# One real gauge potential is one too many

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**Abstract**

To single one out of the infinitely many, empirically indistinguishable gauge potentials of classical electrodynamics, and to deem it 'more real' than the rest is not trivial. Only two routes are open to one who might attempt to do so. The first leads to a slippery slope: if one singles out a potential solely by requiring it to admit well behaved propagations, and on the strength of this behavior one subscribes to its reality, one inevitably subscribes to the reality of infinitely many. As for the second, it seems to be barred from the beginning. But if, for reasons of metaphysical economy, one insisted on taking it, it would lead to a 'truncated theory' that is physically and empirically inferior to the complete.

## 1 Introduction

From the revival of the Hole Argument we have learned that if a theory involves a proliferation of in-principle unobservable yet regarded as real entities, or if the time evolution of those entities is indeterministic, and we attempt a literal interpretation of that theory, then we have gone too far into the tangle of metaphysics. Classical electromagnetism is a gauge theory that involves an infinity of in-principle unobservable, empirically indistinguishable, and, in a sense, mathematically equivalent gauge potentials[1]. Endeavors to provide a literal interpretation of the theory bring about both proliferation of unobservable but considered to be real potentials, and indeterministic time evolution.

[1] Obviously, potentials that belong to different gauges differ from each other in that they satisfy different dynamical equations. Yet, their gauge equivalence guarantees that they yield the same electric and magnetic fields. This is sufficient for physicists (Jackson among them) to proclaim them "fully equivalent" (2002, p.917).

In a recent paper, Mattingly (2006) attempted to rid classical electromagnetism of its metaphysics by suggesting that we can distinguish a particular potential, the so-called Liénard-Wiechert (LW potential henceforth), and subscribe to its reality. With this move, he contends, we make a metaphysical concession (like any other, the LW potential cannot be observed independently of the electromagnetic field-strengths), but this is only a minimal price to pay because we gain a lot. Not only do we "eliminate reference to gauge dependent quantities"[2] (and thus avoid both ontological proliferation and indeterminism[3]), but also we bring in a causal imagery that accounts for effects like the Aharonov-Bohm (A-B henceforth)[4]. An inspired attempt no doubt, but more is required: sound reasons that would validate the differentiation of an unobservable entity from an infinity of observationally indistinguishable entities, and would justify our insistence on its reality. Try as one might, I will argue, it is impossible to find any legitimate reasons for such a move.

To advance this argument, I will first introduce the LW potential along with a larger family of potentials, the $\alpha$-Lorenz, to which it belongs. While doing that, I will call attention to the fact that had we dubbed 'real' any of the $\alpha$-Lorenz potentials, we would have eliminated reference to gauge dependent quantities, and we would have brought in causal imagery similar to that afforded by the LW potential. Surely, there are differences between the LW potential and the rest: the latter potential alone has components that propagate on the light cone only, and this potential alone admits a manifestly Lorentz covariant expression. Yet, given their similarities, a question naturally arises: are these differences physically significant? Or, put differently, do these differ-

[2]Mattingly (2006), p. 250.

[3]Indeterminism in this case is brought about by gauge freedom. It is of a peculiar kind and reminiscent of indeterminism in the hole argument. Roughly, the idea why indeterminism is involved in this case is the following. Electromagnetic fields propagate deterministically. Yet, gauge freedom allows us to choose, at any given space-time point, any one of the infinitely many potentials the theory permits. Thus, propagation of potentials implicates indeterminism because we may begin with any one of those potentials but, for no reason related to any causal interceptions, end up with any other.

[4]To facilitate the reader who is not familiar with the A-B effect, here is a summary of it. Beam-electrons shot from an electron gun pass from two slits and end up on a screen at some distance from the slits. On the screen they produce an interference pattern. The interference pattern of the beam-electrons is shifted when a magnetic field is produced inside a solenoid that is located right outside and between the slits. The peculiar thing about the effect is that the wave-functions of the beam-electrons do not interact with the magnetic field, but, according to the mathematical theory of the effect, they couple with the so-called gauge potentials. It is important to note that the magnetic field is confined within the solenoid, and it is zero in all the regions that are accessible to the beam-electrons.

ences suffice to differentiate the LW potential from the rest, and justify ascription of reality to it alone? In general, the answer to this question is negative, I will argue.

Then I will point out that if one insisted on ascribing reality to a potential, only two routes would be open to them. On the first route, classical electrodynamics, with its gauge freedom and ontological commitments, is taken for granted, but a potential is singled out and is deemed more real than the rest. On the second route one assumes that all electromagnetic effects and interactions supervene on a unique potential. The first route leads to a slippery slope, I will insist. Since the LW potential's special propagation characteristics and manifest Lorentz covariance do not constitute adequate justification for ascription of reality to it alone, if we insisted on its reality, we would be forced to subscribe to the reality of the infinitely many potentials of the $\alpha$-Lorenz gauge. As for the second route, it seems to be blocked from the beginning: since there is no difference of physical significance between the LW potential and the rest of the $\alpha$-Lorenz gauge, ascription of reality to the LW potential alone cannot be justified. Nonetheless, while on the second route one could still appeal to reasons of metaphysical economy and forge ahead. But then, I will show, one would also have to sacrifice part of the empirical content of the complete theory of electrodynamics.

# 2 Potentials

## 2.1 The Liénard-Wiechert Potential[5]

Begin with electric charges and currents, and the geometric structure of Minkowski space-time. Assume that a kind of 'disturbance' or potential will be produced by them and will propagate at the speed of light $c$. Using these elements only, define or construct through an integral equation the following 4-vector potential:

$$A^{\alpha}(x) = \frac{4\pi}{c}\int d^4x' D_r(x-x')J^{\alpha}(x'), \tag{1}$$

where $D_r(x-x')$ the retarded Green's function

$$D_r(x-x') = \frac{1}{2\pi}\theta(x_0-x'_0)\delta[(x-x')^2], \tag{2}$$

with propagation speed $c$. With $\tau$ being the charge's proper time, and $V^{\mu}(\tau)$ its four-velocity, the current $J^{\alpha}(x')$ is defined by

$$J^{\alpha}(x') = ec\int d\tau V^{\alpha}(\tau)\delta^{(4)}[x'-r(\tau)], \tag{3}$$

[5] The subsequent description of the LW potential relies on Jackson (1975).

When $\tau_0$ is defined by the light-cone condition $[x - r(\tau_0)]^2 = 0$, the potential takes the form

$$A^\alpha(x) = \frac{eV^\alpha(\tau)}{V \cdot [x - r(\tau)]}|_{\tau=\tau_0}, \tag{4}$$

which is also known as the LW potential.

Obviously the LW potential represents a field that emanates directly from electric charges and currents, and propagates continuously in space-time at the speed of light. It is related to the electromagnetic field strengths of classical electrodynamics through the equation

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha, \tag{5}$$

and one may go as far as to claim that the LW potential generates or produces the field strengths. Also, it couples directly with quantum mechanically described charged fields, it is manifestly Lorentz covariant and it is a quantity that is not manifestly U(1)-gauge dependent. Finally, once defined, its time evolution is determined. In short, it is a mathematical entity that displays many of the characteristics one routinely associates with physical objects that causally affect other physical objects.

