Time reversal in classical electromagnetism

Frank Arntzenius[†] and Hilary Greaves[*]

November 9, 2007

Abstract

Richard Feynman has claimed that anti-particles are nothing but particles 'propagating backwards in time'; that time reversing a particle state always turns it into the corresponding anti-particle state. According to standard quantum field theory textbooks this is not so: time reversal does not turn particles into anti-particles. Feynman's view is interesting because, in particular, it suggests a nonstandard, and possibly illuminating, interpretation of the CPT theorem.

In this paper, we explore a classical analog of Feynman's view, in the context of the recent debate between David Albert and David Malament over time reversal in classical electromagnetism.

1 Introduction

> A backwards-moving electron when viewed with time moving forwards appears the same as an ordinary electron, except it's attracted to normal electrons - we say it has positive charge. For this reason it's called a 'positron'. The positron is a sister to the electron, and it is an example of an 'anti-particle'. This phenomenon is quite general. Every particle in Nature has an amplitude to move backwards in time, and therefore has an anti-particle. (Feynman, 1985):98

Note that Feynman is not making any claims about backwards causation. He is merely claiming that if you time reverse a sequence of particle states you get a sequence of corresponding anti-particle states. According to standard quantum field theory textbooks this is not so: the *charge conjugation* operator turns particles into antiparticles, but

[†] University College, Oxford. *Email:* frank.arntzenius@philosophy.ox.ac.uk.
[*] Merton College, Oxford OX1 4JD. *Email:* hilary.greaves@philosophy.ox.ac.uk.

time reversal does not. So we read Feynman as suggesting that the *real* time reversal operation (whatever that may mean — on which more below!) is not the operation that is usually given that name. Or, at least, that is the view that we are interested in comparing to the standard view, and that is the view we will call 'Feynman's view'.

Feynman's remarks, of course, were made in the context of quantum field theory. Meanwhile, in classical electromagnetism: David Albert (Albert, 2000) has argued that classical electromagnetism is not time reversal invariant, because (according to him) there is no justification for flipping the sign of the magnetic field under time reversal. David Malament (Malament, 2004) has replied in defense of the standard view of time reversal, according to which the **B** field does flip sign and the theory is time reversal invariant.

Malament's discussion may leave one with the feeling that one only has to appreciate both (i) the four-dimensional formulation of classical electromagnetism and (ii) what we *mean*, or ought to mean, by 'time reversal', and the standard transformation $\mathbf{B} \xmapsto{T} -\mathbf{B}$ will follow. This, however, is incorrect: there is an alternative to Malament's account, consistent with both (i) and (ii). It is an account according to which the magnetic field does *not* flip sign under time reversal (the *electric* field does), but the theory is time reversal invariant anyway; it is the classical analog of Feynman's view.

This paper has two main aims: (i) to explore the 'classical Feynman' view, with the hope that this may later illuminate important issues in quantum field theory, and, relatedly, (ii) to explore a novel conception of time reversal, distinct from the usual notions of 'active' and 'passive' time reversal, that we think is implicit in Malament's work and deserves further attention.

The structure of the paper is as follows. In section 2 we discuss the standard account of what time reversal is, and why one should care about it. Section 3 is a critical review of the existing debate concerning time reversal in classical electromagnetism: the standard 'textbook' account, Albert's objection, and Malament's reply. One of the things this discussion throws up is the contrast between Malament's notion of time reversal, which we call 'geometric' time reversal, on the one hand, and the familiar notions of 'active' and 'passive' time reversal on the other; we articulate the 'geometric' notion in the course of discussing Malament's reply. In section 4 we articulate the 'Feynman' account, in terms of geometric time reversal. Section 5 investigates the possibility of 'deflating' the apparent dispute between the 'Malament' and 'Feynman' accounts, and regarding them as equivalent descriptions of the same underlying reality. Section 6 is the conclusion.

2 Time reversal and the direction of time

Let's start with the more-or-less standard account of what time reversal is, and why one should be interested in it.

Suppose we describe a world (or part of a world) using some set of coordinates x, y, z, t. A **passive time reversal** is what happens to this description when we describe the same world but instead use coordinates x, y, z, t' where $t' = -t$. An **active time reversal** is the following: keep using the same coordinates, but change the world in such a way that the description of the world in these coordinates changes exactly as it does in the corresponding passive time reversal. (So active and passive time reversal have exactly the same effect on the coordinate dependent descriptions of worlds.)

Suppose now that we have a theory which is stated in terms of coordinate dependent descriptions of the world, i.e. a theory which says that only certain coordinate dependent descriptions describe physically possible worlds. Such a theory is said to be **time reversal invariant** iff time reversal turns solutions into solutions and non-solutions into non-solutions. (Since active and passive time reversals have the same effect on the coordinate dependent descriptions of worlds, it follows that coordinate dependent theories will be invariant under active time reversal iff they are invariant under passive time reversal.)

Why might one be interested in the time reversal invariance of theories? Because failure of time reversal invariance of a theory indicates that time has an objective direction according to that theory. Why believe that? Well, suppose that we start with a coordinate dependent description of a world (or part of a world) which our theory allows. And suppose that after we do a passive coordinate transformation our theory says that the new (coordinate dependent) description of this world is no longer allowed. This seems odd: it's the same world after all, just described using one set of coordinates rather than another. How could the one be allowed by our theory and the other not? Indeed, this does not make much sense unless one supposes that the theory, as stated in coordinate dependent form, was true in the original coordinates but not in the new coordinates. And that means that according to the theory there is some objective difference between the x, y, z, t coordinates and the x, y, z, t' coordinates (where $t' = -t$). So time has an objective direction: that is, there is an objectively preferred temporal orientation. And if we want to write our theory in a coordinate independent way we are going to have to introduce a representation of this temporal orientation into our formalism.

Let's now clarify and modify this standard account a little bit. Let's start by asking a question that is rarely asked in physics texts, namely, what determines how things transform under a time reversal transformation? Well, space-time has some coordinate independent structure, and it is inhabited by coordinate independent quantities. We often describe that structure and those quantities in a coordinate dependent manner, but the structure of space-time itself is a coordinate independent geometric structure, and the quantities that inhabit space-time are coordinate independent quantities. This coordinate independent structure and those coordinate independent quantities determine what the coordinate dependent representations of that structure and of those quantities look like, and therefore determine how those coordinate dependent representations transform under space-time transformations. That's all there is to it.

Now, what we have just said might seem rather obvious, rather vague, and hence rather useless. However, there are a few important lessons to be learned from what we have said that are not always heeded.

Firstly, it means some quantities transform non-trivially (i.e. do not remain identical) under time reversal. (Why it is worth noting this will become clear when we discuss David Albert's views on time reversal.)

Secondly, it means that it is not arbitrary how a quantity transforms under time reversal: how a quantity transforms under time reversal is determined by the (geometric) nature of the quantity in question, not by the absence or presence of a desire to make some theory time reversal invariant. For instance, one might think that one can show that some theory which, prima facie, is not time reversal invariant in fact is time reversal invariant, simply by making a judicious choice for how the fundamental quantities occurring in the theory transform under time reversal. However, if one changes one's view as to what the correct time reversal transformations are for the fundamental quantities occurring in a theory, then one is thereby changing one's view as to the geometric nature of those fundamental quantities, and hence one is producing a new, and different, theory of the world rather than showing that the original theory was time reversal invariant. That is to say, in such a circumstance one faces a choice: this theory with these quantities and these invariances or that theory with those quantities and those invariances. If the competing theories are empirically equivalent then one should make such a choice in the usual manner: on the basis of simplicity, naturalness etc.

Thirdly, even if a coordinate dependent formulation of a theory is not invariant under a passive time reversal, this does not yet imply that space-time must have an objective temporal orientation. For coordinate system x, y, z, t and coordinate system x, y, z, t' where $t' = -t$ not only differ in their temporal orientation, they also differ in their space-time handedness. So failure of invariance of the theory under time reversal need not be due to the existence of an objective temporal orientation, it could be due to the existence of an objective space-time handedness. That is to say, one might be able to form two rival coordinate independent theories, one of which postulates an objective temporal orientation but no space-time handedness, while the other postulates an objective space-time handedness but no temporal orientation. In order to decide which is the better theory, one will have to look at other features of the theories (such as other invariances).

