

Against *Pointillisme* about Mechanics

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Abstract

This paper forms part of a wider campaign: to deny *pointillisme*. That is the doctrine that a physical theory's fundamental quantities are defined at points of space or of spacetime, and represent intrinsic properties of such points or point-sized objects located there; so that properties of spatial or spatiotemporal regions and their material contents are determined by the point-by-point facts.

More specifically, this paper argues against *pointillisme* about the concept of velocity in classical mechanics; especially against proposals by Tooley, Robinson and Lewis. A companion paper argues against *pointillisme* about (chrono)-geometry, as proposed by Bricker.

To avoid technicalities, I conduct the argument almost entirely in the context of "Newtonian" ideas about space and time, and the classical mechanics of point-particles, i.e. extensionless particles moving in a void. But both the debate and my arguments carry over to relativistic physics.

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Contents

1	Introduction	2
2	The wider campaign	3
2.1	Connecting physics and metaphysics	3
2.1.1	Avoiding controversy about the intrinsic-extrinsic distinction	5
2.1.2	Distinction from three mathematical distinctions	6
2.2	Classical mechanics is not <i>pointilliste</i> , and can be perdurantist	7
2.2.1	Two versions of <i>pointillisme</i>	7
2.2.2	Two common claims	8
2.2.3	My contrary claims	9
2.3	In more detail	10
2.3.1	Four violations of <i>pointillisme</i>	10
2.3.2	For perdurantism	12
3	Velocity as intrinsic?	13
3.1	Can properties represented by vectors be intrinsic to a point?	13
3.2	Orthodox velocity is extrinsic but local	14
3.2.1	A question and a debate	14
3.2.2	The verdict	16
3.3	Against intrinsic velocity	18
3.3.1	A common view—and a common problem	18
3.3.2	Tooley’s proposal; and his arguments	20
3.3.3	Tooley’s further discussion	26
4	“Shadow velocities”: Lewis and Robinson	28
4.1	The proposal	29
4.2	Criticism: the vector field remains unspecified	31
4.3	Avoiding the presupposition of persistence, using Hilbert’s ε symbol	32
4.4	Comparison with Robinson and Lewis	36
5	References	39

1 Introduction

This paper forms part of a wider campaign: to deny *pointillisme*. That is the doctrine that a physical theory's fundamental quantities are defined at points of space or of spacetime, and represent intrinsic properties of such points or point-sized objects located there; so that properties of spatial or spatiotemporal regions and their material contents are determined by the point-by-point facts.²

I will first describe this wider campaign (Section 2). Then I will argue against *pointillisme* as regards the concept of velocity in classical mechanics (Sections 3 and 4). A companion paper (Butterfield 2006) argues against *pointillisme* about (chrono)-geometry. In both cases, the main debate is about whether properties of a point that are represented by vectors, tensors, connections etc. can be intrinsic to the point; typically, *pointillistes* argue that they can be. In both papers, I focus on contemporary *pointillistes* who try to reconcile *pointillisme* with the fact that vectorial etc. properties seem extrinsic to points and point-sized objects, by proposing some heterodox construal of the properties in question.

The concept of velocity in mechanics provides two illustrations of the lure of *pointillisme*, and this tendency to reconcile it with vectorial properties by reconstruing physical quantities.

(a): Tooley and others argue that for the sake of securing that a particle's instantaneous velocity is intrinsic to it at a time, we should not construe velocity in the orthodox way as a limit of average velocities—but instead reconstrue it along lines they propose. Again, my view is that there is no need for such heterodoxy: instead, we can and should reject *pointillisme*: (Section 3).

(b): Robinson and Lewis argue that for the sake of securing a perdurantist account of identity over time (persistence), we should postulate vectorial properties numerically equal to velocity, but free of velocity's presupposition of the notion of persistence. I maintain that if we reject *pointillisme*, the perdurantist has no need of such novel properties: (Section 4).

In similar vein, Bricker (1993) proposes to reconcile *pointillisme* with modern geometry's need for vectorial and tensorial properties by re-founding geometry in terms of non-standard analysis, which rehabilitates the traditional idea of infinitesimals (Robinson 1996). In (Butterfield 2006), I reply that once the spell of *pointillisme* is broken, these heterodox foundations of geometry are unmotivated.

So in both papers I defend orthodoxy about the foundations of both geometry and mechanics. But I do not mean to be dogmatic. As to geometry, I of course agree that there are several heterodox mathematical theories of the continuum that are technically impressive and philosophically suggestive. Butterfield (2006) gives some references; but I do not discuss details, since these theories offer no support for my target, *pointillisme*. More precisely: these theories do not suggest that fundamental quantities represent

²I think David Lewis first used the art-movement's name as a vivid label for this sort of doctrine: a precise version of which he endorsed.

intrinsic properties of points or point-sized bits of matter; because either they do not attribute such quantities to points, or they even deny that there are any points.³ So the upshot is that although I am open to suggestions about heterodox treatments of the continuum, these treatments do not support *pointillisme*. Accordingly, I find the philosophical doctrine of *pointillisme* an insufficient reason for rejecting the orthodox treatment.

Similarly in this paper for mechanics: though with the difference that mathematicians have not developed a handful of heterodox theories of mechanics, or of velocity, as they have of the continuum—one of which Tooley and the other authors might hope to invoke, to provide their heterodox new foundations of mechanics.⁴ So the upshot is the same as for geometry: I will again find *pointillisme* an insufficient reason for rejecting orthodoxy—though I am open to suggestions! In this spirit, I will end by offering a sort of peace-pipe to Robinson and Lewis. I will formulate a cousin of their proposal—a cousin that is not *pointilliste*—and compare it with their original (Section 4.3).

I will conduct the discussion almost entirely in the context of “Newtonian” ideas about space and time, and the classical mechanics of point-particles, i.e. extensionless point-masses moving in a void (and so interacting by action-at-a-distance forces). This restriction keeps things simple: and at no cost, since both the debate and my arguments carry over to relativistic physics. The restriction also has another merit. Broadly speaking, of the various physical theories, it is the classical mechanics of point-particles that *pointillisme* fits best: other theories have further anti-*pointilliste* features. So it is worth emphasising that even for classical point-particles, *pointillisme* fails.

2 The wider campaign

As I mentioned, this paper is part of a wider campaign, which I now sketch. I begin with general remarks, especially about the intrinsic-extrinsic distinction among properties (Section 2.1). Then I state my main claims; first in brief (Section 2.2), then in more detail (Section 2.3).

2.1 Connecting physics and metaphysics

My wider campaign aims to connect what modern classical physics says about matter with two debates in modern analytic metaphysics. The first debate is about *pointillisme*; but understood as a metaphysical doctrine rather than a property of a physical

³Broadly speaking, the second option seems more radical and worse for *pointillisme*; though in such theories, the structure of a set of points is often recovered by a construction, e.g. on a richly structured set of regions.

⁴As we shall see, the proposals by Tooley and others have slight mathematical aspects; but these aspects do not amount to any such theory.

theory. So, roughly speaking, it is the debate whether the world is fully described by all the intrinsic properties of all the points and-or point-sized bits of matter. The second debate is whether an object persists over time by the selfsame object existing at different times (nowadays called ‘*endurance*’), or by different temporal parts, or stages, existing at different times (called ‘*perdurant*’).

Endeavouring to connect classical physics and metaphysics raises two large initial questions of philosophical method. What role, if any, should the results of science have in metaphysics? And supposing metaphysics should in some way accommodate these results, the fact that we live (apparently!) in a quantum universe prompts the question why we should take classical physics to have any bearing on metaphysics. I address these questions in my (2004: Section 2, 2006a: Section 2). Here I just summarize my answers.

I of course defend the relevance of the results of science for metaphysics; at least for that branch of it, the philosophy of nature, which considers such notions as space, time, matter and causality. And this includes classical physics, for two reasons.

First, much analytic philosophy of nature assumes, or examines, so-called ‘common-sense’ aspects and versions of these notions: aspects and versions which reflect classical physics, especially mechanics, at least as taught in high-school or elementary university courses. One obvious example is modern metaphysicians’ frequent discussions of matter as point-particles, or as continua (i.e. bodies whose entire volume, even on the smallest scales, is filled with matter): of course, both notions arose in mechanics in the seventeenth and eighteenth centuries.

Second, classical physical theories, in particular mechanics, are much more philosophically suggestive, indeed subtle and problematic, than philosophers generally realize. Again, point-particles and continua provide examples. The idea of mass concentrated in a spatial point (indeed, different amounts at different points) is, to put it mildly, odd; as is action-at-a-distance interaction. And there are considerable conceptual tensions in the mechanics of continua; (Wilson (1998) is a philosopher’s introduction). Unsurprisingly, these subtleties and problems were debated in the heyday of classical physics, from 1700 to 1900; and these debates had an enormous influence on philosophy through figures like Duhem, Hertz and Mach—to mention only figures around 1900 whose work directly influenced the analytic tradition. But after the quantum and relativity revolutions, foundational issues in classical mechanics were largely ignored, by physicists and mathematicians as well as by philosophers. Besides, the growth of academic philosophy after 1950 divided the discipline into compartments, labelled ‘metaphysics’, ‘philosophy of science’ etc., with the inevitable result that there was less communication between, than within, compartments.⁵

Setting aside issues of philosophical method, *pointillisme* and persistence are clearly large topics; and each is the larger for being treatable using the very diverse methods

⁵Thus I see my campaign as a foray into the borderlands between metaphysics and philosophy of physics: a territory that I like to think of as inviting exploration, since it promises to give new and illuminating perspectives on the theories and views of the two communities lying to either side of it—rather than as a no-man’s-land well-mined by two sides, ignorant and suspicious of each other!

and perspectives of both disciplines, metaphysics and physics. So my campaign has to be selective in the ideas I discuss and in the authors I cite. Fortunately, I can avoid several philosophical controversies, and almost all technicalities of physics.⁶

But it will clarify the purposes of this paper to give at the outset some details about how I avoid philosophical controversy about the intrinsic-extrinsic distinction among properties, and about how this distinction differs from three that are prominent in mathematics and physics.

2.1.1 Avoiding controversy about the intrinsic-extrinsic distinction

My campaign does not need to take sides in the ongoing controversy about how to analyse, indeed understand, the intrinsic-extrinsic distinction. (For an introduction, cf. Weatherson (2002, especially Section 3.1), and the symposium, e.g. Lewis (2001), that he cites.) Indeed, most of my discussion can make do with a much clearer distinction, between what Lewis (1983, p. 114) dubbed the ‘positive extrinsic’ properties, and the rest. This goes as follows.

Lewis was criticizing Kim’s proposal, to analyze extrinsic properties as those that imply *accompaniment*, where something is accompanied iff it coexists with some wholly distinct contingent object, and so to analyze intrinsic (i.e. not extrinsic) properties as those that are compatible with being unaccompanied, i.e. being the only contingent object in the universe (for short: being *lonely*). Lewis objected that loneliness is itself obviously extrinsic. He also argued that there was little hope of amending Kim’s analysis. In particular, you might suggest that to be extrinsic, a property must either imply accompaniment or imply loneliness: so Lewis dubs these disjuncts ‘positive extrinsic’ and ‘negative extrinsic’ respectively. But Lewis points out that by disjoining and conjoining properties, we can find countless extrinsic properties that are neither positive extrinsic nor negative extrinsic; (though ‘almost any extrinsic property that a sensible person would ever mention is positive extrinsic’ (1983, p. 115)).

This critique of Kim served as a springboard: both for Lewis’ own preferred analysis, using a primitive notion of naturalness which did other important work in his metaphysics (Lewis 1983a); and for other, metaphysically less committed, analyses, developed by Lewis and others (e.g. Langton and Lewis 1998, Lewis 2001).

But I will not need to pursue these details. As I said, most of my campaign can make do with the notion of positive extrinsicity, i.e. implying accompaniment, and its negation. That is, I can mostly take *pointillisme* to advocate properties that are intrinsic in the weak sense of not positively extrinsic. So this makes my campaign’s claims, i.e. my denial of *pointillisme*, logically stronger; and so I hope more interesting.

⁶I note that among the philosophical issues my campaign avoids are several about persistence, such as: (a) the gain and loss of parts (as in Theseus’ ship); (b) the relation of “constitution” between matter and object (as in the clay and the statue); (c) vagueness, and whether there are vague objects. Agreed, there are of course connections between my claims and arguments, and the various issues, both philosophical and physical, that I avoid: connections which it would be a good project to explore. But not in one paper, or even in one campaign!

