# Time–Entanglement Between Mind and Matter

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**Abstract**

This contribution explores Wolfgang Pauli's idea that *mind and matter are complementary aspects of the same reality.* We adopt the working hypothesis that there is an undivided timeless primordial reality (the primordial "one world"). Breaking its symmetry, we obtain a contextual description of the holistic reality in terms of two categorically different domains, one tensed and the other tenseless. The tensed domain includes, in addition to tensed time, nonmaterial processes and mental events. The tenseless domain refers to matter and physical energy. This concept implies that mind cannot be reduced to matter, and that matter cannot be reduced to mind.

The non-Boolean logical framework of modern quantum theory is general enough to implement this idea. Time is not taken to be an *a priori* concept, but an archetypal acausal order is assumed which can be represented by a one-parameter group of automorphisms, generating a time operator which parametrizes all processes, whether material or nonmaterial. The time-reversal symmetry is broken in the nonmaterial domain, resulting in a universal direction of time for the material domain as well.

## 1. Concepts of Time

### 1.1 Two Philosophical Conceptions of Time

Discussing questions related to time, we have to distinguish between two distinct conceptions of time:

- *Tensed* concepts are generated by relating events to the present. They include the properties of pastness, nowness, and futurity.
- *Tenseless* concepts are generated by the relations "earlier than", "simultaneous with", and "later than".

The question whether time consists only of relations of simultaneity, earlier and later, or whether it also carries the characteristics of futurity, nowness and pastness is an old controversy in philosophy. This dispute goes back to Heraclitus, who took the past, present and future to be irreducible, and Parmenides, who opposed the view of Heraclitus and insisted hat there is no metaphysical difference between past and future. More recently, the Cambridge philosopher McTaggart (1908) distinguished between two modes of perception in terms of what he called A-series and B-series:

- the A-series relates events in terms of past, present and future,
- the B-series relates events in terms of "earlier than" and "later than".

McTaggart used the strange argument that tenses, though essential to time, are inherently self-contradictory. He concluded that the Heraclitean concept of an A-series involves an unavoidable contradiction and that only a tenseless discourse is free from contradiction. But since a Parmenidean B-theory lacks the tensed features of time, McTaggart maintained that no fully consistent account of time is possible and that our experience of events as taking place in time is unreal. Of course, despite such arguments, many philosophers have remained convinced of the reality of time.

Instead of discussing which of the two viewpoints is the "correct" one, it is more constructive to consider *A-theories* and *B-theories* as different representations of time, and to ask for their respective domain of validity and their interdependence (compare Denbigh 1981, chapter 4, §2). While A-theories refer to the inner experience of time, B-theories refer to changes in the external world. Very much in outline we may characterize these two views as follows:

- An A-theory refers to a domain where the notions "now" and "coming into being" are central. Here, A-series are of primary importance. The consciousness of time as reflecting the "inner flow" of mental events belongs to this domain.
- In a B-theory events are partially[1] ordered in terms of "earlier than" and "later than", but without any reference to past, present or future. A famous example of a B-theory of time is the special theory of relativity.[2]

Even if one considers A-theories as irrelevant for physical sciences, which do not address issues of becoming, they do not loose their legitimacy for a general theory of time. In particular, the B-theoretical structure of physical time is not incompatible with the A-theoretical structure of psychological time.

[1] This restriction is necessary since in relativity theory there are temporally incomparable events. For a lucid discussion, compare Gödel (1949), and in particular the drafts published in Gödel (1995), pp. 202–259.

[2] A convinced advocate of the B-theory of time is Grünbaum, who eliminates all talk of past, present and future, and claims that "becoming" is merely a subjective feature of consciousness. Compare Grünbaum (1973), chapter 10.

It is intriguing how many philosophers debate whether the tenseless theory or the tensed theory of time reflects the world as it really is. This philosophical debate often confuses two valid, but distinct modes of description which are not necessarily contradictory. Time, as it is used in physics omits indexical elements such as the now in the interest of achieving context-independent and time-independent first principles. Yet, notions which have no proper place as physical first principles may be essential for understanding the concept of time. They may even be crucial to understand the mind–matter problem.

## 1.2 Time in Physics

### *1.2.1 Time in Galilei- and Lorentz-Relativistic Physics*

Both Galilei-relativistic classical and quantum physics are based on Newton's idea of an absolute time. In his *Principia*, Newton (1960, Scholium, p. 6.) distinguished carefully between "absolute time" and "common time":

> "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year."

While Newton's concept of an absolute time was not generally accepted, it was vital for the mathematical formulation of physical laws. Later, in mathematical physics, Newton's absolute time turned into a parameter time $t \in \mathbb{R}$, which is a coordinate like those parametrizing three-dimensional space.

In his theory of relativity Einstein (1916, §9) keeps the common time which can be measured by clocks associated with local observers, but he rejects Newton's notion of an absolute time:

> "Every reference-body (co-ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event."

In the special theory of relativity there is no unique simultaneity relation – every inertial system has its own proper time. Moreover, there is no absolute past, present or future, therefore the now becomes relativized with respect to a frame of reference. This lends strong support to a B-theoretical interpretation of time. In 1908 Minkowski realized that in physics spacetime is more fundamental than time or space separately:[3]

[3] Address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne 1908. English translation quoted from Minkowski (1923), p. 75.

> "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

In Minkowski's spacetime world-lines are pictured as existing *a priori.* We are moving along a world-line, encountering our pre-determined future as it becomes present. According to Grünbaum (1973, p. 329 and pp. 318–319), "coming into being is only coming into awareness":

> "Instead of allowing for the transient division of time into the past and future by the shifting Now of experienced time, the theory of relativity conceives of events as simply being and sustaining relations of earlier and later, but not as 'coming *into* being': we conscious organisms then 'come across' them by 'entering' into their absolute future, as it were. And upon experiencing their immediate effects, we regard them as 'taking place' or 'coming into being'."