More generally, what we want to know is what structure space-time has, and what quantities characterize the state of its contents. If we have in our possession an empirically adequate coordinate dependent theory, then what we should do is manufacture the best corresponding coordinate independent theory that we can, and see what space-time structure and what quantities this coordinate independent theory postulates. In fact, in the end the issue of what the correct time reversal transformation is is a bit of a red herring. What we are really interested in is what space-time structure there is and what quantities there are (and of course we are interested in the equations that govern their interactions). But the invariances and non-invariances of empirically adequate coordinate dependent formulations of theories are useful for figuring that out.

The above discussion was perhaps a bit abstract. So let us turn to a specific case which has been the subject of a fair amount of debate and controversy, namely that of classical electromagnetism.

3 Classical electromagnetism: the story so far

3.1 The standard textbook view

Let's start with the standard textbook account of time reversal in classical electromagnetism. The interaction between charged particles and the electromagnetic field is governed by Maxwell's equations and the Lorentz force law. In a particular coordinate system x, y, z, t, Maxwell's equations can be written as

$$\nabla \cdot \mathbf{E} = \rho \quad (1)$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j} \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (4)$$

and the Lorentz force law can be written as:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (5)$$

Now let us ask how the quantities occurring in these equations transform under time reversal. According to the standard account the active time reverse of a particle that is moving from location A to location B is a particle that is moving from B to A. So, according to the standard view, the ordinary spatial velocity $\mathbf{v}$ must flip over under active time reversal. Obviously, the current $\mathbf{j}$ will also flip over under active time reversal, while the charge density ρ will be invariant under time reversal.

Next let us consider the electric and magnetic fields. How do they transform under time reversal? Well, the standard procedure is simply to assume that classical electromagnetism is invariant under time reversal. From this assumption of time reversal invariance of the theory, plus the fact that $\mathbf{v}$ and $\mathbf{j}$ flip under time reversal while ρ is invariant, it is inferred that the electric field $\mathbf{E}$ is invariant under time reversal, while the magnetic field $\mathbf{B}$ flips sign under time reversal. Summing up, we have:

$$\mathbf{v} \stackrel{T}{\longmapsto} -\mathbf{v}; \tag{6}$$

$$\mathbf{j} \stackrel{T}{\longmapsto} -\mathbf{j}; \tag{7}$$

$$\mathbf{E} \stackrel{T}{\longmapsto} \mathbf{E}; \tag{8}$$

$$\mathbf{B} \stackrel{T}{\longmapsto} -\mathbf{B}; \tag{9}$$

$$\rho \stackrel{T}{\longmapsto} \rho; \tag{10}$$

$$\nabla \stackrel{T}{\longmapsto} \nabla; \tag{11}$$

$$t \stackrel{T}{\longmapsto} -t. \tag{12}$$

It follows from this time reversal transformation, as straightforward inspection of Maxwell's equations and the Lorentz force law can verify, that time reversal turns solutions into solutions and non-solutions into non-solutions.

3.2 Albert's proposal

David Albert ((Albert, 2000), chapter 1) takes issue with the textbooks' account of time reversal in classical electromagnetism. The point of contention is whether or not the magnetic field flips sign under time reversal. The standard account, we have seen, says that it does: $\mathbf{B} \stackrel{T}{\longmapsto} -\mathbf{B}$. Albert suggests, however, that by 'time reversal' one ought to mean 'the *very same thing*' happening in the opposite temporal order; it follows (according to Albert) that the magnetic field (on a given timeslice) will be invariant under time reversal; and it follows from *that* (given Maxwell's equations) that the theory is not time reversal invariant. (Albert is happy with a non-trivial time reversal operation for, say, *velocity*. But that is because velocity is just temporal derivative of position, so of course *it* flips sign under time reversal. Albert's point is that the magnetic field is not the temporal derivative of anything.)

The difference in direction of argument between Albert and the textbooks is worth highlighting. In the textbooks' account reviewed above, the desideratum that the theory should be time reversal invariant enters as a *premise*. One finds some transformation on the set of instantaneous states that has the feature that, if it were the time reversal transformation, then the theory would be time reversal invariant, and one concludes that this is the time reversal operation. Albert is insisting on the opposite direction of argumentation: one should *first* work out which transformation on the set of instantaneous states implements the idea of 'the same thing happening backwards in time'; then and only then one should compare one's time reversal operation to the equations of motion, and find out whether or not the theory is time reversal invariant. He is further insisting that, in the case of electromagnetism, this has not adequately been done.

Albert has a point here. One should, indeed, be wary of taking the textbooks' strategy to extremes: it is not difficult to show that, under very general conditions, *any* theory, including ones that are (intuitively!) not time reversal invariant, can be made to come out 'time reversal invariant' if we place *no* constraints on what counts as the 'time reversal operation' on instantaneous states.[1]

So something in Albert's objection seems to be right. We do not, however, endorse his account of time reversal in electromagnetism. We will come back to this after discussing an alternative account, due to David Malament.

3.3 Malament's proposal

Malament seeks to 'justify' the usual textbook time reversal operation for classical electromagnetism, and for the **B** field in particular.

[1]Here is an example. Suppose that we have a single particle in one dimension. Let r denote its position; let its instantaneous state space be given by $(0, \infty]$. Let its equation of motion be given by

$$\frac{dr}{dt} = -kr, \tag{13}$$

where $k > 0$ is a positive constant. This theory is (intuitively) not time reversal invariant: it says that the particle's position coordinate always *decreases*. However, if we are really willing to let the time reversal operation be whatever is required to secure time reversal invariance, the intuition of asymmetry can easily be violated: simply let the time reversal operation be $r \mapsto \frac{1}{r}$.

At first sight, one might think that this is done as soon as one thinks relativistically, and conceives of the **E** and **B** fields as components of the Maxwell-Faraday tensor F^{ab}. A moment's thought, however, shows that this is not the case. The electric field is read off from the space-time components of F^{ab}, while the magnetic field is read off from the space-space components:[2]

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}. \tag{14}$$

If the Maxwell-Faraday tensor F^{ab} itself (as a tensor) is invariant under time reversal, then it will be the *electric* field, not the magnetic field, that flips sign when we perform a passive time reversal (since the former appears as the time-space components of the Maxwell-Faraday tensor, whereas the latter appears as the space-space components). To justify the standard textbook transformation, we need to justify a sign flip for F^{ab}: $F^{ab} \overset{T}{\longmapsto} -F^{ab}$. This is the task that Malament takes up.

Malament's treatment of electromagnetism embodies a particular conception of what it *means* to 'justify' a time reversal operation, and, relatedly, a third conception (alongside active and passive time reversal) of what time reversal *is*. We will first state these explicitly (but somewhat abstractly), then let our exposition of Malament's treatment of electromagnetism illustrate them:

- To give a **justification** of a non-trivial time reversal operation $X \overset{T}{\longmapsto} X'$ for a state description X is to postulate a particular fundamental ontology for the theory, and to explain how the representation relation between X and the objects of the fundamental ontology depends on temporal orientation, in such a way that it follows that if we flip the temporal orientation but hold the remainder of the fundamental objects fixed, the state description changes as $X \overset{T}{\longmapsto} X'$.
- **Geometric time reversal:** To time-reverse a kinematically possible world, hold all the fundamental quantities fixed [with the exception of the temporal orientation, if that is a fundamental object], and flip the temporal orientation.