From a different perspective, the LW potential is merely one of the infinitely many gauge potentials of classical electrodynamics. It is the solution to Maxwell's equations in the Lorenz gauge, which is expressed by the condition

$$\partial_\mu A^\mu = \nabla \cdot \mathbf{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0. \tag{6}$$

From this perspective, the LW potential is physically indistinguishable from, and gauge equivalent to all the other gauge potentials that solve Maxwell's equations.

## 2.2 The $\alpha-$Lorenz Potentials[6]

Begin with electric charges and currents, and the geometric structure of Minkowski space-time. Assume that a kind of 'disturbance' or potential will be produced by them; certain of the components of this potential will propagate at the speed of light $c$, while the propagation of the rest will involve a speed $\alpha c$, where $\alpha$ is an arbitrary positive constant. Using these elements only, define or construct through integral equations scalar

[6] For the description of the $\alpha-$Lorenz potentials in this section I follow Brown & Crothers (1989). The interested reader may also refer to Yang (1976) and Jackson (2002), or Yang (2005) for the most recent exposition.

and vector potentials as follows:

$$\phi_{\alpha L}(\mathbf{x},t)=\int_{t_0}^{t_1}dt'\int_V d^3x' G(\mathbf{x},t|\alpha c|\mathbf{x}',t')\rho(\mathbf{x}',t) \tag{7}$$

and

$$\mathbf{A}_{\alpha L}(\mathbf{x},t)=\frac{1}{c}\int_{t_o}^{t_1}dt'\int_V d^3x'\mathbf{\Theta}(\mathbf{x},t||\mathbf{x}',t')\cdot\mathbf{J}(\mathbf{x}',t). \tag{8}$$

$G(\mathbf{x},t|\alpha c|\mathbf{x}',t')$ is a scalar Green's function with propagation speed $\alpha c$, while $\mathbf{\Theta}(\mathbf{x},t||\mathbf{x}',t')$ is a vector Green's function, and $\rho(\mathbf{x}',t)$ and $\mathbf{J}(\mathbf{x}',t)$ are the charge and current densities respectively. The vector Green's function can be resolved into two components:

$$\mathbf{\Theta}(\mathbf{x},t||\mathbf{x}',t')=\mathbf{\Theta}(\mathbf{x},t|c|\mathbf{x}',t')+\tilde{\mathbf{\Theta}}_\alpha(\mathbf{x},t||\mathbf{x}',t'), \tag{9}$$

the first of which propagates at $c$, while the propagation speed of the second is not fixed[7].The second component, however, can be rewritten as a product of two propagators, one corresponding to propagation speed $c$ and another corresponding to propagation speed $\alpha c$[8]. Substituting (9) into (8) we get

$$\begin{aligned}\mathbf{A}_{\alpha L}(\mathbf{x},t)=&\frac{1}{c}\int_{t_o}^{t_1}dt'\int_V d^3x'\mathbf{\Theta}(\mathbf{x},t|c|\mathbf{x}',t')\cdot\mathbf{J}(\mathbf{x}',t)+\\&+\frac{1}{c}\int_{t_o}^{t_1}dt'\int_V d^3x'\tilde{\mathbf{\Theta}}_\alpha(\mathbf{x},t||\mathbf{x}',t')\cdot\mathbf{J}(\mathbf{x}',t).\end{aligned} \tag{10}$$

The potentials given by (7) and (10) are called $\alpha-$Lorenz potentials[9].

Obviously the $\alpha$-Lorenz potentials represent fields that emanate directly from electric charges and currents, whose various components propagate continuously in space-time. The scalar component, described by equation (7), propagates at speed $\alpha c$, but its vector part, described by equation (10), has two components: the first propagates at the speed of light, whereas the second "propagates like an expanding smoke-ring"[10].

[7] This is precisely the meaning of the double line in $\tilde{\mathbf{\Theta}}_\alpha(\mathbf{x},t||\mathbf{x}',t')$.

[8] Yang (2005) gives an explicit expression of $\tilde{\mathbf{\Theta}}_\alpha(\mathbf{x},t||\mathbf{x}',t')$ in terms of Green's functions, $\alpha c$ and $c$, namely $\int d^3x''dt''G(\mathbf{x},t|c|\mathbf{x}'',t'')G(\mathbf{x}'',t|\alpha c|\mathbf{x}',t)$ (equation 3.33, with $v=\alpha c$).

[9] Some (e.g. Jackson (2002), Yang (2005)) also call them the potentials of the velocity or $v$-gauge.

[10] Brown & Crothers (1986, p.2955). The details of the propagation of the second component of the $\alpha$-Lorenz potential's vector part depend on whether $\alpha<1$ or $>1$. In the first case the outer boundary propagates at the speed of light and the inner at speed $\alpha c$, whereas in the second the speeds are reversed. In either case, the second part of the vector-component spreads in space-time in a way similar to that of expanding smoke-rings.

The fact that $\alpha$ is an arbitrary positive constant allows for the propagation speeds of those components to differ from almost zero, to equal to that of light, to as fast as almost instantaneous[11]. However, for $0 < \alpha \leq 1$, all components are either $c$-retarded or $\alpha c$-retarded; that is, they propagate continuously and at speeds that do not violate any special relativistic or causal requirements. Thus, they too display the essential characteristics one routinely associates with physical objects that causally affect other physical objects.

Like the LW potential, the potentials of (7) and (10) are related to the electromagnetic field-strengths through the equations

$$\mathbf{E}(\mathbf{x}, t) = -\boldsymbol{\nabla}\phi_{\alpha L}(\mathbf{x}, t) - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}_{\alpha L}(\mathbf{x}, t) \tag{11}$$

and

$$\mathbf{B}(\mathbf{x}, t) = \boldsymbol{\nabla} \times \mathbf{A}_{\alpha L}(\mathbf{x}, t). \tag{12}$$

Also, like the LW potential, they couple directly with quantum mechanically described charged fields, and they are not manifestly U(1)-gauge dependent. Finally, unlike the LW potential, they lack manifest Lorentz covariance; but as we shall see shortly, this difference turns out to be irrelevant to any arguments a propos the alleged reality of any of them.

From a different perspective, the potentials of (7) and (10) are merely solutions to Maxwell's equations[12] in the so-called $\alpha-$Lorenz gauge, which is expressed by the condition

$$\boldsymbol{\nabla} \cdot \mathbf{A}_{\alpha L} + \frac{1}{\alpha^2 c}\frac{\partial \phi_{\alpha L}}{\partial t} = 0. \tag{13}$$

From this perspective, then, the $\alpha$-Lorenz potentials are infinitely many physically indistinguishable from and gauge equivalent to all the other potentials that solve Maxwell's equations.