### *1.2.2 Is Time a Parameter or a Dynamical Variable?*

In the traditional formulation of Galilei-relativistic quantum mechanics time plays essentially the same role as in classical Galilei-relativistic point mechanics. The parameter $t$ in the time-dependent Schrödinger equation refers to an *external* clock, which is not part of the system described by the Hamiltonian in the Schrödinger equation. Conventionally, time is represented by a real number (a "c-number"). Equivalently, one can represent time in traditional quantum mechanics by a sharp value of a classical observable.[4]

Since the advent of relativity theory we know that time is not an absolute notion. This strongly suggests that the time coordinate should be treated on the same footing as other dynamical variables.[5] In such an *extended quantum mechanics* the time operator $T$ still refers to an external clock time. It is *not* the canonical conjugate to the Hamiltonian of the non-extended formulation, so that Pauli's objection against the existence of a time operator is inapplicable.[6] In the special case where $T$ is a classical observable, such an extended formulation is equivalent to the usual formulation with an external time parameter $t$.[7]

From a first-principle viewpoint it is usually assumed that classical observables arise only as emergent quantities in a higher-level description.

[4] An observable is called classical if it commutes with every other observable.

[5] In quantum mechanics this idea goes back to Dirac (1927).

[6] Pauli (1933) (Ziff. 8, p. 140) remarked that the existence of a selfadjoint time operator conflicts with the semibounded character of the spectrum of the Hamiltonian.

[7] Nevertheless, this reformulation is conceptually meaningful, for example for the description of unstable quantum systems in terms of the scattering theory of Lax and Phillips. Compare Flesia and Piron (1984), Horwitz and Piron (1993), Strauss et al. (2000). The basic reference for the classical Lax–Phillips scattering theory is Lax and Phillips (1967).

Therefore the most basic algebra of observables is taken as a *simple* algebra, implying that it has a trivial center. Accordingly, we expect that in a genuinely fundamental theory the time operator is *not* a classical observable.

### *1.2.3 In Fundamental Physical Theories There is No Now*

All really fundamental physical dynamical laws are invariant under time translation and time reversal. Moreover, the concept of the "now" – the brief interval that divides the past from the future – is absent in all fundamental mathematical formulations, both in classical physics and in quantum physics. That is, in a context-independent *ontic* description *there is no physical basis for the the distinction between past and future.*

Carnap (1963, pp. 37–38) reports that this problem of the *now* worried Einstein seriously:

> "He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation. ... Einstein thought ... that there is something essential about the Now which is just outside of the realm of science."

Referring to the static four-dimensional spacetime of relativity, Einstein wrote in a letter of condolence to the sister and the son of his lifelong friend Michele Besso:[8]

> "For us believing physicists, the distinction between past, present, and future is only a stubbornly persistent illusion."

Yet, tensed time is a fundamental dimension of psychological reality. We know immediately and from our own most personal experience that time *flows.* It proceeds persistently from the past through the moment of present into the future. In an operational interpretation of physical theories this poses no profound problems since the *epistemic* physical reality is characterized in terms of events which are localized in space and time *with respect to an observer.* In particular, the present and the past are characterized by procedures associated with experiments performed by experimenters.

Weyl (1922, p. 3, p. 5; 1949, p. 116) tried to reconcile the tenseless world of fundamental physics with our tensed experience:

> "Pure consciousness" is the seat of that which is philosophically a priori. ... Time is the primitive form of the stream of consciousness. It is a fact, however obscure and perplexing to our minds, that

[8] Letter of March 21, 1955. Translated from Einstein and Besso (1979), p. 312.

> the contents of consciousness do not present themselves simply as being (such as conceptions, numbers, etc.), but as *being now* filling the form of the enduring present with a varying content. So that one does not say this *is* but this is *now*, yet now no more. If we project ourselves outside the stream of consciousness and represent its content as an object, it becomes an event happening in time, the separate stages of which stand to one another in the relations of *earlier* and *later*."
>
> "The objective world simply is, it does not *happen*. Only to the *gaze of my consciousness*, crawling upward along the life line of my body, does a certain section of this world come to life as a fleeting image in space which continuously changes in time."

According to Weyl, any explanation of the flow of time must incorporate the concept of a conscious observer. In a similar way, Augustine suggested long ago that time may be a dimension of the soul, not of the outer world:[9]

> "But even now it is manifest and clear that there are neither times future nor times past. Thus it is not properly said that there are three times, past, present, and future. Perhaps it might be said rightly that there are three times: a time present of things past; a time present of things present; and a time present of things future."

Grünbaum (1971, p. 68) concludes that what is necessary to characterize a physical event as belonging to the present is "the occurrence of states of *conceptualized awareness*". Grünbaum does not presuppose that conceptualized awareness necessarily requires a biochemical substratum.

### *1.2.4 The Primacy of Mental Time*

The time which is phenomenally known to us is unidirectional and displays a qualitative difference between "before" and "after". All engineering science is formulated in terms of a unidirectional time. In mechanical and electrical input–output systems, output values do not depend on future input values (principle of "retarded causality"). This one-way direction of time which we experience in our everyday lives, and which we find in phenomenological physical laws, has been called *time's arrow* by Eddington (1928, p. 68). The nature and origin of a temporal asymmetry in the physical world is a perplexing problem: What is the *origin* of the arrow of time? Why do all processes show the *same* arrow of time? Many different variants of the physical origin of the direction of time have been proposed. Of course it is impossible to give a fair review of even the most relevant approaches.[10] We recollect just the most popular ideas:

[9] Quoted from Augustine (1994), book eleven, chapter XX.

[10] Important contributions and references to the original papers can be found in standard references such as Reichenbach (1956), Gold (1967), Grünbaum (1973), Davies (1974), Denbigh (1981), Hollinger and Zenzen (1985), Horwich (1987), Halliwell et al. (1994), Savitt (1995), Price (1996), Zeh (1999), Barbour (1999).

