Entanglement, joint measurement, and state reduction

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Abstract

Entanglement is perhaps the most important new feature of the quantum world. It is expressed in quantum theory by the joint measurement formula. We prove the formula for projection valued observables from a plausible assumption, which for spacelike separated measurements is a consequence of causality. State reduction is simply a way to express the joint measurement formula after one measurement has been made, and its result known.

Keywords: Entanglement, joint measurement, state reduction, causality, measurement problem

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1 Introduction

Entanglement is perhaps the most important new feature of the quantum world. It is expressed in Hilbert space quantum theory by the joint measurement formula (JMF). I prove that the JMF is equivalent to the conjunction of two assumptions. One is NOEFFECT: A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on measurement probabilities for the other member. (The measurement is *nonselective* if we do not use its result to condition measurement probabilities for the other member.)

For projection valued observables, the JMF is equivalent to NOEFFECT alone. We make no effort here to define causality. But we do assume that it includes NOEFFECT.[1] Thus causality implies the JMF for spacelike separated measurements of projection valued observables. The JMF implies violations Bell's inequality, and thus violations of locality. Thus, without assuming the JMF in Hilbert space quantum theory, *causality implies nonlocality.*

"No signaling" theorems have eliminated the worry that the nonlocality in quantum theory violates causality (Jordan, 1983; Zanchini and Barletta, 1991). Our result shows that not only does nonlocality not violate causality, it is required to preserve causality.

We prove that the state reduction formula (SRF) is an immediate corollary of the JMF: state reduction is simply a way to express the JMF after one measurement has been made, and its result known. We then prove the von Neumann-Lüders projection postulate from the SRF. Thus the "postulate" is a theorem, a consequence of the JMF.

All this sheds new light on entanglement, joint measurement, state reduction, nonlocality, and causality in quantum theory.

The paper is organized as follows. Section 2 reviews the postulates of Hilbert space quantum theory, without the JMF or SRF. Section 3 describes my approach to the JMF. Section 4 describes my approach to the SRF. Section 5 describes Masanao Ozawa's approach to the JMF and SRF and compares our two approaches. Section 6 argues that there is no measurement problem. Section 7 gives the example showing that NOEFFECT $\not\Rightarrow$ JMF.

[1] Note added 09/03/28. In the published version, the last two sentences were replaced by "A violation of NOEFFECT in spacelike separated measurements would allow superluminal communication."

2 QT-

To prepare for a discussion of the JMF and SRF, we review the postulates of Hilbert space quantum theory, excluding the JMF and SRF. We call the theory *QT-*. For more details, see Kraus, 1983 and Busch *et al.*, 1991.

A *quantum system* $\mathbf{S}$ is represented by a complex Hilbert space $\mathsf{H}_{\mathbf{S}}$, which in this paper will be finite dimensional. A *preparation* of $\mathbf{S}$ is represented by a *state*, a density operator σ on $\mathsf{H}_{\mathbf{S}}$. A *measurement* of $\mathbf{S}$ is represented by an *observable*, a positive operator valued measure (POVM) $\mathcal{S}$. Let $\mathcal{S}$ map the measured value s to E_s, $0 \leq E_s \leq \mathrm{I}$. According to the *measurement formula*, the probability of result s for an $\mathcal{S}$ measurement on state σ is $\Pr(s) = \mathrm{Tr}(E_s\sigma)$.

If $\mathbf{S}$ is isolated, then σ evolves unitarily according to *Schrödinger's equation*: $\sigma \to U_{\mathbf{S}}\, \sigma\, U_{\mathbf{S}}^\dagger$. Important: for now, "isolated" excludes "entangled with another system". The extent to which Schrödinger's equation applies to a quantum system entangled with another will be the focus of §5.

Let $\mathbf{P}$ be another quantum system. Then $\mathbf{S}+\mathbf{P}$ is represented by $\mathsf{H}_{\mathbf{S}} \otimes \mathsf{H}_{\mathbf{P}}$. Thus the states τ of $\mathbf{S}+\mathbf{P}$ are density operators on $\mathsf{H}_{\mathbf{S}} \otimes \mathsf{H}_{\mathbf{P}}$, and the observables are POVMs whose values are positive operators on $\mathsf{H}_{\mathbf{S}} \otimes \mathsf{H}_{\mathbf{P}}$. A measurement of $\mathcal{S}$ on $\mathbf{S}+\mathbf{P}$ is represented by the POVM which maps s to $E_s \otimes \mathrm{I}$. Then from the measurement formula, $\Pr(s) = \mathrm{Tr}[(E_s \otimes \mathrm{I})\tau]$. The systems $\mathbf{S}$ and $\mathbf{P}$ *do not interact* if the unitary evolution operator of $\mathbf{S}+\mathbf{P}$ factors: $U_{\mathbf{S}+\mathbf{P}} = U_{\mathbf{S}} \otimes U_{\mathbf{P}}$.