We emphasise that this 'geometric' notion of time reversal does *not* coincide either with active, or with passive, time reversal. We

[2] Roman subscripts and superscripts indicate that we are using the abstract index notation: F^{ab} is a rank two tensor, not a component of such a tensor in a particular coordinate system. When we wish to refer to coordinate-dependent components of tensors, we use Greek indices, as in $F^{\mu\nu}$.

can see that geometric and passive time reversal are distinct by, for instance, noting that geometric time reversal generates non-trivial transformations of *coordinate-independent* models — for example, as we will see below, according to Malament the geometric time reversal of the Maxwell-Faraday *tensor* (that is, the tensor itself, *not* its components in any coordinate system) is $F_{ab} \mapsto -F_{ab}$. Hence, one can perform a geometric time reversal while holding the coordinate system fixed, and thus induce a nontrivial transformation on the coordinate-dependent description of the model *in one and the same coordinate system* — something that is obviously impossible under the 'passive' notion of time reversal, which just is a change of coordinate system. We can see that geometric and active time reversal are distinct by noting that geometric time reversal does not, while active time reversal does, move material objects around on the manifold. A further point, and at least part of Malament's own motivation for introducing this notion of time reversal (*ibid.*, p2), is that the geometric notion is applicable in curved spacetimes, in which there may not be any conformal mapping that reverses temporal orientation (as required for the active and passive notions of time reversal).

Malament's treatment of electromagnetism. Malament's account is as follows. There are two fundamental types of objects in a classical electromagnetic world. There are charged particles, and there is the electromagnetic field. Now, the dynamics happens to be such that it will be convenient, mathematically, to represent the motions of particles by means of four-velocities, where the four-velocity at any point on the worldline is tangent to the worldline at that point. The crucial fact now is that a world-line does not have a unique tangent vector at a point: at each point on a world-line, there is a continuous infinity of four-vectors that are tangent to the world-line at the point in question. We can narrow things down somewhat by stipulating that four-velocities are to have unit length, but this still does not quite do the trick: one can associate *two* unit-length four-vectors that are tangent to the world-line at the point in question (if $v^a \in T_p$ is one, then $-v^a$ is the other; see figure 1).

Next, how should we conceive of an electromagnetic field at a point p in spacetime? According to Malament, we should think of the electromagnetic field at p as a quantity which, for any tangent line L at p and charge q, determines what 4-force a (test) particle with charge q and tangent line L at p would experience. More formally, Malament conceives of the electromagnetic field F (*not* F^{ab}) at a point p as a map from pairs $\langle L, q \rangle$ at p to four-vectors at p.

How do Malament's fundamental quantities (tangent lines, maps from tangent lines to 4-vectors) relate to the standard quantities (4-vectors, Maxwell-Faraday tensor) occurring in our three equations? The relation is simple: *relative to a choice of temporal orientation*, one

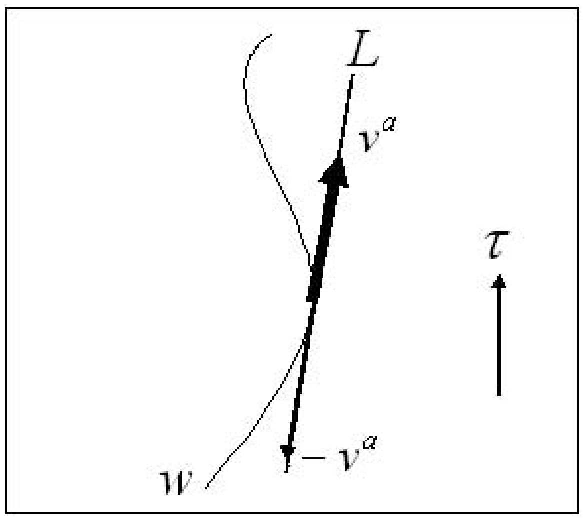


Figure 1: w is the worldline of a particle of mass m and charge q. L is the tangent line to w at the point p. Until we have specified a temporal orientation τ, we have left it open whether the four-velocity is v^a or $-v^a$. $v^b \nabla_b v^a$ is the four-acceleration; it is independent of temporal orientation. The electromagnetic field F maps $< L, q >$ to the four-force $m v^b \nabla_b v^a$.

can associate a unique unit-length tangent vector with each location on a timelike world-line, namely, the one that is 'future'-directed according to that temporal orientation. So, given a temporal orientation, we can represent any given tangent line by a unique unit length four-vector, i.e. a four-velocity. Given such a representation, the electromagnetic field can be represented by a linear map from four-vectors to four-vectors. And that just means that, given a temporal orientation we can represent the electromagnetic field as a rank 2 tensor, which we can identify as the standard representation of the electromagnetic field by the Maxwell-Faraday tensor F^{ab}.

So, given a temporal orientation, Malament can formulate classical electromagnetism using the usual covariantly-formulated equations: the Maxwell equations,

$$\nabla_{[a}F_{bc]} = 0, \tag{15}$$
$$\nabla_n F^{na} = J^a, \tag{16}$$

and the Lorentz force law,

$$qF^a{}_b V^b = mv^b \nabla_b v^a. \tag{17}$$

Using the geometric conception of time reversal, it is then straightforward to see how the quantities in these equations transform under time reversal. Recall that on the geometric conception, to 'time reverse' is to leave all the *fundamental* quantities fixed, and to flip temporal orientation. We then hold fixed (also) our conventions about how non-fundamental quantities are derived from the fundamental ones in an orientation-relative way, and we see which transformations for the non-fundamental quantities result. Now, on Malament's picture, four-velocity is not fundamental: it is defined only relative to a choice of temporal orientation. If v^a is the four-velocity, i.e. is the unit-length future-directed tangent, to a given worldline at some point p relative to our original choice of temporal orientation, then $-v^a$ will be the four-velocity relative to the opposite choice of temporal orientation. Similarly, if F^{ab} correctly maps four-velocities four-forces relative to our original orientation, then, in order to represent *the same map from tangent lines to four-forces* relative to the opposite choice of temporal orientation, we will have to flip the sign of the tensor, to compensate for the sign flip in four-velocity: $F^{ab} \mapsto -F^{ab}$. We have now given justifications for Malament's time reversal operations for v^a and F^{ab}:

$$v^a \overset{T}{\longmapsto} -v^a; \tag{18}$$
$$F^{ab} \overset{T}{\longmapsto} -F^{ab}. \tag{19}$$

Electric and magnetic fields. As Malament notes, the frame-independent formulation suffices to write down the dynamics of the theory and establish their time-reversal invariance. Like Malament, however, we wish to make contact with Albert and the textbooks; to do this, we need to consider decompositions of our four-dimensional F^{ab} into electric and magnetic fields.

Following Malament ((2004):pp.16-17), we make the following two definitions:

- A **volume element** ϵ_{abcd} on M is a completely antisymmetric tensor field satisfying the normalization condition $\epsilon_{abcd}\epsilon^{abcd} = -24$.
- A **frame** η_a is a future-directed, unit, timelike vector field that is constant ($\nabla_a \eta^b = 0$).

We can now decompose the electromagnetic field into electric and magnetic fields, relative to a given frame and volume element:

$$E^a := F^a{}_b \eta^b; \tag{20}$$

$$B^a := \frac{1}{2}\epsilon^{abcd}\eta_b F_{cd}. \tag{21}$$

Note that the electric field E^a and is defined relative to temporal orientation and frame; the magnetic field B^a is defined relative to temporal orientation, frame and volume element. The volume element itself is a more subtle case; we follow Malament in stipulating that it, too, flips sign under time reversal.[3]

It follows that (as Malament explains) the time reversal transformation acts as follows:

$$\tau \xmapsto{T} -\tau; \tag{22}$$

$$\eta_a \xmapsto{T} -\eta_a; \tag{23}$$

$$\epsilon_{abcd} \xmapsto{T} -\epsilon_{abcd}; \tag{24}$$

$$v^a \xmapsto{T} -v^a; \tag{25}$$

$$F^{ab} \xmapsto{T} -F^{ab}; \tag{26}$$

$$E^a \xmapsto{T} E^a; \tag{27}$$

$$B^a \xmapsto{T} -B^a. \tag{28}$$

[3]The point here is just that we choose to mean, by 'time reversal', 'flip the temporal orientation and hold the spatial handedness fixed' (so the total orientation, represented by the sign of the volume element, has to flip), rather than 'flip the temporal orientation and hold the total orientation fixed' (in which case the spatial handedness would have to flip).

