

Holism, Physical Theories and Quantum Mechanics^{*}

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Abstract

Motivated by the question what it is that makes quantum mechanics a holistic theory (if so), I try to define for general physical theories what we mean by ‘holism’. For this purpose I propose an epistemological criterion to decide whether or not a physical theory is holistic, namely: a physical theory is holistic if and only if it is impossible in principle to infer the global properties, as assigned in the theory, by local resources available to an agent. I propose that these resources include at least all local operations and classical communication. This approach is contrasted with the well-known approaches to holism in terms of supervenience. The criterion for holism proposed here involves a shift in emphasis from ontology to epistemology. I apply this epistemological criterion to classical physics and Bohmian mechanics as represented on a phase and configuration space respectively, and for quantum mechanics (in the orthodox interpretation) using the formalism of general quantum operations as completely positive trace non-increasing maps. Furthermore, I provide an interesting example from which one can conclude that quantum mechanics is holistic in the above mentioned sense, although, perhaps surprisingly, no entanglement is needed.

Key words: holism, supervenience, classical physics, quantum mechanics, entanglement

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1 Introduction

Holism is often taken to be the idea that the whole is more than the sum of the parts. Because of being too vague, this idea has only served as a guideline or intuition to various sharper formulations of holism. Here I shall be concerned with the one relevant to physics, i.e., the doctrine of *metaphysical holism*, which is the idea that properties or relations of a whole are not determined or cannot be determined by intrinsic properties or relations of the parts¹. This is taken to be opposed to a claim of supervenience (Healey, 1991), to reductionism (Maudlin, 1998), to local physicalism (Teller, 1986), and to particularism (Teller, 1989). In all these cases a common approach is used to define what metaphysical holism is: via the notion of *supervenience*². According to this common approach metaphysical holism is the doctrine that some facts, properties, or relations of the whole do not supervene on intrinsic properties and relations of the parts, the latter together making up the *supervenience basis*. As applied to physical theories, quantum mechanics is then taken to be the paradigmatic example of a holistic theory, since certain composite states (i.e., entangled states) do not supervene on subsystem states, a feature not found in classical physical theories.

However, in this paper I want to critically review the supervenience approach to holism and propose a new criterion for deciding whether or not a physical theory is holistic. The criterion for whether or not a theory is holistic proposed here is an *epistemological* one. It incorporates the idea that each physical theory (possibly supplemented with a property assignment rule via an interpretation) has the crucial feature that it tells us how to *actually* infer

¹ This *metaphysical holism* (also called *property holism*) is to be contrasted with *explanatory* and *meaning holism* (Healey, 1991). The first is the idea that explanation of a certain behavior of an object cannot be given by analyzing the component parts of that object. Think of consciousness of which some claim that it cannot be fully explained in terms of physical and chemical laws obeyed by the molecules of the brain. The second is the idea that the meaning of a term cannot be given without regarding it within the full context of its possible functioning and usage in a language.

² The notion of supervenience, as used here, is meant to describe a particular relationship between properties of a whole and properties of the parts of that whole. The main intuition behind what particular kind of relationship is meant, is captured by the following impossibility claim. It is not possible that two things should be identical with respect to their subvenient or subjacent properties (i.e., the lower-level properties), without also being identical with respect to their supervening or upper-level properties. The first are the properties of the parts, the second are those of the whole. The idea is that there can be no relevant difference in the whole without a difference in the parts. (Cleland (1984) uses a different definition in terms of modal logic.)

properties of systems and subsystems.

The guiding idea of the approach here suggested, is that some property of a whole would be holistic if, according to the theory in question, there is no way we can find out about it using *only* local means, i.e., by using only all possible non-holistic resources available to an agent. In this case, the parts would not allow for inferring the properties of the whole, not even via all possible subsystem property determinations that can be performed, and consequentially we would have some instantiation of holism, called *epistemological holism*. The set of non-holistic resources is called the *resource basis*. I propose that this basis includes at least all local operations and classical communication of the kind the theory in question allows for.

The approach suggested here thus focuses on property measurement instead of on the supervenience of properties. It can be viewed as a shift from ontology to epistemology³ and also as a shift that takes into account the full potential of physical theories by including what kind of property inferences or measurements are possible according to the theory in question. The claim I make is that these two approaches are crucially different and that each have their own merits. I show the fruitfulness of the new approach by illustrating it in classical physics, Bohmian mechanics and orthodox quantum mechanics.

The structure of this paper is as follows. First I will present in section 2 a short review of the supervenience approach to holism. I especially look at the supervenience basis used. To illustrate this approach I consider what it has to say about classical physics and quantum mechanics. Here I rigorously show that in this approach classical physics is non-holistic and furthermore that the orthodox interpretation of quantum mechanics is deemed holistic. In the next section (section 3) I will give a different approach based on an epistemological stance towards property determination within physical theories. This approach is contrasted with the approach of the previous section and furthermore argued to be a very suitable one for addressing holism in physical theories.

In order to show its fruitfulness I will apply the epistemological approach to different physical theories. Indeed, in section 4 classical physics and Bohmian mechanics are proven not to be epistemologically holistic, whereas the orthodox interpretation of quantum mechanics is shown to be epistemologically holistic without making appeal to the feature of entanglement, a feature that was taken to be absolutely necessary in the supervenience approach for any holism to arise in the orthodox interpretation of quantum mechanics. Finally

³ This difference is similar to the difference between the two alternative definitions of determinism. From an *ontological* point of view, determinism is the existence of a single possible future for every possible present. Alternatively, from an *epistemological* point of view, it is the possibility in principle of inferring the future from the present.

in section 5 I will recapitulate, and argue this new approach to holism to be a fruit of the rise of the new field of quantum information theory.

2 Supervenience approaches to holism

The idea that holism in physical theories is opposed to supervenience of properties of the whole on intrinsic properties or relations of the parts, is worked out in detail by Teller (1986) and by Healey (1991), although others have used this idea as well, such as French (1989)⁴, Maudlin (1998) and Esfeld (2001). I will review the first two contributions in this section.

