. Thus the geometry of the constraint surface will be dictated by the two form $\omega = \mathbf{d}\theta = \mathbf{d}(\theta_{|\Sigma})$ where $\Sigma = \{x \in \Gamma : H = 0\}$.

This two form will in general be closed and it will also be degenerate since it has a null direction associated with the Hamiltonian constraint. The integral curves of this vector are the gauge orbits of ω on Σ. However, since this null vector field on the constraint surface is generated by the Hamiltonian we could also argue that $\omega(X) = 0$ is the equation of motion.[20] Since the integral curves of the kernel of the presymplectic form can be shown to be unique solutions we have the strange

[20] This can be explicitly seen for the case of the simple pendulum system used by Rovelli ([2004]) to illustrate both extended mechanics (§3.1 pp. 104-5) and Jacobi's theory (§3.2 pp.109-11) – n.b. he refers to the latter non-standardly as *relativistic* mechanics.

situation in timeless mechanics where the gauge orbits correspond to the physical histories! The question of how we are to interpret such a perplexing description of mechanics, where degeneracy and dynamics are so closely interwoven, is far from trivial and shall occupy us for the remained of this paper. To go forward, however, we must go back and reconsider the connection between presymplectic geometry and local symmetry groups.

4.4 Degeneracy, indeterminacy and triviality

In our initial discussion of presymplectic geometry we associated the degeneracy encountered with a group of local or gauge symmetries arising on the tangent bundle to some configuration-velocity space, $T\mathcal{C}$. These symmetries were taken to be such that they allow for multiple points to be associated with the same value of the Lagrangian and thus ensured that the Legendre map, $\mathcal{FL} : T\mathcal{C} \to T^*\mathcal{C}$, was not an isomorphism (a bijective homomorphism) since in such a situation it will generically neither be injective nor surjective. In the case of reparameterisation invariant theory the relevant symmetry group is of course that of reparameterisations. It can be seen to be different to the generic gauge group considered in §3.4 in two important respects. First, since it relates curves that differ in terms of parameterisation it is strictly a symmetry of the action rather than the Lagrangian. Second, although it also leads to a Legendre transformation that is again not bijective (since it is not injective) the action of the reparameterisation group is such that the conjugate momenta are not effected by rescaling the parameter. Thus, distinct points on the tangent bundle which can be mapped from one to another by the action of the reparameterisation group will correspond to single points on the cotangent bundle. The structure of our phase space is therefore such that paths through it are invariant under reparameterisations. The degeneracy present does not then lead to the type of pernicious underdetermination which was encountered in the construction of presymplectic mechanics considered in §3.4. Rather it takes us between vector fields that are equivalent up to scaling by a multiplicative factor corresponding to the parameterisation. Our primary motivation for the application of the symplectic reduction procedure is therefore removed since there is no possibility of pernicious indeterminism.

We still, however, have the problem of representing change within the presymplectic constraint surface (Σ, ω) – one would like to be able to associate the Hamiltonian with a unique vector field and therefore be able to establish a unique flow with which we can associate evolution. The most obvious way to do this would be to find an underlying symplectic manifold within the timeless theory – thus it may be worth trying to symplectically reduce such theories even without a pressing theoretical need to. However, as pointed out above, timeless theories have a geometry such that what we would normally call the gauge orbits (since they are the sets of points connected by parameterisation rescalings) are also the usual candidates for the solutions in phase space (since they are generated by the Hamiltonian). Thus, the reduction procedure whereby we quotient out the orbits of ω, will leave us with a reduced phase space, $\Pi_R = \Sigma / Ker(\omega)$, without any meaningful notion of evolution – it consists of unconnected points each of which can only gain meaning when referred back to the entire history on the constraint surface to which they correspond. Moreover, since the space is equipped only with a trivial Hamiltonian function there is no sense in which the reduced phase space symplectic form, ω_R, found in reparameterisation invariant theories of mechanics can play any meaningful role – even in generating maps between points in the reduced space. Thus, representationally Π_R alone is only equipped to describe trivial universes consisting of one static configuration (Maudlin ([2002]) makes a similar point). Furthermore, since ω_R is defined only in virtue of the constraint surface via $\omega = \pi^*\omega_R$ there is a sense in which it could be said to have no more than a purely formal existence.[21]