Anyway, my campaign (even in this paper) makes some novel proposals about positive extrinsicality: namely, I distinguish temporal and spatial (positive) extrinsicality, and propose degrees of (positive) extrinsicality.

2.1.2 Distinction from three mathematical distinctions

Both the murky intrinsic-extrinsic distinction, and the clearer distinction between positive extrinsics and the rest, are different distinctions from three that are made within mathematics and physics, especially in those parts relevant to us: viz. pure and applied differential geometry. The first of these distinctions goes by the name ‘intrinsic’/‘extrinsic’; the second is called ‘scalar’/‘non-scalar’, and the third is called ‘local’/‘non-local’. They are as follows.

(i): The use of ‘intrinsic’ in differential geometry is a use which is common across all of mathematics: a feature is intrinsic to a mathematical object (structure) if it is determined (defined) by just the object as given, without appeal to anything extraneous—in particular a choice of a coordinate system, or of a basis of some vector space, or of an embedding of the object into another. For example, we thus say that the intrinsic geometry of a cylinder is flat; it is only as embedded in \mathbb{R}^3 that it is curved.

(ii): Differential geometry classifies quantities according to how they transform between coordinate systems: the simplest case being scalars which have the same value in all coordinate systems. (Nevermind the details of how the other cases—vectors, tensors, connections, spinors etc.—transform.)

(iii): Differential geometry uses ‘local’ (as vs. ‘global’) in various ways. But the central use is that a mathematical object (structure) is local if it is associated with a point by being determined (defined) by the mathematical structures defined on *any* neighbourhood, no matter how small, of the point. In this way, the instantaneous velocity of a point-particle at a spacetime point, and all the higher derivatives of its velocity, are local since their existence and values are determined by the particle’s trajectory in an arbitrarily small neighbourhood of the point. Similarly, an equation is called ‘local’ if it involves only local quantities. In particular, an equation of motion is called ‘local in time’ if it describes the evolution of the state of the system at time t without appealing to any facts that are a finite (though maybe very small) time-interval to the past or future of t .

I will not spell out *seriatim* some examples showing that the two philosophical distinctions are different from the three mathematical ones. Given some lessons in differential geometry (not least learning to distinguish (i) to (iii) themselves!), providing such examples is straightforward work. Suffice it to make two comments; the second is relevant to this paper.

(1): It would be a good project to explore the detailed relations between these distinctions. In particular, the mathematical distinction (i) invites comparison with Vallentyne’s (1997) proposal about the intrinsic-extrinsic distinction. Besides, there are

yet other distinctions to explore and compare: for example, Earman (1987) catalogues some dozen senses of ‘locality’.

(2): Instantaneous velocity, conceived in the orthodox way as a limit of average velocities, has implications about the object at other times, for example that it persists for some time. (I will discuss this in more detail below, especially Section 3.2.) So most philosophers say that instantaneous velocity is an extrinsic property. I agree. But emphasising its extrinsicness tends to make one ignore the fact that it is mathematically local, i.e. determined by the object’s trajectory in an arbitrarily small time-interval (cf. (iii) above). It is this locality that prompts me to speak of instantaneous velocity (and other local quantities) as ‘hardly extrinsic’. And in pure and applied differential geometry, it would be hard to over-estimate the importance of—and practitioners’ preference for!—such local quantities and local equations involving them. Similarly, the fact that we often find that differential equations of very low order determine the temporal course of quantities of interest, is very important—and very fortunate.⁷

2.2 Classical mechanics is not *pointilliste*, and can be perdurantist

2.2.1 Two versions of *pointillisme*

To state my campaign’s main claims, it is convenient to first distinguish a weaker and a stronger version of *pointillisme*, understood as a metaphysical doctrine. They differ, in effect, by taking ‘point’ in *pointillisme* to mean, respectively, spatial, or spacetime, point.

Taking ‘point’ to mean ‘spatial point’, I shall take *pointillisme* to be, roughly, the doctrine that the instantaneous state of the world is fully described by all the intrinsic properties, at that time, of all spatial points and-or point-sized bits of matter.

As I said in Section 2.1, my campaign can mostly take ‘intrinsic’ to mean ‘lacking implications about some wholly distinct contingent object’; in other words, to mean the negation of Lewis’ ‘positive extrinsic’ (i.e. his ‘implying accompaniment’). But for this version of *pointillisme*, I will take ‘intrinsic’ to mean ‘*spatially* intrinsic’. That is, attributing such a property to an object carries no implications about spatially distant objects; but it *can* carry implications about objects at other times. (Such objects might be other temporal parts of the given object.) So I shall call this version, ‘*pointillisme* as regards space’.

On the other hand: taking ‘point’ to mean ‘spacetime point’, I shall take *pointillisme* to be, roughly, the doctrine that the history of the world is fully described by all the intrinsic properties of all the spacetime points and-or all the intrinsic properties at all the various times of point-sized bits of matter (either point-particles, or in a continuum). And here I take ‘intrinsic’ to mean just the negation of Lewis’ ‘positive

⁷Of course, we sometimes need equations of higher order than we at first think and hope. For an important case of this in population biology, cf. Colyvan and Ginzburg (2003).

extrinsic’. That is, it means ‘both spatially and temporally intrinsic’: attributing such a property carries no implications about objects at other places, or at other times. I shall call this stronger version, ‘*pointillisme* as regards spacetime’.

So to sum up: *pointillisme* as regards space vetoes spatial extrinsicality; but *pointillisme* as regards spacetime also vetoes temporal extrinsicality.

On either reading of *pointillisme*, it is of course a delicate matter to relate such metaphysical doctrines, or the endurance-perdurance debate, to the content of specific physical theories. Even apart from Section 2.1’s questions of philosophical method, one naturally asks for example, how philosophers’ idea of intrinsic property relates to the idea of a physical quantity. For the most part, I shall state my verdicts about such questions case by case. But one main tactic for relating the metaphysics to the physics will be to formulate *pointillisme* as a doctrine relativized to (i.e. as a property of) a given physical theory (from Section 2.3 onwards). Anyway, I can already state my main claims, in terms of these two versions of *pointillisme*. More precisely, I will state them as denials of two claims that are, I think, common in contemporary metaphysics of nature.

2.2.2 Two common claims

Though I have not made a survey of analytic metaphysicians, I think many of them hold two theses, which I will dub (FPo) (for ‘For *Pointillisme*’) and (APe) (for ‘Against perdurantism’); as follows.

(FPo): Classical physics—or more specifically, classical mechanics—supports *pointillisme*: at least as regards space, though perhaps not as regards spacetime. There are two points here:—

(a): Classical physics is free of various kinds of “holism”, and thereby anti-*pointillisme*, that are suggested by quantum theory. Or at least: classical mechanics is free. (With the weaker claim, one could allow, and so set aside, some apparently anti-*pointilliste* features of advanced classical physics, e.g. anholonomies in electromagnetism and the non-localizability of gravitational energy in general relativity: features rich in philosophical suggestions (Batterman 2003, Belot 1998, Hofer 2000)—but not for this paper!)

(b): The concession, ‘perhaps not as regards spacetime’, arises from the endurance-perdurance debate. For it seems that *pointillisme* as regards spacetime must construe persistence as perdurance; (while *pointillisme* as regards space could construe it as endurance). And a well-known argument, often called ‘the rotating discs argument’, suggests that perdurance clashes with facts about the rotation of a continuum (i.e. a continuous body) in classical mechanics. So the argument suggests that classical mechanics must be understood as “endurantist”. Besides, whether or not one endorses the argument, in classical mechanics the persistence of objects surely *can* be understood as endurance—which conflicts with *pointillisme* as regards spacetime.

(The considerations under (a) and (b) are usually taken as applying equally well to

non-relativistic and relativistic classical mechanics: an assumption I largely endorse.)

I also think that many metaphysicians would go further and hold that:

(APe): Classical mechanics does indeed exclude *pointillisme* as regards spacetime: their reason being that this *pointillisme* requires perdurance and that they endorse the rotating discs argument. So they hold that in classical mechanics the persistence of objects *must* be understood as endurance, and that this forbids *pointillisme* as regards spacetime.

2.2.3 My contrary claims

I can now state the main position of my wider campaign. Namely, I *deny* both claims, (FPo) and (APe), of Section 2.2.2. I argue for two contrary claims, (APo) (for ‘Against *Pointillisme*’) and (FPe) (for ‘For perdurantism’), as follows.

(APo): Classical mechanics does *not* support *pointillisme*.

By this I do not mean just that:

(a) it excludes *pointillisme* as regards spacetime.

Nor do I just mean:

(b) it allows one to construe the persistence of objects as endurance.

(But I agree with both (a) and (b).) Rather, I also claim: classical mechanics excludes *pointillisme* as regards space. That is: it needs to attribute spatially extrinsic properties to spatial points, and-or point-sized bits of matter. (But this will not be analogous to the kinds of “holism” suggested by quantum theory.)

(FPe): Though (as agreed in (APo)) classical mechanics excludes *pointillisme* as regards spacetime (indeed, also: as regards space): classical mechanics is *compatible* with perdurance. That is: despite the rotating discs argument, one *can* be a “perdurantist” about the persistence of objects in classical mechanics. The reason is that once we reject *pointillisme*, perdurance does not need persistence to supervene on temporally intrinsic facts. In fact, perdurantism can be defended by swallowing just a small dose of temporal extrinsicality.

So to sum up my wider campaign, I claim that:—

(APo): Classical mechanics denies *pointillisme*, as regards space as well as spacetime. For it needs to use spatially extrinsic properties of spatial points and-or point-sized bits of matter, more than is commonly believed.

(FPe): Classical mechanics permits perdurantism. It does not require temporally extrinsic properties (of matter, or objects), in the sense of requiring persistence to be endurance: as is commonly believed. A mild dose of temporal extrinsicality can reconcile classical mechanics with perdurance.

To put the point in the philosophy of mind’s terminology of ‘wide’ and ‘narrow’ states, meaning (roughly) extrinsic and intrinsic states, respectively: I maintain that classical mechanics:

(APo): needs to use states that are spatially wide, more than is commonly believed;

and

(FPe): does not require a specific strong form of temporal width, viz. endurance. With a small dose of temporal extrinsicity, it can make do with temporally quite narrow states—and can construe persistence as perdurance.

2.3 In more detail ...

So much by way of an opening statement. I will now spell out my main claims in a bit more detail: (APo) in Section 2.3.1 and (FPe) in Section 2.3.2.

2.3.1 Four violations of *pointillisme*

I will begin by stating *pointillisme* as a trio of claims that apply to any physical theory; and making two comments. Then I list four ways in which (chrono)-geometry and classical mechanics violate *pointillisme*: three will form the main topics of this paper and its companion.

The trio of claims is as follows:

(a): the fundamental quantities of the physical theory in question are to be defined at points of space or of spacetime;

(b): these quantities represent intrinsic properties of such points;

(c): models of the theory—i.e. in physicists’ jargon, solutions of its equations, and in metaphysicians’ jargon, possible worlds according to the theory—are fully defined by a specification of the quantities’ values at all such points.

So, putting (a)-(c) together: the idea of *pointillisme* is that the theory’s models (or solutions or worlds) are something like conjunctions or mereological fusions of “ultralocal facts”, i.e. facts at points.

Two comments. First: the disjunction in (a), ‘at points of space or of spacetime’, corresponds to Section 2.2’s distinction between *pointillisme* as regards space, and as regards spacetime. Nevermind that it does not imply the convention I adopted in Section 2.2, that *pointillisme* as regards spacetime is a *stronger* doctrine since it vetoes temporally extrinsic properties, *as well as* spatially extrinsic ones. The context will always make it clear whether I mean space or spacetime (or both); and whether I mean spatially or temporally extrinsic (or both).

Second: Though I have not made a systematic survey, there is no doubt that *pointillisme*, especially its claims (a) and (b), is prominent in recent analytic metaphysics of nature, especially of neo-Humean stripe. The prime example is the metaphysical system of David Lewis, which is so impressive in its scope and detail. One of his main metaphysical theses, which he calls ‘Humean supervenience’, is a version of *pointillisme*. I will return to this in Section 4.