## 3 Potential candidates for reality

If one insisted that gauge potentials must be interpreted realistically, what justification would be needed for attributing reality to one or more of the potentials introduced above? And if more than one potential fit the bill, would it still be possible to argue *on physically relevant grounds* that only one among them is truly the real one? Even if the answer to

[11] Note that for $\alpha = 1$ the resulting potential is no other than the LW. It is also worth pointing out that at the limit $\alpha \to \infty$, the resulting potential is no other than the Coulomb.

[12] The proof of this fact is far from trivial. The interested reader may refer to Yang (1976, 2002) and Brown & Crothers (1989) for detailed proofs and further discussion.

the first question turned out to be 'yes', I am positive that the second question would have to be answered in the negative.

In attributing reality to mathematical entities, the first consideration one usually takes into account is observational evidence. Given that no potential can be observed independently of field-strengths, and that there are infinitely many potentials which yield the exact same electromagnetic observables, there are no empirical means which we might use in order to distinguish any one of them from any other. Focusing on the potentials of the $\alpha$-Lorenz gauge that I introduced above, the unobservability of the potentials entails that no matter what the value of $\alpha$[13], the parts of the potentials whose propagation speed differs from that of light do not affect the observables of electromagnetism. Roughly and qualitatively speaking[14], gauge covariance guarantees that contributions from these parts cancel out, and in the end the resulting electromagnetic observables are independent of $\alpha$. Thus, if the decision had to be grounded on empirical considerations, dubbing one potential 'real' would be tantamount to dubbing them all 'real'.

Since appeal to directly observable and measurable properties is of no use, other means must be employed. This, however, entails that the kind of reality to be appealed to is, in a sense, weaker than the reality attributed to mathematical expressions, e.g. spinors, which describe actual observable entities, e.g. electrons. In the case of electrons, the correspondence between theoretical and physical entities is straightforward; in the case of potentials it is not. For this reason, arguments in favor of an interpretation of gauge potentials as real causally efficacious objects can only be a posteriori and rely on indirect evidence.

The argument that I am challenging in this paper rests on the following idea. If through "non-arbitrary analysis of what is real in a world accurately described by classical electrodynamics"[15] we manage to find a unique potential, which is "purely intrinsically and locally definable in terms only of the charge and the geometric structure of spacetime"[16], this potential must correspond to the reality underlying electromagnetic effects like the A-B. This, I take it, means that a potential must be definable (or constructible) not through Maxwell's differential equations, but through formulae involving only charge distribution and the structure of

[13] Since gauge potentials are *not* observable, all the potentials of the $\alpha$-Lorenz gauge could be included in the present discussion; the propagation speed of the observables would always be within the limits set by Einstein's principle of light.

[14] Once again, I refer the interested reader to the aforementioned references for detailed proofs and further discussion.

[15] Mattingly (2006), p. 251.

[16] Op. cit.

space-time[17]. The mathematical entities that enter the definition or construction of the potential must correspond directly to physical entities or quantities that are locally definable[18] and propagate in accordance with causal requisites, i.e. continuously and at speeds less than or at most equal to the speed of light.

The potential that is thus constructed must too be locally definable, and propagate in accordance with causal requisites; thus there is no room for instantaneous propagation. But in addition, this potential must be non-gauge-dependent, propagate deterministically and be compatible with special relativity and quantum mechanics in all respects. Since the constraints of special relativity dictate that real objects propagate continuously and at speeds that do not exceed the speed of light, the theoretical description of any purportedly real potential must be so constrained too. Finally, the mathematical expression of the potential must not violate the theory's Lorentz covariance, and it must allow for quantization. With the requirements in place, and, consequently, restricting our attention to potentials of the $\alpha$-Lorenz gauge with $0 < \alpha \leq 1$[19], the question that opened this section becomes: among these potentials, are there any that fulfill all these requirements?

For the definition or construction of the LW potential and the other potentials of the $\alpha$-Lorenz gauge, one uses charge and current distributions, and the structure of space-time alone. The components of each potential, LW or otherwise, emanate directly from charges and propagate continuously at finite luminal or sub-luminal speeds. Thus the behavior of any potential in $\alpha-$Lorenz gauge is in accordance with what we might call 'perfectly acceptable causal requirements'. Moreover, each of these potentials is manifestly gauge-independent and it propagates deterministically.

At first glance, though, the LW potential seems to be privileged in that all its components propagate on the light-cone, and therefore its propagation speed is invariant, whereas the propagation speed of the components of the other $\alpha$-Lorenz gauge potentials is explicitly frame-depenendent. In addition, the LW potential is also unique in that its

[17] Assumed throughout to be Minkowski.

[18] I take it that locally definable objects may be localized, but they may also be spatially extended.

[19] In what follows, I limit my discussion to $0 < \alpha \leq 1$. This restriction does not in any way affect what I have to say about the $\alpha$-Lorenz potentials from now on, and it will not allow our attention to be diverted to issues that are not directly relevant to my argument. The reader who is interested in these issues may refer to Yang & Kobe (1986) and Yang (2002).

Thus, from now on, '$\alpha$-Lorenz gauge' means 'restricted $\alpha$-Lorenz gauge with $0 < \alpha \leq 1$.

mathematical expression is manifestly Lorentz covariant. This implies that so far as the LW potential is concerned, Lorentz covariance of the theory is guaranteed and so is its quantization. Hence, among the potentials of the $\alpha-$Lorenz gauge only the LW potential appears to fulfill all the requirements we imposed.

I argue, however, that the characteristics which are unique to the LW potential do not constitute differences of physical significance. Therefore, on the proviso that my claim is well-substantiated, it follows that if one insisted on interpreting the LW potential as 'real' just because it fulfills these requirements, one would have to interpret as real the infinitely many potentials of the $\alpha$-Lorenz gauge as well. Let us turn to all the challenges my claim must meet, one at a time.

## 3.1 The insignificance of the speed of light

To begin with, let us muse over the difference in propagation speeds. By itself, the fact that $\alpha$-Lorenz potentials with $\alpha \neq 1$ have components whose propagation speeds differ from that of light is irrelevant to questions concerning their reality. Given the invariance of the speed of light in special relativity, one might be tempted to press that this is the 'only real speed' allowed for by the theory. By the same token, since the propagation speed of the LW potential is equal to that of light, one might argue that if a potential were real, the LW potential must be it.

The fact of the matter, however, is that this idea is misguided. For, consider an observable actual particle, or a collection of such particles[20]. More likely than not, their speeds with respect to any inertial frame are slower than that of light, and therefore not Lorentz invariant. In this case, even if one asserted that the only 'real' speed in special relativity is the speed of light, one would be hard pressed to argue against the reality of the actual moving particles and ground their argument on the non-reality of their speed. Thus, the fact that the exact value of $\alpha c-$Lorenz is always frame-dependent for $\alpha \neq 1$ cannot and does not render them less real than light, for the same reason that frame-dependence of the speed of an actual particle cannot and does not render it less real than light.