Note that the electric field, E^a, is invariant under time reversal, while the magnetic field, B^a, flips sign. This is exactly the time reversal operation suggested by standard textbooks in classical electromagnetism. So, Malament's proposal provides a justification, based on his geometric conception of time reversal, for the standard view.

3.4 Albert revisited

We noted that, as soon as one thinks of the **E** and **B** fields as derived from a more fundamental Maxwell-Faraday tensor, either **E** or **B** must flip sign under time reversal. On Albert's account, neither flips sign. But, of course, Albert is perfectly aware of the four-dimensional formulation of electromagnetism. So why does he say what he says?

Well, on Albert's view, *pace* any arguments for interpreting electromagnetism in terms of a Minkowski spacetime, spacetime is in fact Newtonian, velocities are good old spatial 3-vectors, and so are the electric and magnetic fields.[4] The dynamics governing the development of the **E** and **B** fields, and the particle worldlines, happens to be 'pseudo-Lorentz invariant': that is, there exist simple transformations on the **E** and **B** fields such that, *if* those were the ways **E** and **B** transformed under Lorentz transformations, *then* the theory would be Lorentz invariant. This is perhaps surprising — there's no *a priori* reason to expect the dynamics to have this feature of 'pseudo Lorentz invariance', if one thinks that spacetime is Newtonian. But then, there's no *a priori* reason why the dynamics in a Newtonian spacetime *shouldn't* be pseudo Lorentz invariant, either. Similarly: it follows from this pseudo Lorentz invariance that observers will never be able to discover, merely by means of 'mechanical experiments' (i.e. observations of particle worldlines), what their absolute velocity is, or pin down the **E** and **B** fields uniquely. So *if* one thought that all features of reality must be empirically accessible to the human machine with its coarse-grained perceptive capacities, one would be very suspicious of Albert's view; but why, Albert might well ask in reply, should one think that?

What should one make of all this? Well, while we agree that Albert's view is internally coherent, we regard it as insufficiently motivated, for the following reason. A straightforward application of Ockham's razor prescribes that, faced with a choice between two empirically equivalent theories, one of which is strictly more parsimonious than the other as far as spacetime structure goes, one should (*ceteris paribus*) prefer the more parsimonious theory. In other words, one should commit to the *minimum* amount of spacetime structure needed

[4]To our knowledge, Albert has not stated this view in print. Our attribution of it to him is based on conversations between Albert and one of us over a period of several years. We also do not know whether he still holds the view in question.

to account for the empirical success of one's theories. Now, on Albert's view, spacetime is equipped with a preferred foliation and a standard of absolute rest; further, it must also be equipped with an objective temporal orientation, in order to account for the non-time-reversal invariance of classical electromagnetism. On the Minkowskian view, spacetime has none of this structure. If other things are equal, this gives us a reason to prefer a Minkowskian view; further, as far as we can see, other things *are* equal. We conclude that, insofar as classical electromagnetism is to be trusted at all, spacetime is Minkowskian rather than Newtonian, it is the unified electromagnetic field, rather than the **E** and **B** fields separately, that is fundamental, and that Albert's view of time reversal is false.

We will say no more about Newtonian interpretations. What is more interesting, for the purposes of our paper, is that even *given* a Minkowskian interpretation of relativity, the ontology, and hence the time reversal operation, for classical electromagnetism remains underdetermined. Malament has suggested one candidate ontology; we turn now to alternatives.

4 The 'Feynman' proposal

In this section, we turn to the view of time reversal that will correspond to Feynman's view of antiparticles. Our discussion here will not differ from our discussion of Albert's or Malament's proposals in terms of what time reversal is or how non-trivial time reversal operations are justified; that is, we are still thinking in terms of geometric time reversal. The 'Feynman' proposal is simply a different proposed ontology, a different view as to what fundamental quantities there in fact are out there in nature. It provides an geometric justification for a *third* time reversal operation for the electric and magnetic fields, distinct from both Albert's and Malament's.

Fundamental ontology. The distinctive feature of the 'Feynman' proposal is the suggestion that there is a fundamental, temporal orientation-independent fact as to the sign of the four-velocity of a given particle. That is, we change our hypothesis about the fundamental properties possessed by particles: rather than supposing that particles' worldlines are mere sets of spacetime points, and hence intrinsically *undirected*, we now suppose that particles' worldlines are intrinsically directed: each worldline comes equipped with an arrow, and there is an objective, temporal-orientation-independent fact about which way the arrow on any given worldline points. In that case, we no longer have Malament's motivation for saying that the electromagnetic field is a map from *tangent lines* to four-vectors. So, on the 'Feynman' proposal, we take the electromagnetic field to be (fundamentally!) a map from *four-vectors* to four-vectors, or, equivalently, a rank 2 tensor field. Thus, the electromagnetic field, independent of a temporal orientation, corresponds to a unique rank 2 tensor: the Maxwell-Faraday tensor F^{ab}.

The **electric and magnetic fields**, E^a and B^a, are then defined from F^{ab}, relative to a frame and volume element, just as they are on Malament's proposal.

Time reversal. The corresponding time reversal transformation is:

$$\tau \overset{T}{\longmapsto} -\tau \tag{29}$$

$$\epsilon_{abcd} \overset{T}{\longmapsto} -\epsilon_{abcd} \tag{30}$$

$$\eta^a \overset{T}{\longmapsto} -\eta^a \tag{31}$$

$$F^{ab} \overset{T}{\longmapsto} F^{ab} \tag{32}$$

$$v^a \overset{T}{\longmapsto} v^a \tag{33}$$

$$E^a \overset{T}{\longmapsto} -E^a \tag{34}$$

$$B^a \overset{T}{\longmapsto} B^a. \tag{35}$$

Note that this is not the textbook time-reversal transformation. The Feynman proposal has the consequence that the *electric* field flips sign under time reversal, and that the magnetic field does not — but it, too, has the consequence that the theory is time reversal invariant.[5]

More on the 'Feynman' proposal. Certain features of the time-reversal operation sanctioned by the 'Feynman' proposal seem rather odd, however; let's take a closer look. Consider, for example, a particle travelling between Harry and Mary (see figure 2). Suppose that, prior to time reversal, the particle's four-velocity happens to be 'future'-directed, and points from Harry's worldline to Mary's. Then, the following two observations can be made about the time-reversed situation. First, in the time-reversed situation the particle's four-velocity will be 'past'-directed. (This follows from the fact that the four-velocity itself dows not change, while the description of a given

[5]The time reversal invariance of this theory is easy to see, by looking at the Lagrangian $L = -\frac{1}{4}F_{ab}F^{ab} - qv_aA^a$. Under 'Feynman' time reversal, all four of the objects appearing in this Lagrangian — the Maxwell-Faraday tensor F^{ab}, the charge q, the four-velocity v^a and the four-potential A^a — are invariant under time reversal. So of course the Lagrangian itself (a scalar field on M) is invariant under time reversal, and, consequently, there will never be a set of field configurations and particle worldlines that is dynamically permitted relative to one temporal orientation and not the other.