When we apply (a)-(c) to classical mechanics, there are, I believe, four main ways in which *pointillisme* fails: or more kindly expressed, four concessions which *pointillisme* needs to make. The first three violations (concessions) occur in the classical mechanics

both of point-particles and of continua; the fourth is specific to continua. The first two violations are discussed in the companion paper (2006); the third is the topic of this paper.

(1): The first is obvious and minor. Whether matter is conceived as point-particles or as continua, classical mechanics uses a binary relation of occupation, ‘... occupies ...’, between bits of matter and spatial or spacetime points (or, for extended parts of a continuum: spatial or spacetime regions). And this binary relation presumably brings with it extrinsic properties of its relata: it seems an extrinsic property of a point-particle (or a continuum, i.e. a continuous body) that it occupy a certain spatial or spacetime point or region; and conversely.

(2): Classical mechanics (like other physical theories) postulates structure for space and-or spacetime (geometry or chrono-geometry); and this involves a complex network of geometric relations between, and so extrinsic properties of, points. This concession is of course more striking as regards space than time: three-dimensional Euclidean geometry involves more structure than does the real line. This is the main topic of (2006).

(3): Mechanics needs of course to refer to the instantaneous velocity or momentum of a body; and this is temporally extrinsic to the instant in question, since for example it implies the body’s existence at other times. (But it is also local in the sense of (iii), Section 2.1.2.) So this second violation imposes temporal, rather than spatial, extrinsicality; i.e. implications about other times, rather than other places.

This is the main topic of this paper. But I should stress that this third violation is *mitigated* for point-particles. For a *pointilliste* can maintain that the persistence of point-particles supervenes on facts that, apart from the other violations (i.e. about ‘occupies’ and (chrono)-geometry), are *pointillistically* acceptable: viz. temporally intrinsic facts about which spacetime points are occupied by matter. In figurative terms: the void between distinct point-particles allows one to construe their persistence in terms of tracing the curves in spacetime connecting points that are occupied by matter. I develop this theme in my (2005). On the other hand: for a continuous body, the persistence of spatial parts (whether extensionless or extended) does *not* supervene on such temporally intrinsic facts. This is the core idea of the rotating discs argument, mentioned in Section 2.2.2.

To sum up: the rotating discs argument means that *pointillisme* fits better with point-particles than with continua. To put the issue in terms of Section 2.2’s two forms of *pointillisme*: the strong form of *pointillisme*, *pointillisme* as regards spacetime, fails for the classical mechanics of continua, even apart from the other concessions mentioned.

(4): Finally, there is a fourth way that the classical mechanics of continua violates *pointillisme*: i.e., a fourth concession that *pointillisme* needs to make. Unlike the rotating discs argument, this violation seems never to have been noticed in recent analytic metaphysics; though the relevant physics goes back to Euler. Namely, the classical mechanics of continua violates (the weaker doctrine of) *pointillisme* as regards

space, because it must be formulated in terms of spatially extended regions and their properties and relations. But in this paper, I set this fourth violation aside entirely; my (2006a) gives details.

So to sum up these four violations, I claim (APo): classical mechanics violates *pointillisme*. This is so even for the weaker doctrine, *pointillisme* as regards space. And it is especially so, for the classical mechanics of continua rather than point-particles.

2.3.2 For perdurantism

I turn to Section 2.2.3's second claim, (FPe): that once *pointillisme* is rejected, perdurantism does not need persistence to supervene on temporally intrinsic facts, and can be defended for classical mechanics provided it swallows a small dose of temporal extrinsicity.

Now I can identify this small dose. It is the extrinsicity of Section 2.3.1's third violation of *pointillisme*; in particular, the presupposition of persistence by the notion of a body's instantaneous velocity. Thanks to the rotating discs argument, 'body' here means especially 'point-sized bit of matter in a continuum'. For as we noted in Section 2.3.1, for point-particles we can construe persistence as perdurance without having to take this dose.

Elsewhere (2004, 2004a) I argue that for a "naturalist" perdurantist, this dose is small enough to swallow. For this paper (especially Section 4) I only need to state the argument's two main ideas:

(i): If the perdurantist rejects *pointillisme*, she can reject instantaneous temporal parts, i.e. believe only in temporal parts with some non-zero duration.

(ii): She can thereby avoid the rotating discs argument. For the argument urges that facts temporally intrinsic to instants cannot distinguish two obviously different states of motion for a continuous body. But the anti-*pointilliste* perdurantist has access to non-instantaneous facts, and so can "thread the worldlines together".

But I should also stress that I do not claim to *refute* endurantism, even for so limited and sharply-defined a class of objects as the point-particles and continua of classical mechanics. The metaphysical debate about persistence is too entangled with other debates in the philosophy of time, and in ontology and semantics, for me to claim that. I claim only that in classical mechanics at least, perdurantism is tenable. In fact, I think that in classical mechanics, the cases for endurantism and perdurantism are about equally strong: the honours are about even. That is an ecumenical conclusion—but one worth stressing since for continua, perdurantism has got such a bad press, thanks to the rotating discs argument.

3 Velocity as intrinsic?

3.1 Can properties represented by vectors be intrinsic to a point?

Classical mechanics represents the properties that encode the structure of space or spacetime, and the properties of matter such as velocity, momentum etc., using mathematical entities such as vectors, tensors, connections etc. (Of course, so do all physical theories.) So the question arises: can properties that are so represented be intrinsic to a point? This question is central to *pointillisme*, and to our other topic, persistence: and will be at the centre of this paper.

But my discussion will be simplified by two drastic restrictions. First, I will consider only properties represented by vectors, which I will for short call *vectorial properties*: not those represented by other mathematical entities such as tensors and connections. Though drastic, this restriction is natural, in that:

(i) vectors are about the simplest of the various mathematical entities that classical mechanics (like other theories) uses to represent properties and relations—so they are the first case to consider;

(ii) the restriction is common in the literature: of the authors I discuss in this paper and its companion, all consider only vectorial properties, except for Bricker (1993) who also considers tensors.

Second, I will concentrate on instantaneous velocity. For the *pointilliste* authors I will criticize (mainly Tooley, Robinson and Lewis) do so; though both they and I will also briefly comment on momentum, force and acceleration.

As announced in Section 1, the discussion will illustrate how strongly some contemporary metaphysicians are attracted by *pointillisme*. They reconcile the apparent extrinsicity of a vectorial property, specifically velocity, with *pointillisme* by proposing to reconstrue the property. Thus in Section 3.3, Tooley and others will reject the orthodox idea of instantaneous velocity as a limit of average velocities, and reconstrue it in order to make it an intrinsic property. And in Section 4, Robinson and Lewis will make a similar proposal. My own view will of course be that there is no need for such heterodoxy: instead, we can and should reject *pointillisme*.

So the plan of battle for the rest of this paper is as follows. I will undertake four projects: the first two in this Section, and then two in Section 4:—

(1): I will endorse the view that instantaneous velocity is extrinsic, in particular temporally extrinsic. But I will also emphasise that because it is local ((iii) of Section 2.1.2), it is hardly extrinsic; (Section 3.2).

(2): I will criticize the heterodox view of Tooley and others that we should reconstrue velocity so as to make it intrinsic; (Section 3.3).

(3): I will criticize the view of Lewis and Robinson that a moving object has a vectorial property numerically equal to velocity, but free of velocity's presupposition of the notion of persistence; (Sections 4.1 and 4.2).

(4): I end by offering a peace-pipe to Robinson and Lewis. I formulate a cousin of their proposal—a cousin that is not *pointilliste*—and end by comparing it with their original; (Section 4.3).

So while the second and third projects are critical, the first and fourth are more positive. In particular, Section 4.3 reflects Section 1’s admission that heterodox treatments of the continuum and of physical quantities are of course worth developing.

3.2 Orthodox velocity is extrinsic but local

3.2.1 A question and a debate

To lay out the ground, let us begin by asking: Is a particle’s velocity intrinsic to it at the time in question? The first thing to say is of course that this question, and similar ones e.g. whether a particle’s velocity counts as part of its instantaneous state, have a long history. Although ‘intrinsic’ is a philosophical term of art (especially nowadays, cf. Section 2.1.1), and although instantaneous velocity was first rigorously defined only with the advent of the calculus, the vaguer notion of “velocity at a time” was involved in all debate about the nature of motion from Zeno’s time onwards.

I will not go in to details details about this long history. Here it must suffice to say that its “highlights” include: Zeno, Ockham’s ‘at-at’ theory of motion, medieval impetus theory, the apparent resolution of Zeno’s paradoxes provided by the calculus (at least as rigorized by Cauchy and Weierstrass, if not by its seventeenth century inventors), the philosophical discussion of that resolution by analytic philosophers like Russell, and the recent development, in mathematics, of heterodox theories of the continuum—for example, the two modern vindications of the idea of infinitesimals, non-standard analysis and smooth infinitesimal analysis. (Sources include: for the history, Mancosu (1996, Chapters 4f.), Leibniz (2001), Arthur (2006); for the recent work on infinitesimals, Robinson (1996), Bell (1998).)

So I set aside both the history and contemporary infinitesimals; and I postpone heterodox philosophical treatments of velocity to the next Subsection. Here I will give the details of the orthodox answer to our question. Namely, as announced in Section 2.3.1: instantaneous velocity is extrinsic, in particular temporally extrinsic, but local. Though this answer is, I submit, straightforward, and the underlying mathematics is elementary, it is worth pausing over; for it has been the topic of some recent debate (between Albert, Arntzenius and Smith).

So let us consider the orthodox notion of instantaneous velocity for a point-particle. This is the limit of the particle’s average velocity as the time-interval around the point in question tends to zero. To be precise, we will require the two one-sided limits and the two-sided limit to all exist and be equal: we say, in an obvious notation,

$$\text{If } \lim_{\varepsilon \rightarrow 0^+} \frac{\mathbf{q}(t + \varepsilon) - \mathbf{q}(t)}{\varepsilon} = \lim_{\varepsilon \rightarrow 0^+} \frac{\mathbf{q}(t - \varepsilon) - \mathbf{q}(t)}{\varepsilon} =$$

$$\lim_{\varepsilon \rightarrow 0+, \delta \rightarrow 0+} \frac{\mathbf{q}(t + \varepsilon) - \mathbf{q}(t - \delta)}{\varepsilon + \delta}, \quad \text{then } \mathbf{v}(t) := \text{this common limit.} \quad (3.1)$$

If we now ask our question—is velocity, defined by eq. 3.1, intrinsic to the particle at t ?—intuition pulls in two directions.

On the one hand, one wants to say Yes, because an attribution of velocity (even of a specific value \mathbf{v}) implies no categorical information about the particle’s velocity, or location, at other times. Having instantaneous velocity \mathbf{v} at a point \mathbf{q} at time t is compatible with *any* values for the particle’s instantaneous velocity and its location, at any other instant t' in time, as near as you please to t . Indeed in classical mechanics, the compatibility is not just logical, but also nomic, since classical mechanics gives no upper bound to either the speed or the acceleration of a particle.

To put the same point in other words: there is no time-interval around t (in particular, no minimal time-interval) for which the course of values of the particle’s location and-or velocity, or some proposition about the possibilities for these courses of values, are equivalent to the particle’s instantaneous velocity at t . So the instantaneous velocity is surely not a property of the particle during a time-interval around t —which suggests that it is intrinsic to the particle at t .

But on the other hand, one wants to say No, for two reasons. First, the particle’s velocity is relative to a frame of reference. This surely makes it a relation between the particle and an object stationary in (or perhaps in some other way representing) the frame. (Or at least, the velocity is an extrinsic property that the particle has because of such a relation.) Though this reason is important, it tends to be ignored in discussions about whether velocity is intrinsic; so it will be clearest to postpone it to Section 3.3.1.B, where I briefly discuss the two authors who mention it.

The second reason for answering No *is* recognized in the literature. It is that the particle’s instantaneous velocity \mathbf{v} at t codes a lot of information about what its velocity and location is at nearby times—but not “categorical information”. The information is conditional or hypothetical information about average velocities (and consequently, locations). The information is given precisely by eq. 3.1. And it is given roughly, by saying that for nearby times the collection of average velocities must be so “well-behaved” as to have a single limit, \mathbf{v} , as the times get closer to t ; or in spacetime terms, by saying that the nearby history of locations (the local segment of the worldline) must be smooth enough to have at t a tangent vector (a 4-velocity determined by \mathbf{v}).