Obviously the analogy is not exact. Particles are not only observable but also causally efficacious beyond reasonable doubt, and their time-evolution is fixed. Gauge potentials, on the other hand, are not independently observable and the indeterminism associated with them is of a peculiar kind that surpasses even quantum indeterminism. As for their causal efficacy, it is under consideration and we may or may not be

[20] To avoid complications that are not relevant to the argument here, consider these particles to behave classically.

able to establish it beyond reasonable doubt. But assume for the sake of the argument that potentials *were* causally efficacious objects; the fact that some parts of some of them would propagate at a non-invariant speed could not and would not entail that they are less real than the ones propagating at the invariant speed of light. Therefore, despite the fact that the description of the propagation of particular potentials in the $\alpha$-Lorenz gauge requires the additional structure of inertial frames, our analogy guarantees that the difference is irrelevant to the question of their reality[21].

Insisting on the difference between the propagations speeds, one might still object that since the exact value of $\alpha$ is inertial-frame-dependent, more than the generic structure of Minkowski space-time is required for the construction of the $\alpha$-Lorenz potentials with $\alpha \neq 1$. On the other hand, they might urge, since the propagation speed of the LW potential's components is the same with that of light, only the LW potential and this alone can be defined or constructed without appeal to inertial frames. Therefore, they would conclude, only the LW potential meets all the conditions that are necessary for reality.

I believe that this kind of argument is based on a negligence. Without doubt, the exact value of $\alpha$, which appears explicitly in equations (7) and (10), can only be given relative to an inertial frame. For this reason, the exact description of the propagation of the components of any particular potential in the $\alpha$-Lorenz gauge (for $\alpha \neq 1$) requires more than the generic structure of space-time: it requires the structure associated with particular inertial frames. But so does the description of the exact form of the individual components of the LW potential. Let me explain.

The LW potential is not an invariant of special relativity. Rather, its form is that of a four-vector, as is the form of the charge/current density that appears in equation (1), and of the velocity that appears in equation (4). Their Lorentz covariant formulation implies that the exact values of each of their four components depend on our point of view; that is, they depend on inertial frames.

To clarify this point further, consider the expression of the component $A^\alpha$, given by equation (4). This is proportional to $V^\alpha$, the corresponding component of the charge's four-velocity. But four-velocity is Lorentz covariant, and thus the actual value of each of its components is inertial-frame-dependent too. This shows that the frame-dependence of the exact value of $\alpha$ might constitute an argument against the reality of the $\alpha$-

[21] Note, for later convenience, that one might argue that if gauge potentials are the reality underlying electromagnetic effects, they should propagate like light. However, I believe that whether this contention is legitimate or not depends on one's ontological commitments. For this reason I postpone its discussion until the next section.

Lorenz potentials (with $\alpha \neq 1$) as much as the frame-dependence of the exact value of $V^{\alpha}$. If one of them is not real, neither is the other, and vice-versa.

### 3.2 Lorentz covariance and its significance

A plausible objection that could be raise is that the fact that the overall expression of the LW potential is manifestly Lorentz covariant still counts as a difference of physical significance; for, first of all, it renders it the only potential that is "uniquely" definable in terms of charge and the generic structure of space-time alone. Strictly speaking, of course, the only uniquely definable quantities of a covariant theory are its invariants. Yet, manifest Lorentz covariance guarantees that the overall form of four-vectors or higher order tensors is the same in all inertial frames. Thus, they too may be thought of as uniquely definable in the sense that once defined, their form remains invariant under Lorentz transformations.

On the other hand, one may insist, the overall expression of the other $\alpha$-Lorenz potentials is not Lorentz covariant and, therefore, explicit appeal to an inertial frame is necessary to even formulate equations (7) and (10). This is as good indication as any that the LW potential is the only one which is defined or constructed once and for all; whereas the other $\alpha$-Lorenz potentials need to be re-defined or re-constructed on each specific inertial frame from scratch.

In effect, however, the form of equations (7) and (10) is ultimately frame-independent too. The fact that their specific form is acquired only relative to a specific frame is in exact analogy to the fact that the exact value of, say, the velocity of a point-particle can be specified only relative to an inertial frame. In the same way that no one specific inertial frame is required for the definition or construction of the generic form of a particle's four-velocity, no such frame is required for the construction or definition of the scalar potential in (7) or the vector potential in (10) either.

To understand why this is the case, consider the $\alpha$-Lorenz gauge condition (13). This can be generalized as follows[22]:

$$G^{\mu\nu}\partial_{\mu}A_{\nu}^{\alpha L}=0, \tag{14}$$

where $G^{\mu\nu}$ is any $4\times 4$ matrix, $\partial_{\mu}$ is the usual derivative in Minkowski space-time, and $A_{\nu}^{\alpha L}=(\frac{1}{\alpha^{2}}\phi_{\alpha L,}\mathbf{A}_{\alpha L})$. This generalization brings to light the fact that $\alpha$-Lorenz potentials constitute a Lorentz invariant class. This, in turn, entails that Lorentz transformations map $\alpha$-Lorenz potentials onto other $\alpha$-Lorenz potentials. Thus, once $\alpha$-Lorenz potentials

[22]For this formulation I am indebted to the Editor of this journal, Professor 't Hooft.

have been defined, through equations (7) and (10), they have been specified for all inertial observers and no more than the generic characteristics of space-time are required in order to identify their form in any inertial frame.

To leave no doubt that this indeed is the case, and to relate equations to the desired causal imagery, let us make the same point using a more familiar expression of equation (14). We may express equation (13) in a particular inertial frame, say $K'$, as follows[23]:

$$\nabla' \cdot \mathbf{A}'_{\alpha L} + \frac{1}{\alpha^2 c}\frac{\partial \phi'_{\alpha L}}{\partial t'} = 0. \tag{15}$$

Taking the time-like unit vector in $K'$ to be $n'^{\mu} = (1; \mathbf{o})$, equation (15) can be rewritten as

$$\nabla' \cdot \mathbf{A}'_{\alpha L} + \frac{1}{\alpha^2 c}(n'_{\mu}\partial'^{\mu})(n'_{\nu}A'^{\nu}) = 0, \tag{16}$$

and therefore as

$$\partial'_{\mu}A'^{\mu} + (\frac{1}{\alpha^2 c} - 1)(n'_{\mu}\partial'^{\mu})(n'_{\nu}A'^{\nu}) = 0. \tag{17}$$

Equation (17) is generally covariant; thus in a frame $K$, with time-like unit vector $n^{\mu} = (\gamma; \gamma\boldsymbol{\beta})$, it becomes:

$$\partial_{\mu}A^{\mu} + (\frac{1}{\alpha^2 c} - 1)(n_{\mu}\partial^{\mu})(n_{\nu}A^{\nu}) = 0, \tag{18}$$

where $A^{\mu}$ is simply the usual Lorentz-transformed $A'^{\mu}$[24]. This goes to show that once we acquire the explicit form of an $\alpha$-Lorenz potential in any particular inertial frame, we can deduce its form in any other inertial frame using only the Lorentz transformation properties of four-vectors. This is exactly what happens with four-vectors, in general, and the LW potential in particular. Namely, once we acquire its explicit form in any particular inertial frame, we can deduce in any other inertial frame using only the Lorentz transformation properties of four-vectors. Therefore, equations (7) and (10) do not require reference to any particular inertial frame, and they are, in effect, frame independent.