- *The causal arrow of time:*

  In engineering science it is generally assumed that causes always precede their effects. This view has also been adopted by a number of contemporary philosophers (Suppes 1970, Mellor 1998). An antithetical position is the *causal theory of time*. The basic idea is to define temporal order by ranking events from earlier to later. Kant (*Critique of Pure Reason*, B 249) postulated that an event $A$ counts as temporally prior to event $B$ provided that $A$ is causally prior to $B$.[11] Related versions of a causal theory of time were developed by a number of more recent philosophers (Reichenbach 1956, 1958; Grünbaum 1973, chapter 7.) But these approaches can hardly be considered as successful. Moreover, "causation" is generally an ill-defined concept in first-principle physics. For this reason, Russell (1913) proposed that the notion of cause is unnecessary for science, and can therefore be eliminated.

- *The manipulative arrow of time:*

  The idea that the manipulation of a cause will result in the manipulation of an effect is deeply embedded in experimental science. This conviction is the cornerstone of the various philosophical accounts of the *manipulative approach to causation* (Gasking 1955, Price 1992, Menzies and Price 1993). Yet, this approach leads to an unacceptable anthropocentric conception of causation (Hausman 1998).

- *The arrow of increasing disorder:*

  The attempt to explain the arrow of time as the direction in which disorder increases goes back to Boltzmann (1877). Boltzmann's ideas are as controversial today as they were more than hundred years ago, yet they are still defended (Lebowitz 1993a,b). Boltzmann's $H$-theorem is based on the unjustifiable assumption that the motions of particles are uncorrelated before collision. If we assume that the root of temporal asymmetry lies in such initial conditions, then the asymmetry of the boundary conditions remains unexplained.

- *The radiative arrow of time:*

  The status of the retarded nature of electromagnetic radiation was the topic of a famous exchange between Ritz and Einstein (1909). Ritz regarded it as "law-like", as one of the roots of the second law of thermodynamics, while Einstein considered it as "fact-like", following from the principle of probability increase. But this "fact-like" temporal asymmetry is purely a matter of asymmetric boundary conditions, which are not explained.

[11] For a modern attempt to make Kant's version of the causal theory of time coherent compare Carrier (2003).

- *The cosmological arrow of time:*

  A currently much discussed idea is that the time asymmetry of thermodynamics and electrodynamics is a result of the expansion of the universe. It has been claimed "that all the important aspects of time asymmetry encountered in the different major topics of physical science may be traced back to the creation or end of the universe" (Davies 1974, p. 197). Even if all arrows of time could be reduced to the cosmological arrow (what has not been shown), one still has to explain why the universe is expanding. So far, only *ad hoc* arguments have been presented why the initial entropy of the universe should be low.

- *Weak anthropic principle:*

  It has been speculated that in a universe, which expands and then eventually contracts, intelligent life (and therefore the perception of time) can only exist during the expanding phase (Hawking 1988, chapter 9).

None of the physical approaches has answered the riddle of the origin of the asymmetry and the direction of time in a really convincing manner. All ideas presented are at best tentative. A fully satisfactory explanation of time asymmetry would have to explain the coincidences of the various reasons for the direction of time (i.e., the introspective experience of the flow of time, the arrow of becoming, the thermodynamic arrow, the electromagnetic radiation arrow, or the cosmological arrow).

According to Grünbaum, any attempt to determine the "direction" of time on the basis of physics (and not on the basis of common-sense notions) is bound to fail (Grünbaum 1973, chapter 10). Since we have immediate access to tensed temporal relations, our experience of the time direction has to be considered as primitive and non-inferential. That is, the temporal direction of events is the order of events based on the *awareness* of before and after. In the following I accept the thesis by Denbigh (1970, p. 243) that

> "the criterion of *before* and *after* which is offered by consciousness has a primacy over any criterion offered by science."

Note that this view does not imply that the now is private to each individual observer.

## 1.3 The Timeless Viewpoint in Depth Psychology

Some psychologists claimed that the idea of a time flux is bound to the functioning of the *conscious* mind. For example, Freud argued that our abstract idea of time seems to be derived from the operation of the perceptual-conscious system:[12]

[12] Translated from Freud (1940a), pp. 27–28, and from Freud (1940b), p. 80.

> "As a result of certain psychoanalytic discoveries, we are today in a position to embark on a discussion of the Kantian axiom that time and space are 'necessary forms of thought'. We have learnt that unconscious mental processes are in themselves 'timeless'."
>
> "...There is nothing in the id which can be compared to negation, and we are astonished to find in it an exception to the philosophers' assertion that space and time are necessary forms of our mental acts. In the id there is nothing corresponding to the idea of time, no recognition of the passage of time ..."

Also Jung maintained that in the deeper layers of the unconscious there is no time at all:[13]

> "...the unconscious has no time. There is no trouble about time in the unconscious. Part of our psyche is not in time and not in space. They are only an illusion, time and space, and so in a certain part of our psyche time does not exist at all."

Jung called the factors responsible for the organization of unconscious psychic processes *archetypes*. He described space and time as manifestations of archetypal elements of the collective unconscious (Jung 1969, par. 840):

> "[Space and time] are, therefore, essentially psychic in origin, which is probably the reason that impelled Kant to regard them as *a priori* categories. But if space and time are only apparently properties of bodies in motion and are created by the intellectual needs of the observer, then their relativization by psychic conditions is no longer a matter for astonishment but is brought within the bounds of possibility."

Von Franz (1966, p. 222) concluded:

> "All these factors seem to suggest that the time flux, as a subjective psychological experience, is bound to the functioning of our conscious mind but becomes relative (or possibly even nonexistent) in the unconscious."

Meier (1950, 1975, 1988) suggested that the relation between mind and matter should be understood as permanently synchronistic. In his monograph on synchronicity, Jung (1969, par. 938, footnote 70) commented:

> "I must again stress the possibility that the relation between body and soul may yet be understood as a synchronistic one. Should this conjecture ever be proved, my present view that synchronicity is a relatively rare phenomenon would have to be corrected."

[13] Jung (1975), par. 684. Compare also Jung (1958), par. 782, 792, 814.