These diverse intuitions are clearly in evidence in the recent debate between Albert, Arntzenius and Smith. I will not arbitrate this debate in detail: that would require extended quotation and textual exegesis. But a short summary will suffice to bring out the diverse intuitions; and I submit, to make clear that the right answer to our question is ‘No: velocity is extrinsic though local’.

The debate runs as follows:—

(i): Albert (2000, pp. 9-10, 17-18) and Arntzenius (2000, Section 3, especially pp. 192-195) do not address exclusively our question. Their attention is predominantly on the similar question, whether a particle’s instantaneous velocity as defined by eq. 3.1

should count as part of its instantaneous state. They argue that it should not, since they require that an object’s instantaneous state should not imply, in virtue of logic and definition alone, any constraints on its instantaneous states at other times. And velocity clearly does imply such constraints: roughly, that for some time-interval around t , maybe very short, the particle’s average velocities are suitably “well-behaved”; cf. the ‘No’ answer above.

(ii): Smith (2003, pp. 269-280; especially pp. 274-277) replies that instantaneous velocity *should* count as part of the instantaneous state at t , essentially because for any *other* time the state (in particular the location and velocity) could be anything; cf. the ‘Yes’ answer above.⁸

(iii): In his brief reply, Arntzenius (2003) emphasises that his and Albert’s main idea is the requirement reported in (i), that an instantaneous state should not imply, by just logic and definitions, any constraints on instantaneous states at other times. And he argues that velocity’s implications about other times surely make it extrinsic.

(iv): In a yet briefer rejoinder (2003a), Smith: (a) is sceptical about metaphysicians’ intrinsic-extrinsic distinction; and (b) rejects Albert’s and Arntzenius’ requirement, since ‘physicists have not chosen to adhere to that requirement, and in the absence of a good reason ... we ought to stick with physics here’ (2003a, p. 283).

3.2.2 The verdict

Surveying this debate, I think the verdict is clear. Ultimately, it is of course just a verbal matter whether to impose Albert’s and Arntzenius’ requirement as part of the meaning of ‘instantaneous state’; though *ceteris paribus*, one is well-advised to follow the usage of the discipline concerned—and so, in this case, to join the physicists in not imposing it.

But similarly, ‘intrinsic’ and ‘extrinsic’ are established philosophical terms. And there is no doubt that an attribution of instantaneous velocity (either the determinable or a determinate value) has implications for other times (not least that the object exists then), and so is extrinsic, indeed positive temporally extrinsic; (cf. Section 2.1.1 and the ‘No’ answer above). In short: once the meaning of ‘extrinsic’ is settled, it is uncontentious that velocity is extrinsic.

On the other hand, the “grain of truth” in the ‘Yes’ answer is that velocity is local in the sense of (iii) Section 2.1.2: whether the particle has a velocity at t , and if so what it is, is determined by its positions at times in an arbitrarily short time interval around t . Again this is uncontentious.⁹

One might add to this verdict that the physicists’ usage, that velocity is part of the instantaneous state, fits dynamics, as well as kinematics (i.e. as well as velocity’s

⁸Smith (pp. 264-268) also corrects the common view that Russell in his (1903) took the calculus to vindicate the idea of instantaneous velocity as intrinsic to the object at the time. In fact, Russell’s version of the “at-at” theory of motion denies that there are instantaneous states of motion (and more generally: of change). Tooley (1988: Sections 1, 2.2) makes the same point.

⁹But terminology varies. Bricker (1993, p. 289) calls such a property ‘neighbourhood-dependent’; similarly, Arntzenius (2000, p. 193) suggests calling them ‘neighbourhood properties’.

being local). For in a deterministic theory like mechanics, the laws of motion are to determine all later states from the present state ('initial data'); and since these laws are second-order in time, the initial data must include the velocity, as well as the position.¹⁰

Though uncontentious, this verdict is important for the debate about whether persistence is endurance or perdurance; and in particular, for my pro-perdurantist claim (FPe) (Section 2.3.2). Thus one way in which velocity is extrinsic is its implication that the object exists at other times; and this has prompted a consensus that the perdurantist's reply to the rotating discs argument cannot appeal to different velocities (or angular velocities). But extrinsicality, though usually discussed as an all-or-nothing affair, comes in degrees (Lewis 1983, p. 111): a property is more extrinsic, the more that its ascription implies about the world beyond the property's instance. Since velocity is local, it is hardly extrinsic; and this means that a perdurantist who swallows this small dose of temporal extrinsicality can invoke velocity in her reply to the rotating discs argument. More precisely, an anti-*pointilliste* perdurantist who vetoes instantaneous temporal parts can do so. (Butterfield (2004, Sections 4.2.2 and 7.4; 2004a, Sections 2.2.2, 4.5) gives details.)

Finally, two technical remarks which will be needed in Section 3.3. (1): In eq. 3.1 and the ensuing discussion, we have implicitly assumed that the particle exists throughout a time interval around t , not least because textbooks of analysis define continuity and differentiability at a point t only for functions defined on a neighbourhood of t . But philosophers, concerned with logical as well as nomic possibilities, sometimes consider particles that come in and out of existence (e.g. Tooley and others discussed in Section 3.3). So it is worth noticing that the usual definitions of continuity and differentiability at a point $t \in \mathbb{R}$ can be carried over to any function defined on a subset of \mathbb{R} that has t as a limit point from both above and below. For example, one can talk about the continuity and differentiability at zero of a function defined on zero together with the reciprocals of integers, i.e. defined on $\{1/n : n \text{ an integer}\} \cup \{0\}$.

(2): The orthodox account of velocity can also be liberalized, as regards one-sided limits. There is no reason to insist that a "rate of motion" deserves the name 'velocity' only if both one-sided limits exist and are equal to the two-sided limit (cf. eq. 3.1). Thus it is common practice to call the limit from above (eq. 3.1's first term) $v_+(t)$; and to call the limit from below (eq. 3.1's second term) $v_-(t)$; and to talk of one-sided velocities in a case where $v_+(t) \neq v_-(t)$. Combining this idea with (1) above, we see that a one-sided limit at a point t only requires the function's domain to have t as a limit point from that side.

So much for the orthodox notion of velocity. From now on, I will consider, and rebut, heterodox views:—

(1): in Section 3.3, a view of velocity as intrinsic, which has been advocated without

¹⁰This point is not specific to Newton's laws, with force proportional to acceleration d^2q/dt^2 . Suitably generalized (to substitute momentum for velocity), it applies to both Lagrangian and Hamiltonian formulations of classical mechanics; and to relativistic generalizations.

regard to the debate about persistence; and

(2): from Section 4, a view that accepts orthodox velocity but proposes that to understand persistence we need to postulate another vectorial quantity, always equal in value to orthodox velocity, but free of its presupposition of persistence.

3.3 Against intrinsic velocity

3.3.1 A common view—and a common problem

3.3.1.A The view Several authors have sketched a heterodox view of instantaneous velocity as intrinsic: Tooley (1988, p. 236f.), Bigelow and Pargetter (1989, especially pp. 290-294; 1990, pp. 62-82)¹¹ and Arntzenius (2000: pp. 189, 196-201). As we shall see, a dozen or so philosophers have commented on this heterodox view, and often sympathetically. But so far as I know, these authors' arguments for their view have not received detailed scrutiny—or rebuttal. So that will be my purpose until Section 4.

These authors' proposals seem to be mutually independent: the three later authors do not cite the previous work. But they share a common view, as follows:

(i): Velocity should be an intrinsic property of an object at a time that (together with the position, and the regime of impressed forces) causes, and so explains, the object's position at later times.

(ii): Causation should be understood in a broadly neoHumean way, as a contingent relation between 'distinct existences'. This means that the orthodox notion of velocity, being a "logical construction" out of the object's positions at other times (cf. Section 3.2) cannot do the job required by (i).

But the proposals differ in detail. For example, Tooley presents his proposal by applying Lewis' (1970) tactic for functional definition of theoretical terms to a version of "Newton's laws of motion". Or to use another jargon: he "Ramsifies" some accepted laws of motion. I presume that by thus "piggy-backing" on the accepted laws, Tooley's approach can readily secure that its intrinsic velocity is a vector; (though Tooley does not go into this). On the other hand, Bigelow and Pargetter develop the view without using details about the laws of motion: instead, they appeal to the metaphysics of universals and the logic of relations to argue that their intrinsic velocity is vectorial.

Another difference concerns how to treat vectorial quantities other than velocity. Tooley proposes to treat force in a similar way to velocity but is content with an orthodox account of acceleration as "just" $d^2\mathbf{q}/dt^2$ (1988, p. 249). But Bigelow and Pargetter (1989, pp. 294-295) propose heterodoxy for both force and acceleration; though not for higher derivatives of position, which, they assert, play no explanatory role.¹²

¹¹I will only cite the former, since the latter is almost identical.

¹²For all three, the shared view (i) and (ii) exemplifies a characteristically Australian realism, and endorsement of inference to the best explanation. Thus Tooley elsewhere proposes a functional definition of causation (1987, Chapters 1.2, 8); and even proposes that a spacetime point causes later

There are also differences as regards connections to other topics. Thus Bigelow, Pargetter and Arntzenius, but not Tooley, suggest the view is a descendant of some medieval views (about impetus and flux). And Arntzenius (2000, pp. 198-201) connects the view to time-reversal, and so to Albert's heterodox allegation (2000, pp. 14-15, 20-21) that electromagnetism is not time-reversal invariant.¹³

In this Subsection, I will focus on Tooley, and so speak of 'Tooleyan velocities'. My reasons for this focus are that:—

(i): So far as I know, his arguments for the view are the most developed; but as I mentioned, they seem not to have received detailed scrutiny.

(ii): This focus enables me to avoid Bigelow and Pargetter's contentious metaphysical territory of universals.¹⁴

3.3.1.B The problem The view that velocity is intrinsic faces an obvious problem, which I mentioned in Section 3.2.1. Namely, velocity being intrinsic conflicts with velocity being relative to a frame of reference. For the latter surely makes velocity a relation between objects, viz. the given one and an object stationary in (or in some other way representing) the frame; or perhaps, a corresponding extrinsic property: in any case, not intrinsic.

Among advocates of the view, only Tooley, so far as I know, addresses this problem; and among commentators, only Zimmerman (1998, pp. 276-277). In Section 3.3.3, I shall criticize Tooley's response to this problem; which is in any case brief. Here I want just to emphasise two points which force the problem on *any* advocate of intrinsic velocity.

(1): The problem is not specific to special relativity; (as Tooley's response and Zimmerman's discussion both suggest). Classical mechanics no less than relativity can be formulated without postulating absolute rest, so that velocity is indeed relative to a frame of reference. (For the idea, think of the galilean transformations. For a rigorous formulation along these lines, one uses a neoNewtonian conception of spacetime: for philosophical expositions cf. e.g. Sklar (1974: Chapter III.D.3, pp. 202-206), Earman (1989, Chapter 2.4, p. 33).)

(2): The problem takes us back to the distinction between the philosophical and the mathematical notions of intrinsic, where the latter means in particular, coordinate-independent (cf. (i) of Section 2.1.2). To be precise, this distinction clarifies the problem—and shows that any advocate of intrinsic velocity faces it. Thus it is not

ones, and that in the context of relativity, this is a linear non-branching relation, providing a version of relativity theory with an absolute simultaneity (1997, pp. 338-344, 354-355: for discussion, cf. Dainton 2001, pp. 278-281). Bigelow and Pargetter elsewhere defend a realist view of forces as a species of causation (Bigelow, Ellis and Pargetter, 1988).

¹³Earman (2002), Arntzenius (2004) and Malament (2004) are replies to Albert.