If for some state σ, $\Pr(s) = \mathrm{Tr}(E_s\sigma)$ for every observable $\mathcal{S}$ and every result s, then σ is the state of $\mathbf{S}$. For the $\mathrm{Tr}(E_s\sigma)$ uniquely determine the state σ. We say that "probabilities determine states".

For reference we list several identities which we will use without comment: $\mathrm{Tr}(XY) = \mathrm{Tr}(YX)$, $\langle s_1 \otimes p_1 | s_2 \otimes p_2 \rangle = \langle s_1 | p_1 \rangle \langle s_2 | p_2 \rangle$, $X \otimes Y = (X \otimes \mathrm{I})(\mathrm{I} \otimes Y)$, and $(X \otimes Y)|s \otimes p\rangle = X|s\rangle \otimes Y|p\rangle$. The *partial trace* operator $\mathrm{Tr}_{\mathbf{P}}$ maps operators on $\mathbf{S}+\mathbf{P}$ to operators on $\mathbf{S}$ (Cohen-Tannoudji *et al.*, 1997). We have the partial trace identities $\mathrm{Tr}(X) = \mathrm{Tr}[\mathrm{Tr}_{\mathbf{P}}(X)]$ and $\mathrm{Tr}_{\mathbf{P}}[(X \otimes \mathrm{I})Y] = X\mathrm{Tr}_{\mathbf{P}}(Y)$ (Kraus, 1983; Busch *et al.*, 1991). Using these identities and "probabilities determine states", we see that if the state of $\mathbf{S}+\mathbf{P}$ is τ, then the state of $\mathbf{S}$ is $\mathrm{Tr}_{\mathbf{P}}(\tau)$:

$$\Pr(s) = \mathrm{Tr}[(E_s \otimes \mathrm{I})\tau] = \mathrm{Tr}\left\{\mathrm{Tr}_{\mathbf{P}}\left[(E_s \otimes \mathrm{I})\tau\right]\right\} = \mathrm{Tr}\left[E_s\, \mathrm{Tr}_{\mathbf{P}}(\tau)\right]. \qquad (1)$$

3 Joint Measurement

In this section and the next we prove results about joint measurement, state reduction, causality, and nonlocality in the theory QT- defined in §2.

Joint Measurement Formula. *Prepare* $\mathbf{S}+\mathbf{P}$ *in state* τ *at time* t_1*, after which* $\mathbf{S}$ *and* $\mathbf{P}$ *do not interact. Let* $U_{\mathbf{S}}$ *be the unitary evolution operator for* $\mathbf{S}$ *from* t_1 *to* $t_{\mathbf{S}} \geq t_1$*. At time* $t_{\mathbf{S}}$ *measure observable* $\mathcal{S}$ *of* $\mathbf{S}$*, with result* s*. Define* $U_{\mathbf{P}}$*,* $t_{\mathbf{P}}$*,* $\mathcal{P}$*, and* p *similarly. (The time order of the two measurements is irrelevant.*[2]*) Then*

$$\Pr(s \,\&\, p) = \mathrm{Tr}\left[\left(U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes U_{\mathbf{P}}^{\dagger} E_p U_{\mathbf{P}}\right) \tau\right]. \tag{JMF}$$

(We are only stating, not assuming, JMF.)