Returning to the causal imagery, the meaning of the preceding discussion is that if gauge potentials *were* causally efficacious and observable objects, what would appear as an $\alpha$-Lorenz potential in one inertial frame would also appear as an $\alpha$-Lorenz potential in any other, albeit

[23]For this approach I am grateful to Professor J. D. Jackson.

[24]Note, for later convenience, that in this case we can, and do, treat gauge potentials as four-vectors.

with different propagation characteristics. The scalar component, originally given by equation (7), will still be retarded, but the actual value of its propagation speed will most likely be different; so will the second part of the vector component, originally given by equation (10). The exact form and propagation speed of these components depends, of course, on their initial form and propagation speed. But the fact of the matter is that they can be found using a manifestly Lorentz covariant equation, equation (18), and the usual Lorentz transformation of four-vectors, no more.

Reiterating my previous example, the situation here is no different from the situation of a quantity like the four-velocity of, say, a point-particle. As with $\alpha$-Lorenz potentials, to define this quantity there is no need to appeal to any one inertial frame in particular. Yet again, as with $\alpha$-Lorenz potentials, the specific values of the components of that four-velocity can only be obtained relative to a specific inertial frame. Under Lorentz transformations the components of the four-velocity will change in the way components of vectors change; but also, in the case of the $\alpha$-Lorenz potentials the components will change in a way dictated by a Lorentz covariant equation. So, really, $\alpha$-Lorenz potentials only behave in a way analogous to that of a "uniquely" definable quantity. The only difference between them is that whereas the "uniquely" definable four-velocity transforms like a four-vector under Lorentz transformations, $\alpha$-Lorenz potentials follow a more complicated transformation rule. But again, this is the most one can expect from compound objects that do not propagate at the speed of light.

Naturally, the question arises: even if the previous objection is overcome, isn't manifest Lorentz covariance *per se* physically significant? After all, one may rightly claim, the mathematical expression of physically significant entities is usually of that type. In addition, and most decisively, Lorentz covariance of the entire theory is also at stake. Manifest Lorentz covariance of these expressions is therefore essential for methodological reasons, even if not for ascription of reality. My answer, of course, is that no, the physical significance of manifest Lorentz covariance cannot be thus supported, and here are the reasons.

To answer the first complaint first, it is true that in theories which admit special relativistic formulation the mathematical expressions of physically significant objects, entities and quantities are usually manifestly Lorentz covariant. In other words, the mathematical expressions of these objects, etc. usually take the form of invariants, four-vectors or higher-order tensors. An example of a manifestly Lorentz covariant object is provided by the electromagnetic field-strengths. On the other hand, these theories may also involve objects, etc. whose mathematical expressions are given relative to an inertial frame. A typical example of a non-covariant quantity is three-velocity.

But the idea that real objects, etc. can be represented only by manifestly Lorentz covariant expressions is misguided. To begin with, manifest Lorentz covariance is not sufficient for reality. One may combine physically significant scalars and tensors of a theory to successfully construct manifestly Lorentz covariant expressions with no physical counterparts or interpretations. Moreover, manifest Lorentz covariance is not necessary either. There exist quantities (like the three-velocity itself or the height of a building or the electric field relative to the lab) which, though not manifestly Lorentz covariant, admit a physical and often realistic interpretation.

One might object, of course, that all the aforementioned quantities can be expressed in terms of covariant quantities: four-velocity, four-vector and the electromagnetic-field-tensor respectively. Hence, since physically significant objects etc. *can* be expressed in some Lorentz covariant form or another, the mathematical expressions of all physically significant objects etc. *must* be expressible in covariant form too. In this manner, by using the fact that the LW potential is the only one among the $\alpha$-Lorenz potentials that is manifestly Lorentz covariant, one might advance the case of its reality. But this would be, at best, a weak argument from necessity (which could not secure reality), or, at worse, an argument involving an unwarranted leap from 'can' to 'must'. I am afraid that the latter is the case.

In order to get my point across, let us consider the following example. Assume that the object of our interest is an arbitrary distribution of light and matter. The result of a Lorentz transformation would be yet another distribution where photons still move at the speed of light, but material particles move at speeds different from their original ones. Assuming that the Lorentz transformation rule for the shape of the entire distribution could be found, it would be *very surprising* if it turned out to be anything like a tensor transformation rule. And yet, it is preposterous to claim that the distribution is not real just because its mathematical description does not transform like a tensor under Lorentz transformations. Along the same lines, it would be preposterous to claim that a theoretical entity, like a gauge potential, cannot represent a real object just because its expression is not manifestly Lorentz covariant; and it would be unwarranted to claim that a potential is real just because its expression is.

A plausible response is that a distribution can be perceived of as a conglomeration of objects, ideally elementary, whose mathematical descriptions are necessarily Lorentz covariant. Thus the point is not whether there exist compound objects whose description is not manifestly Lorentz covariant; rather, the point is what happens when fundamental objects are concerned. When it comes to that, although manifest Lorentz covariance may not to be necessary, strictly speaking, it may still be desirable as a methodological constraint. Put differently, manifest Lorentz covariance of mathematical expressions of objects etc. is physically significant because it safeguards the Lorentz covariance of the theory as a whole.

The requirement for Lorentz covariance of theories, however, is rather subtle. For this reason, before I tackle this last objection, I have to first say a few things in order to clarify the meaning and implications of the requirement of Lorentz covariance.

When the relation between Lorentz covariance of a theory and manifest Lorentz covariance of expressions in it is examined, sometimes a mistaken conflation of the two notions takes place. What is really at stake is the entire theory's Lorentz covariance. Thus relevant question is not whether the theoretical description of its objects is manifestly Lorentz covariant. Rather, the pertinent question is whether lack of manifest Lorentz covariance in the theoretical description of these objects jeopardizes the overall covariance, the covariance of the theory as a whole. The answer, of course, is no, it does not.

In general, although manifest Lorentz covariance of the expressions of a theory guarantees the covariance of the theory as a whole, lack of manifest Lorentz covariance of certain expressions does not always destroy the theory's covariance. When it comes to gauge potentials, their general behavior under Lorentz transformations is idiosyncratic, yet well documented. The potentials of the Lorenz gauge, and therefore the LW potential, behave like four-vectors. All other potentials do not. But the generic form of their Lorentz transformations is such that the theory's Lorentz covariance is guaranteed[25]. There is an extensive literature on the issue, and the conclusion, time and again, has been that the lack of manifest Lorentz covariance of different gauges does not affect the covariance of the theory as a whole[26]. The lack of manifest Lorentz covariance of the $\alpha$-Lorenz potentials is no different, but let me persist on this point a little longer.