Before discussing the specific way in which part and whole are related, Healey (1991) clears the metaphysical ground of what it means for a system to be composed out of parts, so that the whole supervenience approach can get off the ground. I take this to be unproblematic here and say that a whole is composed if it has component parts. Using this notion of composition, holism is the claim that the whole has features that cannot be reduced to features of its component parts. Both Healey (1991) and Teller (1986) use the same kind of notion for the reduction relation, namely *supervenience*. However, whereas Teller only speaks about relations of the whole and non-relational properties of the parts, Healey uses a broader view on what features of the whole should supervene on what features of the parts. Because of its generality I take essentially Healey's definition to be paradigmatic for the supervenience approach to holism⁵. In this approach, holism in physical theories means that there are physical properties or relations of the whole that are not supervenient on the intrinsic physical properties and relations of the component parts. An essential feature of this approach is that the *supervenience basis*, i.e., the properties or relations on which the whole may or may not supervene, are only the *intrinsic* ones, which are those which the parts have at the time in question in and out of themselves, regardless of any other individuals.

We see that there are three different aspects involved in this approach. The

⁴ French (1989) uses a slightly different approach to holism where supervenience is defined in terms of modal logic, following a proposal by Cleland (1984). However, for the present purposes, this approach leads essentially to the same results and I will not discuss it any further.

⁵ The exact definition by Healey (1991, p.402) is as follows. '*Pure physical holism*: There is some set of physical objects from a domain D subject only to processes of type P , not all of whose qualitative, intrinsic physical properties and relations are supervenient upon the qualitative, intrinsic physical properties and relations of their basic physical parts (relative to D and P)'. The definition by Teller (1986) is a restriction of this definition to solely relations of the whole and intrinsic non-relational properties of the parts.

first has to do with the metaphysical, or ontological effort of clarifying what it means that a whole is composed out of parts. I took this to be unproblematic. The *second* aspect gives us the type of dependence the whole should have to the parts in order to be able to speak of holism. This was taken to be supervenience. *Thirdly*, and very importantly for the rest of this paper, the supervenience basis needs to be specified because the supervenience criterion is relativized to this basis. Healey (1991, p.401) takes this basis to be ‘just the qualitative, intrinsic properties and relations of the parts, i.e., the properties and relations that these bear in and out of themselves, without regard to any other objects, and irrespective of any further consequences of their bearing these properties for the properties of any wholes they might compose.’ Similarly Teller (1986, p.72) uses ‘properties *internal* to a thing, properties which a thing has independently of the existence or state of other objects.’

Although the choice of supervenience basis is open to debate because it is hard to specify precisely, the idea is that we should not add global properties or relations to this basis. It is supposed to contain only what we intuitively think to be *non-holistic*. However, as I aim to show in the next sections, an alternative basis exists to which a criterion for holism can be relativized. This alternative basis, the *resource basis* as I call it, arises when one adopts a different view when considering physical theories. For such theories allow not only for presenting us with an ontological picture of the world (although possibly only after an interpretation is provided), but also they allow for specific forms of property assignment and property determination. The idea then is that these latter processes, such as measurement or classical communication, have intuitively clear *non-holistic features*, which allow for an epistemological analysis of whether or not a whole can be considered to be holistic or not.

However, before presenting this new approach, I discuss how the supervenience approach treats classical physics and quantum mechanics (in the orthodox interpretation). In treating these two theories I will first present some general aspects related to the structure of properties these theories allow for, since they are also needed in future sections.

2.1 *Classical physics in the supervenience approach*

Classical physics assigns two kinds of properties to a system. State independent or fixed properties that remain unchanged (such as mass and charge) and dynamical properties associated with quantities called dynamical variables (such as position and momentum) (Healey, 1991). It is the latter we are concerned with in order to address holism in a theory since these are subject to the dynamical laws of the theory. Thus in order to ask whether or not classical physics is holistic we need to specify how parts and wholes get assigned the

dynamical properties in the theory⁶. This *ontological issue* is unproblematic in classical physics, for it views objects as bearers of determinate properties (both fixed and dynamical ones). The *epistemological issue* of how to gain knowledge of these properties is treated via the idea of *measurement*. A measurement is any physical operation by which the value of a physical quantity can be inferred. Measurement reveals this value because it is assumed that the system has the property that the quantity in question has that value at the time of measurement. In classical physics there is no fundamental difference between measurement and any other physical process. Isham (1995, p.57) puts it as follows: ‘Properties are intrinsically attached to the object as it exists in the world, and measurement is nothing more than a particular type of physical interaction designed to display the value of a specific quantity.’ The bridge between ontology and epistemology, i.e., between property assignment (for any properties to exist at all (in the theory)) and property inference (to gain knowledge about them), is an easy and unproblematic one called measurement.

The specific way the dynamical properties of an object are encoded in the formalism of classical physics is in a state space Ω of physical states x of a system. This is a phase space where at each time a unique state x can be assigned to the system. Systems or ensembles can be described by *pure states* which are single points x in Ω or by *mixed states* which are unique convex combinations of the pure states. The set of dynamical properties determines the position of the system in the phase space Ω and conversely the dynamical properties of the system can be directly determined from the coordinates of the point in phase space. Thus, a *one-to-one correspondence* exists between systems and their dynamical properties on the one hand, and the mathematical representation in terms of points in phase space on the other. Furthermore, with observation of properties being unproblematic, the state corresponds uniquely to the outcomes of the (ideal) measurements that can be performed on the system. The specific property assignment rule for dynamical properties that captures the above is the following.

A physical quantity \mathfrak{A} is represented by a function $A : \Omega \rightarrow \mathbb{R}$ such that $A(x)$ is the value A possesses when the state is x . To the property that the value of A lies in the real-valued interval Δ there is associated a Borel-measurable subset

$$\Omega_{A \in \Delta} = A^{-1}\{\Delta\} = \{x \in \Omega | A(x) \in \Delta\}, \quad (1)$$

of states in Ω for which the proposition that the system has this property is true. Thus dynamical properties are associated with *subsets* of the space of

⁶ This presentation of the structure of properties in classical physics was inspired by Isham (1995).

states Ω , and we have the one-to-one correspondence mentioned above between properties and points in the state space now as follows: $A(x) \in \Delta \Leftrightarrow x \in \Omega_{A \in \Delta}$. Furthermore, the logical structure of the propositions about the dynamical properties of the system is identified with the *Boolean σ -algebra* \mathcal{B} of subsets of the space of states Ω . This encodes the normal logical way (i.e., Boolean logic) of dealing with propositions about properties⁷.