For given $t_{\mathbf{P}}, \mathcal{P}, t_{\mathbf{S}}$, and $\mathcal{S}$ let the POVM representing the joint measurement map the result (s, p) to $E_{s \,\&\, p}$. Then according to the measurement formula, $\Pr(s \,\&\, p) = \mathrm{Tr}(E_{s \,\&\, p}\tau)$ for all s, p, and τ. Thus the JMF for the measurement is equivalent to

$$\forall s, p \quad E_{s \,\&\, p} = U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes U_{\mathbf{P}}^{\dagger} E_p U_{\mathbf{P}}. \tag{2}$$

The (nonselective) probability of s is $\sum_p \Pr(s \,\&\, p) = \mathrm{Tr}\left[\left(\sum_p E_{s \,\&\, p}\right) \tau\right]$. If the $\mathcal{P}$ measurement is not made, then according to Theorem 7,

$$\Pr(s) = \mathrm{Tr}\left[E_s \left(U_{\mathbf{S}} \mathrm{Tr}_{\mathbf{P}}(\tau) U_{\mathbf{S}}^{\dagger}\right)\right] = \mathrm{Tr}\left[\left(U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes \mathrm{I}\right) \tau\right].$$

NOEFFECT from §1 asserts that the two probabilities are equal:

> A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on measurement probabilities for the other member.

Thus according to NOEFFECT,

$$\forall s \quad \sum_p E_{s \,\&\, p} = U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes \mathrm{I}. \tag{NOEFFECT}$$

Similarly,

$$\forall p \quad \sum_s E_{s \,\&\, p} = \mathrm{I} \otimes U_{\mathbf{P}}^{\dagger} E_p U_{\mathbf{P}}. \tag{NOEFFECT}$$

Consider also the assertion that $E_{s \,\&\, p}$ is the product of its marginals:

$$\forall s, p \quad E_{s \,\&\, p} = \left(\sum_i E_{s \,\&\, i}\right)\left(\sum_j E_{j \,\&\, p}\right). \tag{PRODMARG}$$

[2] Note added after publication, 06/07/02. H. Zbinden, et. al (Phys. Rev. A 63, 022111 (2001)) report an EPR type experiment in which the two measurements were made by devices at rest in different inertial frames. In each frame the measurement preceded, from the point of view of that frame, the other measurement. The results conformed to the JMF.

Theorem 1. *For given $t_{\mathbf{P}}, \mathcal{P}, t_{\mathbf{S}}$, and $\mathcal{S}$,*

$$\text{JMF} \Leftrightarrow (\text{NOEFFECT \& PRODMARG}).$$

Proof. We use the JMF in the form Eq. (2).

JMF $\Rightarrow$ NOEFFECT. Sum Eq. (2) over p and use $\sum_p E_p = \mathrm{I}$. (This is the no signaling theorem of Jordan, 1983.)

JMF $\Rightarrow$ PRODMARG. Multiply the two NOEFFECT equations, which we have just shown follow from the JMF, and use Eq. (2) to obtain PRODMARG.

(NOEFFECT & PRODMARG) $\Rightarrow$ JMF. Multiply the two NOEFFECT equations and use PRODMARG to obtain Eq. (2). □

Corollary 2. *If $\mathcal{P}$ and $\mathcal{S}$ are projection valued, then* JMF $\Leftrightarrow$ NOEFFECT.

Proof. From the theorem, it is sufficient to prove that if $\mathcal{P}$ and $\mathcal{S}$ are projection valued, then NOEFFECT $\Rightarrow$ PRODMARG. For a projection valued $\mathcal{S}$, the E_s are orthogonal projections. Thus the $U_{\mathbf{S}}^\dagger E_s U_{\mathbf{S}} \otimes \mathrm{I}$ on the right side of the first NOEFFECT equation are orthogonal projections. Sums of these projections are projections. Every POVM on a product space with projection valued marginal measures satisfies PRODMARG (Davies, 1976, Th. 2.1, Eq. 2.7). □

The example E' of §7 shows that for general POVMs, NOEFFECT $\not\Rightarrow$ JMF.

The implication NOEFFECT $\Rightarrow$ JMF for projection valued observables is of special interest. As noted in §1, for spacelike separated measurements causality implies NOEFFECT. Thus,

Corollary 3. In QT-, *causality implies the JMF for spacelike separated measurements of projection valued observables.*

The JMF predicts violations of Bell's inequality for some spacelike separated measurements of projection valued observables. It thus predicts violations of locality. Thus,

Corollary 4. In QT-, *causality implies nonlocality.*

We have only been discussing measurement *values*; nothing has been said about postmeasurement *states*.

The commonly assumed *projection postulate* describes these states. Its simplest form is concerned with the measurement of a nondegenerate projection valued observable. The postulate states that after the observable is measured, the postmeasurement state of the measured system is the eigenvector of the observable associated with the measured eigenvalue. But this is not always so. Consider, for example, a momentum measurement on a neutron made by observing a recoil proton. Or a photon polarization measurement which destroys the photon in a photographic plate.