It could be argued (Belot [2007], p.78) in this context that points in the reduced phase space should be taken to describe entire dynamic solutions and therefore that the space is not representationally trivial. In normal circumstances it is reasonable to interpret the reduced phase space, Π_R, resulting from the application of symplectic reduction as a space of instantaneous initial data states, $\mathcal{I}$. This follows from the fact that for any curve γ_{PS} in the space of gauge invariant solutions to the Euler-Lagrange equations $\mathcal{S}_R$ we can define a set of isomorphisms between Π_R and $\mathcal{S}_R$ such that for each value of the curve's parameterisation there will be a map uniquely picking out a point in Π_R with corresponding value of the Hamiltonian flow parameter.[22] However, for the case of nonrelativistic[23] timeless theory there is only a single canonical isomorphism defined between points in the reduced phase space and the *unparameterised* gauge invariant solutions, γ_{UPS}. Thus we can see why one might think the representational role of Π_R should be modified such that it becomes identical to that of $\mathcal{S}_R$. But such a move has highly nontrivial consequences for how we must interpret the *unreduced* phase space and is therefore difficult to countenance. In particular, if $x_R \in \Pi_R$ is a solution then given a point on the constraint manifold in the unreduced phase space, $x \in \Sigma$, we must interpret the relevant 'gauge' orbit, $[x] : \Sigma \to \mathbb{R}$, as an equivalence class of solutions. This interpretation cannot hold since these orbits are equivalent to solutions themselves rather than equivalence classes of solutions. Thus, in nonrelativistic timeless theory at least, the representational role of the reduced phase space cannot be in describing entire histories – we cannot treat it as a primitive arena for representing our fundamental ontology. Rather, any status it can be given as a history space is purely parasitic on the pull-back map to the unreduced space and it is fallacious to argue that the isomorphism that exists between $\mathcal{S}_R$ and Π_R must confer representational equivalence between these two very different mathematical structures.

It would seem therefore that we have established two examples of mechanical theory within which the presence of a first class constraint does not indicate that a symplectic reduction is appropriate. This means that Dirac's theorem

[21] Rovelli's ([2005]) treatment introduces ω_R as ω_{ph} (p.111) but fails to make any use of it.

[22] The geometric structure of such a reduced space of solutions as well as its connection with the Hamiltonian framework is extensively discussed in (Belot [2007]).

[23] In this respect general relativity would seem to be identical to nonrelativistic theory. Belot's argument (which was designed for application to GR) is explicitly re-examined for the case of relativistic theory in (Thébault [2011], §3) and (Thébault [in preparation]).

(first class constraints generate gauge symmetry) does not hold for the timeless theories considered and is therefore not generally valid in its original form.[24]

5 Representing change and observables in timeless mechanics

The essential point established by our argument thus far is that the unreduced phase space of a timeless system (i.e. one in which the Hamiltonian is a constraint) is such that we cannot interpret it using the convectional machinery of constrained Hamiltonian mechanics. Although, as in the generic case, points not on the constraint surface must be classified as inaccessible states, it has been demonstrated that, unlike in the generic case, the difference between points connected by the orbits generated by the constraint on the constraint surface itself cannot be classified as purely unphysical gauge without trivialising the theory. Thus, the geometric structure of timeless theories leads us into an acute *problem of representing change* since we cannot avail ourselves of the conventional temporal machinery provided by a reduced phase space. The definition of a Dirac-Begmann observable also becomes ambiguous within timeless theory since by application of the third condition from §3.4 observable functions must be equivalent to single points on our reduced phase space – and this would seem to trivialise them. Furthermore, the first condition (constancy along gauge orbits on the constraint manifold) can only be satisfied in the case of phase space functions which are constant along entire histories of the system and it is difficult to see how such functions—*perennials* in the terminology of Kuchař ([1992])—could be used to represent dynamic physical quantities since they cannot change along the solutions defined by the Hamiltonian on the constraint surface. Thus we are also presented with a *problem of representing observables*. This section will outline and evaluate two methodologies each designed to meet our two problems for the case of nonrelativistic theory.

5.1 The emergent time strategy

That the Hamiltonian constraint in reparameterisation invariant theories should be thought of as generating genuine change is a position that has been notably defended by Kuchař ([1991]) and Barbour ([1994]); more recently it has been outlined explicitly in (Barbour and Foster [2009]). We shall call it the Kuchař-Barbour-Foster (KBF) position with regard to change. In keeping with our discussion in §4.4 it is an explicitly non-reductive strategy since it involves us treating the differences between points on the integral curves corresponding to the Hamiltonian vector field as genuine physical change. Parallel, although logically independent, to this position with regard to change is the view that observable functions need not commute with the Hamiltonian – we shall call this view the

[24] Rather we should say that first class constraints indicate the presence of gauge symmetries but need not necessarily be identified as the relevant generators. This point is in full agreement with (Barbour and Foster [2009]).

KBF position with regard to observables. This explicitly non-reductive strategy characterises observables as full functions on the unreduced phase space which are allowed to break all three of the Dirac-Bergmann criteria. Essential to the practical viability of this position is the possibility of quantifying the change of an observable in a gauge invariant manner and we shall here outline the methodology for doing this uniquely by using an *emergent* notion of time following (Barbour and Foster [2009]).