In order to address holism we need to be able to speak about properties of composite systems in terms of properties of the subsystems. The first I will call *global* properties, the second *local* properties⁸. It is a crucial and almost defining feature of the state space of classical physics that the local dynamical properties *suffice* for inferring *all* global dynamical properties. This is formalised as follows. Consider the simplest case of a composite system with two subsystems (labeled 1 and 2). Let the tuple $\langle \Omega_{12}, \mathcal{B}_{12} \rangle$ characterize the state space of the composite system and the Boolean σ -algebra of subsets of that state space. The latter is isomorphic to the logic of propositions about the global properties. This tuple is determined by the subsystems in the following way. Given the tuples $\langle \Omega_1, \mathcal{B}_1 \rangle$ and $\langle \Omega_2, \mathcal{B}_2 \rangle$ that characterize the subsystem state spaces and property structures, Ω_{12} is the Cartesian product space of Ω_1 and Ω_2 , i.e.,

$$\Omega_{12} = \Omega_1 \times \Omega_2, \tag{2}$$

and furthermore,

$$\mathcal{B}_{12} = \mathcal{A}(\mathcal{B}_1, \mathcal{B}_2), \tag{3}$$

where $\mathcal{A}(\mathcal{B}_1, \mathcal{B}_2)$ is the smallest σ -algebra generated by σ -algebras that contain Cartesian products as elements. This algebra is defined by the following three properties (Halmos, 1988): (i) if $\mathcal{A}_1 \in \mathcal{B}_1$, $\mathcal{A}_2 \in \mathcal{B}_2$ then $\mathcal{A}_1 \times \mathcal{A}_2 \in \mathcal{A}(\mathcal{B}_1, \mathcal{B}_2)$, (ii) it is closed under countable conjunction, disjunction and taking differences, (iii) it is the smallest one generated in this way. The σ -algebra \mathcal{B}_{12} thus contains by definition all sets that can be written as a countable conjunction of Cartesian product sets such as $\Lambda_1 \times \Lambda_2 \subset \Omega_{12}$ (with $\Lambda_1 \subset \Omega_1$, $\Lambda_2 \subset \Omega_2$), also called *rectangles*.

⁷ The relation of *conjunction* of propositions corresponds to the set-theoretical *intersection* (of subsets of the state space), that of *entailment* between propositions to the set-theoretical *inclusion*, that of *negation* of a proposition to the set-theoretical *complement* and finally that of *disjunction* of propositions corresponds to the set-theoretical *union*. In classical physics the (countable) logic of propositions about properties is thus isomorphic to a Boolean σ -algebra of subsets of the state space.

⁸ Note that *local* has here nothing to do with the issue of locality or spatial separation. It is taken to be opposed to global, i.e., restricted to a subsystem.

The above means that the Boolean σ -algebra of the properties of the composite system is in fact the product algebra of the subsystem algebras. Thus propositions about global properties (e.g., global quantity B having a certain value) can be written as disjunctions of propositions which are conjunctions of propositions about local properties alone (e.g., subsystem quantities A_1 and A_2 having certain values). In other words, the truth value of all propositions about B can be determined from the truth value of disjunctions of properties about A_1 and A_2 . The first and the latter thus have the same extension.

On the phase space Ω_{12} all this gives rise to the following structure. To the property that the value of B of a composite system lies in Δ there is associated a Borel-measurable subset of Ω_{12} , for which the proposition that the system has this property is true:

$$\{(x_1, x_2) \in \Omega_{12} \mid B(x_1, x_2) \in \Delta\} \in \mathcal{B}_{12}, \quad (4)$$

where (x_1, x_2) are the pure states (i.e., points) in the phase space of the composite system and x_1 and x_2 are the subsystem states that each lie in the state space Ω_1 or Ω_2 of the respective subsystem. The important thing to note is that this subset lies in the product algebra \mathcal{B}_{12} and therefore is determined by the subsystem algebras \mathcal{B}_1 and \mathcal{B}_2 via Eq. (3).

From the above we conclude, and so is concluded in the supervenience approaches mentioned in the Introduction, although on other non-formal grounds, that classical physics is not holistic. For the global properties supervene on the local ones because the Boolean algebra structure of the global properties is determined by the Boolean algebra structures of the local ones. Thus all quantities pertaining to the global properties defined on the composite phase space such as $B(x_1, x_2)$ are supervening quantities.

For concreteness consider two examples of such supervening quantities $B(x_1, x_2)$ of a composite system. The first is $q = \|\vec{q}_1 - \vec{q}_2\|$ which gives us the global property of a system that specifies the distance between two subsystems. The second is $\vec{F} = -\vec{\nabla}V(\|\vec{q}_1 - \vec{q}_2\|)$ which gives us the property of a system that says how strong the force is between its subsystems arising from the potential V . This could for example be the potential $\frac{m_1 m_2 G}{\|\vec{q}_1 - \vec{q}_2\|}$ for the Newtonian gravity force. Although both examples are highly non-local and could involve action at a distance, no holism is involved since the global properties supervene on the local ones. As Teller (1986, p.76) puts it: ‘Neither action at a distance nor distant spatial separation threaten to enter the picture to spoil the idea of the world working as a giant mechanism, understandable in terms of the individual parts.’

Some words about the issue of whether spatial relations are to be considered holistic, are in order here. Although the spatial relation of relative distance

of the whole indicates the way in which the parts are related with respect to position, whereby it is not the case that each of the parts has a position independent of the other one, it is here nevertheless not regarded a holistic property since it is supervening on spatial position. We have seen that the distance q between two systems is treated supervenient on the systems having positions \vec{q}_1 and \vec{q}_2 in the sense expressed by Eq. (4). However, the argumentation given here requires an *absolutist* account of space so that position can be regarded as an intrinsic property of a system. But one can deny this and adopt a *relational* account of space and then spatial relations become monadic and positions become derivative, which has the consequence that one has to incorporate spatial relations in the supervenience basis⁹.