¹⁴Besides, I will reply *en passant* to their three principal arguments for distinguishing intrinsic velocity from orthodox velocity, since they are similar to arguments of Tooley's which I discuss. Another discussion of Tooleyan velocities which emphasises causation more than I will, but whose verdict, like mine, is broadly negative, is Le Poidevin (2006).

enough for the advocate to point out that rigorous formulations of mechanics, whether classical or relativistic, associate a mathematically intrinsic notion of 4-velocity to an object such as a point-particle, viz. the tangent vector to the object’s worldline. For that fact does not imply that the 4-velocity is philosophically intrinsic, and thereby fit (by the advocate’s lights) to do the jobs of causing and explaining later positions. And even if the 4-velocity is philosophically intrinsic, that would not imply that a 3-velocity, i.e. an ordinary spatial velocity of the sort all the advocates discuss, is philosophically intrinsic and so “fit for work”. For to define the 3-velocity from the 4-velocity, one has to choose a frame of reference—in either a neoNewtonian or a relativistic theory: and so face the problem.¹⁵

So this problem is substantial. Nevertheless, metaphysicians seem to still consider the view a live option; (e.g. Zimmerman (1998, pp. 275-278) and Sider (2001, pp. 35, 39, 228). And I will also set the problem aside, apart from briefly reporting Tooley’s response (in Section 3.3.3). After all, the view is so far only a sketch: none of these authors makes their heterodox account of velocity part of a mathematically elaborated theory of motion (and-or causation).¹⁶ But my setting this problem aside is not just an act of charity. There will be plenty else to comment on—and criticize!

3.3.2 Tooley’s proposal; and his arguments

3.3.2.A Tooley’s proposal As I mentioned, Tooley proposes that velocity is an intrinsic property that is functionally defined by the laws of motion, in the manner of Lewis (1970: especially Section IV). So, roughly speaking: Tooley says that the velocity of object o at time t is that unique intrinsic property of o at t that is thus-and-thus related to other concepts, as spelled out in the usual formulas of kinematics and dynamics. More precisely, Tooley gives a kinematic formula and a dynamical one, which he labels T_1 and T_2 , Writing $q(t), v(t)$ for the position and velocity at t of the

¹⁵These comments apply equally to classical and relativistic mechanics. For the present topic, they only differ in the metrical properties attributed to a 4-velocity: in a neoNewtonian theory it has only a temporal length, while in a relativistic theory, it has a spatiotemporal length. Incidentally, Zimmerman’s comment (1998, pp. pp. 276-277) amounts to: (i) my (2), but applied only to relativity theory; (ii) the suggestion that 4-acceleration is a ‘much better candidate for an intrinsic state of motion’ (p. 267). Le Poidevin (2006: Section 6) also makes the suggestion (ii). I agree with (ii), not least because in both theories, 4-acceleration has a coordinate-independent length. But it would take us too far afield to assess whether Tooley and his ilk could or should adapt the proposal and arguments in Section 3.3.2 to acceleration instead of velocity: their project, not mine! Similarly, as regards adapting them to 4-velocity, rather than 3-velocity.

¹⁶Tooley (1988, pp. 231-2) briefly considers whether his account could be formalized using non-standard analysis’ infinitesimals; but concludes that it cannot be.

object o with mass m , and $F(t)$ for the force impressed on o at t , these formulas are:¹⁷

$$T_1 : \quad q(t_2) = q(t_1) + \int_{t_1}^{t_2} v(t) dt; \quad (3.2)$$

$$T_2 : \quad v(t_2) = v(t_1) + \int_{t_1}^{t_2} F(t)/m dt. \quad (3.3)$$

So velocity is to be implicitly defined in Ramsey-Lewis style, as the unique intrinsic property with the functional role enjoyed by the term v in T_1 & T_2 .

As Tooley interprets this proposal, it differs both mathematically and philosophically from the orthodox account of eq. 3.1. Let us first address the mathematical difference; which is minor.

Eq. 3.1 requires q to be differentiable. But T_1 & T_2 (indeed T_1) implies only that q is continuous (since the integral $\int_{t_1}^{t_2} v dt$ is a continuous function of its limits)¹⁸; not that it is differentiable, since v in T_1 is *not* defined as $\frac{dq}{dt}$. Thus Tooley considers the case of a particle initially at rest at the origin, with $v(t) = 0$ for all $t < 0$ and $v(t) = 1$ for all $t > 0$, so that $q(t) = 0$ for all $t < 0$ and $q(t) = t$ for all $t > 0$. Because of the corner at $t = 0$, q is not differentiable at $t = 0$; and however we might choose to define $v(0)$, v will be discontinuous at 0. Yet T_1 holds good: in particular, v is integrable.

There are good mathematical questions hereabouts. For example: how “well-behaved” must v be in order to be integrable (on either the Riemann or the Lebesgue definition)? And as a consequence: how well-behaved must the integral i.e. q be? These questions are addressed in integration theory. But Tooley does not pursue them;¹⁹ and nor will I. For us it is enough to note that Tooley’s T_1 & T_2 is a mathematically mild generalization of eq. 3.1’s orthodox account: in short, the difference is that whereas orthodoxy takes position as primitive and velocity as its derivative, T_1 & T_2 takes velocity as primitive and position as its integral.

3.3.2.B Tooley’s arguments Turning to philosophy, Tooley gives six arguments for his proposal, which together bring out how he interprets it; (his Sections 4.1-4.6, with further discussion and replies in Sections 5 and 6). He admits that the arguments vary in strength, and that they need a variety of deniable premises. These premises are typical of contemporary analytic metaphysics of nature. They concern such topics as:

(a) whether there can be “action at a temporal distance”: i.e. roughly, a cause at t_1 of an effect at t_2 , without causally relevant states of affairs at all the times between

¹⁷Cf. Tooley (1988, pp. 238-239). For brevity, I have suppressed universal quantifiers and a variable for the object o ; and I have simplified T_2 so as to assume a constant mass; Tooley’s T_2 tries to accommodate relativity’s velocity-dependence of mass by writing $m(t)$. Both Tooley and I simplify to one spatial dimension: the generalization to three spatial dimensions, $\mathbf{q}(t)$, $\mathbf{v}(t)$ etc. is trivial.

¹⁸Tooley makes a slip here (p. 238), saying that T_1 & T_2 do not even imply that q is continuous: no matter.

¹⁹But Tooley does deploy the above example in one of his philosophical arguments for his proposal; cf. (2) in Section 3.3.2.B below.

t_1 and t_2 ; or

(b) whether motion can be discontinuous.

And as one might expect, the arguments also depend on less explicit, but again deniable, premises (“intuitions”), articulating a broadly neoHumean view of causation; (cf. (ii) in Section 3.3.1.A).

I will not try to state all Tooley’s arguments; but will concentrate on the arguments and premises that seem most important. (This will cover three arguments given by Bigelow and Pargetter; cf. footnote 14.) This means I will focus on the arguments of his Sections 4.2, 4.4, 4.5 and 4.6. But it will be clearer to discuss his 4.5 before his 4.4; for 4.4 and 4.6 share a common premise, that motion could be discontinuous. It will also be clearest to reply to the arguments *seriatim*, so as to avoid having to refer back to arguments.

But I can already state the general tenor of my reply. I will criticize Tooley’s (and Bigelow and Pargetter’s) arguments as either:

(i): not justifying a crucial premise, or “intuition” about a thought-experiment; and-or

(ii) under-estimating the resources of the orthodox account.

I would add that (i) and (ii) both arise from the arguments and thought-experiments being by and large too far removed from the details of mechanics. But of course a philosopher of physics *would* say that to a metaphysician!

(1): *Tooley’s Section 4.2*:—

The argument of Tooley’s Section 4.2 assumes a denial of “action at a temporal distance” ((a) above), which he calls the ‘principle of causal continuity’. We do not need the exact formulation of the principle, but only this consequence of it:

If there is a causally sufficient condition of some state of affairs, however complex the condition and however gappy it may be [i.e. spread across disconnected intervals or instants of time—JNB], there must also be some instantaneous state of affairs which is also a causally sufficient condition of the state of affairs in question. (p. 242)

Tooley accepts that the principle of causal continuity is not a necessary truth, but holds that it is ‘reasonable’ to believe it true of ‘our world’ (1988, p. 242). Presumably he would say the same about this consequence. In any case, he then says

In either a Newtonian or a relativistic world ... the state of the world at an instant cannot be a causally sufficient condition of later states unless velocity (or, alternatively, something to which velocity is definitionally related, such as momentum) characterizes the instantaneous states of objects. If therefore the principle of causal continuity is accepted, the Russellian [i.e. orthodox] analysis of velocity must be rejected. (p. 242)

In the first sentence here, Tooley is of course referring to the fact noted in Section 3.2’s verdict on the Albert-Arntzenius-Smith debate: that since in Newtonian or relativistic

mechanics the laws of motion are second-order in time, the initial data of a solution must include the velocity or momentum, so that it is natural to call velocity part of the instantaneous state. So far, so uncontentious: at least in so far as we go along with Tooley in regarding the determination of the later state by the present one (the initial data) as a case of causal sufficiency.

But the second sentence *is* contentious. Here, Tooley assumes that an orthodox velocity cannot count as part of the instantaneous state—at least if ‘instantaneous state’ is understood as causally sufficient for a later state. Thus his view is like that of Arntzenius and Albert (cf. Section 3.2), though more explicit and more detailed in its commitment to a neoHumean view of causation. So my reply is: why should we accept this assumption? So far, I see no reason: especially in the light of Section 3.2’s verdict that orthodox velocity, though extrinsic, is local and part of the instantaneous state.²⁰

(2): *Tooley’s Section 4.5*:—

The argument of Tooley’s Section 4.5 uses Section 3.3.2.A’s example of a particle which is at rest and then moves with velocity $v(t) = 1$ at all $t > 0$. Tooley argues that though orthodoxy dictates that $v(0)$ is undefined, the intuitive verdict about the case is that $v(0) = 0$. For since (as he argues elsewhere: 1987, pp. 207-212) cause and effect cannot be simultaneous, the motion’s cause, viz. an instantaneous impulsive force acting at $t = 0$, can only have an effect (viz. $v = 1$) later. Tooley then also assumes that his functional definition of velocity using T_1 & T_2 will imply this result, i.e. that if the particle’s movement is due to an impulsive force acting instantaneously, then $v(0) = 0$ (p. 246, paragraph 4). Finally, he says that in a world in which impulsive forces act “at a temporal distance”, e.g. with a time delay of one second, and in which an appropriate²¹ instantaneous impulse is impressed on the particle at time $t = -1$ s., the velocity at $t = 0$ *would* be 1: $v(0) = 1$. And again he assumes that his functional definition will imply this result (p. 246-247).

I reply that while Tooley’s judgments about the intuitive values of velocity, in the light of various postulated causal stories, may well be defensible in an elaborated theory of causation and motion, they are debatable (i) in general and (ii) in the context of his T_1 & T_2 .

(i): Philosophers with other views of causation might well disagree. And not only philosophers who are sceptical about causal talk: philosophers who “believe in” causation, but see little connection between causation and classical mechanics, in particular velocity, might well disagree.

(ii): In particular, these judgments do not just follow from T_1 & T_2 , or from a func-

²⁰The same reply works against Bigelow and Pargetter’s similar, but more free-wheeling, argument. They go so far as to say that the extrinsicity of orthodox velocity amounts to action at a temporal distance! Thus they allege that the orthodox description of an impact, e.g. a meteor striking Mars, requires the meteor’s past positions to exert a force now; so they remark incredulously that ‘this requires the meteor to have a kind of ‘memory’—what it does to Mars depends not only on its current properties but also on where it has been’ (1989, p. 296). I submit that Section 3.2’s discussion and verdict scotches this argument.

²¹Namely, the mass of the particle times 1 unit of velocity.

tional definition obtained from them. For the transition from orthodoxy, eq. 3.1, to treating velocity as primitive and position as its integral, i.e. $T_1&T_2$, cannot settle disputed matters of causation—causal relations are too misty a subject to be settled by such a mathematically mild generalization.²²

The arguments of Tooley’s Sections 4.4. and 4.6 both assume that a particle’s spatial trajectory could be discontinuous (not just non-differentiable) so that it “jumps about” in space.

(3): *Tooley’s Section 4.4:*—

In Section 4.4, Tooley adds to discontinuous motion the idea of a world in which a particle’s present position gives only probabilities for its later positions, and then asks us to consider a particle which just happens to have a spatial trajectory that is throughout some time interval a differentiable function of time. In short, we are to consider what Tooley dubs ‘accidentally orderly movement in a probabilistic world’ (p. 243).

Tooley now urges the intuition that such a particle would *not* have a velocity at any time in the interval in question, because ‘the velocity of an object at a time should be *causally* relevant to its positions at later times’ (p. 244).