From above we have that a generic timeless Hamiltonian will be of the form:

$$H = Nh \tag{18}$$
$$h(p,q) = 0 \tag{19}$$

If we take a function on phase space $g(p,q)$ which we would like to interpret as corresponding to some physical quantity then, since the full phase space is a symplectic manifold, we can define the Poisson bracket of this function with the Hamiltonian function, $\{g, H\}$. This is equivalent to the Lie derivative of the function with respect to the Hamiltonian vector field, $\mathcal{L}_{X_H}(g)$. Since the Lie derivative is an operation on scalar functions that gives us the change of the function along a vector field $\mathcal{L}_{X_H}(g)$ is equivalent to a real number representing the rate of change of g along the Hamiltonian vector field with respect to an arbitrary parameter τ:

$$\frac{\delta g}{\delta \tau} = \{g, H\} \tag{20}$$

Thus an infinitesimal change in the function along the vector field is equivalent to:

$$\begin{aligned} \delta g &= \delta\tau\{g, H\} && (21)\\ &= \delta t\{g, h\} && (22) \end{aligned}$$

where we have introduced the temporal increment $\delta t = N d\tau$. Crucially, we have from the invariance of the canonical action that $N d\tau$ must be invariant under reparameterisations. Since the Poisson bracket must be a real number δg must itself also be a reparameterisation invariant quantity. However, it cannot yet be taken to represent the change in a physical quantity; we have not made any restriction to the constraint surface so we have not excluded change that takes us from accessible to inaccessible states. To resolve this we introduce the weak inequality and the infinitesimal change of a dynamic variable along a physical history can be then represented as:

$$\delta g \approx N d\tau\{g, h\} \tag{23}$$

We can put this result in the context of our geometric discussion since we have that: i) The Hamiltonian can be taken to generate an equivalence class of vector fields, X_{Nh} upon phase space[25]; ii) The integral curves of each of the vector fields will correspond to the same set of solutions only with a differently scaled

[25] We get an equivalence class rather than a unique field because the multiplier N is arbitrary.

parameter τ marking out change along them; iii) A reparameterisation is then the map between one vector field and another (between one solution and another) by re-scalings τ. Such a change is between different objects both generated by H but is not strictly generated by H itself. Thus it should come as no surprise that there is a viable methodology for gauge invariantly using the vector fields associated with the unreduced Hamiltonian to solve our problem of representing both change and observables in timeless theory.

Although we now have a valid methodology for representing the change of a function along a timeless solution there does still seem to be a problem. If we were to consider astronomers in two nonidentical isolated sub-systems each using these equations to describe the dynamics of their solar system, they would end up arriving at two different measures of change since each will have to make an arbitrary choice in the form of the lapse and parameter τ. However, if we make the restriction that we are dealing with closed systems of fixed energy then we are justified in fixing the form of the lapse in accordance with Jacobi's theory – i.e. such that $N = \sqrt{\frac{T}{(E-V)}}$. This Jacobi lapse allows us to define a uniquely distinguished and reparameterisation invariant *Newtonian temporal increment*[26]:

$$\delta t = \sqrt{\frac{T}{(E-V)}}d\tau \tag{24}$$

Furthermore, this Newtonian temporal increment is such that it can be defined based purely upon change in the configuration variables as:

$$\delta t = \sqrt{\frac{\delta q_i.\delta q_i}{2(E-V)}} \tag{25}$$

and we can therefore represent the change in a function along a solution without reference to the parameterisation. This means that we can treat time as something which naturally *emerges* from the dynamics and is thus ontologically secondary to the change of configuration variables.

5.2 The correlation strategy

An alternative, and perhaps more radical, methodology for representing change and observables in timeless mechanics places emphasis on the idea of correlations and may be traced back through a linage featuring famous names such as DeWitt ([1967]), Bergmann ([1961]), and (arguably) Einstein ([1916]). Here we will present a particular implementation of the correlation strategy which follows on from Rovelli's ([1990], [1991], [2002], [2004]) complete and partial observables methodology and is due to Dittrich ([2006], [2007]) and Thiemann ([2007]). We shall focus initially on this correlation strategy as addressing the problem of representing observables in isolation from the problem of representing change and

[26] As pointed out by Barbour ([1994], §4) this privileged time measure derivable from dynamics of a closed system is equivalent to the astronomers notion of ephemeris time.

shall designate the position outlined as the Rovelli-Ditterich-Thiemann (RDT) observables position.

An essential element of this scheme is the move away from a representation of change in an observable as the variation of a phase space function along a history. Rather, we focus upon the configuration variables themselves (the partial observables) and assert that the quantities we should be interested in endowing with physical meaning are the relations between configuration variables (the gauge invariant complete observables).[27] Change in an observable can then be represented as the reparameterisation invariant specification of the value of one configuration variable with respect to another – as correlations between partial observables. The complete observables are the families of correlation functions which individually give the value of one of the partial observables when the other (the clock variable) is equal to some real number.