On an absolutist account of space the spatial relation of relative distance between the parts of a whole is shown to be supervenient upon local properties, and it is thus not to be included in the supervenience basis¹⁰. A relationist account, however, does include the spatial relations in the supervenience basis. The reason is that on this account they are to be regarded as *intrinsically* relational, and therefore non-supervening on the subsystem properties. Cleland (1984) and French (1989) for example argue spatial relations to be non-supervening relations. Furthermore, some hold that all other intrinsic relations can be regarded to be supervenient upon these. The intuition is that wholes seem to be built out of their parts if arranged in the right spatial relations, and these spatial relations are taken to be in some sense *monadic* and therefore not holistic¹¹.

Thus we see that issues depend on what view one has about the nature of space (or space-time). Here I will not argue for any position, but merely note that if one takes an absolutist stance towards space so that bodies are considered to have a particular position, then spatial relations can be considered to be supervening on the positions of the relata in the manner indicated by the decomposition of Eq. (4). This discussion about whether spatial relations are to be regarded as properties that should be included in the supervenience basis clearly indicates that the supervenience criterion must be relativized to the supervenience basis. As we will see later on this is analogous to the fact that

⁹ A more subtle example than the relative distance between two points would be the question whether or not the relative angle between two directions at different points in space is a supervening property, i.e., whether or not the relative angle is to be considered holistic or not. This depends on whether or not one can consider local orientations as properties that are to be included in the supervenience basis.

¹⁰ Teller (1987) for example takes spatial relations to be supervening on intrinsic physical properties since for him the latter include spatiotemporal properties.

¹¹ Healey (1991, p.409) phrases this as follows: ‘Spatial relations are of special significance because they seem to yield the only clear example of qualitative, intrinsic relations required in the supervenience basis in addition to the qualitative intrinsic properties of the relata. Other intrinsic relations supervene on spatial relations.’

the epistemological criterion proposed here work must be relativized to the resource basis.

As a final note in this section, I mention that because of the one-to-one correspondence in classical physics between physical quantities on the one hand and states on the state space on the other hand, and because composite states are uniquely determined by subsystem states (as can be seen from Eq.(2)), it suffices to consider the state space of a system to answer the question whether or not some theory is holistic. The supervenience basis is thus determined by the state space (supplemented with the fixed properties). However, this is a special case and it contrasts with the quantum mechanical case (as will be shown in the next subsection). The supervenience approach should take this into account. Nevertheless, the supervenience approach mostly limits itself to the quantum mechanical state space in determining whether or not quantum mechanics is holistic. The epistemological approach to be developed here uses also other relevant features of the formalism, such as property determination, and focuses therefore primarily on the structure of the assigned properties and not on that of the state space. This will be discussed in the following sections.

2.2 *Quantum physics in the supervenience approach*

In this section I will first treat some general aspects of the quantum mechanical formalism before discussing how the supervenience approach deals with this theory.

In quantum mechanics, just as in classical physics, systems are assigned two kinds of properties. On the one hand, the fixed properties that we find in classical physics supplemented with some new ones such as intrinsic spin. On the other hand, dynamical properties such as components of spin (Healey, 1991). These dynamical properties are, again just as in classical physics, determined in a certain way by values observables have when the system is in a particular state. However, the state space and observables are represented quite differently from what we have already seen in classical physics. In general, a quantum state *does not* correspond uniquely to the outcomes of the measurements that can be performed on the system. Instead, the system is assigned a specific Hilbert space \mathcal{H} as its state space and the physical state of the system is represented by a state vector $|\psi\rangle$ in the pure case and a density operator ρ in the mixed case. Any physical quantity \mathfrak{A} is represented by an observable or self-adjoint operator \hat{A} . Furthermore, the spectrum of \hat{A} is the set of possible values the quantity \mathfrak{A} can have upon measurement.

The pure state $|\psi\rangle$ can be considered to assign a probability distribution $p_i = |\langle\psi|i\rangle|^2$ to an orthonormal set of states $\{|i\rangle\}$. In the case where one of

the states is the vector $|\psi\rangle$, it is completely concentrated onto this vector. The state $|\psi\rangle$ can thus be regarded as the analogon of a δ -distribution on the classical phase space Ω , as used in statistical physics. However the radical difference is that the pure quantum states do not (in general) form an orthonormal set. This implies that the pure state $|\psi\rangle$ will also assign a positive probability to a different state $|\phi\rangle$ if they are non-orthogonal and thus have overlap. This is contrary to the classical case, where the pure state $\delta(q - q_0, p - p_0)$ concentrated on $(p_0, q_0) \in \Omega$ will always give rise to a probability distribution that assigns probability zero to every other pure state, since pure states on Ω cannot have overlap. Furthermore, the probability that the value of an observable \hat{B} lies in the real interval X when the system is in the quantum state ρ is $Tr(\rho P_{\hat{B}, X})$ where $P_{\hat{B}, X}$ is the projector associated to the pair (\hat{B}, X) by the spectral theorem for self-adjoint operators. This probability is in general not concentrated in $\{0, 1\}$ even when ρ is a pure state. Only in the special case that the state is an eigenstate of the observable \hat{B} is it concentrated in $\{0, 1\}$, and the system is assigned the corresponding eigenvalue with certainty. From this we see that there is no one-to-one correspondence between values an observable can obtain and states of the quantum system.

Because of this failure of a one-to-one correspondence there are *interpretations* of quantum mechanics that postulate *different* connections between the state of the system and the dynamical properties it possesses. Whereas in classical physics this was taken to be unproblematic and natural, in quantum mechanics it turns out to be problematic and non-trivial. But a connection must be given in order to ask about any holism, since we have to be able to speak about possessed properties and thus an interpretation that gives us a property assignment rule is necessary. Here I will consider the well-known *orthodox interpretation* of quantum mechanics that uses the so called *eigenstate-eigenvalue link* for this connection: a physical system has the property that quantity \mathfrak{A} has a particular value if and only if its state is an eigenstate of the operator \hat{A} corresponding to \mathfrak{A} . This value is the eigenvalue associated with the particular eigenvector. Furthermore, in the orthodox interpretation measurements are taken to be ideal *von Neumann measurements*, whereby upon measurement the system is projected into an eigenstate of the observable being measured and the value found is the eigenvalue corresponding to that particular eigenstate. The probability for this eigenvalue to occur is given by the well-known Born rule $\langle i | \rho | i \rangle$, with $|i\rangle$ the eigenstate that is projected upon and ρ the state of the system before measurement. Systems thus have properties *only* if they are in an eigenstate of the corresponding observables, i.e., the system either already is or must first be projected into such an eigenstate by the process of measurement. We thus see that the *epistemological* scheme of how we gain knowledge of properties, i.e., the measurement process described above, serves also as an *ontological* one defining what properties of a system can be regarded to exist at a given time at all.