Tooley also points out that the same example threatens any view that takes velocity to be determined by (supervenient upon) the history of positions; e.g. a liberalization of eq. 3.1 which required only that the one-sided limit, from earlier times, of average velocities, should exist. For the accidentally orderly history of position in Tooley’s imagined probabilistic world could match exactly a particle’s history in a deterministic (say, classical mechanical) world. And the latter particle, says Tooley, *does* have a velocity—it is part of the instantaneous state and causally relevant to later positions. So if we accept Tooley’s intuition that the accidentally orderly particle lacks a velocity, then the worlds match as to the particles’ positions but differ as to their velocity: and not just as to what the value of velocity is, but as to whether there is a value.

I make two replies, analogous to those for Tooley’s Section 4.5 ((2) above). First,

²²Bigelow and Pargetter’s argument for Tooleyan velocity differing from orthodox velocity is different. They present a case where both are defined but differ, for an instant, in value (1989, pp. 292-293). Suppose two perfectly rigid spheres, B and C, are at rest and touching; then B is struck along the line between their centres by a third such sphere, A, at velocity v . Supposing the spheres are of equal mass, ‘theory tells us that A will stop, B will not budge, and C will move off with velocity v ’ (1989, p. 293); (think of a “Newton’s cradle”). Bigelow and Pargetter assert that at the instant of impact, B has a Tooleyan velocity v : ‘the velocity of A is transferred from A, through B, to C’ (ibid.).

I reply that this is a case where everyday or philosophers’ intuitions are too far removed from the details of mechanics: in two ways.

(i): That ‘A will stop, B will not budge, and C will move off’ is only an approximate description, based on the idealization of perfect rigidity. This is a very strong idealization: by assuming forces, and finite impulses, are transmitted instantaneously through bodies, it forbids any account of processes within bodies. And this can be philosophically misleading; for example, in the rotating disc argument (Butterfield 2004: Section 5.5.2; 2004a: Section 3.2), and in the metaphysics of causation (Wilson 2004).

(ii): Even if we assume perfect rigidity, it does not follow that B has a velocity, even a heterodox one for just an instant. It only follows that a finite impulse is transmitted through B.

intuitions clash. I have no hesitation in judging, *contra* Tooley, that the particle moving accidentally in an orderly way has a velocity—even if I try to forget my previous inclination to the orthodox view! Nor am I alone; cf. Smith (2003, p. 279 fn 14). So far, this is a stalemate. But second, as in (2)(ii) above: Tooley’s T_1 & T_2 , and a functional definition obtained from them, do not imply that the accidentally ordered particle lacks a velocity. Why should the unique realizer of that functional role have the causal properties and relations Tooley intuitively wants?²³

(4): *Tooley’s Section 4.6*:—

In Section 4.6, the imagined discontinuous motion is much more extreme. Tooley asks us to

consider the following case. The world contains a rather unusual force field that causes objects to “flash” in and out of existence. Specifically, any object [it had better be a point-particle!—JNB] that enters this field exists only at points whose distance from the center of the field, measured in terms of a certain privileged unit of length, is given by an irrational number. Thus, if an object is moving along, and enters the field, it blinks in and out of existence an infinite number of times in any interval, however short. ... Given that the object is progressing through the field, it is natural to describe it as being in motion. Moreover, given that Achilles would, we can suppose, pass through a given field more quickly than the tortoise, it seems natural to say that objects have different velocities as they move through the field ... [But] the standard account of velocity [will not] assign a velocity to objects that are flashing along through [this] peculiar force field. In contrast, if velocity is a theoretical property of an object at a time, there would seem to be no reason why objects could not possess a velocity as they move along, blinking in and out of existence. (p. 247-248)

In reply, I think many philosophers of physics will find this thought-experiment so physically unrealistic that they will be happy to discount any “intuitions” about whether there is motion and velocity in it. Fair enough, say I. But there is also a more specific response, which builds on my earlier comments.

We noted at the end of Section 3.2 that the orthodox account of velocity could be readily generalized to attribute velocity at a time that was a limit point (or even a one-sided limit point) of a domain of definition of a position function q . In Tooley’s thought-experiment, this is of course exactly what occurs—all the time! Thus Tooley is presumably imagining the simplest sort of case where the discontinuous worldline of the particle is a dense subset of a smooth curve in spacetime. To put the case heuris-

²³Bigelow and Pargetter’s argument for the same conclusion, that an object can have orthodox velocity without Tooleyan velocity, is that movie-images and spots of light have orthodox velocity but no Tooleyan velocity, for lack of an appropriate causal link (1989, p. 293-294). I reply: you are at liberty to introduce a notion of velocity stronger than the orthodox one by requiring some causal link, and your notion may mesh better with everyday use of ‘velocity’. But that hardly counts as a criticism of the orthodox notion.

tically in terms of counterfactuals: the particle would have had the smooth curve as its worldline, and so an orthodox velocity, were it not being “flashed out of existence” when at rational distances from the center of the field. For this sort of case, the orthodox account can be readily generalized: the discontinuous worldline determines a unique smooth curve, so that at each time when the particle exists it can be attributed unambiguously the velocity associated with that point on the curve. (The same comment could of course be made using the discontinuous spatial trajectory rather than the worldline.)

Again, there are good mathematical questions hereabouts: viz. about the conditions under which “bad” curves, e.g. discontinuous ones, have “good” e.g. differentiable extensions. But Tooley does not pursue these questions; and nor will I. He just says, as I quoted above, that his kind of account of velocity as a theoretical property would surely attribute a velocity to the “flashing” particle. So be it, say I. But I deny that this merit is thanks to features specific to Tooley’s account (such as velocity being a cause or effect, or being functionally defined). The reason his account could, or would, attribute velocity is the simple mathematical one: that even the orthodox account can easily be generalized to do this; and since Tooley’s T_1 & T_2 is a mathematically mild generalization of the orthodox account, it also can be thus generalized. Nothing specific to Tooley’s account seems relevant.²⁴

3.3.3 Tooley’s further discussion

I turn to Tooley’s further discussion in his Sections 5, 6. I shall consider two of his topics. (Footnote 24 replied *en passant* to a third.)

(1): *Tooley’s Section 5.1*:—

The first is his account’s explanation of why the orthodox account works as well as it does (his 5.1). Tooley thinks it likely that in the actual world the two accounts will always coincide, in the sense that the orthodox and Tooleyian velocities will:

- (a) be defined in all the same cases and
- (b) be equal.

He argues for (a) by saying:

(i): T_2 ‘ensures that an object’s velocity at a time is causally relevant to its velocity at later times, and this means that the sort of situation where an object has an [orthodox] velocity, but fails to have a [Tooleyian] velocity—namely, cases of accidentally orderly movement—cannot arise in our world’ (p. 249).

(ii): On both accounts, velocity changes require forces. This

... together with plausible hypotheses about how forces depend upon other factors, such as the distance between the objects in question, entails that

²⁴In Section 6.1, Tooley admits that for the particle’s velocity function, which is defined on the dense subset of the worldline, to be integrable, as T_1 demands, his account needs to interpret T_1 as using the Lebesgue rather than Riemann theory of integration. Fair comment: but the orthodox account can equally adopt Lebesgue integration.

velocity, in our world, cannot change in a discontinuous fashion. Accordingly, the sorts of cases where an object can have a [Tooleyan] velocity, but fail to have an [orthodox] velocity [i.e. cases like the particle which is at rest and then moves with velocity $v(t) = 1$ for all $t > 0$, in his Section 4.5.—JNB], cannot occur in our world (p. 249).

Finally Tooley argues for (b) by saying that since both orthodox velocities and his velocities satisfy T_1 , they must be equal whenever both are defined.

In reply: I applaud Tooley's seeking an argument why the orthodox account works as well as it does: better an argument than just postulating a law of nature that the two accounts always coincide.²⁵ But I find Tooley's argument unpersuasive as regards (a) and (b): only a much more elaborated theory of causation and motion could sustain the inferences needed.

Thus, as to (i): why believe that accidentally orderly movement is the only way to have an orthodox but not Tooleyan velocity?. Tooley's (ii) obviously does not purport to be more than a sketch. But whatever the 'plausible hypotheses' might be, there are basic problems about the strategy of the argument. Why should discontinuous changes in velocity be the only way to have a Tooleyan velocity but not an orthodox one? After all, the latter requires non-differentiability of position, not discontinuity of its derivative. Besides, discontinuous changes in forces mean discontinuities in acceleration: which need not spell discontinuities in velocity.

Finally, even if (a) were established, (b) would not follow just from the fact that both orthodox and Tooleyan velocities satisfy T_1 . After all, integration is an averaging operation and so "loses information". So even if $q(t)$ is differentiable, so that indeed $q = \int dq/dt dt$, there are still countless other functions $v \neq dq/dt$ such that $q = \int v dt$: for example, v could be a "scarring" of a smooth dq/dt by inserting some discontinuities.

(2): *Tooley's Section 6.2*:—

The second topic is the problem I emphasised at the outset (Section 3.3.1.B) as confronting all advocates of Tooleyan velocities: the conflict between Tooleyan velocities being intrinsic, and velocity being relative to a frame of reference.

Tooley treats this briefly in his Section 6.2 (p. 251). He sees it as a matter of reconciling his proposal with special relativity. He admits that 'philosophical criticisms of scientific theories do not have an extraordinarily impressive track record', but conjectures that a heterodox interpretation of special relativity which adds a relation of absolute simultaneity may be tenable, for example because it better allows a tensed account of the nature of time. (He develops this conjecture in his (1997); cf footnote 12.)

In reply, I will not repeat Section 3.3.1.B's endorsement of the problem. But I would make two further comments, specifically about Tooley's answer to it. First, I would be much less willing than Tooley to let metaphysical views determine the interpretation

²⁵As Arntzenius (2000: p. 196) and Bigelow and Pargetter (1989, p. 294) do: admittedly, in much briefer discussions.

of physical theories, and in particular to adopt a tensed account of time. Second I emphasise that relativity’s denial of absolute simultaneity is irrelevant to the problem. As noted in (1) of Section 3.3.1.B, classical mechanics no less than relativity can be formulated with velocity being relative to a frame of reference. So the problem arises already on Tooley’s chosen “home-ground” of classical mechanics.

So to sum up this and the previous Subsection’s critique of Tooleyan velocities: I have argued that Tooley and other authors (i) do not justify crucial premises of their arguments for these velocities, and (ii) under-estimate the resources of the orthodox account.

4 “Shadow velocities”: Lewis and Robinson

I turn to a proposal of Lewis and Robinson that is similar in some ways to Section 3.3’s proposed intrinsic velocity. In short, they propose that a moving object has a vectorial property (i.e. a property represented by a vector) which is intrinsic to the object, and whose vector is equal to the velocity vector. But this property is not itself velocity: hence this Section’s title. For velocity presupposes the persistence of the moving object; and this property is to be intrinsic, not merely (as we have emphasised, especially in Section 3.2) “almost intrinsic”. In fact, Robinson (1989) floats the proposal but does not endorse it; Lewis (1999) endorses it. (As we shall see, this difference between them turns on our central question, familiar since Section 3.1: can vectorial properties be intrinsic to a point?)

But unlike Tooley, Lewis and Robinson are concerned about persistence. Specifically, they take their proposal to provide the perdurantist, especially the advocate of Humean supervenience (Lewis 1986, pp. ix-x; 1994, pp. 225-226), with a reply to the rotating disc argument. I discuss persistence as a context for their proposal elsewhere (2004, Section 4.3). So in this paper, I will mostly leave aside this context: my pro-perdurantist claim (FPe) in Sections 2.2.3 and 2.3.2 will suffice. (I also leave aside how even a simple quantity such as mass defined on the points and regions of a continuous body causes trouble for *pointillisme*, in particular Humean supervenience: cf. Butterfield (2006) or Hawthorne (2006: Section 2).)

I will first present the proposal, in Section 4.1. Then in Section 4.2 I will criticize it. Then the last two Subsections try to offer a peace-pipe to Lewis and Robinson. In Section 4.3, I use Hilbert’s ε operator to define a quantity which is like their proposed quantity, in being analogous to velocity yet not presupposing persistence. I will call it ‘velocity’. Finally in Section 4.4, I briefly ask whether velocity satisfies Lewis’ and Robinson’s goals: but again my conclusion is negative—it would not.