A simple example will illustrate the important elements of this scheme. We can consider a system described by two configuration variables (partial observables) q_1 and q_2 which together with their conjugate momenta obey a Hamiltonian constraint of the form $H[q_1, q_2, p_1, p_2] = 0$. The phase space, $(q_1, q_2, p_1, p_2) \in \Gamma$, will as usual have a symplectic structure. We can use the relevant symplectic form to define the action of the Hamiltonian vector field on an arbitrary function, $X_H(f) = \omega(X_f, X_H) = \{f, H\}$. The flow, α_H^τ, generated by this vector field can then be defined for every $x \in \Gamma$ and we can see this flow as acting on a phase space function, $\alpha_H^\tau(f)(x)$, such that it takes us along the solutions.[28] For our system therefore we calculate $\alpha_H^\tau(q_1)(q_1, q_2, p_1, p_2)$ and $\alpha_H^\tau(q_2)(q_1, q_2, p_1, p_2)$ We then designate one of our variables as a *clock variable* and seek to invert an expression of the form $T_x(\tau) = \alpha_H^\tau(q_1)(x)$ such that solving $T_x(\tau) = s$ for $s \in \mathbb{R}$ will give us an expression for τ in terms of s and q_1. In general this inversion will only be possible for a specific interval – thus the clock variables are typically going to be at best locally well defined and so are unlikely to be continuous on phase space and this means that the scheme will be difficult to implement in practice. We can then insert the inverted expression into the second flow equation $\alpha_H^\tau(q_2)(x)$ by substituting for τ, and produce an expression which (within the interval specified) gives us the value of q_2 when q_1 takes the value s. This *complete observable* represents a family of functions (one for each s) each of which expresses the correlation between our two partial observables without reference to parameterisation.

Importantly, not only are complete observables families of reparameterisation invariant objects but the functions on phase space that each correlation defines will commute with the Hamiltonian constraint. This means that they explicitly fulfil the second condition for a Dirac-Bergmann observable and demonstrates the fundamental difference between the RDT and KBF positions with regard to observables. We can consider the extent to which the complete observables satisfy the other two criteria. The first condition was that Dirac-Bergmann observables are functions which are constant along the orbits generated by the constraint on

[27] There is some debate as to how we should interpret the partial observables see (Thiemann ([2007] p. 78), (Rickles [2008] pp.154-68) and (Rovelli [2007]).

[28] See (Dittirich [2007] eq. 2.5, 2.6 and 2.7) for explicit formulas.

the constraint surface. By definition the flows generated by the Hamiltonian constraint in the phase space and the integral curves of the relevant null vector field will coincide on the constraint surface. Since each of the correlations that make up a complete observable are defined for a specific value of the flow parameter these functions do not vary along this flow and are therefore constant along gauge orbits. But it must be noted that the sense in which these functions satisfy this condition is somewhat different from the generic case in two senses. First, in a typical gauge theory an observable would be constant along gauge orbits but it would also vary between them – it is this variation off the orbits that we would normally consider physical change. Second, the sense in which they are constant on gauge orbits is almost trivial – they are each defined for a particular value of the flow parameter so in effect they establish the correlation at a particular point along an orbit. Clearly such a specification is valid all the way along the orbit only in the same strange sense that 'in Sydney in 2011 AD, Caesar crossed the Rubicon in 49 BC' is a valid statement concerning modern Australian history.

Application of the third Dirac-Bergmann condition is more acutely problematic. Since the functions that define them do not vary between gauge orbits complete observables are each equivalent to single points rather than functions on reduced phase space. This means that if we take the symplectic reduction ontologically seriously (i.e. treat the reduced phase space as primitive) we will only be left with a single correlation specified by each complete observable rather than an entire family of correlation functions since it is only through the pull back to the constraint manifold that these correlations are defined. It would seem, therefore, that there is some motivation for setting aside the Dirac-Bergmann notion of an observable altogether – complete observables are defined in such a way that it is no longer fully appropriate and the RDT position should be seen as a distinct alternative rather than a innovative application of the orthodoxy.

We can now finally turn the the problem of change. Here we appear to have a problem since Rovelli and Dittrich hold both that evolution generated by the Hamiltonian is gauge[29] and that the entire orbit it generates is what should be considered physically real.[30] If we dispense with the first proposition (which clearly must contradict the non-reductive stance taken by these authors) and focus on the second, then a coherent but highly radical position emerges. In particular, if we consider the implications of the change in the notion of the physical state that seems to have been made, then it appears that the RDT position with regard to change in nonrelativistic reparameterisation invariant mechanics amounts to a denial of the need for any fundamental concept of time at all.