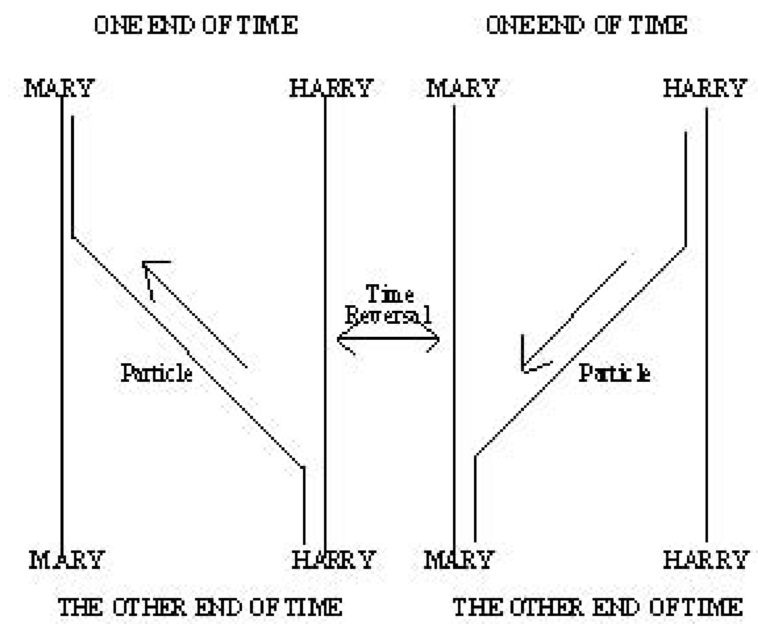


Figure 2: The time-reverse of a particle traveling from Mary to Harry, according to the Feynman view, is (still) a particle traveling from Mary to Harry.

temporal direction on the manifold as ‘future’/‘past’ does change when we flip the temporal orientation.) Second, the four-velocity *will still point from Harry to Mary.* On the ‘Feynman’ proposal, that is, we are asked to make sense of a notion of ‘time reversal’ according to which the time-reverse of a particle traveling from Harry to Mary is not a particle traveling from Mary to Harry. This seems an odd feature of the ‘Feynman’ view.

However, let us suppose that it is not the case that the four-velocities of all particles point in the same temporal direction. That is, let us suppose that, relative to a fixed choice of temporal orientation, some particles have future-directed four-velocities, and others have past-directed four-velocities. Suppose, then, that we have a model of electromagnetism which consists of a single particle of charge q, moving in an electromagnetic field F^{ab} with four-velocity v^a. One can then trivially produce another model by keeping the electromagnetic field F^{ab} the same and the trajectory the same, while flipping the sign of the charge ($q/mapsto - q$) and of the four-velocity ($v^a \mapsto -v^a$). (One can see that this operation does indeed turn models into models by inspecting Maxwell's equations and the Lorentz force law, or, alternatively, by inspecting the Lagrangian. The only changes in any of these quantities are in the signs of q and v^a, which always occur together, so that the changes cancel; so, changing the sign of the charge and of the four-velocity must turn a solution into a solution, and a non-solution into a non-solution.)

Let us put this another way: a particle with charge q and four-velocity v^a behaves, in a given electromagnetic field, exactly as if it is a particle with charge $-q$ and velocity $-v^a$: it follows exactly the same trajectory, so that, given only access to the results of 'mechanical experiments', the two possible situations cannot be distinguished in any way. This observation opens the door for the following hypothesis: particles that we have regarded as belonging to different types, related by the 'is the antiparticle of' relation — electrons and positrons, say — are really of the same type as one another. In particular, they have the same electric charge as one another. Things appear otherwise only if we erroneously assume that all four-velocities must point in the same temporal direction as one another. In other words, we can achieve parsimony in particle types at the cost of the 'extravagance' of endowing particle worldlines with an intrinsic direction; the Feynman proposal is that we do so. If this hypothesis is right, then it is indeed true that an anti-particle is nothing but a particle traveling in the opposite direction of time.

5 Structuralism: A Third Way?

We have been assuming so far that the Malament and Feynman proposals represent distinct alternatives, at most one of which can be correct. One can have a different time reversal operation for the same formalism, we said, only if one makes a different postulate about the *fundamental ontology*; but if one does that, then (we said) one has changed one's theory, in the clear sense that one has changed one's hypothesis about the fundamental nature of the world.

Be that as it may, one might still (on the other hand) have the gut feeling that the 'disagreement' between the Malament and Feynman ontologies is not a genuine one; that the two 'rival theories' are, in some sense, saying the same thing in different ways.

Clearly, one cannot fully hold onto both of these ideas: one says that the Malament and Feynman proposals are distinct, the other says they are not. In the present section, however, we will sketch a third set of hypotheses about the fundamental nature of a classical electromagnetic world that does justice to the basic principles behind both ideas. It will do justice to the just-mentioned gut feeling, in that it will provide a way of regarding the claim that worldlines have arrows on them and that four-velocities can be past-directed (as Feynman says), and the claim that worldlines have no intrinsic arrows and four-velocities are always future directed (as Malament says), as equivalent descriptions of the same underlying situation. However, it will also do justice to our earlier insistence that this business of formulating alternative descriptions is not ontologically innocent, because it will be a third, rival, suggestion for what the fundamental nature of electromagnetic reality might be, rather than a claim that the *original* Malament and Feynman theories are equivalent.

The 'third way' is *structuralism.* In the broader context, structuralism arises as an attempt to steer the correct course between (on the one hand) an excessively deflationary positivism, according to which empirical equivalence is supposed straightforwardly to entail equivalence of meaning, and (on the other) an excessively realistic position, according to which every difference in notation (the use of the boldface letter **D** rather than **E** for the electric field, say) is taken to correspond to a difference in postulated physical reality. The sort of 'structuralism' we are interested in typically proceeds – either on a case-by-case basis (i.e. applying the structuralist strategy where and only where it happens to seem appropriate) or as a sweeping claim about the possibility of knowledge, reference and/or the nature of reality – by reifying, at the *fundamental* level, *relations*, but not monadic properties. This (fundamental reification of relations only) will be our tactic here too.

5.1 Structures: the debate recast

Before setting out the relationist's attempt to deflate the debate between the Malament and Feynman views, it will serve the interests of clarity if we recast the moves that have been made so far in a more formal framework.

In the beginning , we were representing classical electromagnetic worlds using one-parameter families of *standard Newtonian structures.* A standard Newtonian structure is a mathematical entity of the form

$$\mathcal{S}_{Newt} = \langle \Sigma \times T, P, \mathbf{x}, m, q_s, \mathbf{E}, \mathbf{B} \rangle, \tag{36}$$

where:

- $\Sigma \times T$ is a Newtonian spacetime: that is, Σ is a Euclidean three-space, and $T \sim (\mathbb{R}, +)$ is the set of times.
- P is a set of particles. (In the first instance, P is structureless; structure is added by the functions $\mathbf{x}, m, q_s$ below.)
- $\mathbf{x} : P \times T \to \Sigma$ is an assignment of a three-position to each particle at each point in time.
- $m : P \to M$ is an assignment of a (determinate) mass property, such as $9.11 \times 10^{-31} kg$, to each particle. The space M of

mass properties has the structure $M \sim (\mathbb{R}_0^+, +)$: that is, M is isomorphic to the nonnegative part of the real line, where 'isomorphism' is understood in the restricted sense of 'preserving addition'. (The structure of the space of mass properties is not as rich as that of the reals; in contrast to real numbers, one cannot multiply two masses to obtain a third.)

- $q_s : P \to Q_s$ is an assignment of a (determinate) charge property, such as $-1.6022 \times 10^{-19}C$, to each particle. The space Q_s of 'standard' charge properties has the structure $Q_s \sim (\mathbb{R}, +)$, i.e. Q_s is isomorphic (in the same restricted sense) to the real line. (The subscript 's' (and corresponding adjective 'standard') is for contrast with the later case of 'Feynman' charges.)
- $\mathbf{E}$ is a three-vector field — the electric field. (Formally: $\mathbf{E} : \Sigma \times T \to T\Sigma$, with $(\mathbf{E}(\mathbf{x}, t) \in T_x\Sigma)$ for all $x \in \Sigma, t \in T$.)
- $\mathbf{B}$ is another three-vector field — the magnetic field. (Formally: $\mathbf{B} : \Sigma \times T \to T\Sigma$, with $\mathbf{B}(\mathbf{x}, t) \in T_x\Sigma$ for all $x \in \Sigma, t \in T$.)

Albert's view, described in section 3.2, amounts to the claim that structures of this form $\mathcal{S}_{Newt}$ contain no element of conventionality; that is, that such structures 'carve electromagnetic reality at the joints'.

