

The renormalization of charge and temporality in quantum electrodynamics

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abstract

In this article it is intended a closer look at the renormalization procedure used in quantum electrodynamics to cope with the divergent integrals that appear in higher-order calculations within the theory. The main focus will be the charge renormalization that reveals, in a clearer way than the mass renormalization, structural limitations present in quantum electrodynamics that are even more aggravating than the ones usually pointed at when considering the renormalization procedure. In this way we see that the possibility of charge renormalization is due to limitations of the theory in the temporal description of the interactions.

1 Introduction

The appearance of divergent integrals in higher-order calculations in quantum electrodynamics where the so-called radiative corrections are taken into account has been seen as, at least, indicating that the theory fails for high energies. As J. Schwinger stated, “electrodynamics unquestionably requires revision at ultra-relativistic energies” (Aramaki 1989, 93). Even considering the accuracy of the theory at lower energies, Schwinger considered that the renormalization procedure, that permits avoiding the infinities in the results of the calculations, ultimately has to be excluded from physics (Cao 1993, 50). Regarding this problem the position of P. Dirac was even less sympathetic: “I am very dissatisfied with the situation, because this so-called “good theory” does involve neglecting infinities which appear in its equations” (Kragh 1990, 184).

In general the position of leading physicists was very critical regarding quantum electrodynamics, and some pinpointed to structural problems that go beyond the high energy behaviour of quantum electrodynamics. N. Bohr considered that the whole program only made sense taking into account the weakness of the coupling constant, which means applying the theory only in situations where the electron interacts weakly with the electromagnetic field (Rueger 1992, 317). There were others, like L. Landau, that considered that the limitations of quantum electrodynamics were even more drastic, because they would be due to very basic structural problems in the design of the theory: “for them the very concept of a local field operator and the postulation of any detailed mechanism for interaction in a microscopic spacetime region were totally unacceptable” (Cao 1993, 47).

In this article I’ll present another aspect which reveals once again the limitations of the renormalization procedure, now related to the even more intrinsic limitation of any quantum theory in what regards the temporal description of physical processes.

In section 2 the historical emergence of the problem of infinities in quantum electrodynamics is considered, as the ‘provisory’ solution attained in the late forties with the completion of a renormalization program. In section 3 some details of the renormalization program are considering using as a ‘case study’ the calculation of the self-energy of the electron, and considering in particular the role of the cut-off procedure that provides a ‘regularization’ of the theory previous to the renormalization proper. In section 4 it is addressed not only the dubious mathematical procedure related with the calculation of the photon’s self-energy, but also the even more dubious

procedure – from a physical point of view – of attaching the infinite constant that pops out in the photon self-energy calculation to the charge of the electrons ‘connected’ by the photon. In particular the charge renormalization procedure is considered in a second-order radiative correction to the calculation of the Møller scattering amplitude. In this way the limitations of the theory regarding the temporal description of physical processes and its relation to the possibility of a charge renormalization procedure are made clear.

2 The emergence and submergence of infinities in quantum electrodynamics

When in 1929-30, W. Heisenberg and W. Pauli presented in two papers a relativistic quantum theory of the interaction between the quantized Maxwell and Dirac fields, they moved from Heisenberg’s first view that the self-energy of the electron didn’t constitute a problem and the infinite Coulomb self-energy could be neglected, to a more circumspect position recognizing that this problem might even render the theory inapplicable (Darrigol 1984, 484-486).

The fact that the self-energy problem could not simply be traced back to a similar situation occurring already in classical electrodynamics was soon revealed by J. R. Oppenheimer (1930) that found out a new (infinite) contribution to the self-energy without any classical counterpart.

The situation of quantum electrodynamics during the thirties didn’t improve. On the contrary.