Rovelli ([2002]) distinguishes the 'physical phase space' as the 'space of orbits generated by the constraints on the constraint surface' (p3) and Dittrich ([2007]) similarly defines the physical state as an 'equivalence class of phase space points' which 'can be identified with an n-dimensional gauge orbit' (p 1894). For a theory where the Hamiltonian is itself a constraint this constitutes a redefinition

[29]See Rovelli ([2004] p. 127) and Dittrich ([2007] p.1892). Thiemann's ([2007] p.75) position with regard to this point is more nuanced and is specifically targeted to the case of general relativity.

[30]See (Rickles [2008] pp.182-6) and (Dittrich [2007] p.1894).

of the structure of our dynamics such that the basic ontological entity is an entire history rather than an instantaneous configuration. In typical gauge theories points on the constraint surface connected by a gauge orbit are classified as the same state because the difference between them is taken to be unphysical – we can then proceed to a symplectically reduced phase space within which we can characterise the change between two instantaneous states without problem. This interpretation of change drawn from the complete observables scheme on the other hand leads us to classify two such points as the same state because the word 'state' is redefined such that in includes all points on the orbit. This is not to classify time or evolution as gauge since that would indicate that the trivial reduced phase space of single initial data points was the arena of true physical significance. Rather, it is to adopt a position such that any notions of evolution and time in a conventional sense are redundant within reparameterisation invariant theory. Adoption of a correlation strategy has then the capacity for radical philosophical implications for the nature of time in physical theory – the next section will examine these in more detail as well as considering the emergent time strategy in a more philosophical context.

6 Interpretational implications

The more strictly analytical objective of this paper has been to demonstrate that, unlike standard gauge theories, timeless nonrelativistic theories are such that the constraints cannot be considered as gauge generators without trivialisation and that a reduced phase space with a symplectic geometry cannot be considered as both a viable and autonomous representative structure. In this context we have examined two strategies for representing observables and change in the unreduced phase space and considered some of the implications of each scheme. What now concerns us are the interpretational consequences we should attach to our conclusions. In particular, it is interesting to consider how we should place the existence of: 1) gauge theories with phase spaces such that passage to a representatively viable reduced space is not available; and 2) our two strategies for representing change without an explicit notion of time; in the context of the debates over both relationalism/substantivalism with respect to time and reductionism/nonreductionism with respect to the interpretation of gauge theories.

6.1 The relationalist vs substantivalist dispute with regard to time

The long standing relationalist/substantivalist dispute with regard to space and motion in nonrelativistic mechanics contains many important lessons for the parallel dispute with regard to time. In particular, modern treatments in terms of analytical mechanics allow us to precisely characterise a number of refinements to the traditional binary distinction – we will very briefly introduce the ideas key for our purpose, a more exhaustive analysis can be found in (Rickles [2008]).

Let us define a *substantivalist* as someone who is committed to the existence of space (or space-time) as an entity in its own right, over and above the relations that hold between material bodies. The position of *straightforward substantivalism* then involves a commitment to the existence of distinct spatial (or space-time) models which differ only by the application of an element of the Euclidean (Galilean) group of global symmetry transformations. The difference between the two models is cashed out in Haecceitistic terms since it rests upon the non-qualitative cross-identification between spatial points as the means of differentiation. In terms of the Hamiltonian formulation of mechanics (where the models are curves in phase space) this is to insist that sequences of points in phase space which are related by symmetry transformations can represent distinct sequences of instantaneous states despite being distinguished only by which individuals (in this case the instantaneous states) play which role in the relevant structure – straightforward substantivalism thus involves taking a Haecceitist line with regard to sequences of points within phase space as well as spatial points within spatial models.

A *sophisticated substantivalist* is someone who maintains the commitment to the ontological fundamentality of space (or space-time) but does not allow Haecceitism and therefore does not insist that models related by symmetry transformations are distinct – rather the individual spatial points are multiply realised within symmetry related models. Since we do not allow non-qualitative determinants of cross-identification between spatial points across models we disavow the differentiation between these models along Haecceitist lines. Furthermore, within a Hamiltonian framework (where again the instantaneous states are the individuals and phase space curves the models) sophisticated substantivalism involves the view that although space is a fundamental entity, symmetry related sequences of points represent the same structure since they only differ as to which individuals play which roles. We can therefore understand sophisticated substantivalism in terms of an anti-Haecceitist position with regard to sequences of points within phase space – with the equivalence class of points connected by the relevant symmetry transformation constituting realisations of the same possibility.

The *relationalist* on the other hand, wants to deny that space (space-time) is a fundamental entity and is therefore committed to disavowing any ontological distinction between models which differ only with regard to space (space-time) symmetries. This leaves open two options with regard to the relevant possibility spaces of the Hamiltonian re-formulation of Newtonian mechanics; either endorse anti-Haecceitism and stick with the original phase space—this is what Rickles ([2004]) calls *unsophisticated relationalism*—or move to a quotient space where all points related by elements of the relevant symmetry group are reduced to single points. This second option is what we will call *reductive relationalism* and, like reductionism with respect to gauge theory, is notably advocated by Belot ([1996],[1999], [2000]).