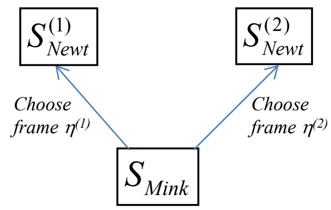
Then we noticed that we could have Lorentz invariance if we allowed $\mathbf{E}$ and $\mathbf{B}$ to transform nontrivially under Lorentz transformations; but we took it that this required regarding $\mathbf{E}$ and $\mathbf{B}$ as non-fundamental, and as defined in terms of something more fundamental only relative to a choice of frame. We therefore introduced a class of structures that (for want of a better name) we will call *Minkowski structures*, i.e. mathematical entities of the form

$$\mathcal{S}_{Mink} = \langle \mathbf{M}, g; P, v^a, m, q, F_{ab} \rangle, \tag{37}$$

where:

- $(\mathbf{M}, g)$ is a Minkowski spacetime;
- v^a is an assignment of a four-vector (four-velocity) field to each particle (vanishing except on the particle's worldline). (Formally: $v^a : P \times M \to TM$, with $v^a(p, x) \in T_xM$ for all $p \in P, x \in M$.)
- F_{ab} is a two-form field: the Maxwell-Faraday tensor field. (Formally: $F_{ab} : M \to \Lambda T(0,2)M$, with $F_{ab}(x) \in \Lambda T_x(0,2)M$ for all $xinM$.)
- Other elements of the structure are as above.

And we noted that, given a Minkowski structure, we could represent it by a Newtonian structure relative to a choice of frame η^a or, equivalently, a choice of simultaneity convention; but we recognized that the choice of frame or simultaneity convention was arbitrary, that it did not latch onto anything of metaphysical privilege, and, hence, that different Newtonian structures obtainable from the same Minkowskian structure were to be regarded as different ways of representing the same underlying reality:



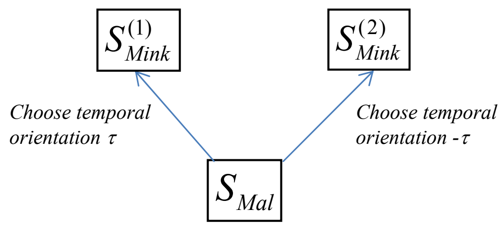
But, we noticed, the idea that *Minkowski* structures were fundamental seemed to force upon us a nonstandard time reversal operation, according to which the **E** field, but not the **B** field, flips sign. *Then* we (Malament) noticed that we could recover the standard time reversal operations if we allowed F_{ab} to transform nontrivially under time reversal (specifically, if F_{ab} picked up a sign flip under time reversal); but we took it that this required regarding F_{ab} (and, in consequence, also v^a) as non-fundamental, and as defined in terms of something more fundamental only relative to a choice of temporal orientation. We therefore introduced the notion of a *Malament structure*, i.e. a mathematical entity of the form

$$\mathcal{S}_{Mal} = \langle \mathbf{M}, g; P, w_u, m, q_s, f_m \rangle, \qquad (38)$$

where:

- P is a set of particles.
- $w_u : P \to W_u$ is an assignment of an undirected worldline to each particle. (The set W_u of undirected worldlines can be identified with the set of images of inextendible timelike curves in M.)
- $f_m : L_u \times Q_s \to TM$ is the (Malament) electromagnetic field. Here, L_u is the set of undirected tangent lines; it can be identified with the set $TM\backslash\sim$ of equivalence classes under the equivalence relation: $v^a_{(1)} \sim v^a_{(2)}$ iff $v^a_{(1)} = \lambda v^a_{(2)}$ for some $\lambda \in \mathbb{R}$. We have $f_m(l_u, q) \in T_xM$ whenever l_u is a line in T_xM.
- Other elements of the structure are as above.

We then noted that, given a Malament structure, we could represent it by a unique Minkowski structure relative to a choice of temporal orientation; but we recognized that the choice of temporal orientation was arbitrary, that it did not latch onto anything of metaphysical privilege, and, hence, that different standard Minkowskian structures obtainable from the same Malament structure were to be regarded as different ways of representing the same underlying reality:



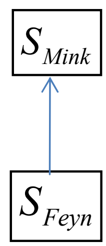
'Feynman''s point was then that there was an alternative to Malament structures, apparently at least as defensible, although this alternative did not recover the standard time reversal operations: we could hypothesize instead that the more fundamental reality was well-represented by mathematical entities of the form

$$\mathcal{S}_{Feyn} = \langle \mathbf{M}, g; P, w_d, m, q_f, f_f \rangle, \tag{39}$$

where

- $w_d : \mathbf{P} \rightarrow W_d$ is an assignment of a *directed* worldline to each possible particle. (The space W_d of directed worldlines can be identified with a set of equivalence classes of inextendible timelike curves, under the equivalence relation that relates all and only pairs of curves whose parameters increase in the same time sense as one another.)
- $q_f : \mathbf{P} \rightarrow Q_f$ is an assignment of a determinate *Feynman charge property* to each possible particle. The space Q_f of 'Feynman' charge properties has the structure $Q_f \sim (\mathbb{R}_0^+, +)$, corresponding to our earlier remark that, for Feynman, 'all charges are positive'.
- $f_f : L_d \times Q_f \rightarrow TM$ is the (Feynman) electromagnetic field. Here, L_d is the set of directed tangent lines; it can be identified with the set $TM \backslash \sim$ of equivalence classes under the equivalence relation: $v^a_{(1)} \sim v^a_{(2)}$ iff $v^a_{(1)} = \lambda v^a_{(2)}$ for some $\lambda > 0$. We have $f_f(l_d, q) \in T_x M$ whenever l_d is a (directed) line in $T_x M$.

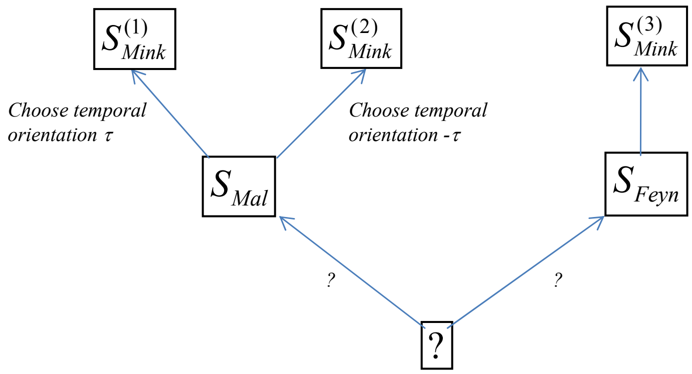
To complete the summary of our account thus far: We then noted that, given a Feynman structure, a representation convention can be set up according to which there is a unique standard 4D structure that represents the given Feynman structure, even *without* the selection of any conventional temporal orientation, or indeed any conventional pieces of structure:



The 'structuralist' wants to continue this pattern: whereas the advocate of (say) the fundamentality of standard Minkowskian structures regards a large class of Newtonian structures as differing from one another only on choices of convention ('choice of frame'), not on matters of fundamental ontology (which latter are given by S_{Mink}); and whereas the advocate of the fundamentality of Malament structures regards a class of two standard Minkowskian structures as differing from one another only on choices of convention (in this case, temporal orientation), while the fundamental ontology is given by S_{Mal}; so the 'structuralist' wants to regard the elements of a class that contains *both Malament and Feynman structures* as differing from one another only on choices of convention. Malament and Feynman structures, according to the structuralist, will be equally good representors of some more fundamental underlying reality.

So far so good. It *seems*[6] reasonable, however, to require that we say more directly what the nature of this underlying reality is, rather than just 'it's something that can equally well be represented by this Malament or this Feynman structure.' That is, it seems reasonable to demand that we 'fill in the question-marks' in the following diagram:

[6] *Perhaps* there are limits to how far this demand can be pushed. Perhaps, that is, we eventually reach a level at which we are compelled to recognize the existence of conventionality, *but* we cannot describe the representation relations, or give a more direct description of the underlying, convention-independent reality. Interesting questions concern whether or not this happens and, if so, where it happens, and why it happens where it does.