## 2. A Unitary Ground of Mind and Matter

### 2.1 Mind–Matter Holism

Quantum theory describes the material world in a basically holistic way. Generalizing this result beyond the material world, we may ponder upon a holistic conception concerning mind and matter. Pauli (1994, p. 260) suggested that the mental and the material domain are governed by common ordering principles, and should be understood as "complementary aspects of the same reality":

> "The general problem of the relation between psyche and physis, between the inner and the outer, can, however, hardly be said to have been solved by the concept of 'psychophysical parallelism' which was advanced in the last century. Yet modern science may have brought us closer to a more satisfying conception of this relationship by setting up, within the field of physics, the concept of complementarity. It would be most satisfactory of all if physis and psyche could be seen as complementary aspects of the same reality."

From his numerous psychological studies, Jung conjectured that there was a holistic reality – the *unus mundus* – underlying both mind and matter (Jung 1970, par. 767):

> "Undoubtedly the idea of the unus mundus is founded on the assumption that the multiplicity of the empirical world rests on an underlying unity, and that not two or more fundamentally different worlds exist side by side or are mingled with one another. Rather, everything divided and different belongs to one and the same world, which is not the world of sense but a postulate whose probability is vouched for by the fact that until now no one has been able to discover a world in which the known laws of nature are invalid. That even the psychic world, which is so extraordinarily different from the physical world, does not have its roots outside the one cosmos is evident from the undeniable fact that causal connections exist between the psyche and the body which point to their underlying unitary nature."

But Jung (1975, par. 7) also admitted that any details of such a concept are still lacking:

> "Body and mind are the two aspects of the living being, and that is all we know. Therefore I prefer to say that the two things happen together in a miraculous way, and we had better leave it at that, because we cannot think of them together. For my own use I have coined a term to illustrate this being together; I say there is a peculiar principle of synchronicity active in the world so that things happen together somehow and behave as if they were the same, and yet for us they are not. Perhaps we shall some day discover a new

kind of mathematical method by which we can prove that it must be like that. But for the time being I am absolutely unable to tell you whether it is the body or the mind that prevails, or whether they just coexist."

## 2.2 Pre-Established Harmony

The idea of psycho-physical parallelism goes back to Leibniz. He thought of body and soul as two synchronized clocks:[14]

> "Imagine two clocks or watches in perfect agreement. That can happen in three ways:
>
> (1) The first consists in a mutual influence.
>
> (2) The second is to have a skillful worker continually adjust them and keep them in agreement.
>
> (3) The third is to manufacture these two time-pieces with so much art and accuracy that their agreement is guaranteed thereafter.
>
> Now substitute the *soul* and *body* for these two timepieces; their agreement can be obtained through one of these three ways. The *way of influence* is that of popular philosophy; but as we cannot conceive of material particles which can pass from one of these substances to another, we must abandon this idea. The way of the *continual* assistance of the Creator is that of the system of occasional causes; but I hold that this introduces Deus ex Machini in a natural and ordinary occurrence where, according to reason, it ought not intervene except as it operates in all other natural things. Thus there remains only my hypothesis, that is, the way of *Harmony.* From the beginning God has made each of these two Substances of such a nature that each by following its own laws, given to it with its being, still agrees with the other, just as though there were a mutual influence or as though God always took a hand in it beyond his general supervision of things. There is nothing further I have to prove, unless you wish to ask that I prove God is skillful enough to use this prearranged scheme, examples of which we see even among men. Now assuming that he can, you do see that this way is most admirable and most worthy of God. You suspected that my explanation would be opposed by the very different idea we have of the mind and body; but you see now that nobody has better established their independence. For while people are compelled to explain the communication of mind and body by a sort of miracle, there is cause for many people to fear that the distinction between soul and body might not be as real as they believe, since they have to go so far in order to maintain it. I shall not be vexed if learned persons sound out the thoughts I have just explained to you."

[14] *Second explanation of the system of the communication of substances*, 1715. Quoted from Wiener (1951), pp. 118–119.

According to Leibniz, such a *pre-established harmony* of the soul and the body is the key to understanding the relationship of mind and body. While Leibniz's idea that psychic and physical aspects are perfectly synchronized without any causal interconnections is radically at variance with classical physics, it fits well into the theoretical framework of quantum theory. Leibniz considered neither space nor time as a fundamental feature of reality. Temporal relations are taken to provide a convenient short-hand for keeping track of the relations among the timeless properties of the world. According to Leibniz the mind and the body are of radically distinct nature, without any direct causal effect on each other:[15]

> "The soul follows its own peculiar laws and the body also follows its own laws and they agree in virtue of the *pre-established harmony* between all substances, since they are all representations of one and the same universe."

### 2.3 A Timeless Holistic Reality as Point of Departure

Although the descriptions of matter and the descriptions of mind use some concept of time each, *time is neither purely physical nor purely mental.* While the experience of time is closely linked to the concepts "now" and "coming into being", the omnipresent features of "nowness" and a "flow of time" have no status whatsoever in physics.[16] This situation suggests to start from a *timeless* description of the unus mundus. Breaking the primordial symmetry of the holistic reality, we obtain contextual descriptions in terms of two disjoint domains, one tensed and the other tenseless.

The *nonmaterial, tensed domain* includes, in addition to tensed time, entities such as the experiential world of perceptions, all kinds of subjective conscious, subconscious or unconscious experience, explicit and tacit knowledge, mental processes and personal memory. However, we do not restrict the tensed domain to the inner world of private thoughts and experiences. *We relate the tensed domain to a mental world which we consider as fundamental to the nature of existence and being.* According to this view, "mind" operates as a principle beyond individual consciousness and is not restricted to the "human mind". We do not consider the differentiation of the mental domain into individual mental egos. Nevertheless, we expect that in the proposed approach subjectively different "nows" are synchronized so that we do not encounter the problem that "unsynchronized 'nows' unequivocally belong to different worlds" (Franck 2000).

[15] *Monadologie*, thesis 78. Quoted from Wiener (1951), p. 549.

[16] Compare Grünbaum (1967); Denbigh (1970); Grünbaum (1973), chapter 10; Denbigh (1978); Denbigh (1981).