With these distinctions in hand, and the existence of a connection between reductionism and relationalism already apparent, we can turn our attention to the ontological status of time within our timeless theories of nonrelativistic mechanics. We can define a temporal substantivalist as someone who asserts the existence of time as a basic entity in its own right over and above the relations that exist between the instantaneous states of material systems (be they relationally defined or not). Such a position is a natural reformulation of the Newtonian concept of absolute time; in particular, it seems to implement that notion of time defined in the influential Scholium section of his Principia.[31] Now, it could be argued that, at least as nonrelativistic mechanics is concerned, substantivalist time is inherently connected to the use of an external temporal dimension and on this basis a substantivalist would have a very hard time dealing with Jacobi's theory. However, what is essential to temporal substantivalism—under our reading of it at least—is that time can be asserted as a basic entity parameterising change that is not parasitic on the motion of the bodies that are doing the changing. Thus, Jacobi's theory does not in principle exclude temporal substantivalism since change is parameterised (albeit non-uniquely) in terms of τ. Moreover, unlike its Newtonian counterpart (as well as parameterised particle mechanics) Jacobi's theory offers a level playing field for matching the temporal substantivalist against their relationalist foe since it is a mechanical framework free from the fundamental *presumption* of preferred parameterisation or external time that would inherently favour a substantivalist reading.

A straightforward (i.e. Haecceitist) temporal substantivalist reading of Jacobi's theory could then proceed as follows. Just as the reality of space indicates that there is a real but non-qualitative difference between two sequences of instantaneous states related by a spatial symmetry transformation, the reality of time indicates that there is a real but non-qualitative difference between two sequences of instantaneous states related by a temporal symmetry transformation. In the first case this difference is represented by sequences of points in velocity-configuration/phase space differing only with regard to the application on an element of the Galilei group of global space-time symmetries. In the second it is represented by two sequences of points in velocity-configuration/phase space differing only with regard to an application of an element of the reparameterisation group. In each case this non-qualitative difference can be understood precisely in Haecceitistic terms because it is established via inter-structure cross-identification of individual instantaneous states (they play different roles in the different structures). That these models are connected by an element of the local symmetry group of time reparameterisations does not mean that they fail to be distinct because, even though such a symmetry means that there can be no empirical difference between worlds which differ only with respect to their parameterisation, our acceptance of Haecceitism allows us to say that there is an ontological difference. Thus, the straightforward substantivalist type position with respect to time in Jacobi's theory leads us to endow parameterisation of solutions with a stamp of physical reality.

Correspondingly, Jacobi's theory, at least as formulated in §5.3, leaves open the conceptual space for a sophisticated (i.e. anti-Haecceitist) form of temporal

[31]'Absolute, true, mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, common time, is something sensible and external (whether accurate or unequal) measure of duration by which the means of motion, which is commonly used instead of true time' (Newton [1687]).

substantivalism whereby time is still asserted as a basic ontological entity but the fundamental temporal structure of a sequence of instantaneous states is multiply realised in terms of the different parameterisations of a solution – a single fundamental notion of time is understood as being represented by the equivalence class of parameterisations. We do not have an inflation of possibility within the representation of histories since the difference between two parameterisations of a solution is understood to be merely of the excluded Haecceitist variety (it is only which instantaneous states play which roles that is different).

A temporal relationalist can be defined as someone who treats time as a non-fundamental or derived entity. Such an anti-Newtonian position is typically seen to have originated with the work of Descartes, Leibniz and perhaps also Huygens (Barbour [unpublished]). but is contained in the most direct form within the ideas of Mach.[32] With regard to Jacobi's theory temporal relationalism should be understood as an insistence that the parameterisation of a solution is unphysical since it is only the relation between two instantaneous states which should matter, not how this relation is 'abstracted' in terms of parameterisation.[33] Thus, just as the spatial relationalist was committed to two points in velocity-configuration/phase space which are connected by spatial symmetries representing the same thing, the temporal relationalist is committed to two parameterisations of a solution within the relevant space representing the same thing. This would seem, *prima facie*, to leave open the option for either an unsophisticated variant of temporal relationalism whereby we merely utilise anti-Haecceity to *mop-up* the excess possibilities entailed by the multiplicity of parameterisations or a reductive variant whereby we we quotient out the relevant symmetry group to leave a space with the requisite reduced set of possibilities.