It is in this attempt at more direct description of the nature of reality that the emphasis on relations arises. The idea is to articulate a fifth type of structure, that of 'relationist structure', to hypothesize that *that* captures the fundamental nature of electromagnetic reality better than any of the four alternatives we have articulated so far, and to show how a given structure of this fifth type can be represented by a Malament, Feynman, Minkowskian or Newtonian structure relative to the selection of a certain number of arbitrary, but well-understood, conventions.

5.2 Relational structures

Suppose that the more fundamental story is as follows. Let M, g, P, w_u and m be (respectively) a manifold, metric, set of particles, assignment of undirected worldlines to particles, and assignment of mass properties to particles, as before. But, in place of a space of monadic charge properties (Q_m or Q_f) and an ascription (q_m, q_f respectively) of these monadic properties to particles, we have a binary *relation* $q_r : P \times P \to \mathbb{R} \cup \{\infty\}$, satisfying the following constraints:

'Reflexivity': $\forall p \in P, q_r(p,p) = 1$.
[**'Antisymmetry':** $\forall p_1, p_2 \in P, q_r(p_1,p_2) = q_r(p_2,p_1)^{-1}$.
[**'Transitivity':** $\forall p_1, p_2, p_3 \in P, q_r(p_1,p_2) \cdot q_r(p_2,p_3) = q_r(p_1,p_3)$;

Heuristically: in terms of Malament structures, q_r corresponds to a 'charge ratio' relation; while, in terms of Feynman structures, the absolute value of q_r corresponds to the charge ratio, while the sign of q_r encodes whether or not the worldlines of the two particles have the same temporal direction as one another. But it is crucial to note that neither of these translation schemata forms part of the relationist account *per se*. According to the relationist, there is just q_r.

We are then dealing with *relational structures*: mathematical entities of the form

$$S_{rel} = \langle \mathbf{M}, g; P, w_u, m, q_r, f_r \rangle, \tag{40}$$

where

- q_r is as above.
- $f_r : L_u \times P \to TM$ is a map assigning a four-vector in T_xM to every pair (l_u, p) such that l_u is a line in T_xM (for some $x \in M$).
- Other elements of the structure are as above.

A relational structure represents an electromagnetic world as containing point particles $p \in P$ that have monadic mass properties[7], and that bear a 'charge-ratio'-like *relation* to one another; the electromagnetic field is accordingly reconceived as f_r rather than f_m or f_f, so that it makes no reference to monadic charge properties.

5.3 Malament and Feynman structures as conventional representors of a relational reality

We now wish to explore the ('structuralist') suggestion that it is the relational structures that best 'carve electromagnetic reality at its joints', and that Malament and Feynman structures arise as convenient mathematical tools which, however, require us to make some choices of arbitrary convention that need not be made by the pure relational approach. Specifically, we wish to explore the nature of the representation relation between (represented) relational structures and (representing) Malament or Feynman structures.

The following definition will prove useful: Say that a particle $p \in P$ *has zero charge* iff for some $p' \in P$, $q_r(p, p') = 0$.[8]

Suppose, then, that we are given a relational structure, i.e. an entity of the form (40). We first wish to represent this via a Malament structure. To do so, we proceed as follows:

1. Let Q_m be a space with the structure $Q_m \sim (\mathbb{R}, +)$. (This structure suffices to define a notion of multiplication by an arbitrary real number on Q_m.)

[7] A more thorough-going structuralism, of course, would treat mass, as well as charge, in a relational way. We omit this complication for brevity.

[8] This definition has the consequence that if, intuitively, *all* particles have zero charge, none will count as having zero charge according to the definition. This consequence is unwanted, but does not create any problems. In such cases, the indifference of the particles to the EM field will be encoded in f_r (which would everywhere take zero vectors as its values).

2. Define a function $q_m : P \to Q_m$ as follows:
   - Choose arbitrary $\tilde{p} \in P$ such that $\tilde{p}$ has nonzero charge. (The existence of some such particle, providing that P is nonempty, is guaranteed by the axioms governing q_r; cf. footnote 8. If P is empty, then, of course, any function with domain P is trivial.)
   - Choose arbitrary nonzero charge $\tilde{q} \in Q_m - \{0\}$.
   - Define a function $q_m : P \to Q_m$ as follows:
     (a) $q_m(\tilde{p}) = \tilde{q}$.
     (b) For all $p' \in \mathbf{P_r}, q_m(p') = q_m(p) \cdot q_r(p', p)$.
3. Define a map $f_m : L_u \times Q_m \to TM$ as follows:

$$\forall l_u \in L_u, \forall q \in Q_m, f_m(l_u, q_m) = \frac{q}{\tilde{q}} f_r(\langle l_u, \tilde{p} \rangle). \tag{41}$$

4. Form the Malament structure $\langle \mathbf{M}, g; P, w_u, m, q_m, f_m \rangle$.

We note that, given a relational structure, we have the following arbitrary choice of convention to make, in order to determine the Malament structure that would represent it: the charge $q_m(\tilde{p}) \in Q_m - \{0\}$ for an arbitrarily selected charged particle $\tilde{p}$.

To represent our given relational structure using a Feynman structure, on the other hand, we would proceed as follows:

1. Let Q_f be a space with structure $Q_f \sim (\mathbb{R}_0^+, +)$. (This structure suffices to define a notion of multiplication by an arbitrary nonnegative real number on Q_f.)
2. If all particles in $\mathbf{P_r}$ have zero charge, set $q_m(p) = 0 \in Q_f$, for all $p \in \mathbf{P_r}$. If some particle in $\mathbf{P_r}$ has nonzero charge, then:
   - Choose arbitrary $\tilde{p} \in P$ with nonzero charge.
   - Choose arbitrary nonzero charge $\tilde{q} \in Q_f - \{0\}$.
   - Define $q_f : P \to Q_f$ as follows:
     (a) $q_f(\tilde{p}) = \tilde{q}$.
     (b) For all $p' \in P, q_f(p') = q_f(p) \cdot |q_r(p', \tilde{p})|$.
3. Construct the ascription w_d of *directed* worldlines to particles, as follows. First, note that the set W_d has two natural pieces of structure. (i) If $w_1, w_2 \in W_d$, say that w_1 *is codirected with* w_2 iff w_1 'points in the same temporal direction as' w_2.[9] Codirectedness

[9]Rigorously: $w_1 \sim w_2$ iff, for any continuous nowhere-vanishing timelike vector field τ^a on M and any $s_1, s_2 \in \mathbb{R}$,

$$\left(\eta_{ab}\left(\frac{dw_1}{ds}\right)^a|_{s_1}\tau^b(w_1(s_1))\right)\left(\eta_{cd}\left(\frac{dw_1}{ds}\right)^c|_{s_2}\tau^d(w_1(s_2))\right) > 0. \tag{42}$$

is then an equivalence relation on W_d, partitioning W_d into two mutually exclusive and jointly exhaustive classes. (ii) If $w_3, w_4 \in W_d$, *or* if $w_3 \in W_d$ and $w_4 \in W_u$, say that w_3 *is coextensive with* w_4 iff w_3, w_4 occupy the same set of points of M. Coextensiveness (in the first sense) is also an equivalence relation on W_d, this time partitioning W_d into uncountably many equivalence classes of two elements each. Then:

- Select an arbitrary directed worldline w that is coextensive with the undirected worldline $w_u(p)$ that our relational structure ascribes to p; let $w_d(p) = w$.
- For all other particles $p' \in \mathbf{P}$:
  - If $q_r(p', p) > 0$, let $w_d(p')$ be the unique element of W_d that is coextensive with $w_u(p')$ and codirected with $w_d(p)$.
  - If $q_r(p', p) < 0$, let $w_d(p')$ be the unique element of W_d that is coextensive with $w_u(p')$ and *not* codirected with $w_d(p)$.