The next section addresses postmeasurement states.

4 State Reduction

Since probabilities determine states, we can reformulate NOEFFECT:

> A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on the state of the other member.

But the SRF says that if we make a *selective* measurement, conditioning the state of $\mathbf{S}$ on the $\mathcal{P}$ measurement result, then we must *reduce* the state of $\mathbf{S}$:

State Reduction Formula. *Prepare* $\mathbf{S}+\mathbf{P}$ *in state* τ *at time* t_1*, after which* $\mathbf{S}$ *and* $\mathbf{P}$ *do not interact. At* t_1 *measure observable* $\mathcal{P}$ *of* $\mathbf{P}$*, with result* p*. Let* $U_{\mathbf{S}}$ *be the unitary evolution operator of* $\mathbf{S}$ *over the time of the* $\mathcal{P}$ *measurement. Let* σ_p *be the state of* $\mathbf{S}$ *after the* $\mathcal{P}$ *measurement, conditioned on* p*. Then*

$$\sigma_p = U_{\mathbf{S}} \frac{\mathrm{Tr}_{\mathbf{P}}[(\mathrm{I} \otimes E_p)\tau]}{\mathrm{Tr}\,[(\mathrm{I} \otimes E_p)\tau]} U_{\mathbf{S}}^{\dagger}. \qquad \text{(SRF)}$$

(We are only stating, not assuming, SRF.)

Remarks. (i) The SRF makes *no* assumptions about the state of $\mathbf{P}$ after the $\mathcal{P}$ measurement, even that $\mathbf{P}$ still exists. (ii) Since we do not assume that Schrödinger's equation applies to a system entangled with another, we cannot interpret the SRF as giving the evolution of $\mathbf{S}$ during the $\mathcal{P}$ measurement. (iii) It is *classical* information, i.e., p, which allows us to reduce the state of $\mathbf{S}$ to σ_p. (iv) From the SRF, $\sum_p \mathrm{Pr}(p)\sigma_p = U_{\mathbf{S}} \mathrm{Tr}_{\mathbf{P}}(\tau) U_{\mathbf{S}}^{\dagger}$, the unreduced state.

Theorem 5. JMF $\Rightarrow$ SRF.

Proof. Measure $\mathcal{S}$ immediately after the $\mathcal{P}$ measurement. From the JMF,

$$\mathrm{Pr}(s \,\&\, p) = \mathrm{Tr}\,\{(U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes E_p)\tau\}. \qquad (3)$$

Thus for every $\mathcal{S}$ and every s,

$$\begin{aligned} \mathrm{Pr}\,(s \,|\, p) &= \frac{\mathrm{Pr}\,(s \,\&\, p)}{\mathrm{Pr}\,(p)} = \frac{\mathrm{Tr}\left\{\left(U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes E_p\right)\tau\right\}}{\mathrm{Tr}\,[(\mathrm{I} \otimes E_p)\,\tau]} \\ &= \frac{\mathrm{Tr}\left\{\mathrm{Tr}_{\mathbf{P}}\left[\left(U_{\mathbf{S}}^{\dagger} E_s U_{\mathbf{S}} \otimes \mathrm{I}\right)(\mathrm{I} \otimes E_p)\,\tau\right]\right\}}{\mathrm{Tr}\,[(\mathrm{I} \otimes E_p)\,\tau]} \\ &= \mathrm{Tr}\left\{E_s\left(U_{\mathbf{S}} \frac{\mathrm{Tr}_{\mathbf{P}}[(\mathrm{I} \otimes E_p)\tau]}{\mathrm{Tr}\,[(\mathrm{I} \otimes E_p)\tau]} U_{\mathbf{S}}^{\dagger}\right)\right\}. \end{aligned} \qquad (4)$$

Since probabilities determine states, the SRF follows. □

(For more on this kind of reasoning to obtain state reduction, see Svetlichny, 2002.)

Conversely, given the SRF, a rearrangement of Eq. (4) proves Eq. (3). Thus

> *State reduction is simply a way to express the JMF after one measurement has been made, and its result known.*

K. Kraus makes a similar statement: "[State reductions] provide a convenient 'shorthand' description of correlation measurements. We may thus conclude that, contrary to widespread belief, [state reductions] can be perfectly well understood, *if* quantum mechanics is assumed to be valid also for measuring instruments." (Kraus, 1983, p. 99; my emphasis.) Our proof of the SRF does not assume that quantum mechanics is valid for measuring instruments. Thus Kraus' *if* clause is unnecessary. In view of §6, this is an important improvement.