Let me now go back to the supervenience approach to holism and ask what it says about quantum mechanics in the orthodox interpretation stated above. According to all proponents of this approach mentioned in the Introduction quantum mechanics is holistic. The reason for this is supposed to be the feature of *entanglement*, a feature absent in classical physics. In order to discuss the argument used, let me first present some aspects of entanglement. Entanglement is a property of composite quantum systems whereby the state of the system cannot be derived from any combination of the subsystem states. It is due to the tensor product structure of a composite Hilbert space and the linear superposition principle of quantum mechanics. In the simplest case of two subsystems, the precise definition is that the composite state ρ cannot be written as a convex sum of products of single particle states, i.e., $\rho \neq \sum_i p_i \rho_i^1 \otimes \rho_i^2$, with $p_i \in [0, 1]$ and $\sum_i p_i = 1$. In the pure case, an entangled state is one that cannot be written as a product of single particle states. Examples include the so-called *Bell states* $|\psi^-\rangle$ and $|\phi^-\rangle$ of a spin-1/2 particle. These states can be written as

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle_z - |10\rangle_z), \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle_z - |11\rangle_z), \quad (5)$$

with $|0\rangle_z$ and $|1\rangle_z$ eigenstates of the spin operator $\hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z$, i.e., the spin up and down state in the z -direction respectively. These Bell states are eigenstates for total spin of the composite system given by the observable $\hat{S}^2 = (\hat{S}_1 + \hat{S}_2)^2$ with eigenvalue 0 and $2\hbar^2$ respectively.

According to the orthodox interpretation, if the composite system is in one of the states of Eq. (5), the system possesses one of two global properties for total spin which are completely different, namely eigenvalue 0 and eigenvalue $2\hbar^2$. The question now is whether or not this spin property is holistic, i.e., does it or does it not supervene on subsystem properties? According to the supervenience approach it does not and the argument goes as follows. Since the individual subsystems have the same reduced state, namely the completely mixed state $\frac{1}{2}\mathbb{1}$, and because these are not eigenstates of any spin observable, no spin property at all can be assigned to them. So there is a difference in global properties to which no difference in the local properties of the subsystems corresponds. Therefore there is no supervenience and we have an instantiation of holism¹². It is the feature of entanglement in this example that is held responsible for holism. Maudlin (1998) even defines holism in quantum mechanics in terms of entanglement and Esfeld (2001, p.205) puts it as follows: ‘The entanglement of two or more states is the basis for the discussion on holism in quantum

¹²This is the exact argument Maudlin (1998) uses. Healey (1991) and Esfeld (2001) also use an entangled spin example whereas Teller (1986, 1989), French (1989) and Howard (1989) use different entangled states or some consequence of entanglement such as violation of the bipartite Bell inequalities.

physics.’ Also French (1989, p.11), although using a different approach to supervenience (see footnote 4), shares this view: ‘Since the state function [...] is not a product of the separate state functions of the particles, one cannot [...] ascribe to each particle an individual state function. It is *this*, of course, which reveals the peculiar non-classical holism of quantum mechanics.’

I would now like to make an observation of a crucial aspect of the reasoning the supervenience approach uses to conclude that quantum mechanics endorses holism. In the above and also in other cases the issue is treated via the concept of entanglement of quantum states. This, however, is a notion primarily tied to the structure of the state space of quantum mechanics, i.e., the Hilbert space, and not to the structure of the properties assigned in the interpretation in question. There is no one-to-one correspondence between states and assigned dynamical properties, contrary to what we have already seen in the classical case. Thus questions in terms of states, such as ‘is the state entangled?’ and in terms of properties such as ‘is there non-supervenience?’ are different *in principle*. And although there is some connection via the property assignment rule using the eigenvalue-eigenstate link, I claim them to be relevantly different. Holism is a thesis about the structure of properties assigned to a whole and to its parts, not a thesis about the state space of a theory. The supervenience approach should carefully ensure that it takes this into account. However, the epistemological approach of the next section naturally takes this into account since it focuses directly on property determination. It probes the structure of the assigned properties and not just that of the state space.

The reason that in the supervenience approach one immediately and solely looks at the structure of the state space is because in its supervenience basis only the properties the subsystems have in and out of themselves at the time in question are regarded. This means that using the eigenstate-eigenvalue rule for the dynamical properties one focuses on properties the system has in so far as the state of the system implies them. Only eigenstates give rise to properties, other states do not. A different approach, still in the orthodox interpretation, would be to focus on properties the system can possess according to the possible property determinations quantum mechanics allows for. It is the structure of the properties that can be possibly assigned at all, which is then at the heart of our investigations. In this view one could say that the physical state of a system is regarded more generally, as also Howard (1989) does, as a set of *dispositions* for the system to manifest certain properties under certain (measurement) circumstances, whereby the eigenstates are a special case assigning properties with certainty. This view is the one underlying the epistemological approach which will be proposed and worked out next.

3 An epistemological criterion for holism in physical theories

Before presenting the new criterion for holism I would like to motivate it by going back to the spin-1/2 example of the last section. Let us consider the example, which according to the supervenience approach gives an instantiation of holism, from a different point of view. Instead of solely considering state descriptions, let us look at what physical processes can actually be performed according to the theory in question in order to gain knowledge of the system. I call this an *epistemological stance*. I will show next that it then *is* possible to determine, using only non-holistic means (to be specified later on) whether or not one is dealing with the Bell state $|\psi^-\rangle$ or $|\phi^-\rangle$ of Eq. (5). How? First measure on each subsystem the spin in the z -direction. Next, compare these results using classical communication. If the results have the same parity, the composite system was in the state $|\phi^-\rangle$ with global spin property $2\hbar^2$. And if the results do not have the same parity, the system was in the state $|\psi^-\rangle$ with global spin property 0.