So far the debate seems to resemble closely that for space/space-time. However there are two new and interesting complications that we must consider. The first stems from the fact that the reparameterisation symmetry of Jacobi's theory is, unlike the global symmetries that feature in the space/spacetime debate, manifestly local. The locality of the symmetry means that a straightforward substantivalist who sticks with Haecceity and an unreduced possibility space could be left open to pernicious indeterminism in their ontology of the type discussed in §5.4. Such a development has been key to the perceived derailment of straightforward substantivalism for the case of general relativity which features local space-time symmetries [34] and may be expected for this case also. Our straightforward temporal substantivalist is understood to be committed to Haecceitism in that that they admit cross-identification of temporally relabelled instantaneous states between histories as represented by curves related by reparameterisation. Thus, the differently parameterised curves are taken to represent ontologically distinct

[32]'It is utterly beyond our power to measure the changes of things by time. Quite on the contrary, time is an abstraction, at which we arise by means of the change of things' (Mach [1883]).

[33]Such a definition is in full accordance with the notion of a 'Leibnizian relationalist' with respect to time found in (Pooley and Brown [2001]).

[34]This is in fact the essence of the hole argument, see (Rickles [2008] §4-5) for a more extensive discussion.

structures. Such an ontological distinction between objects differing by the application of the action of a local symmetry group has the potential to generate ontological indeterminism since the two curves may initially coincide and then diverge. Since Jacobi's theory is an empirically deterministic theory this potential for ontological indeterminism seems highly problematic and could be taken to drive us away from the straightforward variant of temporal substantivalism on the grounds of the commitment to Haecceitism involved.

However, the case of Jacobi's theory is particularly interesting because although pernicious indeterminism *is* possible within the velocity-configuration space of Jacobi's theory—since the velocities are dependent on parameterisation—it *is not* possible within the phase space since reparameterisations are symmetries on the canonical momenta. This means that provided they confine themselves to the constraint manifold, a temporal substantivalist can stick to a completely literal reading of phase space (and the Haecceitism that we are presuming goes along with it) such that each point represents a distinct instantaneous state and each solution representing a distinct dynamical history. Thus even though Jacobi's theory can be classified as a gauge theory in that it features first class constraints, it has a phase space that can unproblematically accommodate a non-reductive interpretation without any recourse to anti-Haecceitism. In this respect it constitutes a notable counter-example to accounts of the interpretation of gauge theories (such as that presented by Belot and Earman [1999], [2001]) which are presumed by their authors to hold generically.

The second point that marks the substantivalism vs relationalism dispute with regard to time in Jacobi's theory distinct from both the case of global symmetries in Newtonian mechanics and local symmetries in generic gauge theories is that the reductionist position is no longer available. As discussed extensively above, the structure of Jacobi's theory is such that the application of symplectic reduction will lead to a reduced phase space which has a trivial dynamical structure such that it can only be made sense of by reference back to the unreduced space. This renders a reductionist reading of the theory inadequate since to get off the ground it would require the utilisation of exactly the otiose structure (*gauge* related points on the constraint manifold) the elimination of which was its supposed benefit. Moreover, the reductionist desire to construct a reduced phase space which can be interpreted along literal lines manifestly fails since on its own the relevant reduced space can only be read as representing isolated instantaneous states corresponding to dynamically trivial universes. Thus, with regard to time in Jacobi's theory at least, any viable form of relationalism is going to have to be non-reductive – does this then mean that it must be anti-Haecceitist? In order to answer this question let us then consider the relationalist credentials of our two non-reductive strategies for representing change and observables. In particular, it is interesting to consider how we should interpret their presentation of dynamics upon the constraint manifold in terms of the ideas of possibility spaces and Haecceity that we have been discussing.