The *tenseless domain* refers to physical objects. Since "mass and energy are both but different manifestations of the same thing",[17] all concepts related to energy are taken to refer to the tenseless domain. Any higher-level or emergent features in the tensed or in the tenseless domain will not be discussed in this essay, so that particular relations between human consciousness and human brain are not addressed.

In the following we explore, on the basis of the most fundamental first principles of quantum theory, Leibniz's idea that mind and matter are categorically distinct, and have no direct causal effect on each other, although they are perfectly correlated.[18] Our point of departure is the hypothesis that there is a *timeless* holistic reality which can be described in terms of the non-Boolean logical structure of modern quantum theory. Neither time, nor mind, nor matter and energy are taken to be *a priori* concepts. Rather, it is assumed that these concepts emerge by a contextual breaking of the holistic symmetry of the unus mundus. This symmetry breaking is not unique; there may be different separations, leading to complementarity descriptions of the unus mundus which do not use the concepts "mental" and "material".

### 2.4 A Quantum Theoretical Approach

In the following we analyze the aforesaid problems in the language of quantum theory. We adopt only the most fundamental structures of quantum theory, but to simplify matters we use the familiar Hilbert-space formalism of traditional quantum mechanics and the Kolmogorov–Rényi probability theory. This is presumably inappropriate since these formalisms refer to either a non-Boolean or a Boolean statistical description, but not to an individual description. In an explorative study this inconsistency may be tolerated. A properly elaborated discussion should, however, start with a basic non-statistical C*-formalism and the non-probabilistic concept of individual chaotic function in the sense of Wiener. If desired, epistemic W*-algebraic descriptions can then be introduced by an appropriate GNS-construction. Such a procedure is mathematically more involved, but I see no major difficulty to accomplish it in a more definitive discussion.

We assume that the description of the timeless holistic reality is invariant under the action of the group representing the primordial symmetry of the unus mundus. The crucial problem is then to explain *the emergence of time* from a timeless theory. The main idea is that time arises from the *entanglement* of the decomposition of the timeless universe of discourse

[17] Einstein's formulation, taken from the soundtrack of the film *Atomic Physics* (1948), accesssible at http://www.aip.org/history/einstein/voice1.htm.

[18] We do not, however, follow Leibniz into the peculiarities of his system.

into a tensed nonmaterial and a tenseless material part.[19] The mind–matter distinction will be described by a tensor-product decomposition of the Hilbert space $\mathcal{H}$ of the primordial timeless reality. In a quantum theoretical framework, the basic requirements for a description of a mind–matter distinction, applied to the unus mundus, can be summarized as follows:

1. The primordial, timeless universe of discourse can be quantum theoretically described in terms of a separable Hilbert space $\mathcal{H}$. This universe of discourse is conceived as a strictly closed system without any external interactions or external correlations. In this framework, mind and matter are not yet differentiated.

2. The Hilbert space $\mathcal{H}$ of the timeless universe of discourse can be represented as a tensor product $\mathcal{N} \otimes \mathcal{M}$, where the Hilbert space $\mathcal{N}$ refers to the *tensed* nonmaterial domain, while the Hilbert space $\mathcal{M}$ refers to the *tenseless* material domain.

3. The archetypal structure element responsible for the acausal orderedness of the two domains of mind and matter is given by the symmetry of the continuum.[20] This symmetry is represented by the additive group of real numbers, realized by a one-parameter group of automorphisms of the algebra $\mathfrak{B}(\mathcal{H})$ of all bounded operators acting in the Hilbert space $\mathcal{H}$. Such an automorphism can be implemented by the unitary group $\{e^{-2\pi i \tau G} | \tau \in \mathbb{R}\}$, where $\tau$ is an order parameter and $G$ is the selfadjoint generator of the symmetry group of the primordial unity. Since an automorphism does not change any structural relation, but simply connects equivalent descriptions, the action of the unitary group $\{e^{-2\pi i \tau G} | \tau \in \mathbb{R}\}$ only refers to a change of viewpoint.

4. The concept of pre-established harmony is implemented by a thorough entanglement of the quantum state of the timeless universe of discourse with respect to the tensorization $\mathcal{N} \otimes \mathcal{M}$.

[19] Most discussions of the emergence of time are related to cosmological studies; compare the review by Isham (1994). Usually, such studies of the "problem of time" use an *ad hoc* semiclassical approximation which promotes time to a classical observable. According to our proposal, the emergence of time is not primarily related to cosmology. It is a pure quantum phenomenon, leading to a non-classical time.

[20] Jung (1969, par. 870) suggested that the natural numbers were the most primitive element of order in the human mind. But according to Pauli (1994, p. 160) "a more general concept 'archetype' ... ought to be formulated in such a way that 'primitive mathematical intuition' comes within its scope – an intuition manifesting itself for instance in arithmetic in the idea of the infinite series of the integers, and in geometry in the idea of the continuum." The later Frege, after admitting his failure to derive arithmetic from pure logic, proposed that arithmetic must have a geometrical foundation (compare Frege, *Numbers and Arithmetic*, reprinted in Frege, 1997, pp. 275–277). We adopt this point of view.

5. In this preliminary study we assume that the nonmaterial and the material domain do not interact.[21] In this case the unitary operator $e^{-2\pi i\tau G}$ of the representation of the archetypal symmetry is given by

$$e^{-2\pi i\tau G} = e^{-2\pi i\tau\Lambda} \otimes e^{-2\pi i\tau H/\hbar} \quad , \quad \tau \in \mathbb{R} \quad , \tag{2.1}$$

so that the selfadjoint generator $G$ of the underlying symmetry group of the timeless universe of discourse is of the form

$$G = \Lambda \otimes \mathbf{1} + \mathbf{1} \otimes H/h \quad . \tag{2.2}$$

Here $H$ is the Hamiltonian of the material domain and $h$ is Planck's constant $2\pi\hbar$. We assume that Planck's constant refers only to matter and radiation, so we do not include it in the generator $\Lambda$ for the nonmaterial domain.