4. Define a map $f_f : L_d \times Q_f \to TM$ as follows:

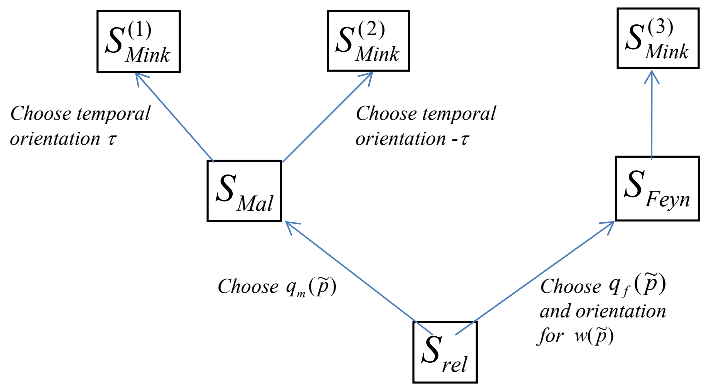
$$\forall l_d \in L_d, \forall q \in Q_f, f_f(l_d, q) = \pm \frac{q}{\tilde{q}} f_r(l_d, \tilde{p}), \tag{43}$$

where the positive sign applies iff the orientation of l_d is the same as that of $w_d(\tilde{p})$.

5. Form the Feynman structure $\langle \mathbf{M}, g; P, w_d, m, q_f, f_f \rangle$.

In this case, we had to make *two* arbitrary choices of convention: the charge $q_f(\tilde{p}) \in Q_f - \{0\}$ of our arbitrarily selected charged particle $\tilde{p}$, and the orientation of its worldline. The superficial appearance that this involves 'more conventionality' than does the construction of a Malament from a relational structure, however, is no more than that: on any reasonable way of quantifying 'degree of conventionality', the selection of an arbitrary element of $Q_m \sim \mathbb{R}$ will count as the introduction of 'just as much convention' as will the selection of an arbitrary element of $Q_f \sim \mathbb{R}_0^+$ and an arbitrary orientation for a given worldline.

To sum up our structuralist program, then: we have written down prescriptions for constructing Malament and Feynman structures from a given relational structure $\langle \mathbf{M}, g; P, w_u, m, q_r, f_r \rangle$. In this way, it can be a consequence of our third candidate ontology, according to which it is the relational structures that best 'carve electromagnetic reality at the joints', that the choice between representation via a Malament structure and representation via a Feynman structure is merely a choice of convention:



Thus (given our earlier accounts of how the ontologies on which Malament and Feynman structures are based give rise to distinct geometrical time reversal operations), we have shown how a relationist can support the claim that answers to questions like whether or not four-velocity flips sign under time reversal, whether time reversal turns particles into antiparticles, and so on, are convention-dependent: questions that have no determinate answers until we implicitly choose our convention (by answering the question, or otherwise).

6 Conclusions and open questions

In this final section, we summarize our conclusions to date, and then indicate some open issues that we would like to resolve.

Summary of conclusions from this paper. We have elaborated the 'geometric' notion of time reversal introduced by Malament, according to which time reversal consists in leaving all [other] fundamental quantities alone, and merely flipping the temporal orientation. This allows us to give an account, as the passive and active notions of time reversal cannot, of how it may come about that a coordinate-independent quantity such as F^{ab} transforms nontrivially under time reversal. We have then discussed four approaches to time reversal in classical electromagnetism in the light of this geometric conception: Albert's, Malament's, the 'Feynman' approach, and the structuralist approach. Only according to Albert is the theory not time reversal invariant; we have rejected Albert's account by appeal to Ockham's Razor.

Theory choice. This does, however, leave us with an apparent case of underdetermination: how might one choose between the Malament, Feynman and Structuralist ontologies, and which seems to be preferable all things considered? We are not sure how best to answer this question; so let us merely list several considerations that *may* tell one way or another.

Firstly: one feeling is that Structuralism is preferable because it eliminates distinctions that seem to be devoid of differences. But it would be better if this 'feeling' could be replaced with argument, and it is difficult to turn the sentiment expressed in the preceding paragraph into an argument for structuralism without falling foul of the point that the choice between structuralism and its alternatives is itself a choice that is, in a very similar sense, 'underdetermined by the physics'.

Secondly: it is not clear that the Feynman account can give a reasonable treatment of *neutral* particles. We skirted over this difficulty in our above discussion, but it is not hard to see, particularly in the context of the attempt to define a Feynman structure to represent a relational reality: in the case of a neutral particle, there is nothing 'in the physics' to determine what the orientation of the particle's worldline should be. To insist that even in this case there must nevertheless be a fact about the worldline's orientation seems ontologically extravagant; to treat neutral particles in Malament's way, while retaining a Feynman treatment of charged particles, though, seems to amount to adopting an ugly hybrid position.

Thirdly: it is not clear that the Structuralist account can give a reasonable treatment of the electromagnetic field, either of cases in which all particles are neutral, or of *vacuum* solutions of the Maxwell equations. Let us take up the second point first. The point here is that if the relationist electromagnetic field f_r *just is* a map from $L_u \times P$ to TM then, if P is empty, f_r is a map with empty domain; thus, the structuralist account does not seem to have the resources to underwrite a genuine physical difference between any one vacuum solution and any other. Going back to the case of neutral particles: similarly, in any case in which all actual particles are neutral, the relationist electromagnetic field must assign the zero four-vector to every pair (l_u, p); thus, again, it cannot underwrite genuine physical differences between solutions of the Maxwell equations that differ radically on the value of the Maxwell-Faraday field F_{ab}. Of course, the structuralist could bite the bullet and say that, indeed, there is no genuine physical difference between such pairs of solutions; whether or not this (bullet-biting) move would lead to trouble is an open question.

Fourthly: the Malament account does not seem to sit particularly well with the idea that, at a rather fundamental level, the Maxwell-Faraday tensor is to be thought of as the curvature of a $U(1)$ connection one-form A_a. If one takes this latter idea seriously, one *seems* to be led to something like the Feynman view: the most fundamental representation of the electromagnetic field is (according to this idea) as a two-form, *not* as a map from tangent lines and either charges or particles to four-vectors. Thus, connection realism seems to lead to the Feynman view of time reversal by default.

One final (and very plausible) possibility is that the underdetermination in question simply cannot be correctly resolved within the confines of classical electromagnetism, and that it is only by viewing classical electromagnetism as the classical limit of a quantum field theory, and thus obtaining further ontological insight as to the nature of charged particles and of the electromagnetic field, that one runs across considerations that favor the true ontological position over others. The investigation of these possibilities is a future project.

Conventionality of spacetime structure? An intriguing issue arises on the supposition that structuralism is indeed correct. In that case, as we have emphasized, the difference between the Malament and Feynman languages is just that — a difference in language; one's choice of language is a convention. In the case of classical electromagnetism, nothing of ontological substance even *threatens* to hang on the choice of convention; in particular, the existence or nonexistence of a preferred temporal orientation does not, since the theory comes out time reversal invariant according to both Malament and Feynman. A more interesting case would be one, if any such there be, in which the time reversal invariance of the theory was (according to structuralism) a convention-dependent matter. Given the standard link between spacetime symmetries and spacetime structure, this would render the question of whether or not a privileged temporal orientation exists a convention-dependent matter. It is not immediately clear whether or not this makes sense. If it does, the details have yet to be worked out; if it does not, this seems to be a strong argument against the structuralist position.

Field theory. Finally, let us return to a comment we made at the outset. The original motivation for this project was the feeling that the existence of a CPT theorem is rather puzzling — why should *charge conjugation* be so intimately related to spacetime symmetries? The point here is that, according to the 'Feynman' proposal, the operation that *ought* to be called 'time reversal' — in the sense that it bears the right relation to spatiotemporal structure to deserve that name — is the operation that is usually called TC; on this proposal, the theorem known as the 'CPT theorem' would be more properly called a PT theorem, and (the thought continues) perhaps this opens the door to new insights into why that theorem should hold. A future project is to investigate a geometrical understanding of the (classical and quantum field-theoretic) 'CPT' theorems, drawing on this suggestion.

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