Thus using local measurements and classical communication the different global properties can be inferred after all. There is *no* indication of holism in this approach, which is different from what the supervenience approach told us in the previous section. Although it remains true that the mixed reduced states of the individual subsystems do not determine the composite state and neither a local observable (of which there is no eigenstate), enough information can be nevertheless gathered by local operations and classical communication to infer the global property. We see that from an epistemological point of view we should not get stuck on the fact that the subsystems themselves have no spin property because they are not in an eigenstate of a spin observable. We can assign them a state, and thus can perform measurements and assign them some local properties, which in this case do determine the global property in question.

From this example we see that this approach to holism does not merely look at the state space of a theory, but focuses on the structure of properties assigned to a whole and to its parts, as argued before that it should do. Then how do we spot *candidates* for holism in this approach? Two elements are crucial. Firstly, the theory must contain global properties that cannot be inferred from the local properties assigned to the subsystems, while, secondly, we must take into account *non-holistic constraints* on the determination of these properties. These constraints are that we only use the resource basis available to local agents (who each have access to one of the subsystems). The guiding intuition is that using this resource basis will provide us with only non-holistic features of the whole. From this we finally get the following criterion for holism in a physical theory:

A physical theory is holistic if and only if it is impossible in principle, for a set of local agents each having access to a single subsystem only, to infer the global properties of a system as assigned in the theory (which can be inferred by global measurements), by using the resource basis available to the agents.

Crucial is the specification of the resource basis. The idea is that these are all non-holistic resources for property determination available to an agent. However, just as in the case of the specification of the supervenience basis, this basis probably cannot be uniquely specified, i.e., the exact content of the basis is open to debate. Here I propose that these resources include at least all *local operations and classical communication* (abbreviated as LOCC)¹³. The motivation for this is the intuition that local operations, i.e., anything we do on the separate subsystems, and classically communicating whatever we find out about it, will only provide us with non-holistic properties of a composite system. However it could be possible to include other, although more debatable, non-holistic resources. A good example of such a debatable resource we have already seen: Namely, whether or not an agent can consider the position of a subsystem as a property of the subsystem, so that he can calculate relative distances when he knows the fixed positions of other subsystems. Another example is provided by the discussion of footnote 9 which suggests the question whether or not an agent can use a shared Cartesian reference frame, or a channel that transmits objects with well-defined orientations, as a resource for determining the relative angle between directions at different points in space.

I believe that the determination of these and other spatial relations should be nevertheless included in the resource basis, for I take these relations to be (spatially) nonlocal, yet not holistic. Furthermore because we are dealing with epistemology in specifying the resource basis, I do not think that including them necessarily implies ontological commitment as to which view one must endorse about space or space-time. Therefore, when discussing different physical theories in the next section, I will use as the content of the resource basis, firstly, the determination of spatial relations, and secondly LOCC (local operations and classical communication). The latter can usually be unproblematically formalized within physical theories and do not depend on for example the view one has about spacetime. I thus propose to study the physical realizability of measuring or determining global properties while taking as a constraint that one uses LOCC supplemented with the determination of spatial relations.

Let me mention some aspects of this proposed approach before it is applied

¹³Note again that *local* has here nothing to do with the issue of locality or spatial separation, but that it is taken to be opposed to global, i.e., restricted to a subsystem.

in the next section. *Firstly*, it tries to formalize the question of holism in the context of what modern physical theories are, taking them to be (i) schemes to find out and predict what the results are of certain interventions, which can be possibly used for determination of assigned properties, and (ii), although not relevant here, possibly describing physical reality. Theories are no longer taken to necessarily present us with an ontological picture of the world specified by the properties of all things possessed at a given time.

Secondly, the approach treats the concept of *property* physically and not ontologically (or metaphysically). I mean by this that the concept is treated analogous to the way Einstein treated space and time (as that what is given by measuring rods and revolutions of clocks), namely as that which can be attributed to a system when measuring it, or as that which determines the outcomes of interventions.

Thirdly, by including classical communication, this approach considers the possibility of determining some intrinsic relations among the parts such as the parity of a pair of bits, as was seen in the previous spin-1/2 example. The parts are considered as parts, i.e., as constituting a whole with other parts and therefore being related to each other. But the idea is that they are nevertheless considered non-holistically by using only the resource basis each agent has for determining properties and relations of the parts.

Fourthly, as mentioned before, the epistemological criterion for holism is relativized to the resource basis. Note that this is analogous to the supervenience criterion which is relativized to the supervenience basis. I believe this relativizing to be unavoidable and even desirable because it, reflects the ambiguity and debatable aspect inherent in any discussion about holism. Yet, in this way it is incorporated in a fair and clear way.

Lastly, note that the epistemological criterion is logically independent of the supervenience criterion. Thus whether or not a theory is holistic in the supervenience approach is independent of whether or not it is holistic in the newly proposed epistemological approach. This is the case because not all intrinsic properties and relations in the supervenience basis are necessarily accessible using the resource basis, and conversely, some that are accessible using the resource basis may not be included in the supervenience one¹⁴.

¹⁴Of the latter case an example was given using the spin 1/2 example, since the property specifying whether the singlet state or the triplet state obtain is not supervening, but can be inferred using only LOCC. Of the first case an example will be given in the next section.

4 Holism in classical physics and quantum mechanics; revisited.

In this section I will apply the epistemological criterion for holism to different physical theories, where I use as the content of the resource basis the determination of spatial relations supplemented with LOCC.

4.1 *Classical Physics and Bohmian Mechanics*

In section 2.1 classical physics on a phase space was deemed non-holistic in the supervenience approach because global properties in this theory were argued to be supervening on subsystem properties. Using the epistemological criterion we again find that classical physics is deemed non-holistic¹⁵. The reason is that because of the one-to-one relationship between properties and the state space and the fact that a Cartesian product is used for combining subsystem state spaces, and because measurement in classical physics is unproblematic as a property determining process, the resource basis allows for determination of all subsystem properties. We thus are able to infer the Boolean σ -algebra of the properties of the subsystems. Finally, given this the global properties can be inferred from the local ones (see section 2.1), because the Boolean algebra structure of the global properties is determined by the Boolean algebra structures of the local ones, as was given in Eq. (3). Hence no epistemological holism can be found.