4.1 The proposal

Recall the rotating disc argument. The perdurantist is challenged to say what distinguishes two rigid congruent utterly homogeneous discs, one rotating and one stationary. It seems that the perdurantist, with her meagre resources, in particular “qualitative” facts intrinsic to temporal instants, cannot do so: she cannot “thread the worldlines together” correctly.

Robinson (1989; p. 405 para 2, p. 406 para 2 to p. 408 para 1) floats the following reply. It combines the idea of a “cousin” of velocity that is intrinsic and does not presuppose persistence, with the idea (endorsed by many philosophers) that persistence is determined (“subvened”) by relations of qualitative similarity and causal dependence between events. To be precise, the reply has five components; as follows:

- (i) a vectorial property at a point can be an intrinsic property of that point;
- (ii) the propagation of continuous matter through spacetime involves such a property at every spacetime point; and
- (iii) these properties distinguish the rotating and non-rotating discs, since the vector that represents the property at a point is timelike, and points in the same direction as the instantaneous four-dimensional velocity vector at that point;
- (iv) the distribution of these properties, from point to point, determines (subvenes) the relations of qualitative similarity between points, and especially the relations of causal dependence between events at those points; and
- (v) the distribution of these properties, by determining the lines of causal dependence, determines the lines of persistence.

But Robinson himself has second thoughts about this proposal. His doubts concern the first component, (i): i.e. the Yes answer to Section 3.1’s question. He thinks the directionality of a vector forbids it from representing an intrinsic property; and he backs this up with an argument about a point and a duplicate of it, which he credits to Lewis in discussion.²⁶

On the other hand, Lewis believed (at least by about 1993) that vectorial properties could be intrinsic to points (1994, p. 226). And in a final short paper on the rotating disc argument (replying to Zimmerman’s critique of perdurantism, 1998), Lewis endorsed Robinson’s proposal; (Lewis 1999, p. 211).²⁷

So the idea of the proposal is that the difference in the properties postulated by (ii)

²⁶Similar doubts are expressed by other contemporary metaphysicians, some sharing a Lewisian approach to the intrinsic-extrinsic distinction. Butterfield (2006: Section 4.1) gives references.

²⁷My (2004, Section 4.3.1) gives more details about how Lewis came around (ca. 1998) to this proposal, after espousing for a while (ca. 1986-1994) a more “stone-walling” reply.

Note also that the proposal is clearly similar in spirit to Tooley’s heterodoxy about velocity, as in Section 3.3. Robinson does not refer to Tooley et al.: the work was of course contemporaneous. But Zimmerman (1998, p. 281, p. 284) and Sider (2001, p. 228) both see the similarity. Zimmerman first discusses reading Robinson’s proposal as the same as Tooley’s (p. 281), and then discusses reading it as just similar (p. 284, note 65). Sider reads the proposals as similar. More specifically, Sider and Zimmerman’s second reading both see Robinson’s proposal as going with an orthodox, or “Russellian at-at”, account of motion. So also (implicitly) does Lewis’ discussion.

and (iii) amounts to a difference in the ‘local arrangement of qualities’ as demanded by Humean supervenience. Thus Lewis (1999, p. 211) begins by approvingly quoting Robinson, suggesting we should

...see the collection of qualities characteristic of the occupation of space by matter as in some sense jointly self-propagating; the fact of matter occupying space is itself causally responsible ... for the matter going on occupying space in the near neighbourhood immediately thereafter. ... [The posited vectors] figure causally in determining the direction of propagation of [themselves as well as] other material properties. (Robinson 1989, p. 406-407.)

Lewis then goes on to formulate the proposal more formally, as a putative law that partially specifies a vector field V . The specification is partial, both in (i) being admitted to be a “first approximation”, and (ii) specifying only the direction but not the length of the vector at each point. But (ii) hardly matters: it will be obvious that Robinson and Lewis could frame their proposal entirely in terms of postulating a timelike direction field (i.e. a specification at each point of continuous matter of a timelike direction), rather than a vector field. But I shall follow them and talk of a vector field.

In giving this formulation, Lewis’ aim is partly to avoid various objections or limitations. In particular, the formulation should not invoke either persistence or causation, since these are meant to supervene on the local arrangement of qualities, taken of course as including facts about the vector field V . Thus the formulation is to avoid circularity objections that had been urged by Zimmerman (1998) against some related proposals.

So in particular: the vector field V cannot simply be the instantaneous (four-dimensional) velocity (orthodox, not Tooleyan!) of the matter at the point in question. For V is to contribute to an analysis of (or at least to a supervenience basis for) persistence and thereby of velocity.

Similarly, since Lewis agrees that causation is crucial to persistence (‘the most important sort of glue that unites the successive stages of a persisting thing is causal glue’: 1999, p. 210), causation cannot be invoked in the course of specifying the vector field V .

Lewis proposes that (for a world with continuous space and time), the specification of V ‘might go something like this’:

Let p be any spacetime point, and let t be any smooth timelike trajectory through spacetime with p as its final limit point. Let each point of t before p be occupied by matter with its vector [i.e. vector of the vector field V] pointing in the direction of t at that point. [So in the jargon of modern geometry, t is an integral curve of V .] Then, *ceteris paribus*, there will be matter also at p . (1999, p. 211.)

Here, the ‘*ceteris paribus*’ clause is to allow for the fact that the point-sized bit of matter might cease to exist before p , because of ‘destructive forces or self-destructive tendencies’ (ibid.).

Lewis also stresses that this proposal is to be read as a law of succession, not of causation. This means, I take it, that the ‘Then, *ceteris paribus*’ is to be read as a material conditional.

4.2 Criticism: the vector field remains unspecified

I claim that Lewis’ proposal fails. It is too weak: it does not go far enough to specify V . For it only says, of any timelike open curve that is an integral curve of V , that the future end-point p of this curve will, *ceteris paribus*, have matter at it.

But every suitably smooth vector field U defined on a open region R of spacetime has integral curves throughout R ; (which are timelike, by definition, if U is). (To be precise: ‘suitably smooth’ is none too demanding: all we need is that U be C^1 , i.e. the partial derivatives of its components exist and are continuous.) So suppose Lewis stipulates, that the field V is to be timelike and C^1 on an open set R which is its domain of definition (say, the spatiotemporal region occupied by continuous matter): which (“giving rope”) we can assume to be a legitimate, in particular non-circular, stipulation. Then his proposal says that, *ceteris paribus*, every point $p \in R$ has matter at it.

But that claim hardly helps to distinguish V from the countless other (timelike smooth) vector fields U . For however exactly one interprets ‘*ceteris paribus*’, the claim is surely true of p regardless of the integral curve one considers it as lying on. So the claim about p does not constrain the vector field. Indeed, if Lewis sets out to specify V on the spatiotemporal region occupied by continuous matter, the claim is thereby assumed to be true for all p in the region, regardless of vector fields. So again, we have said nothing to distinguish V from the countless other vector fields U .

Agreed, Lewis puts forward his proposal as a “first approximation” to specifying V . But so far as I can see, his discussion doesn’t contain any ingredients which would, for continuous matter, help distinguish V from other vector fields.²⁸

I should note here that Zimmerman (1999) makes a somewhat similar objection to Lewis’ proposal. But his exact intent is not clear to me.

He maintains that in some seemingly possible cases of continuous matter, Lewis’ proposal does not specify a unique vector field V —indeed hardly constrains V at all. He

²⁸Nor can I guess how I might have misinterpreted Lewis’ proposal. The situation is puzzling: and not just because Lewis thought so clearly, and my objection is obvious. Also, the objection is analogous to what Lewis himself says (p. 210) against the naive idea that V should point in the direction of perfect qualitative similarity: viz. that ‘in non-particulate homogeneous matter, ... lines of qualitative similarity run every which way’.

An anonymous referee suggests that Lewis’ attempted specification of V might succeed if the property is required to be natural, in Lewis’ sense; or at least, might succeed if the environment around the region R is also sufficiently heterogeneous. I confess I do not see how naturalness and-or a heterogeneous environment will help Lewis; but I discuss the latter in the sequel.

says (p. 214 para 1 and 2) that in possible worlds with a physics of the sort Descartes might have envisaged, i.e. where there is *nowhere* any vacuum, and only *one* kind of (continuous homogeneous) stuff fills all of space: ‘*every* vector field will satisfy [Lewis] law.’

Thus Zimmerman assumes that:

(i): the worlds with which he is concerned are wholly filled with the one kind of stuff; and

(ii): these worlds are thus filled as a matter of law, not happenstance (in the jargon: as a matter of physical or nomic necessity).

He also says (p. 214-5) that he needs to assume (i) and (ii) in order to criticize Lewis’ proposal, together with obvious modifications of it which allow for different types (“colours”) of continuous matter. That is: Zimmerman thinks Lewis’ proposal works, or could be modified to work, for worlds in which:

(i’) continuous matter does not fill all of space and-or comes in various types; or

(ii’) continuous matter of just one type fills all of space, but only as a matter of happenstance.

In view of my own objection, I do not understand why Zimmerman feels he needs to assume (i) and (ii) in order to object to Lewis. He does not explicitly say why he does so. *Maybe* it is to block some Lewisian rejoinder, that would better specify V , by adding constraints of either or both of two kinds:

(i’): constraints about the spatiotemporal relations of the continuous matter in a bounded volume (say, one of our discs) to other matter outside the volume;

(ii’): constraints about the nomic or modal properties of matter.

But it remains unclear how the details of (i’’) and (ii’’) might go.

To sum up: For all I can see, my objection, that V is not distinguished from countless other vector fields, applies to Lewis’ proposal (and thereby: the spirit of Zimmerman’s objection also applies) for the case that Lewis intended it—i.e. the discs of the original rotating discs argument.

4.3 Avoiding the presupposition of persistence, using Hilbert’s ε symbol

Since Section 3.3, my discussion has been critical. Now I try to be more constructive! I propose to make precise the idea that velocity, understood in the orthodox way, is hardly extrinsic. Elsewhere (2004, Section 4.2.2; 2004a, Section 4.5) I make this precise in two related ways. Here I develop a third way.

Namely, I will define a quantity which will be like Robinson and Lewis’ proposal from Section 4.1, in that it is (“usually”) equal to instantaneous velocity, and yet does *not* presuppose persistence. But unlike their proposal, it has no *pointilliste* motivations: in fact, it is adapted from the orthodox definition of velocity. After presenting it, I will end by comparing it with their proposal (Section 4.4).

I will call my new-fangled quantity *velocity*, the ‘w’ being a mnemonic for ‘(log-

ically) weak’ and-or ‘without (presuppositions)’. So my goal is that velocity is to reflect, in the way its values are defined, this lack of presupposition. That is: the values are to be defined in such a way that it is impossible to infer from the value of the velocity of the object o at time t that o in fact exists in a neighbourhood of t , and has a differentiable worldline at t : an inference which, as we have just seen, *can* be made from the value of velocity as orthodoxly understood.

There are three preliminary points to make about this goal: the first philosophical, the second mathematical and the third physical. But only the first represents a limitation of scope for what follows.

First, presupposition is often taken to be a subtler notion than just ‘necessary condition’. So I admit that for a property ascription to lack a presupposition of persistence, it is perhaps *not* enough that the ascription fails to imply that the instance persists. But I will set this aside: I will aim only for the modest goal of avoiding the implication (if not, perhaps, the presupposition *stricto sensu*) of persistence.

Second, I just said that o ’s having an orthodox velocity at t implies o ’s existing in an open neighbourhood of t , and its position in space $\mathbf{q}(t)$ being differentiable at t . Indeed, that is orthodoxy. But as I remarked at the end of Section 3.2.2, one can generalize so as to imply only that o exists at a set of times for which t is a limit point (and that its average velocities go to a common limit at t). But to avoid cumbersome phrasing, I will from now on not repeat this generalization.

Third, as I stressed (against Tooley and others) in Sections 3.3.1 and 3.3.3: orthodox velocity is relative to a frame of reference. And one cannot expect that avoiding an implication of o ’s persistence will also make for avoiding the implication of, and relativity to, a frame. Agreed: and indeed, ascriptions of velocity will be just as obviously relative to a frame (and so in that way extrinsic) as are ascriptions of velocity. But in order to engage better with authors such as Tooley, Robinson and Lewis, I will not emphasise this aspect.