When certain non-covariant but physically significant entities appear in a theory, there is only one way to tell whether their lack of manifest

[25] For a general account of the general behavior of gauge potentials under Lorentz transformations, the interested reader may refer to Bjorken & Drell (1965), especially pp. 73-4.

[26] For further reinforcment of this point, the interested reader may refer to the literature re the Coulomb gauge. See, for example, Zumino (1970) and France (1976).

Lorentz covariance is undesirable: by examining the impact of this deficiency on the observable implications of the theory. In the case at hand, manifest Lorentz covariance would not only be desirable, but it would be required only if its absence entailed observational or physical inconsistencies. Such inconsistencies would be signaled either by dependence of the resulting classical observables on the arbitrary constant $\alpha$, or by a failure to formulate quantum electrodynamics (QED) in that gauge.

Classical electromagnetism's gauge covariance guarantees that the observables of the theory are the same, no matter what the gauge potentials. Therefore, the acid test in this case is provided by quantum theory, and the question that determines the fate of $\alpha$-Lorenz potentials with $\alpha \neq 1$ is: does the explicit dependence of the potentials on $\alpha$ destroy the possibility for quantization? If it did, the potentials of the $\alpha$-Lorenz gauge would destroy the theory's Lorentz covariance as well; but if it did not they would not. In 1990 Baxter presented a self-consistent formulation of QED in the $\alpha$-Lorenz gauge, and proved that the quantum observables resulting from it do not depend on $\alpha$ either. Thus, in the case at hand, not only is the requirement for manifest Lorentz covariance dispensable, but also its alleged desirability is not justified by appeal to the Lorentz covariance of the theory as a whole.

## 3.3 Candidates for reality

Before I turn to the main argument of this paper, a summary of the conclusions I have already established is in order. I begun this section by asking two questions. The first concerned the justification that would be required for attributing reality to one or more of the $\alpha$-Lorenz gauge potentials. Since one cannot appeal to their observability, if, for other reasons, one felt compelled to attribute reality to them, one would have to examine whether there are any among them that behave like real causally efficacious objects. The requirements one may come up with cannot establish reality, but at least one may argue that they are necessary.

Restricting my attention to potentials in the $\alpha$-Lorenz gauge with $0 < \alpha \leq 1$, I have shown that there are infinitely many potentials from which one might choose. All of them emanate directly from charges, are defined or constructed using charge/current distributions and the structure of Minkowski space-time alone, and propagate in accordance with constraints imposed by both special relativity and causal considerations.

I have also shown that despite the fact that only one among them, the LW potential, is expressed in manifestly Lorentz covariant form, manifest Lorentz covariance is relevant to its uniqueness only, but not to any claims about its reality. Put differently, notwithstanding its uniqueness, the LW potential has no more 'real' attributes than the other potentials in the $\alpha$-Lorenz gauge. Since more than one potential fits the bill, the second question is answered in the negative: it is not possible to argue *on physically relevant grounds* that only one among them is the real one. For this reason, adherence to the idea that one of them must be real would amount to no more than an arbitrary choice of some specific value for $\alpha$, which would be as good (or as bad) as the choice of any other value.

## 4 Two routes

The classical theory of electrodynamics involves an infinity of unobservable gauge potentials, which constitute what has become known as its surplus structure. For almost a century, from Maxwell's formulation of the theory until the late 1950s, potentials were considered to be mere mathematical artifacts. The discovery of the A-B effect, however, signaled a change. In the paper that heralded the effect[27], Aharonov and Bohm showed that, after all, potentials are physically significant. For this reason, they claimed, a different interpretation of the electromagnetic potentials and their role in the theory was required. But they hastily suggested a literal reading of the entire theory; a suggestion that opened Pandora's box of interpretational metaphysics, indeterminism and a long debate.

The debate concerning the status of gauge potentials rests mostly on an interpretive dilemma. Either electrodynamics describes a world of fields, and field is all there is; but then our theory falls prey to *un-mediated* action at a distance[28]. Or we embrace locality, granted by unobservable potentials[29], and along with it a literal interpretation of the theory; but then electrodynamics becomes a theory involving heavy metaphysics[30] and a vicious kind of indeterminism[31].

In the literature, both physics and philosophical, there have been

[27] Aharonov & Bohm (1959).

[28] I use here the term 'un-mediated' to denote that the action at a distance involved in effects like the A-B is worse than, say, the more familiar Newtonian version. In the former we deal with effects of zero-valued forces. In the latter we deal with forces that are non-zero but are defined simultaneously throughout space.

[29] Note in passing that the locality granted by potentials in general is of a kind that one might call 'the locality of being there'. The fact that some of these potentials (i.e. of the Coulomb gauge) propagate instantaneously indicates clearly that this kind of locality does not necessarily involve what one might consider as continuous causal propagation.

[30] That is, infinitely many, empirically indistiguishable gauge potentials that correspond to a single set of covariant values of the observable electromagnetic field.

[31] See footnote 3 for a clarification about the kind of indeterminism involved.

various responses, Mattingly's (2006) being the latest. In it he suggests that it is possible to resolve the dilemma by recognizing "the primary role that current plays in the A-B effect and replace our differential equation, $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$, for the vector potential with the sum over definite integrals that expresses a field arising from the current itself"[32]. Once we have recognized the role of that integral, he claims, "we can, if we like, view it as the "reality" underlying"[33] electromagnetic effects like the A-B. The only problem with ascribing reality to that integral is that being merely an expression of a particular gauge potential, the Lorenz, it can never be observed independently of the field-strengths associated with it. However, the cost is minimal, in his view, because we are compensated generously by metaphysical economy (only one unobservable real entity as opposed to infinitely many) and explanatory power (a real object as the cause of the A-B effect).

Though well intended and compelling, the motivation for this move (i.e. gains from it) is hardly enough. The combination of the mathematical indistinguishability of the potentials in the context of electrodynamics[34] with their unobservability implies that there are no a priori reasons to single out one; neither observational nor theoretical arguments suffice. The gains from choosing one would be the same with those from choosing almost any other. So, really, the significant question is: on what grounds can we pick one among them? Or, in other words, how can we justify the choice of the LW potential?

Were we to interpret gauge potentials realistically and avoid the metaphysical inflation that accompanies electromagnetism's gauge freedom, in general only two routes would be open.

On the first route one accepts electromagnetism as we know it but maintains that there exists a unique, real gauge potential which underlies at least some parts of the reality described by the theory. In other words, one is committed to the reality of fields as independent entities, but one also claims that at the same time there exists a single real potential that underlies electromagnetic effects like the A-B, for which the fields alone cannot account. Thus, in this case, one is committed to both the ontology of electrodynamics, modulo gauge potentials, and the reality of a single gauge potential. The problem with this turn, and therefore with the route it leads to, is that one must provide compelling reasons for singling out one gauge potential. Or else, the interpretation offered by this route would be no different from the ontology offered by the literal interpretation; for, subscribing to the reality of one potential

[32] Mattingly (2006), p.250.
[33] Op. cit., p. 251.
[34] See footnote 1.

would entail subscribing to the reality of more, and therefore not only the metaphysical economy of the suggested solution would be blown away, but also indeterminism would sneak back.