Corollary 6. *If* $\mathcal{P}$ *is projection valued, then* NOEFFECT $\Rightarrow$ SRF.

Proof. Measure a projection valued observable $\mathcal{S}$ immediately after the $\mathcal{P}$ measurement. Then Corollary 2 implies Eq. (3), which implies Eq. (4) for projections E_s, which is sufficient to imply the SRF for the $\mathcal{P}$ measurement. □

We close this section with a discussion of the von Neumann-Lüders measurement model. Let $\mathbf{S}$ be a quantum *system* to be measured and $\mathbf{P}$ be a quantum *probe*, which is part of a macroscopic measuring apparatus. Initially $\mathbf{S}$ and $\mathbf{P}$ are separated and unentangled, and in states σ_0 and π_0. The system enters the measuring apparatus, interacts with the probe, and leaves the apparatus. Let $\tau = U(\sigma_0 \otimes \pi_0)\, U^\dagger$ be the state of $\mathbf{S}+\mathbf{P}$ after the interaction, which is called a *premeasurement*. (A premeasurement is *not* a measurement: a premeasurement is reversible and no measured value is created.) Now measure $\mathcal{P}$, with the result p appearing on the measuring apparatus. In the von Neumann-Lüders model, the $\mathcal{P}$ measurement serves as a proxy for an $\mathcal{S}$ measurement.

The model is for projection valued $\mathcal{S}$ with an associated self-adjoint operator $\sum_{ij} s_i |s_{ij}\rangle\langle s_{ij}|$. Let $\mathcal{P}$ be a nondegenerate projection valued observable with an associated self-adjoint operator $\sum_i p_i |p_i\rangle\langle p_i| = \sum_i p_i E_i$. Choose a unitary operator U with $U(|s_{ij}\rangle|p_0\rangle) = |s_{ij}\rangle|p_i\rangle$ for some fixed initial state $|p_0\rangle$ of $\mathbf{P}$. Then for an initial vector state $|s_0\rangle = \sum_{ij} a_{ij}|s_{ij}\rangle$ of $\mathbf{S}$, $U(|s_0\rangle|p_0\rangle) = \sum_{ij} a_{ij}|s_{ij}\rangle|p_i\rangle \equiv |t\rangle$. For a $\mathcal{P}$ measurement on state $|t\rangle$, $\Pr(p_k) = \sum_j |a_{kj}|^2$. For an $\mathcal{S}$ measurement on state $|s_0\rangle$, $\Pr(s_k)$ has the same value. Thus a $\mathcal{P}$ measurement on state $|t\rangle$ with result p_k is also an $\mathcal{S}$ measurement on state $|s_0\rangle$ with result s_k.

The SRF gives the reduced state σ_{s_k} of $\mathbf{S}$ after the $\mathcal{S}$ measurement. To apply it, use the identity $\mathrm{Tr}_{\mathbf{P}}[(\mathrm{I}\otimes X)Y] = \mathrm{Tr}_{\mathbf{P}}[Y(\mathrm{I}\otimes X)]$ (Kraus, 1983, Eq. 5.15):

$$\begin{aligned}
\mathrm{Tr}_{\mathbf{P}}\left\{(\mathrm{I}\otimes E_{p_k})\,\tau\right\} &= \mathrm{Tr}_{\mathbf{P}}\left\{(\mathrm{I}\otimes E_{p_k})|t\rangle\,\langle t|(\mathrm{I}\otimes E_{p_k})\right\} \\
&= \mathrm{Tr}_{\mathbf{P}}\left\{\Sigma_j a_{kj}|s_{kj}\rangle|p_k\rangle\;\Sigma_j \bar{a}_{kj}\langle s_{kj}|\langle p_k|\right\} \\
&= \mathrm{Tr}_{\mathbf{P}}\left\{E_{s_k}|s_0\rangle|p_k\rangle\,\langle s_0|E_{s_k}\langle p_k|\right\} \\
&= E_{s_k}|s_0\rangle\langle s_0|E_{s_k}.
\end{aligned}$$

Substitute this into the SRF:

$$\sigma_{s_k} = U_{\mathbf{S}}\,\frac{E_{s_k}|s_0\rangle\,\langle s_0|E_{s_k}}{\mathrm{Tr}\left\{E_{s_k}|s_0\rangle\,\langle s_0|E_{s_k}\right\}}\,U_{\mathbf{S}}^\dagger.$$

As a vector, the reduced state is $U_{\mathbf{S}}E_{s_k}|s_0\rangle/\|E_{s_k}|s_0\rangle\|$. This is the state given by the *von Neumann-Lüders projection postulate*. Since JMF $\Rightarrow$ SRF, the "postulate" is a *theorem* of QT- + JMF.