Another interesting theory that also uses a state space with a Cartesian product to combine state spaces of subsystems is *Bohmian mechanics* (see e.g. Dürr, Goldstein, & Zanghì (1996)). It is not a phase space but a configuration space. This theory has an ontology of particles with well defined positions on trajectories¹⁶. Here I discuss the interpretation where this theory is supplemented with a property assignment rule just as in classical physics (i.e., all functions on the state space correspond to possible properties that can all be measured). Indeed, pure physical states of a system are given by single points (\vec{q}) of the position variables \vec{q} that together make up a configuration space. There is a one-to-one relationship between the set of properties a system has

¹⁵ Note that in both cases only systems with finite many subsystems are considered.

¹⁶ *Bohmian mechanics*, which has as ontologically existing only particles with well defined positions on trajectories, should be distinguished (although this is perhaps not common practice) from the so-called the *de Broglie-Bohm theory* where besides particles also the wave function has ontological existence as a guiding field. This contrasts with Bohmian mechanics since in this theory the wave function has only nomological existence. Whether or not de Broglie-Bohm theory is holistic because of the different role assigned to the wave function needs careful examination, which will here not be executed.

and the state on the configuration space it is in, as was shown in section 2.1. The dynamics is given by the possibly non-local quantum potential $U_{QM}(\vec{q})$ determined by the quantum mechanical state $|\psi\rangle$, supplemented with the ordinary classical potential $V(\vec{q})$, such that the force on a particle is given by: $\vec{F} \equiv \frac{d\vec{p}}{dt} = -\vec{\nabla}[V(\vec{q}) + U_{QM}(\vec{q})]$. This theory can be considered to be a real mechanics, i.e., a Hamilton-Jacobi theory, although with a specific extra interaction term. This is the quantum potential in which the wave function appears that has only nomological existence. (Although a Hamilton-Jacobi theory, it is not classical mechanics: the latter is a second order theory, whereas Bohmian mechanics is of first order, i.e., velocity is not independent of position).

In section 2.1 all theories on a state space with a Cartesian product to combine subsystem state spaces and using a property assignment rule just as in classical physics were deemed non-holistic by the supervenience approach and therefore we can conclude that Bohmian mechanics is non-holistic in this approach. Perhaps not surprising, but the epistemological approach also deems this theory non-holistic. The reason why is the same as why classical physics as formulated on a phase space was argued above to be not holistic in this approach.

Because Bohmian mechanics and quantum mechanics in the orthodox interpretation have the same empirical content, one might think that because the first is not holistic, neither is the latter. However, this is not the case, as will be shown next. This illustrates the fact that an interpretation of a theory, in so far as a property assignment rule is to be given, is *crucial* for the question of holism. A formalism on its own is not enough.

4.2 Quantum Operations and Holism

In this section I will show that quantum mechanics in the orthodox interpretation is holistic using the epistemological criterion, without using the feature of entanglement. In order to do this we need to specify what the resource basis looks like in this theory. Thus we need to formalize what a local operation is and what is meant by classical communication in the context of quantum mechanics. For the argument it is not necessary to deal with the determination of spatial relations and these will thus not be considered.

Let us first look at a general quantum process \mathcal{S} that takes a state ρ of a system on a certain Hilbert space \mathcal{H}_1 to a different state σ on a possibly different Hilbert space \mathcal{H}_2 , i.e.,

$$\rho \rightarrow \sigma = \mathcal{S}(\rho), \quad \rho \in \mathcal{H}_1, \quad \mathcal{S}(\rho) \in \mathcal{H}_2, \quad (6)$$

where $\mathcal{S} : \mathcal{H}_1 \rightarrow \mathcal{H}_2$ is a *completely positive trace-nonincreasing map*. This is an operator \mathcal{S} , positive and trace non-increasing, acting linearly on Hermitian matrices such that $\mathcal{S} \otimes \mathbb{1}$ takes states to states. These maps are also called *quantum operations*¹⁷. Any quantum process, such as for example unitary evolution or measurement, can be represented by such a quantum operation.

We are now in the position to specify the class of LOCC operations. It is the class of local operations plus two-way classical communication. It consists of compositions of elementary operations of the following two forms

$$\mathcal{S}^A \otimes \mathbb{1}, \quad \mathbb{1} \otimes \mathcal{S}^B, \quad (7)$$

with \mathcal{S}^A and \mathcal{S}^B arbitrary local quantum operations. The class contains the identity and is closed under composition and taking tensor products. As an example consider the case where A performs a measurement and communicates her result α to B , after which B performs his measurement:

$$\mathcal{S}^{AB}(\rho) = (\mathbb{1} \otimes \mathcal{S}_\alpha^B) \circ (\mathcal{S}^A \otimes \mathbb{1})(\rho). \quad (8)$$

We see that B can *condition* his measurement on the outcome that A obtained. This example can be extended to many such rounds in which A and B each perform certain local operations on their part of the system and condition their choices on what is communicated to them.

Suppose now that we have a physical quantity \mathfrak{R} of a bi-partite system with a corresponding operator \hat{R} that has a set of nine eigenstates, $|\psi_1\rangle$ to $|\psi_9\rangle$, with eigenvalues 1 to 9. The property assignment we consider is the following: if the system is in an eigenstate $|\psi_i\rangle$ then it has the property that quantity \mathfrak{R} has the fixed value i (this is the eigenstate-eigenvalue link). Suppose \hat{R} works on $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ (each three dimensions) and has the following complete orthonormal set of *non-entangled* eigenstates:

$$\begin{aligned} |\psi_1\rangle &= |1\rangle \otimes |1\rangle, \\ |\psi_{2,3}\rangle &= |0\rangle \otimes |0 \pm 1\rangle, \\ |\psi_{4,5}\rangle &= |2\rangle \otimes |1 \pm 2\rangle, \\ |\psi_{6,7}\rangle &= |1 \pm 2\rangle \otimes |0\rangle, \\ |\psi_{8,9}\rangle &= |0 \pm 1\rangle \otimes |2\rangle, \end{aligned} \quad (9)$$

with $|0 + 1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, etc.