On the second route one takes one's commitment to the reality of gauge potentials a step further: one asserts that all electromagnetic phenomena are epiphenomena, which transpire only as a result of the existence of a single potential. To use a philosophically loaded word, this amounts to the assertion that all electromagnetic phenomena supervene on that potential. Again, a unique potential must be singled out, or else the metaphysical inflation associated with the literal interpretation wont be diminished. But providing compelling arguments for uniqueness is only the first hurdle on the second route. Given its stricter ontological commitment, the complete theory's gauge freedom is abolished on the second route and one ends up with a truncated theory. The downside of such a radical move is that one has to show that the new truncated theory is empirically equivalent to the old one. Or else one would trade empirical content for metaphysical economy, and this is a price to high to pay.

So far as I understand him, Mattingly is not clear regarding the route that he is willing to (and does) follow. Although he seems to assert that the second route is to be taken[35], as a matter of fact his central argument for the reality of the LW potential appears to proceed on the course of the first[36]. By proposing the second route, I may therefore be accused for stretching Mattingly's argument to the breaking point. My aim in what follows, however, is not only to rebut Mattingly's argument, but also to show that all possible attempts to argue for the reality of the LW potential are untenable.

Let us now explore these two routes.

[35] At least this is how I understand the following assertion. He asks what might have been the reason why all gauge potentials were considered to be important in understanding the ontology of electrodynamics. His response: "a rough and ready answer is that because of an accident of history that defined the vector potential as any field $\mathbf{A}$ satisfying $\nabla \times \mathbf{A} = \mathbf{B}$, and therefore originally masking its [the LW potential's] reality, we were unable to see that just one member of the family [of gauge potentials] is numerically identical to a real, causally efficacious, non-arbitrary, local quantity" (p.252). In other words, he seems to assert that the potential has ontological primacy over the fields.

[36] Or at least this is how I understand his assertion that "only the LW potential of a single charge arises from a non-arbitrary analysis of what is real in a world accurately described by classical electrodynamics" (p. 251). In other words, here he seems to have taken the complete theory of classical electrodynamics for granted, which leaves open to him only the first route.

## 4.1 Slippery slope

In taking this route one accepts that the world is accurately described by classical electrodynamics and its unavoidable gauge covariance. Yet, without being inconsistent, one may still claim that, against the theory's odds, just one among the infinitely many, in-principle unobservable, and thus indistinguishable gauge potentials is "the true vector potential". One may insist further that this is the only potential that underlies electromagnetic effects; at least those that are due to charges and currents[37].

Given its agreeable characteristics, the LW potential presents itself as a good candidate. After all, it is defined through an integral equation that involves only physical quantities, and its four-vector form is manifestly Lorentz covariant. What is more, it does not depend explicitly on gauge-dependent quantities and its time evolution is deterministic. Hence it does have certain characteristics that physical objects have.

Surely, this is a case where having those appropriate characteristics amounts to having characteristics that are necessary for reality but not sufficient: observability is missing and will always be missing from the list. Claiming that the LW potential is real merely because it has *some* necessary characteristics would, therefore, beg the question. Something more is needed. To fill this gap, Mattingly asserts that the LW potential is the only one that "does not require non-local specifications"[38], but instead it propagates causally from the source-charges to the space-time regions where it may interact with other charges or fields. By this, I take it, he means the following.

Assuming a special relativistic world, one expects electromagnetic waves and interactions to propagate at the speed of light. Yet, potentials that result from arbitrary gauge choices have components that require "non-local assignment[s] of values to points in space-time"[39]. The example he refers to is the scalar potential in the Coulomb gauge, which appears to propagate at an infinite speed: it spreads throughout space instantaneously. The observable electromagnetic fields that result from this potential propagate of course at the speed of light. But in order for the fields to do so, the contribution of the scalar component must be cancelled out by another factor: the contribution of the transverse part

[37] This way one avoids the thorny issue of plane electromagnetic waves, which cannot be recovered by retarded potentials in general and the LW potential in particular. Retarded potentials are solutions to Maxwell equations in the presence of charges, while plane waves are solutions to Maxwell equations in the absence of charges. The implication of this difference is that plane waves are not part of a world described by the LW potential.

[38] Op. cit., p.252.

[39] Op. cit. p. 251.

of the vector current, which is produced by the source-charges, and propagates instantaneously too. Hence, in the case of the Coulomb gauge, an instantaneous (or non-local as Mattingly prefers to call it) effect is rendered harmless only because it is cancelled out by another instantaneous (or non-local) effect.

Similarly, the argument goes, in all other gauges except the Lorenz, instantaneous (or non-local) specifications are also required in order to reinstate propagation of fields at the speed of light. Given that Mattingly is "unaware of any other "gauge" that involves neither extra geometric structure that is not part of Minkowski space nor a non-local assignment of values to points in spacetime"[40], he concludes that the Lorenz gauge, or the resulting LW potential for that matter, is unique. Therefore the LW potential can be singled out and viewed as the reality underlying electromagnetic effects and phenomena that are due to charges and currents.

But being unaware of the existence of other potentials that are both "purely intrinsically and locally definable in terms only of the charge and the geometric structure of spacetime"[41], and such that they do not "involve non-local specifications from the very beginning in order to be themselves definable", is hardly a proof that such potentials do not exist. In fact, the existence of the $\alpha-$Lorenz gauge shows that there is a whole family of potentials, infinitely many if truth be told, that have the exact same characteristics which might render the LW potential "unique" and therefore real.

The entire family of the $\alpha-$Lorenz potentials with $0 < \alpha < 1$ are defined intrinsically and locally, in terms only of the charge and the geometric structure of space-time, and no potential in that family involves super-luminal, or, even worse, instantaneous propagations in order to be so defined. From their specific descriptions[42] it is clear that their components propagate continuously and at luminal or sub-luminal speeds. Therefore, if we assumed them to be independent objects in their own right, they would propagate continuously from the sources from which they emanate, and they would affect other charges or fields in $c$-retarded or $\alpha c$-retarded manner. For these reasons they could be considered to be as causally efficacious as their $\alpha = 1$ counterpart. Each of these potentials could therefore play the exact same causal role that the LW potential is called to play. What is more, since gauge potentials are never observed independently of the electromagnetic field-strengths, no one can tell whether their parts that propagate at subluminal speeds are

[40] Op. cit.
[41] Op. cit.
[42] Equations (7) and (10).

not there; nor whether they are.

With infinitely many gauge potentials that behave like causally efficacious objects do, it becomes impossible to choose "the one". Consequently, if one subscribed to the reality of one of these potentials *solely* because it admitted well behaved propagation at speed less than or equal to that of light, one would have to subscribe to the reality of them all. But then the unpalatable metaphysics of the literal interpretation reemerges, and along with it, its indeterminism. Let me clarify this last point.