5 Ozawa's Approach

Masanao Ozawa has published several papers on joint measurement and state reduction (Ozawa, 1997a, 1997b, 1998a, 1998b, 2000a, 2000b, 2000c). He argues, correctly I believe, that existing proofs of the JMF and SRF are inadequate or flawed (Ozawa, 2000a, p. 6; 1998a, p. 616; 1997b, p. 123; 1997a, p. 233). He then offers his own proofs of the JMF (Ozawa, 1997a, Th. 5.1; 2000a, Th. 3) and the SRF (Ozawa 1998a, Eq. 32; 1997b, Eq. 43). Ozawa considers projection valued observables only.

As emphasized in §2, QT- does not assume that Schrödinger's equation applies to a quantum system entangled with another. But we can prove:

Theorem 7. *A unitary evolution of one member of a pair of entangled noninteracting systems has no effect on the state of the other member.*

Proof. Since $\mathbf{S}$ and $\mathbf{P}$ do not interact, the unitary evolution operator of $\mathbf{S}+\mathbf{P}$ factors: $V_{\mathbf{S}+\mathbf{P}} = V_{\mathbf{S}} \otimes V_{\mathbf{P}}$. Let τ be the initial state of $\mathbf{S}+\mathbf{P}$. Then for all E_s,

$$\begin{aligned}
\mathrm{Tr}[E_s(V_{\mathbf{S}}\mathrm{Tr}_{\mathbf{P}}(\tau)V_{\mathbf{S}}^{\dagger})] &= \mathrm{Tr}[(E_s \otimes \mathrm{I})(V_{\mathbf{S}} \otimes \mathrm{I})\tau(V_{\mathbf{S}}^{\dagger} \otimes \mathrm{I})] \\
&= \mathrm{Tr}[(E_s \otimes \mathrm{I})(\mathrm{I} \otimes V_{\mathbf{P}}^{\dagger})(\mathrm{I} \otimes V_{\mathbf{P}})(V_{\mathbf{S}} \otimes \mathrm{I})\tau(V_{\mathbf{S}}^{\dagger} \otimes \mathrm{I})] \\
&= \mathrm{Tr}[(E_s \otimes \mathrm{I})(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^{\dagger}] \qquad (5) \\
&= \mathrm{Tr}\{E_s \mathrm{Tr}_{\mathbf{P}}[(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^{\dagger}]\}.
\end{aligned}$$

Since probabilities determine states, $\mathrm{Tr}_{\mathbf{P}}[(V_{\mathbf{S}} \otimes V_{\mathbf{P}})\tau(V_{\mathbf{S}} \otimes V_{\mathbf{P}})^{\dagger}] = V_{\mathbf{S}}\mathrm{Tr}_{\mathbf{P}}(\tau)V_{\mathbf{S}}^{\dagger}$; the state of $\mathbf{S}$ at a later time is the same as the state given by Schrödinger's equation applied to $\mathbf{S}$ alone. □

(This is the no signaling theorem of Zanchini and Barletta, 1991, Th. 3.)

For projection valued observables, we proved the JMF in Corollary 2 and the SRF in Corollary 6 from the assumption NOEFFECT:

> A nonselective measurement of one member of a pair of entangled noninteracting systems has no effect on the unreduced state of the other member.

(We use the reformulated version following Corollary 4 and add the word "unreduced" for clarity and comparison.)

Ozawa uses a different assumption:

> A selective measurement of one member of a pair of entangled noninteracting systems has no effect on the reduced state of the other member.

(In Ozawa, 1998a see the discussions surrounding Eqs. (5), (6), and (15), and also p. 622.)

One example of Ozawa's use of his assumption is in his proof of the JMF in Ozawa, 1997a, Th. 5.1, when passing from the third to the fourth member in the equation between Eqs. (9) and (10). (Ozawa has confirmed this reading in a private communication.) Another example is in his proof of the SRF in Ozawa, 1998a, Sec. 7.