## 3. The Description of the Tensed Domain

### 3.1 The Tensed Structure of the Nonmaterial Domain

Neither nowness nor consciousness can be identified with any property known to physics, so we relate these phenomena to the nonmaterial domain. Since mental phenomena are closely related to time,[22] we assume that A-series phenomena can be described within the nonmaterial domain. Instead of thinking of time as a "stream that flows" it is more fruitful to adopt the idea that the stock of memories increases. The essence of such a mental time is the now – the past is stored in the present memory, while the future is not. It corresponds to Leibniz's order of succession:[23]

> "*Time is the order of non-contemporaneous things.* It is thus the *universal* order of change in which we ignore the specific kind of changes that have occurred."

The A-series does not refer to physical or physiological clocks, but is synchronized with the time concepts of the material domain via an entanglement between the nonmaterial and the material domain. Therefore, the nonmaterial time system cannot be a classical system (described

[21] This assumption is not crucial. There are special interactions which do neither affect the synchronization of the tensed and tenseless domains nor the conservation laws of the material domain, but nevertheless permit influences between the tensed and the tenseless domain. Such interactions may be important for the problem how mental events relate to physical events, but this question will not be dealt with in this exploratory study.

[22] For example, Augustine (1994, book eleven, chapter XXVI) asks whether spirit itself is time.

[23] *Metaphysical Foundations of Mathematics*, 1715. Quoted from Wiener (1951), p. 202.

by a commutative algebra) since a quantum system cannot be entangled with a classical system.[24] Hence, an appropriate time system has to be characterized by a *noncommutative* algebra. We will generate this noncommutative structure of the nonmaterial time system by a Hilbert-space representation of a two-parameter *non-Abelian time group*.

## 3.2 The Time System in the Nonmaterial Domain

### *3.2.1 The Time Operator of the Nonmaterial Domain*

Any subspace of the Hilbert space $\mathcal{N}$ of the tensed nonmaterial domain characterizes a set of particular *mental events*. We represent mental activities in terms of subspaces $\mathcal{N}', \mathcal{N}'', \ldots$ of the Hilbert space $\mathcal{N}$. With respect to the inclusion relation $\subseteq$ the subspaces $\mathcal{N}', \mathcal{N}'', \ldots$ form an orthomodular lattice $\mathcal{L}(\mathcal{N})$ with the intersection $\mathcal{N}' \bigwedge \mathcal{N}''$ as the greatest lower bound, and the closure $\mathcal{N}' \bigvee \mathcal{N}''$ of the union of $\mathcal{N}'$ and $\mathcal{N}''$ as the least upper bound. It is consistent to interpret the elements of $\mathcal{L}(\mathcal{N})$ as *attributes* where $\mathcal{N}' \subseteq \mathcal{N}''$ means that the attribute $\mathcal{N}''$ is superordinate to the attribute $\mathcal{N}'$.[25] The fact that the lattice $\mathcal{L}(\mathcal{N})$ is not distributive implies that there exist *incompatible* attributes.

The history of nonmaterial processes can be conceived by a family $\{\mathcal{N}_\tau \mid \tau \in \mathbb{R}\}$ of closed subspaces of the Hilbert space $\mathcal{N}$ of the nonmaterial domain. The subspace $\mathcal{N}_\tau \subset \mathcal{N}$ will be used to characterize the set of mental events available in a nonmaterial memory at $\tau$. Here, the real number $\tau$ characterizes the *now*. The chronological order and the forward motion of time will be related to the growth of $\mathcal{N}_\tau$:

$$\tau'' > \tau' \quad \text{if and only if} \quad \mathcal{N}_{\tau''} \supset \mathcal{N}_{\tau'} \quad . \tag{3.1}$$

Evidently, $\mathcal{N}_\tau$ cannot increase as $\tau$ decreases, so that the *remote past* $\bigwedge_{\tau\in\mathbb{R}} \mathcal{N}_\tau$ is a well-determined subspace of $\mathcal{N}$. Since it represents the $\tau$-independent portion of $\mathcal{N}$, we call it the *innate part* $\mathcal{N}_{\text{inn}}$ of $\mathcal{N}$, excluding any features of novelty. It satisfies

$$\mathcal{N}_{\text{inn}} \;:=\; \bigwedge\nolimits_{\tau\in\mathbb{R}} \mathcal{N}_\tau \,\subset\, \mathcal{N}_\tau \,\subset\, \mathcal{N} \quad \text{for every} \quad \tau \in \mathbb{R} \quad . \tag{3.2}$$

We call the orthogonal complement of $\mathcal{N}_{\text{inn}}$ in $\mathcal{N}$ the *acquired part $\mathcal{N}_{acq}$ of $\mathcal{N}$*, containing elements of $\mathcal{N}$ which are due to novelty,

$$\mathcal{N}_{\text{acq}} \;:=\; \mathcal{N} \ominus \mathcal{N}_{\text{inn}} \quad . \tag{3.3}$$

[24] Theorem: Let $\mathfrak{M}_1$ and $\mathfrak{M}_2$ be W*-algebras, and $\mathfrak{M}_1 \bar{\otimes} \mathfrak{M}_2$ their W*-tensor product. Then every pure normal state functional on $\mathfrak{M}_1 \bar{\otimes} \mathfrak{M}_2$ is a product state functional if and only if either $\mathfrak{M}_1$ or $\mathfrak{M}_2$ is commutative. Compare Raggio (1988). For singular states the corresponding theorem is: Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be C*-algebras. Then every pure state functional on the minimal C*-tensor product $\mathfrak{A}_1 \otimes \mathfrak{A}_2$ is a product state if and only if either $\mathfrak{A}_1$ or $\mathfrak{A}_2$ is commutative. Compare Takesaki (1979), theorem 4.14, p. 211.

[25] For details, compare Scheibe (1973), p. 137, and in particular Kanthack and Wegener (1976).

If $\mathcal{N}_\tau$ is independent of $\tau$, then there are no novelty acquirements, so that $\mathcal{N}_{\mathrm{inn}}$ equals $\mathcal{N}$.