To solve inconsistencies of his electron theory related to the existence of negative-energy solutions, Dirac proposed his hole theory. From this a new infinite problem popped out. In Dirac’s hole theory we have an infinite sea filled with negative-energy electrons. This made Dirac consider that the electromagnetic field is generated by “the difference in the electric density from its value when the world is in its normal state (i.e. when every state of negative energy and none of positive energy is occupied)” (Pais 1986, 387). But when considering in more detail the effect an ‘external’ electromagnetic field (that properly considered, could simply result from the presence of a sole electron above the negative-energy sea), Dirac concluded that it had “an effect of polarization of the distribution of negative-energy electrons” (Dirac 1934a, 208). The calculation of the density matrix of the sea electrons, in the simple case of an external electrostatic field, gave a logarithmically divergent result. Dirac considered that “we cannot assume that the theory applies when it is a question of energies greater than the order of $137mc^2$ ” (Dirac 1934a, 211). So, he used a cut-off to render the results finite. With the finite result in his hands, Dirac concluded that “there is no induced electric density except at the places where the electric density producing the field is situated, and at these places the induced electric density cancels a fraction of $1/137$ of the electric density producing the field” (Dirac 1934a, 212).

In a development of this work, Dirac tried to deal with the infinite in the density matrix not with the arbitrary cut-off prescription, but by “finding some natural way of removing the infinities [...] so as to leave finite remainders” (Dirac 1934b, 153). The uniqueness of this subtraction method was soon questioned (Miller 1994, 60). Heisenberg tried to improve Dirac’s method, and in the process, due to his use of a second-quantized version of Dirac’s formalism in which electron and positrons were treated in a symmetrical way, came up with the existence of an “infinite self-energy of the light-quanta” (Heisenberg 1934, 186).

In what regards the self-energy problems there was not much improvement during the thirties. The only solid result was V. F. Weisskopf demonstration that the electron self-energy is ‘only’ logarithmically divergent (Pais 1986, 385).

Things changed drastically in 1947.

H. A. Bethe coming from a conference in Shelter Island, held from March 30 to April 2, 1948, did some calculations on a train going from New York to Schenectady. In the conference Lamb presented his recent experimental results on the shift of the $2^2S_{1/2}$ state relative to the P states in the hydrogen atom. According to Bethe: “Kramers suggested that what one really ought to do was to renormalize the mass of the electron, taking into account its interaction with its own electromagnetic field. Then only those parts of the self-energy which are not contained in the mass of the particle would be observable” (Mehra 2001, 1039).

The idea of renormalization in the case of the electron’s charge was basically present in Dirac’s report to the Solvay conference of 1933 (Dirac 1934a). His ideas are stated more clearly in a letter to N. Bohr written after the preparation of the report: “We then have a picture in which all the charged particles of physics electrons, atomic nuclei, etc. have effective charges slightly less than their real charges, the ratio being about 136/137. The effective charges are what one measures in *all* low energy experiments, and the experimentally determined value of e must be the effective charge of an electron, the real value being slightly bigger” (Schweber 1994, 116).

An equivalent approach regarding the electron self-energy started to emerge in the end of the thirties in Dirac’s own work, and in H. A. Kramers’s investigation of the renormalization of the electromagnetic mass at the classical level as a first step into dealing with the problem at the quantum level. Kramers’s intention was to sidestep the problem by obtaining a consistent model for a finite size electron – that avoided the classical self-energy divergence –, considering from the start the experimental mass of the electron (that contained the mechanical and electromagnetic mass). In this way Kramers “tried to present the theory in such a fashion that the questions of the structure and the finite extension of the particles are not explicitly involved and that the quantity that is introduced as the ‘particle mass’ is from the very beginning the experimental mass” (Kramers 1938, 254).

But mass renormalization was only put to use in quantum electrodynamics in 1947, in the quantum-mechanical (non-relativistic) train-ride calculation of Bethe.

A few months after the conference, J. Schwinger worked on a non-covariant relativistic calculation of the Lamb shift using the mass a charge renormalization recipe, and obtained finite results to order $e^2/\hbar c$. Schwinger, knowing of G. Breit’s suggestion that the electron might have an intrinsic magnetic moment different from the predicted by the Dirac equation – that explained the discrepancy with the experimental results regarding the hyperfine structure of the hydrogen atom –, calculated the so-called anomalous magnetic moment, which accounted for the previous hyperfine discrepancies between theory and experiment (Mehra 2001, 1045). Schwinger published his result in a short note from last December 1947. This note didn’t include the precise results of his Lamb shift calculations due to discrepancies between the Coulomb (Lamb shift) and the magnetic field (anomalous magnetic moment) calculations. The calculations were done using non-covariant hole-theoretic methods. Afterwards, Schwinger developed a covariant formulation of the Heisenberg and Pauli quantum electrodynamics, but “there were a great many ambiguities in the procedure” (Schweber 1994, 333) of identifying the divergent contributions. Also the calculation method was terribly complicated and might become insurmountable in higher-order calculations.