Ozawa agrees that the SRF gives the reduced state σ_p after the $\mathcal{P}$ measurement, but his assumption rules out our view that the reduction occurs with the measurement, a view he rejects (Ozawa, 1997b, p. 123). For him, the reduction occurs earlier, with the *premeasurement*, to a state that we denote σ_p^1. (σ_p^1 is denoted $\rho(t + \Delta t \,|\, \mathbf{a}(t) \in \{p\})$ in Ozawa, 2000a, and $\rho(t + \Delta t \,|\, p)$ in Ozawa, 1998a and 1997b.) (Warning: Ozawa sometimes calls just the premeasurement – which he calls *stage 1* – a "measurement" (Ozawa, 1998a, Eq. (1); 1997b, Eq. (1))).

According to Ozawa, σ_p^1 is the state of $\mathbf{S}$ after the premeasurement, "conditional upon" the result p of the later $\mathcal{P}$ measurement (Ozawa, 2000a, p. 9), or "that leads to the outcome p" in the measurement (Ozawa, 1997b, p. 124). More specifically:

> Suppose the system and probe are spin-$\frac{1}{2}$ particles brought into the singlet state by the premeasurement. After the premeasurement is complete, we can choose to measure the spin of the probe in the z-direction or the x-direction. If we choose the z-direction and the result is "up", then the system was prepared in the "down" eigenstate $\sigma_\downarrow^1$ just after the premeasurement. If we choose the x-direction and the result is "left", then the system was prepared in the "right" eigenstate $\sigma_\rightarrow^1$ just after the premeasurement. [Private communication.]

If, according to Ozawa's assumption, $\mathbf{S}$ evolves unitarily from after the premeasurement until after the probe measurement, and if its state after the probe measurement is σ_p, then its state after the premeasurement is, from the SRF,

$$\frac{\mathrm{Tr}_{\mathbf{P}}\left[(\mathrm{I} \otimes E_p)\tau\right]}{\mathrm{Tr}\left[(\mathrm{I} \otimes E_p)\tau\right]}.$$

This is Ozawa's expression for σ_p^1 (Ozawa, 1998a, Eq. (32); 1997b, Eq. (34)). For him, the SRF describes a unitary evolution of $\mathbf{S}$ from σ_p^1 to σ_p. For me, the SRF does not describe an evolution of $\mathbf{S}$, as stated in the remarks following the SRF.

Bell's inequality is relevant here. The inequality shows that not only is the result p of the probe measurement not known before the measurement, *it does not exist* before the measurement. This even though p would be correlated with the result of a later measurement of $\mathcal{S}$. Mermin explains this clearly (Mermin, 1981 and 1985).

For me, this makes the states σ_p^1 problematic. Furthermore, they are not needed to obtain the SRF: we proved in §4 that the correlations given by the JMF imply that the state of $\mathbf{S}$ after the $\mathcal{P}$ measurement is given by the SRF. State reduction is not a *dynamical* consequence of Schrödinger's equation; it is a *logical* consequence of entanglement.

To reject attributing the state reduction of $\mathbf{S}$ to the $\mathcal{P}$ measurement is to cling to classical notions of causality, instead of fully embracing that remarkable new quantum phenomenon, entanglement.

6 The Measurement Problem

We have been careful to distinguish the probe **P**, a *quantum* system, from the macroscopic apparatus measuring it. We made no assumptions about the apparatus other than the minimal requirement that it display measurement results in accordance with the measurement formula. In particular, we did not model it as a quantum system obeying Schrödinger's equation. Modeling the apparatus in this way leads to the notorious *measurement problem*: the appearance of a definite measured value on the apparatus would be a state reduction of the apparatus, which is inconsistent with Schrödinger's equation.

I argue at length elsewhere that the apparatus cannot be so modelled and thus there is no measurement problem (Macdonald, 2002). Here I support this point of view only with the following quotes.