Let $P_{\mathrm{inn}}$ be the orthogonal projection operator from $\mathcal{N}$ onto the subspace $\mathcal{N}_{\mathrm{inn}}$ and let $P_\tau$ be the orthogonal projection operator from $\mathcal{N}$ onto the subspace $\mathcal{N}_\tau$. Then the orthogonal projectors $E_\tau$ defined by $P_\tau = \mathbf{1} \oplus E_\tau$ form a spectral family acting on the Hilbert space $\mathcal{N}_{\mathrm{acq}}$. We define a family of unitary operators $U(\lambda) \in \mathfrak{B}(\mathcal{N}_{\mathrm{acq}})$ by

$$U(\lambda) \;:=\; \int_{-\infty}^{\infty} e^{2\pi i \lambda \tau}\, dE_\tau \quad , \quad \lambda \in \mathbb{R} \quad , \tag{3.4}$$

generating a unitary one-parameter group $\mathfrak{U} := \{U(\lambda) \,|\, \lambda \in \mathbb{R}\}$, called the *frequency-translation group*, which is assumed to be strongly continuous. The associated selfadjoint Stone generator is called *time operator* $T$,

$$T \;:=\; \int_{\mathbb{R}} \tau\, dE_\tau \quad , \quad U(\lambda) \;=\; e^{2\pi i \lambda T} \quad . \tag{3.5}$$

The unitary one-parameter group $\mathfrak{U} := \{U(\lambda) \,|\, \lambda \in \mathbb{R}\}$ is a representation of the Abelian group $\mathbb{R}$ on the Hilbert space $\mathcal{N}_{\mathrm{acq}}$. It describes the *acausal orderedness* of the unus mundus.

#### *3.2.2 Time–Frequency Complementarity*

In the multiplication representation of the time operator $T$ on the Lebesgue space $\boldsymbol{L}^2(\mathbb{R}, dx)$ of square-integrable complex-valued functions $x \mapsto \Phi(x)$ on the real axis $\mathbb{R}$,

$$\big\{U(\lambda)\,\Phi\big\}(x) \;=\; e^{2\pi i \lambda x}\,\Phi(x) \quad x, \lambda \in \mathbb{R} \quad , \quad \Phi \in \boldsymbol{L}^2(\mathbb{R}, dt) \quad , \tag{3.6}$$

we define a family of unitary shift operators $V(\tau)$ by

$$\big\{V(\tau)\,\Phi\big\}(x) \;=\; \Phi(x-\tau) \quad , \quad x, \tau \in \mathbb{R} \quad , \quad \Phi \in \boldsymbol{L}^2(\mathbb{R}, dt) \quad . \tag{3.7}$$

These operators generate a canonically conjugated unitary one-parameter group $\mathfrak{V} := \{V(\tau) \,|\, \tau \in \mathbb{R}\}$, called the *time-translation group*. The shift operator $V(\tau)$ generates a canonical system of imprimitivities[26]

$$V(\tau)^*\, T\, V(\tau) \;=\; T + \tau \quad , \quad \tau \in \mathbb{R} \quad . \tag{3.8}$$

[26] In the context of regular stochastic processes this imprimitivity relation has been recognized by Hanner (1950; eq. 2.3, p. 163) and by Kallianpur and Mandrekar (1965; p. 560). The time operator itself has been introduced by Tjøstheim (1975, 1976a, 1976b), who also discussed the canonical commutation relation between the time and the frequency operator. Independently, these relations have also been found by Gustafson and Misra (1976).

The frequency-translation group $\mathfrak{U} := \{U(\lambda) \,|\, \lambda \in \mathbb{R}\}$ and the canonically conjugated time-translation group $\mathfrak{V} := \{V(\tau) \,|\, \tau \in \mathbb{R}\}$ are related by Weyl's canonical commutation relations[27]

$$U(\lambda)\, V(\tau) \;=\; e^{2\pi i \lambda \tau}\, V(\tau)\, U(\lambda) \quad , \quad \lambda, \tau \in \mathbb{R} \quad . \tag{3.9}$$

The equivalent formulation in terms of the unitary Weyl operators

$$W(\lambda, \tau) \;:=\; e^{\pi i \lambda \tau}\, V(\tau)\, U(\lambda) \quad , \quad \lambda, \tau \in \mathbb{R} \quad , \tag{3.10}$$

realizes the commutation relations of the noncommutative *Heisenberg group*:

$$\begin{aligned} &W(\lambda, \tau)\, W(\lambda', \tau') \\ &= e^{\pi i (\lambda \tau' - \tau \lambda')}\, W(\lambda + \lambda', \tau + \tau')\,,\ \lambda, \lambda', \tau, \tau' \in \mathbb{R} \quad . \end{aligned} \tag{3.11}$$

Both strongly continuous unitary one-parameter groups $\{U(\lambda) \,|\, \lambda \in \mathbb{R}\}$ and $\{V(\tau) \,|\, \tau \in \mathbb{R}\}$ are realizations of the commutative group $\mathbb{R}$. Stone's theorem allows the representations

$$U(\lambda) \;=\; W(\lambda, 0) \;=\; e^{2\pi i \lambda T} \quad , \quad V(\tau) \;=\; W(0, \tau) \;=\; e^{-2\pi i \tau \Lambda} \quad , \tag{3.12}$$

$$W(\lambda, \tau) \;=\; e^{2\pi i \lambda T - 2\pi i \tau \Lambda} \quad . \tag{3.13}$$