Back in Cornell in early July, Bethe gave a lecture on his non-relativistic calculation of the Lamb shift, which R. P. Feynman attended to. According to Feynman:

He explained that it gets very confusing to figure out exactly which infinite term corresponds to what in trying to make the correction for the infinite change in mass. If there were ... any modification whatever at high frequencies, which would make this correction finite, then there would be no problem at all to figuring out how to keep track of everything ... if in addition this method were relativistically invariant, then we would be absolutely sure how to do it without destroying relativistic invariance (Feynman 1965, 170).

Feynman considered first the case of determining a relativist cut-off for classical electrodynamics. Using his path-integral method, Feynman, following Bethe's idea, replaced a delta function appearing in the interaction term of the action by an invariant function dependent on a cut-off parameter that made all results finite, corresponding to a 'regularization' of the theory. Feynman would then renormalize the mass, putting the mechanical mass and the now finite electromagnetic mass under the umbrella of the experimental mass.

Feynman didn't manage to derive the Dirac equation using his path-integral method. So, by 'guessing' he was able to use an invariant regularization method based on a cut-off with the Dirac theory of the electron (Mehra 1994, 229-234).

Feynman developed is "little theory of electrodynamics in which the interaction is not exact on a delta function" (Schweber 1994, 427) as if it was different from the conventional electrodynamical theory. In his 1948 paper on the relativistic cut-off, Feynman presented his method as "a model, for which all quantities automatically do come out finite" (Feynman 1948, 1430). This doesn't mean that Feynman rejected the renormalization method. In his calculations he performed the mass and charge renormalization, but he saw his method as an "attempt to find a consistent modification of quantum electrodynamics" (Feynman 1949, 778). Because it was inconsistent, the correct physics had to be obtained by making the renormalization of mass and charge, and obtaining expressions independent of the cut-off parameter by making the cut-off parameter go to infinity after renormalization (Feynman 1962, 145).

With this tricky procedure in the edge of the abyss, be it with Schwinger's direct absorption of infinities in phenomenological parameters (the experimental mass and charge), or Feynman's inconsistent cut-off electrodynamics with last minute cut-off independent expressions, it is difficult to consider renormalization as "a sound and remarkable ingenious way of extracting information from an imperfect theory" (Teller 1988, 88), as a closer look can show.

3. The electron self-energy and the meaning of the cut-off regularization

The problems in the calculation of the electron self-energy are all concentrated in this apparently harmless integral:

$$ie_0^2 \Sigma(p) = \frac{e_0^2}{(2\pi)^2} \int d^4k \frac{1}{k^2 + i\epsilon} \frac{2\not{p} - 2\not{k} - 4m_0}{(p-k)^2 - m_0^2 + i\epsilon} \quad (\text{Mandl 1984, 187}).$$

As it stands this integral is divergent. From Lorentz invariance $\Sigma(p)$ can be put in the form

$$\Sigma(p) = A + (\not{p} - m)B + (\not{p} - m)\Sigma_c(p) \quad (\text{Mandl 1984, 189}),$$

where, in particular, $A = \Sigma(p) |_{\mathbf{p} = m}$. This term provides a correction $\delta m = -e_0^2 A$ of electromagnetic origin to the ‘mechanical’ mass of the electron, which in more classical language could be seen as resulting from the interaction of the electron with its own field, and in a more quantum language as resulting from the virtual emission and reabsorption of photons by the electron. For $k \rightarrow \infty$ the integral is logarithmically divergent (this is the famous ultra-violet divergence). A way out is to make a “change in the fundamental laws” (Feynman 1961, 137): the photon propagator $1/k^2$ is multiplied by a relativistic invariant convergence factor, assume by Feynman to be $c(k^2) = -\lambda^2/(k^2 - \lambda^2)$. This change has to be seen as a formal calculational device, a mathematical trick to get ride of the logarithmically divergence in the integral. If we try to see it as a new theory distinct from the one derived from classical electrodynamics we obtain a non-hermitian interaction Lagrangian that implies that probability is not conserved. Also from a physical point of view the use of this convergence factor is equivalent to consider “an additional interaction of the electron-positron field with a vector field whose quanta have mass λ and whose propagators are $-(k^2 - \lambda^2)^{-1}$ ” (Schweber 1961, 519). With this prescription it is possible to calculate the integral, with the result:

$$A = -\frac{3m}{8\pi^2} \ln \frac{\Lambda}{m} \quad (\text{Mandl 1984, 191}),$$

where Λ is a cut-off parameter.

It turns out that the only contribution from the self-energy not renormalizable is the finite integral $\Sigma_c(p)$. From this term the radiative correction (due to the electron self-energy) to the lowest-order calculations is obtained.

If we stopped here, we would have an experimentally measurable radiative correction dependent on an arbitrary cut-off parameter Λ . This problem wouldn’t be visible in what regards δm because we consider it to be ‘absorbed’ in the experimental measurable mass $m_{\text{exp}} = m_{\text{mech}} + \delta m$, that is seen as an amalgamation of the ‘mechanical’ mass and this electromagnetic mass, and whose magnitude – as a phenomenological parameter – is determined not by the theory but from experiments. To get things right, after the renormalization we have to make the cut-off parameter go to infinity, so that the radiative correction term “remains well-defined and finite in this limit and independent of the details of the regularization procedure” (Mandl 1984, 191). This method to overturn the problem of infinities in quantum electrodynamics was summarized by Feynman in one of his QED rules: “(1) Put in an arbitrary cutoff factor $c(k^2) = -[\lambda^2/(k^2 - \lambda^2)]$ for each propagator $1/k^2$. (2) Express everything in $m_{\text{exp}} = m - \delta m$. (3) Take the limit as $\lambda \rightarrow \infty$ and keep m_{exp} fixed.” (Feynman 1962, 143).

It is usually considered that the divergence problem in quantum electrodynamics is due to a failure of the theory at ultra-relativistic energies. This might lead to the idea that the cut-off parameter serves like a “boundary line separating the knowable region from the unknowable” (Cao 1993, 52). But since there is no indication on where to put this cut-off, it seems that “we cover our ignorance by calculating only quantities which are independent of the exact value of the cut-off” (Teller 1988, 87). This procedure would result in a change from the approximative regularized version of the theory to a recovered quantum electrodynamics with renormalized mass and charge. This means changing “the status of the cutoff from a tentative, and tantalizing, threshold energy to a purely formalistic device” (Cao 1993, 53).

Even if Feynman was trying to achieve a consistent regularized theory, and published his method as a provisory one while searching for a “correct form of f_+ [the function that substitutes the delta function appearing in the interaction term] which will guarantee energy conservation” (Feynman 1949, 778) it ended up being what Bethe had envisaged from the beginning: a mathematical calculational device designed to overcome the diverge problem in some integrals. Also from a physical point of view the regularized theory is completely different from quantum electrodynamics. It hasn’t a divergence problem because of the presence of an auxiliary vector field. This field can be seen as a formal mathematical device if, and only if, after the renormalization we recover a cut-off independent theory. If we tried to maintain the regularized theory, so that we didn’t have to deal with the problem of infinites in the calculations with quantum electrodynamics, we would be working not with quantum electrodynamics but with another field theory.

The divergence of the integrals and the use of the cut-off trick don’t reveal where the theory stops being good and a ‘true’ theory should come into play. It reveal structural problems in the design of the theory, that are impossible not to see when we have some integrals, that should (from a physical point of view) provide small radiative corrections to lower-order calculations, and end up blowing apart in a proliferation of infinites.