Developing this idea—values of a quantity like velocity, but which do not imply o ’s persistence nor its worldline’s differentiability—takes us to a familiar philosophical territory: viz., rival proposals for the semantics of empty referring terms. In our case, the empty terms will be expressions for o ’s instantaneous velocity at t ; and, as just discussed, they can be empty because either:

(NotEx): o does not exist for an open interval around t , or

(NotDiff): o does exist for an open interval around t , but its position \mathbf{q} is not differentiable at t ; (roughly: there is a “sharp corner” in the worldline).

(And similarly for acceleration and higher derivatives; but I shall discuss only velocity—tempting though words like ‘wacceleration’ are!)

In fact, it will be clearest to lead up to my proposal for velocity by first considering a simpler one, which is modelled on Frege’s proposal that (to prevent truth-value gaps) empty terms should be assigned some “dustbin-referent”, such as the empty set \emptyset . Thus if one sets out to define a quantity that is like velocity but somehow avoids its presupposition of persistence, one naturally first thinks of a quantity, call it \mathbf{u} , defined to be

(a): equal to the (instantaneous) velocity \mathbf{v} , for those times t at which o has a velocity; and

(b): equal to some dustbin-referent, say the empty set \emptyset , at other times t ; i.e. times such that either (NotEx): o does not exist for an open interval around t ; or (NotDiff): o does exist for an open interval around t , but its position \mathbf{x} is not differentiable at t .

Of course, variations on (b) are possible. One could select different dustbin-referents for the two cases, (NotEx) and (NotDiff), (say, \emptyset and $\{\emptyset\}$) so that \mathbf{u} 's value registered the different ways in which an instantaneous velocity could fail to exist. And instead of using a dustbin-referent, one could say that the empty term just has no "semantic value", or "is undefined": (a contrast with dustbin-referents which would presumably show up in truth-value gaps, and logical behaviour in general).

Agreed, this definition is natural. But it does not do the intended job. For this quantity \mathbf{u} , whether defined using (b) or using the variations mentioned, does not avoid, in the way intended, the presupposition of persistence. For \mathbf{u} 's value (or lack of it, if we take the no-semantic-value option) registers whether or not the presupposed persistence holds true. That is: we *can* infer from the value of \mathbf{u} (or its lack of value) whether (a) o has a velocity in the ordinary sense, or (b) the presupposition has failed in that (NotEx) or (NotDiff) is true. In short: \mathbf{u} 's individual values tell us too much.

But there *is* an appropriate way of assigning semantic values to empty terms, i.e. a way of defining a quantity, *velocity*, that is like velocity but whose values do not give the game away about whether the presupposition has failed, i.e. about whether (NotEx) or (NotDiff) is true. In order not to give the game away, velocity must obviously take ordinary values, i.e. triples of real numbers (relative to some frame of reference), even when the presupposition has failed. But how to assign them?

The short answer is: arbitrarily. The long answer is: we can adapt schemes devised by logicians in which a definite description, whose predicate has more than one instance, is assigned as a referent *any one* of the objects in the predicate's extension. (The first such scheme was devised by Hilbert and Bernays; but we will only need the general idea.) Such a scheme applies to our case, because we can write the definition of velocity in such a way that when the presuppositions fail (i.e. (NotEx) or (NotDiff) is true), the predicate (of triples of real numbers) in the definition is vacuously satisfied by *all* such triples; so that forming a definite description, and applying semantic rules like Hilbert-Bernays', velocity is assigned an *arbitrary* triple of real numbers as value.

Thus we get the desired result: if you are told that the value of velocity for o at t is some vector in \mathbb{R}^3 , say $(1,10,3)$ relative to some axes and choice of a time-unit, you cannot tell whether:

(a): (NotEx) and (NotDiff) are both false (i.e. the presuppositions of velocity hold), and o has velocity $(1,10,3)$; or

(b): Either (NotEx) or (NotDiff) is true, the predicate is vacuously satisfied by all triples, and $(1,10,3)$ just happens to be the triple assigned by semantic rules taken from Hilbert-Bernays' (or some similar) scheme.

The details are as follows. (1): Hilbert and Bernays introduced the notation $(\varepsilon x)(Fx)$ for the definite description ‘the F ’, with the rule that if F had more than one instance, then $(\varepsilon x)(Fx)$ was assigned as referent any such instance, i.e. any element of F ’s extension. (We need not consider their rule in more detail than this; nor their rule for what to say when F has no instances; nor their rules’ consequences for the semantics and syntax of singular terms. For details, cf. Leisenring (1969).)

(2): Next, we observe that the *velocity* of an object o at time t relative to a given frame can be defined with a definite description containing a material conditional whose antecedents are the presuppositions of persistence and differentiability. That is: velocity can be defined along the following lines:—

The velocity of o at time t (relative to a given frame) is the triple of real numbers \mathbf{v} such that:

for some (and so any smaller) open interval I around t :
 $\{[o \text{ exists throughout } I] \text{ and } [o\text{'s position } \mathbf{x}(t) \text{ is differentiable in } I]\} \supset$
 $[\mathbf{v} \text{ is the common limit of average velocities for times } t' \in I, \text{ compared with } t, \text{ as } t' \rightarrow t \text{ from above or below}].$

This *definiens* uses a material conditional. So it will be vacuously true for all triples \mathbf{v} , if the antecedent is false for all open intervals I around t , i.e. if (NotEx) or (NotDiff) is true: in other words, if velocity’s presuppositions of continued existence and differentiability fail.

(3): Now we put points (1) and (2) together. Let us abbreviate the displayed *definiens*, i.e. the open sentence with \mathbf{v} as its only free variable, as $F(\mathbf{v})$. Then I propose to define the *velocity* of o at t by the singular term $(\varepsilon \mathbf{v})(F\mathbf{v})$: which is, by Hilbert-Bernays’ semantic rule:

(a): equal to the (instantaneous) velocity of o , for those times t at which o has a velocity; and

(b): equal to some arbitrary triple of real numbers, at other times t ; i.e. at times such that either (NotEx): o does not exist for an open interval around t ; or (NotDiff): o does exist for an open interval around t , but its position \mathbf{x} is not differentiable at t .

Velocity, so defined, has the desired features: its values do not give the game away about whether (NotEx) or (NotDiff) is true.

That is all I need to say about velocity, for this paper’s purposes; and in particular, for Section 4.4’s comparison with Robinson’s and Lewis’ proposal.

But I end this Subsection by noting that there are of course various technical questions hereabouts, even apart from the logical questions about the ε symbol. For example, a natural question arises from letting o be a point-sized bit of matter in a continuum, and letting the presuppositions of velocity fail for various such bits of matter: some such bits may fail to exist, and some may have a non-differentiable worldline. One then asks: how widely across space, and in how arbitrary a spatial distribution, can these bits fail to exist, or have a non-differentiable worldline—i.e. how

widely and arbitrarily can the presuppositions of velocity fail—while yet the velocity field might not “give the game away”, in that the arbitrary values *can* be assigned at all the points where the presuppositions fail, so as to give a smooth (e.g. continuous or even differentiable) velocity field? This is in effect a question about the scope and limits of regularization of singularities in real vector fields: a good question—but not one for this paper!

4.4 Comparison with Robinson and Lewis

I shall compare velocity with Robinson’s and Lewis’ proposal in two stages; the first more specific than the second.

(1): *Is velocity intrinsic?*—

In the light of Section 3.1’s central question, this is the obvious question to ask about velocity! But in asking this, I will set aside the fact that velocity, like velocity, is relative to a frame. This sets aside the existence of other objects representing the frame, and focuses attention on extrinsicality arising from implications about the other temporal parts of *o* itself. This tactic will make for a less cumbersome comparison with Robinson and Lewis, and authors like Tooley; who, as we have seen, tend to ignore the frame; (cf. the third preliminary point at the start of Section 4.3).

The question whether velocity is intrinsic returns us to Section 2.1.1’s idea of positive extrinsicality, i.e. the idea of implying accompaniment. Recall that according to Lewis (1983) and almost all succeeding authors, this is a species of extrinsicality, since a property like being unaccompanied (more vividly: being lonely) is itself extrinsic; and that this species, positive extrinsicality, is agreed to be a good deal clearer than the genus, extrinsicality. Similarly for the negations: the negation of positive extrinsicality, i.e. not implying accompaniment (i.e. compatibility with being lonely), is weaker than—and a good deal clearer than—intrinsicality. So we might call it ‘weakened intrinsicity’. And as announced in Section 2.1.1, my campaign against *pointillisme* can mostly take *pointillisme* to advocate weakened intrinsic properties.

Once we set aside frame-relativity and any extrinsicality ensuing from that, it is clear that both an ascription to *o* at *t* of some or other value of velocity, and an ascription of a specific value of velocity, are *not* positive extrinsic. For thanks to velocity allowing for (NotEx), each ascription is compatible with *o*’s temporal part at *t* being lonely: i.e. compatible with *o*’s not existing at other times. So the two ascriptions are weakened intrinsic.

Besides, having some velocity or other (again setting aside frames and other objects) is a necessary property. For *o* has a velocity at *t* iff: either

- (1) *o* at *t* is lonely, i.e. (NotEx); or
- (2) *o*’s worldline is not differentiable at *t*, i.e. (NotDiff); or
- (3) *o*’s worldline is differentiable (and so *o* has a velocity).

This disjunction is obviously equivalent to *o*’s merely existing at *t*; so that having some velocity or other is a necessary property.

Now on Lewis' preferred analysis (1983a) and several alternatives (e.g. his "fall-back" analysis in Langton and Lewis (1998)) an intrinsic property is one that does not differ between duplicate objects—where duplication is defined as sharing a certain elite minority of properties. Clearly, on any such analysis, any necessary property is intrinsic: for it will not differ between duplicates. (More generally: on any such analysis, intrinsicity is not hyperintensional. That is, necessarily co-extensive properties are alike in being intrinsic, or not.)

But on the other hand, having a *specific* velocity, say 5 ms^{-1} North, is of course not necessary. If this property applies to o at t , while yet (1) and (2) are both false, then o has a *velocity* 5 ms^{-1} North.

Indeed, I claim this property is extrinsic (though as just established, weakened intrinsic—setting aside frames). The argument is a general one; as follows.

Consider a property P defined along the lines: o is P iff: either (1') o is lonely, or (2') if o is accompanied, then o and some accompanying objects satisfy some condition which is *not* necessary and which involves them all "non-redundantly". Can we conclude that P is intrinsic? Or extrinsic? Or can we make no conclusion?

The intuitive verdict is surely that P is extrinsic. Intuitions apart, P is certainly extrinsic according to several analyses, e.g. by Lewis (1983a), Langton and Lewis (1998), Vallentyne (1997) and Lewis (2001). For example, for the first two analyses the reason is essentially that P is a disjunction whose disjuncts are a positive extrinsic, viz. (2') and what Lewis (1983, p. 114) dubbed a negative extrinsic, i.e. a property implying loneliness, viz. (1').

This common verdict suggests that not only velocity, but any quantity that is analogously defined with a disjunct like (1'), will be extrinsic.

To sum up:— We have established that having a specific value of velocity is a weakened intrinsic property; and that having some velocity or other is a necessary property, and so according to several analyses, intrinsic. On the other hand, having a specific value of velocity seems intuitively to be extrinsic; and certainly is extrinsic, according to several analyses.

(2): *Rounding off*:—

To conclude: already in Sections 2 and 3, I announced my denial of *pointillisme*, and so antipathy to Humean supervenience; and more specifically, my view that velocity was "almost" intrinsic. In Section 4.3, we saw my peace-pipe for the *pointilliste*: at least, for the *pointilliste* who advocates weakened intrinsic properties. Namely, I defined a quantity, *velocity*, which is weakened intrinsic, and in that sense avoids velocity's implication of persistence.

But I am afraid Lewis would not smoke my peace-pipe! For first, he would presumably be unimpressed by velocity's being almost intrinsic. For Humean supervenience is so central to his neoHumean metaphysical system that he sets great store by intrinsicity. So he would probably say that as regards failing to be intrinsic, a miss is as good (i.e. bad!) as a mile.

Similarly, I expect that he would not welcome velocity. I agree that he might be

“envious” of its being well-defined (*modulo* the freedom to assign referents associated with the ε operator), since his own proposal, the intrinsic vector V , is yet to be successfully defined (Section 4.2). But Lewis is an advocate of intrinsicity, not just weakened intrinsicity; and as we have just seen, by Lewis’ lights, velocity is extrinsic.

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