6.2 An ontology of timeless change?

As discussed above the emergent time strategy explicitly makes use of the Hamiltonian constraint as the generator of evolution. A point on the constraint manifold is taken to represent an instantaneous state and the dynamical change between this state and the next is represented in terms of the null vector corresponding to the flow generated by the Hamiltonian at that point. Similarly, an observable is represented by a function of the constraint manifold and the change in an observable is represented by the change in that function along the Hamiltonian flow. Now, it has been argued by Belot and Earman ([1999], [2001]) that for the case of general relativity treating the relevant Hamiltonian constraint in such a manner (in particular allowing for observables that do not commute with the Hamiltonian constraint) is the the hallmark of a *Heraclitean* position that asserts the fundamentality of time within the theory. Conversely, according to this viewpoint, there is an equivalence between treating the Hamiltonian constraint as gauge generating (and therefore implementing the Dirac-Bergmann criteria for observability) and relationalism. Clearly, adopting such a classification scheme for Jacobi's theory would seem to suggest that we should think about the emergent time strategy in terms of temporal substantivalism. Pooley ([2001]) argues that we should adjust this classification scheme such that how we treat the relevant constraints of general relativity is now thought of as a guide to deciding between 'straightforward substantivalism on the one hand and the disjunctive set of sophisticated substantivalism and anti-substantialism relationalism on the other' (p. 15). Thus, under Pooley's scheme the emergent time strategy for understanding change in Jaocbi's theory would be classed as a straightforward substantivalist one with respect to time. However, as has been argued for the case of general relativity (Rickles [2007], p. 170) the assertion of such definite connections between the treatment of the observables/Hamiltonian constraint and substantivalist/relationalist distinctions is not in fact justified. There is more potential for metaphysical underdetermination within the formalism than would appear at first sight. The crucial factor informing Pooley's distinction is the reduction in possibility entailed by how we interpret objects within structures connected by the relevant symmetry. For the case of Jacobi's theory (and actually also in GR itself – see Thébault [in preparation] for detailed argument) this turns on how we understand solutions related by the relevant gauge symmetry and not points connected by the action of the Hamiltonian constraint. In Jacobi's theory one can happily avoid straightforward substantivalism whist still denying that the Hamiltonian constraint generates gauge so long as one describes the *change* of observables (which themselves may fail to respect the Dirac-Bergmann criteria) without reference to parameterisation – it is change in parameterisation that we want to call unphysical not the change that is parameterised! The emergent time strategy is temporally relational since it has removed temporal structure altogether and allows us to describe change, both of observables and states, without reference to parameterisations. Moreover, it has no need for the anti-Haecceitism of unsophisticated relationalism since (within a Hamiltonian formalism) it can make use of a one-to-one representational relationship between points and instantaneous states on the one hand and solutions uniquely parameterised via the Newtonian temporal increment and dynamical histories on the other. As such it is in fact an irresistibly temporally relational mechanical framework since their is simply no temporal entity available for the substantivalist to reify – in effect a reduction of the possibilities entailed by the multiplicity of parameterisations has been enacted. However, this *reduction* is done by use of the Newtonian temporal increment rather than by a direct geometric reduction of the relevant symmetry.

The correlation strategy is distinguished by providing a reparameterisation invariant description of the change of observables which satisfies the second Dirac-Begrmann criterion of commuting with the constraints but does not make explicit recourse to the reduced space *à la* reductionism. However, as discussed at the end of the last section it leads us to a notion of change which constitutes a radical departure from that used in conventional physical theory. The notion of an instantaneous state is dispensed with and the observables are smeared non-locally along an entire solution as constituted by the *gauge* orbit of the Hamiltonian constraint on the constraint surface in phase space. Thus, like in an anti-Haecceitist reading of a possibility space structure in gauge theory there is a representational correspondence between an equivalence class of *gauge* related points and the fundamental individual entity. However, unlike under an anti-Haecceitist viewpoint the correlation scheme does not treat these points as multiple realisations of the same structure but rather as a collected realisation. The fundamentally original manoeuvre is to redefine the idea of a state such that it is closer to the idea of a history than its original meaning. How should we see the correlation scheme in the context of our various forms of relationalism and substantivalism? Clearly it cannot be interpreted in terms of temporally substantivalist ontology since time or even change in the traditional sense do not feature in the relevant formalism. It is also incompatible with reductive relationalism since it utilises the un-reduced phase space, nor can it be interpreted in unsophisticated relationalist terms because it does not make use of anti-Haecceitism.

Rather, we must consider the possibility that the correlation strategy cannot be naturally interpreted in terms of either a relationalist or substantivalist ontology. If we take the issue of primacy between temporal structure and the relations between instantaneous states of a material system to demarcate the distinction between temporal relationalism and substantivalism then clearly a theory in which there are no instantaneous states or temporal structure will transcend our system of classification. If we define temporal relationalism to mean simply 'not temporally substantivalist' then we can happily think of the correlation scheme as relationalist – but if we are to think more constructively about temporal relationalism in terms of its Machian philosophical underpinnings with the concept of time parasitic on relational change, then the correlation scheme is certainly not relationalist with regard to time since even a derived, relational notion of time cannot be found within the formalism. What kind of ontology should we give to the correlation scheme then if not a temporal relationalist one? The most obvious option would be to take a starkly Parmenidean one – time is purely an illusion and not even a derived or emergent phenomena. There is no change or evolution, merely correlations and timeless states corresponding to histories which cannot be temporally decomposed into instants. In the context of nonrelativistic mechanics adopting such a radical notion of timelessness would seem undesirable given the viability of other options and this, together with the issue of practical applicability, would seem to push us away from adopting the correlation strategy. For addressing the problem of representing change and observables in nonrelativistic timeless mechanics the emergent time strategy clearly provides us with a better option since its interpretation consequences are far more palatable. The case of general relativity, however, is another matter, and in that arena radical timelessness may become a necessity. Since a number of complications.[35] within this more powerful theory must be considered in detail before our arguments can be reconstructed, we will defer this discussion to further work.[36]

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