Considering the second-order term of the scattering matrix $S_2(x_1, x_2)$ in configuration space, the mathematical expression of the terms related with the divergent part of the electron’s self-energy are dependent on $\delta(x_1 - x_2)$. This means that “all the divergences in $S_2(x_1, x_2)$ come from terms proportional to $\delta(x_1 - x_2)$ and to its derivatives which differ from zero only in the infinitesimal neighbourhood of the point $x_1 = x_2$ ” (Bogoliubov 1959, 299). The divergence problem doesn’t arise solely from the fact that there is in the theory no upper bound to the energy of the virtual quanta. It results from the ‘coincidence’ in the theory of the inexistence of this natural limit for the energy of virtual quanta and from the local character of the interaction between the fields in quantum electrodynamics.

4 Charge renormalization and temporality in quantum electrodynamics

That the divergence in the calculation of the electron self-energy reveals structural problems related with distinct aspects of the theory doesn’t mean that the theory hasn’t other less noticed limitations not related to the divergence of the integrals and that would be present even if they were not divergent. This can be seen with a close look at the second-order calculation of the photon self-energy. Again we have a divergent integral and again a regularization scheme is used and a renormalization is done.

The (second-order) photon self-energy leads to a modification of the photon propagator: $D_{\mu\nu}'(k) = g_{\mu\nu}D(k) + D(k)\Pi_{\mu\nu}(k)D(k)$, where $\Pi_{\mu\nu}(k)$ is a quadratically divergent integral and $D(k)$ is the bare-photon propagator. Considering the requirement of Lorentz invariance, the second-order tensor $\Pi_{\mu\nu}(k)$ must have the form $\Pi_{\mu\nu}(k) = C(k^2)k_\mu k_\nu + D(k^2)g_{\mu\nu}$ (Jauch 1976, 189). As it stands, for $k^2 = 0$ we have $\Pi_\mu^\mu(0) \neq 0$. This would mean that the propagator we obtain taking into account this second-order correction is not the propagator for a zero mass photon but the propagator for a massive neutral vector boson (Sakurai 1967, 275). To recover our photon we must remember that $\Pi_{\mu\nu}(k)$ must be gauge invariant. Imposing this condition, we must have $\Pi_{\mu\nu}(k)k^\nu = 0$. From this we obtain the ambiguous result that the quadratically divergent integral

$$\Pi_{\mu}^{\mu}(0) = -\frac{2\alpha}{\pi} \frac{1}{i\pi^2} \int \frac{p^2 + 2m^2}{(p^2 + m^2)^2} d^4p \quad (\text{Jauch 1976, 190}),$$

must be identically zero. The only way to skip this situation is to consider that “the integral is, strictly speaking, meaningless, since it is divergent” (Schweber 1961, 552).

The pragmatic view is that we need a ‘functioning’ theory that is gauge invariant and provides a zero mass for the photon in the first terms of the perturbation expansion of the S-matrix that are used in practice. This implies when evaluating $\Pi_{\mu\nu}(k) = (k_{\mu}k_{\nu} - k^2 g_{\mu\nu})C(k^2)$ to take into account the divergent integral $\Pi_{\mu}^{\mu}(0)$ and subtracting it from $C(k^2)$, which leads to a logarithmically divergent integral.

Using a gauge invariant regularization scheme we have $\Pi(k^2) = C + k^2\Pi^f(k^2)$, where

$$C = \frac{\alpha}{3\pi} \log\left(\frac{\Lambda^2}{m^2}\right) \quad (\text{Sakurai 1967, 277}).$$

$\Pi^f(k^2)$ is a finite correction term, and C is logarithmically divergent as $\Lambda \rightarrow \infty$.

With this procedure we obtain a second-order finite (regularized) modified photon propagator.

The next step is to incorporate the regularized constant C in a parameter of the theory whose value is experimentally determined, so that we can take the cut-off limit to infinity and recover quantum electrodynamics from the regularized ‘theory’.