When viewed as objects with independent existence, each of those “unique” potentials propagates deterministically. But at any given time, or space-time point, only the fields are fixed deterministically. Hence at any given time we may pick any other from the potentials of the $\alpha$-Lorenz family with $0 < \alpha < 1$. This is the kind of gauge freedom that spoils determinism in the complete theory; and so it does here as well.

To conclude, being unable to single out a unique potential from the infinitely many of the $\alpha$-Lorenz gauge, subscribing to the reality of just one is tantamount to subscribing to the reality of any other. This, in turn, is as good as subscribing to the reality of them all.

## 4.2 Insurmountable obstacles

With the first route leading to a slippery slope, if one insisted that nevertheless there must be a way, the only route left open to one would be the second. The second route starts with the assumption that all electromagnetic effects, including fields themselves, supervene on the properties of unobservable potentials, but the aim is to narrow down the number of real potentials to one, the LW potential. This unique potential is still be unobservable. So, if all the reasons we can provide for its uniqueness are the same as before, we are left, again, with infinitely many potentials to choose from: all the potentials of the $\alpha$-Lorenz gauge.

The second route seems to be barred from the beginning, unless we can appeal to some characteristic of the LW potential that distinguishes it from the other potentials of the $\alpha$-Lorenz gauge. Given the commitment we are prepared to make, this would be appeal to the fact that it is the only potential whose components propagate on the light-cone.

While on the first route we could not appeal to this property of the LW potential. Being committed to the reality of fields independently of the reality of the potentials, the fact that some potentials appeared to have more structure than the fields was not significant. As long as the properties of this additional structure were analogous to those of observable causally efficacious entities, and as long as they did not alter the observables, it was impossible to decide whether this additional structure was there or not. Thus, we were bound to accept that all those potentials could be perceived of as real.

On the second route, however, one's ontological loyalties lie with the potentials. This comes handy, because we may now use the fact the LW potential propagates on the light-cone. We could argue that if electromagnetic phenomena, in general, and light, in particular, supervene on a single potential, this potential might as well be the only one that propagates exactly like light. Of course this choice is not imposed by physical or theoretical necessity. Rather, it is imposed by an argument for metaphysical economy. Still, this argument could go, the choice in this case is not as arbitrary as it would have been while on the first route; for, now we may assert that only the potential that behaves exactly like light can represent the reality behind light itself. Therefore, assuming that all electromagnetic phenomena transpire only as a result of a unique potential, which, in addition, must propagate exactly like those phenomena, we are able to narrow our options down to one: the LW potential.

This kind of commitment comes at a price. If the idea that the LW potential is the only reality behind electromagnetic phenomena is to be sustained, the truncated theory that is based on this potential alone should deliver the entire empirical content of electromagnetism as we know it. This entails that everything of physical significance, including quantization and the empirical content of QED, must be derivable from the LW potential alone. If it turned out that the truncated theory did not deliver, and if appeal to other potentials was required, the very idea that kept us going on the second route would be overthrown. In other words, the idea that the LW potential is the unique reality behind all electromagnetic effects would turn out to be indefensible.

The first response that comes to mind almost automatically is along the following lines. If the entire physical and empirical content of electromagnetism can be gained from a truncated theory based on the LW potential alone, chances are it can be retrieved from a truncated theory based on any other gauge potential. In such case, electromagnetism's gauge freedom and arbitrariness in choice of gauge remain, and along come metaphysical inflation and indeterminism. However, I would like to leave this fair response aside and offer instead a different rejoinder. I argue that the antecedent is blatantly false. That is, I argue that the truncated theory based on the LW potential does not give back the complete empirical content of electromagnetism, and this constitutes an insurmountable obstacle indeed. Just one example suffices to bring this point home.

As it is well known, the Lorenz gauge condition (given by equation

(1)) is satisfied automatically by Maxwell's equations on the light-cone[43]. This entails that, on the light-cone, we cannot use this condition as a gauge-fixing condition, because it does not eliminate all the unphysical degrees of freedom that are associated with the gauge covariance of electrodynamics. In order to eliminate the remaining gauge freedom on the light-cone and proceed with quantization, a further fixing of the gauge is required. There is only one way we can achieve the required gauge fixing: by appeal to additional gauges and gauge-fixing conditions. But appealing to gauges other than the Lorenz is as good as accepting that the empirical content of the truncated theory is inferior to the empirical content of the complete theory.

It appears as thought the second route has led to a dilemma. On the one hand we have committed to the LW potential as the only reality underlying all electromagnetic phenomena; thus we have achieved the metaphysical economy we strived for. But we cannot quantize the truncated theory, and this means that the truncated theory leaves out a great part of our observable world. On the other hand, if we appeal to gauge potentials other than the LW and thus manage to quantize the truncated theory we get that part of our world back. But appeal to other potentials amounts to admitting that the idea which allowed us to proceed on the second route is false: not all electromagnetic phenomena supervene on the reality of the only potential that propagates like light. Acknowledging what is at stake leaves no doubt as to what to keep and what to let go. The full theory with its gauge freedom and its empirical richness stays; the truncated theory with its metaphysical economy and its empirical poverty goes.

To sum up, when the need to quantize electrodynamics arises, the truncated theory provided by the LW potential proves to be both physically and empirically inferior to the full theory with its gauge covariance. This goes to show that the last stronghold of those who would like to press for the uniqueness and consequently the reality of the LW potential falls. While on the second route, the only means to single out the LW from the other potentials of the $\alpha$-Lorenz gauge leads to a truncated theory which, although metaphysically parsimonious, is physically inferior to the complete.

## 5 Concluding remarks

Electromagnetism is a theory with gauge freedom. Attempting a literal interpretation of it goes down with an inflation of metaphysics and indeterminism. Attempting an interpretation according to which the reality

[43] For this point I am indebted to Kelly Stelle.

that underlies electromagnetic phenomena is a single, unique potential is at least equally bad. The theory of electrodynamics proves *not* to be conducive to such a move.

The upshot of the analysis in this paper is that once we commit to the reality of one potential, we have to commit to the reality of infinitely many. Thus the toll one would have to pay is commitment to the same kind (and 'quantity') of metaphysics that Aharonov and Bohm originally subscribed to, and Mattingly tried to fend off.

None the less, the original hunch of Aharonov and Bohm, namely that gauge potentials are physically significant, turned out to be correct. The success of gauge theories in high energy theoretical physics proves it. An interpretation of this significance is yet to be produced, and the way that will take us there is yet to be discovered. My gut feeling, however, is that an opening is bound to appear if (and possibly only if) we go beyond the 'interpretive dilemma' I mentioned before. It need not be "either theory of real fields and un-mediated action at a distance, or theory of real potentials, and locality". It may as well be a theory of fields, in which potentials play a role different from the one we usually attribute to physical causal agents.

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