In *The Quantum Theory of Measurement*, P. Busch, P. Lahti, and P. Mittelstaedt write: "The quantum theory of measurement is motivated by the idea of the universal validity of quantum mechanics, according to which this theory should be applicable, in particular, to the measuring process. One would expect, and most researchers in the foundations of quantum mechanics have done so, that the problem of measurement should be solvable *within* quantum mechanics. The long history of this problem shows that ... there seems to be no straightforward route to its solution." (Busch *et al.*, 1991, p. 138)

K. Kraus also describes the measuring apparatus as a quantum system (Kraus, 1983, pp. 81, 99). But "There are good reasons to doubt that quantum mechanics in its present form is the appropriate theory of macroscopic systems." (Kraus, 1983, p. 100)

According to A. Leggett, "What is required is to explain how one particular macrostate can be forced by the quantum formalism to be realized. In the opinion of the present author (which is shared by a small but growing minority of physicists) no solution to this problem is possible within the framework of conventional quantum mechanics." (Leggett, 1992, p. 231)

W. Zurek writes, "The key (and uncontroversial) fact has been known almost since the inception of quantum theory, but its significance ... is being recognized only now: macroscopic systems are never isolated from their environment. Therefore they should not be expected to follow Schrödinger's equation, which is applicable only to a closed system." (Zurek, 1991)

J. Bub claims that three information-theoretic constraints (NOEFFECT, "no cloning" of quantum states, and the impossibility of unconditionally secure bit commitment) together imply that "no mechanical theory of quantum phenomena that includes an account of measurement interactions can be acceptable." (Bub, 2004)

J. Bub and I. Pitowsky write, "There is no dynamical explanation for the definite occurrence of a particular measurement outcome, as opposed to other possible measurement outcomes in a quantum measurement process – the occurrence is constrained by the kinematic probabilistic correlations encoded in the projective geometry of Hilbert space, and only by these correlations." (Bub and Pitowski, 2007)

7 NOEFFECT $\nRightarrow$ PRODMARG

Consider the following measurement. A spin-$\frac{1}{2}$ particle **S** moving in the y-direction enters a Stern-Gerlach device oriented in the z-direction. In each output beam ($\pm z$) there is a SG device oriented in the x-direction. Detect **S** leaving one of the x-direction SG devices. Assign a value 0 to the measurement if **S** is detected in a $-x$ beam and a 1 if in a $+x$ beam. Then for every state of **S**, $\Pr(0) = \Pr(1) = \frac{1}{2}$. Think of this triple SG device as a fair coin tosser. The POVM $E_0 = E_1 = \frac{1}{2}\mathrm{I}$ represents the measurement: for every state σ of **S**, $\mathrm{Tr}(E_0\sigma) = \mathrm{Tr}(E_1\sigma) = \frac{1}{2}$.

Let **P** be another spin-$\frac{1}{2}$. Measure both **S** and **P** with triple SG devices. Absent any assumption about the joint measurement probabilities, we can imagine different POVMs giving those probabilities. One possibility is $E_{s\,\&\,p}$ with $E_{0\,\&\,0} = E_{0\,\&\,1} = E_{1\,\&\,0} = E_{1\,\&\,1} = \frac{1}{4}\mathrm{I}\otimes\mathrm{I}$. Another is $E'_{s\,\&\,p}$ with $E'_{0\,\&\,0} = E'_{1\,\&\,1} = \frac{1}{2}\mathrm{I}\otimes\mathrm{I}$ and $E'_{0\,\&\,1} = E'_{1\,\&\,0} = 0$. For every state of $\mathbf{S}+\mathbf{P}$, $E_{s\,\&\,p}$ predicts two *independent* fair coin tosses and $E'_{s\,\&\,p}$ predicts two *correlated* fair coin tosses, 0 with 0 and 1 with 1.

Straightforward calculations show that $E_{s\,\&\,p}$ satisfies both NOEFFECT and PRODMARG. From these, we can see that the JMF implies that $E_{s\,\&\,p}$ represents the joint measurement:

$$\begin{aligned}\Pr(s\,\&\,p) &= \mathrm{Tr}\left[(E_s\otimes E_p)\,\tau\right] = \mathrm{Tr}\left[(E_s\otimes\mathrm{I})\,(\mathrm{I}\otimes E_p)\,\tau\right]\\ &= \mathrm{Tr}\left[\left(\sum_p E_{s\,\&\,p}\right)\left(\sum_s E_{s\,\&\,p}\right)\tau\right] = \mathrm{Tr}(E_{s\,\&\,p}\tau).\end{aligned}$$

The POVM $E'_{s\,\&\,p}$ satisfies NOEFFECT but not PRODMARG. Thus NOEFFECT $\nRightarrow$ PRODMARG.

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