Considering, for example, the second-order correction to Møller scattering due to the self-energy of the photon, the change in the Møller scattering amplitude amounts to

$$\frac{e^2(\bar{u}_1\gamma_{\mu}u_1)(\bar{u}_2\gamma_{\mu}u_2)}{ik^2} \rightarrow e^2(1-C)\frac{e^2(\bar{u}_1\gamma_{\mu}u_1)(\bar{u}_2\gamma_{\mu}u_2)}{ik^2} + ie^2\Pi^f(k^2)(\bar{u}_1\gamma_{\mu}u_1)(\bar{u}_2\gamma_{\mu}u_2) \quad (\text{Sakurai 1967, 278}).$$

The trick is to consider the (infinite) constant $1 - C$ not as a correction factor related to the photon propagator itself, but, as can be seen more clearly in the limit $k^2 \rightarrow 0$, the factor can be seen as a correction to the electron charge ‘attached’ to the two vertices of the Feynman diagram representing the second-order correction to the scattering amplitude. In this way we relate this (infinite) correction to the coupling constant.

In the limit $k^2 \rightarrow 0$ the modified photon propagator is given by $D'_{\text{F}}(k) = (1 - C)D_{\text{F}}(k)$. We renormalize the theory considering that $(1 - C)^{1/2}$ is a correction to the unobservable electron bare charge e_{bare} , so that what is observed is $e_{\text{obs}} = (1 - C)^{1/2}e_{\text{bare}}$.

The distinctive flavour of this renormalization procedure when compared with the mass renormalization is that we are, so to speak, passing the problems of the photon to the electrons.

Looking closely at the second-order self-energy correction to the Møller scattering, the infinite arising in the photon propagator is absorbed by the charge of the electrons ‘located’ at each ‘vertice’ of the interaction. This is possible because the description of the scattering by an S-matrix perturbative approach is done in a way that what appears in between the initial and final asymptotic states is not described as a process occurring in time, but the situation is such that “the S-matrix describes the scattering in the operational spirit of Heisenberg’s matrix mechanics. It gives transition probabilities

which correspond to measurable relative frequencies. But it treats the scattering itself as a black box” (Falkenburg 2007, 131). In this way there is no incompatibility between the charge renormalization and the general procedure followed. But if in some way a temporal description was possible, it is clear that it would be incompatible with the charge renormalization procedure, because we can only have charge renormalization in a (statistical) ‘atemporal’ description of the interaction going on inside the unobservable black box.

This result is of course in agreement with the lower-order calculation of the Møller scattering, which, by itself, reveals the limitations of quantum electrodynamics in the temporal description of physical processes when compared with classical electrodynamics (cf. Valente 2008).

Conclusions

The previous results drive us back to the battery of criticism over quantum electrodynamics.

The fact is that, as Bohr clearly indicated, the theory was design based on different fields that have a small coupling constant. This is the strength and at the same time the weakness of the theory, as it is clearly exemplified in the description of scattering. All the description of the interaction between the fields is done from the point of view of free non-interacting fields, where the interaction is described using a perturbative approach that is mathematically clenched to the use of operators acting on free states.

We see that in part the possibility of renormalization is due to the ‘non-detailed’ approach to the description of interactions built from (apparently) clearly separated concepts: the Dirac and Maxwell fields.

But it is clear that when we go beyond the lowest-order approximations, and the mass and charge renormalization are needed to render the results finite, the theory falls into a conceptual contradiction: it is developed firmly from the idea of two clearly independent fields, but at the same time the more exigent experimentally testable results that are provided by the theory demand considering the mass and charge of the electron as having a non detachable contribution from the Maxwell field. In this way the observed ‘particle’ – the electron – is constructed from contributions from the two fields, and there is really no observational meaning to the so-called bare mass and bare charge.

This point is precisely what made Pauli, already in the thirties, to have a critical view regarding his and Heisenberg’s quantum electrodynamics. Not only he disliked all of the ‘subtraction physics’ that was being tried, but principally he felt the lack of explanatory depth in the theory, that he expected could only be achieved with a unified field theory that didn’t treat the Dirac and Maxwell fields as independent physical entities (Rueger 1992, 314).

The situation is ‘aggravated’ by the fact that besides this conceptual mismatch between the physical ideas used and the physical-mathematical implication of the theory, the procedure of charge renormalization is only possible due to the intrinsic limitations of the theory, that provides only statistical predictions for ensembles of systems, and where the time evolution is not related, as in classical physics, with the evolution in space of a physical system, but with the “wave giving the symbolic representation of its state” (Bohr 1935, 697).

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