¹⁷ See Nielsen & Chuang (2000) for an introduction to the general formalism of quantum operations.

We want to infer whether the composite system has the property that the value of the observable \mathfrak{R} is one of the numbers 1 to 9, using only LOCC operations performed by two observers A and B , who each have one of the individual subsystems. Because the eigenstate-eigenvalue link is the property assignment rule used here, this amounts to determining which eigenstate A and B have or project on during the LOCC measurement. If A and B project on eigenstate $|\psi_i\rangle$ then a quantum operation $\mathcal{S}_i : \rho \rightarrow \frac{S_i(\rho)}{\text{Tr}[S_i(\rho)]}$ is associated to the measurement outcome i , with projection operators $S_i = |i\rangle_A |i\rangle_B \langle\psi_i|$. This is nothing but the well-known projection due to measurement (with additional renormalisation), but here written in the language of local quantum operations¹⁸. The state $|i\rangle_A$ denotes the classical record of the outcome of the measurement that A writes down, and similarly for $|i\rangle_B$. These records can be considered to be *local* properties of the subsystems A and B .

It follows from the theory of quantum operations (Nielsen & Chuang, 2000) that implementing the quantum operation $\mathcal{S}(\rho) = \sum_i \mathcal{S}_i \rho \mathcal{S}_i^\dagger$ amounts to determining the *global* property assignment given by \hat{R} . Surprisingly, this cannot be done using LOCC, a result obtained by Bennett et al. (1999). For the complete proof see the original article by Bennett et al. (1999) or Walgate & Hardy (2002)¹⁹, but a sketch of it goes as follows. If A or B perform von Neumann measurements in any of their operation and communication rounds then the distinguishability of the states is spoiled. Spoiling occurs in *any* local basis. The ensemble of states as seen by A or by B alone is therefore non-orthogonal, although the composite states are in fact orthogonal.

From this we see that a physical quantity, whose corresponding operator has only product eigenstates, gives a property assignment using the eigenvalue-eigenstate link that is not measurable using LOCC. Furthermore, we see that the resource basis sketched before does not suffice in determining the global property assignment given by \hat{R} . Thus according to the epistemological criterion of the previous section quantum mechanics is holistic, although no entanglement is involved. Examples of epistemological holism that do involve entanglement can of course be given. For example, distinguishing the four (entangled) Bell states given by $|\psi^+\rangle$, $|\psi^-\rangle$, $|\phi^+\rangle$ and $|\phi^-\rangle$ (see Eq.(5)) cannot be done by LOCC. Thus entanglement is *sufficient* to prove epistemological holism. However, this is hardly surprising. What is surprising is the fact that

¹⁸ Instead of writing the projection operators as $S_i = |\psi_i\rangle \langle\psi_i|$, I write $S_i = |i\rangle_A |i\rangle_B \langle\psi_i|$ to show explicitly that *only* local records are taken. Since the states $|i\rangle$ can be regarded eigenstates of some local observable, we can regard them to determine a local property using the property assignment rule in terms of the eigenvalue-eigenstate link of the orthodox interpretation.

¹⁹ This result is a special case of the fact that some family of separable quantum operations (that all have a complete eigenbasis of separable states) cannot be implemented by LOCC and von Neumann measurements. This is proven by Chen & Li (2003).

it is *not necessary*, i.e., that here a proof of epistemological holism is given not involving entanglement. Furthermore because of the lack of entanglement in this example it would not allow for a proof of holism in the supervenience approach. Of course, it may well be that the resource basis used in this example is too limited, but I do not see other resources that may sensibly be included in this basis so as to render this example epistemologically non-holistic.

5 Conclusion and outlook

I sketched an epistemological criterion for holism that determines, once the resource basis has been specified, whether or not a physical theory with a property assignment rule is holistic. It was argued to be a suitable one for addressing holism in physical theories, because it focuses on property determination as specified by the physical theory in question (possibly equipped with a property assignment rule via an interpretation). I distinguished this criterion from the well-known supervenience criterion for holism and showed them to be logically independent. Furthermore, it was shown that both the epistemological and the supervenience approaches require relativizing the criteria to respectively the resource basis and the supervenience basis. I argued that in general neither of these bases is determined by the state space of a physical theory. In other words, holism is not a thesis about the state space a theory uses, it is about the structure of properties and property assignments to a whole and its parts that a theory or an interpretation allows for. And in investigating what it allows for we need to try to formalize what we intuitively think of as holistic and non-holistic. Here, I hope to have given a satisfactory new epistemological formulation of this, that allows one to go out into the world of physics and apply the new criterion to the theories or interpretations one encounters.

In this paper I have only treated some specific physical theories. It was shown that all theories on a state space using a Cartesian product to combine subsystem state spaces, such as classical physics and Bohmian mechanics, are not holistic in both the supervenience and epistemological approach. The reason for this is that the Boolean algebra structure of the global properties is determined by the Boolean algebra structures of the local ones. The orthodox interpretation of quantum mechanics, however, was found to instantiate holism. This holds in both approaches, although on different grounds. For the supervenience approach it is the feature of entanglement that leads to holism, whereas using only LOCC resources, one can have epistemological holism in absence of any entanglement, i.e., when there is no holism according to the supervenience approach.

There are of course many open problems left. What is it that we can single

out to be the reason of the holism found? The use of a Hilbert space with its feature of superposition? Perhaps, but not the kind of superposition that gives rise to entanglement, for I have argued that it is not entanglement that we should per se consider to be the paradigmatic example of holism. Should we blame the property assignment rule which the orthodox interpretation uses? I shall leave this an open problem.

The entangled Bell states $|\psi^-\rangle$ and $|\phi^-\rangle$ of section 2.2 could, despite their entanglement, be distinguished after all using only LOCC, whereas this was not possible in the set of nine (non-entangled) product states of Eq.(9). These two quantum mechanical examples show us that we can do both more and less than quantum states at first seem to tell us. This is an insight gained from the new field of *quantum information theory*. Its focus on what one *can* or *cannot do* with quantum systems, although often from an engineering point of view, has produced a new and powerful way of dealing with questions in the foundations of quantum mechanics that can lead to fundamental new insights or principles. I hope the new criterion for holism in physical theories suggested in this paper is an inspiring example of this.

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