The generator $\Lambda$ is called the *frequency operator*. It is unitarily equivalent to the time operator $T$. Both $\Lambda$ and $T$ are unbounded selfadjoint operators with the simple, absolutely continuous spectrum $\mathbb{R}$. On an appropriate domain they fulfill the relations

$$e^{2\pi i \tau \Lambda}\, f(T)\, e^{-2\pi i \tau \Lambda} \;=\; f(T + \tau) \ , \quad \tau \in \mathbb{R} \quad , \tag{3.14}$$

$$e^{-2\pi i \lambda T}\, g(\Lambda)\, e^{2\pi i \lambda T} \;=\; g(\Lambda + \lambda) \ , \quad \lambda \in \mathbb{R} \quad , \tag{3.15}$$

$$T\Lambda - \Lambda T \;=\; (i/2\pi)\, \mathbf{1} \quad . \tag{3.16}$$

## 3.3 The Basic Time Translation/Scaling Group

### *3.3.1 The Extended Affine Group*

We achieve a conceptually deeper understanding if we replace the two-parameter Heisenberg group (describing time and frequency translations) by the only other two-dimensional simply-connected Lie group, the two-parameter affine group (acting as a scale-translation group). For $x, s, \tau \in \mathbb{R}$, the assignment $x \mapsto e^s x + \tau$ is a permutation $\mathbb{R} \to \mathbb{R}$, so that the mapping $\{e^{2\pi i \lambda T} \,|\, \lambda \in \mathbb{R}\} \to \{e^{2\pi i \lambda (e^s T + \tau)} \,|\, \lambda \in \mathbb{R}\}$ is an automorphism of

[27] Weyl (1927). Compare also Weyl (1928), chapter IV, section D, §45 (in the English edition: chapter IV, section D, §14).

the Abelian group generated by the time operator $T$. That is, the time operator $T$ is invariant under the group of affine transformations of the real line $\mathbb{R}$ preserving the orientation. The transformed time operator $e^s T + \tau$ refers to a change in origin $\tau \in \mathbb{R}$ and to a change of scale by the factor $e^s > 0$. Since $T$ and $e^s T$ lead to equivalent descriptions, it makes no sense to speak of a "rate" of the flow of time.

While the non-Abelian two-parameter Heisenberg group is isomorphic to $\mathbb{R}^2$ with addition as the group operation, the non-Abelian two-parameter affine group is isomorphic to the group of all linear transformations $x \to e^s x + \tau$ of the real line $\mathbb{R}$. If we complement the group of affine transformations $T \to e^s T + \tau$ by the time-reversal transformation $T \to -T$ we get the non-Abelian *extended affine group*

$$\{g(s,\tau,j) \,|\, s,\tau \in \mathbb{R}, j = \pm 1\} \quad , \tag{3.17}$$

which we consider as the *fundamental group of time*. Its group operations are given by

$$g(s,\tau,j) \circ g(s',\tau',j') \;=\; g(s+s', \tau + j e^s \tau', jj') \quad , \tag{3.18}$$

where $g(0,0,+1)$ is the identity and $g(0,0,-1)$ represents the time reversal operation. The extended affine group has two connected components. The connected component containing the identity is the *affine group* $\{g(s,\tau,1) \,|\, s,\tau \in \mathbb{R}\}$. It is a continuous, infinite, non-unimodular non-Abelian locally compact group. It is the semidirect product of the *scaling group* $\{g(s,0,1) \,|\, s \in \mathbb{R}\}$ (which describes the scalings $T \to e^s\, T$) and the *translation group* $\{g(0,\tau,1) \,|\, \tau \in \mathbb{R}\}$ (which describes the translations $T \to T + \tau$). Both these subgroups are isomorphic to the commutative additive group $\mathbb{R}$ of the real numbers.

### *3.3.2 Hilbert-Space Realization of the Time Group*

The extended affine group $\{g(s,\tau,j) \,|\, s,\tau \in \mathbb{R}, j = \pm 1\}$ can be represented on the Hilbert space $\mathcal{N}_{\text{acq}}$ by operators $G(s,\tau,j)$, realizing the group relations (3.18) by the commutation relation

$$G(s,\tau,j)\, G(s',\tau',j') \;=\; G(s+s', \tau + j e^s \tau', jj') \quad , \tag{3.19}$$

We can obtain a more explicit representation by studying the scaling subgroup $\{Y(s) \,|\, s \in \mathbb{R}\}$ with $Y(s) := G(s,0,1)$, the translation group $\{V(\tau) \,|\, \tau \in \mathbb{R}\}$ with $V(\tau) := G(0,\tau,1)$, and the time-reversal group $\{G(0,0,j) \,|\, j = \pm 1\}$. The scaling group $\{Y(s) \,|\, s \in \mathbb{R}\}$ and the translation group $\{V(\tau) \,|\, \tau \in \mathbb{R}\}$ are strongly continuous commutative unitary one-parameter groups on the Hilbert space $\mathcal{N}_{\text{acq}}$, fulfilling the commutation relations

$$\begin{aligned} Y(s)Y(s') \;=\; Y(s+s') \quad , \quad V(\tau)V(\tau') \;=\; V(\tau+\tau') \quad , \\ V(\tau)Y(s) \;=\; Y(s)V(e^s\tau) \quad . \end{aligned} \tag{3.20}$$

Stone's theorem allows the introduction of selfadjoint generators $\Lambda$ and $B$ by

$$V(\tau) \;=\; e^{-2\pi i\tau\Lambda} \quad , \quad Y(s) \;=\; e^{2\pi isB} \quad , \quad \tau, s \in \mathbb{R} \quad . \tag{3.21}$$

The operator $B$ is unitarily equivalent to the operator $\Lambda$; both of them have the purely continuous spectrum $\mathbb{R}$. The group relation $e^{-2\pi i\tau\Lambda}e^{2\pi isB} = e^{2\pi isB}e^{-2\pi ie^s\tau\Lambda}$ implies, on an appropriate domain, the following commutation relation for the generators:

$$\big[B, \Lambda\big]_- \;=\; (i/2\pi)\,\Lambda \quad . \tag{3.22}$$

In terms of the selfadjoint operators $\Lambda$ and $T$ one can express the selfadjoint generator $B$ of the unitary scaling group by

$$B \;=\; \tfrac{1}{2}\,(\Lambda T + T\Lambda) \quad . \tag{3.23}$$

The canonical commutation relation $[T, \Lambda]_- = (i/2\pi)\,\mathbf{1}$ implies

$$\big[B\,,\,T\big]_- \;=\; -\,(i/2\pi)\,T \quad , \tag{3.24}$$

so that the unitary one-parameter scaling group $\{\exp(2\pi isB)\,|\,s \in \mathbb{R}\}$ scales the operators $T$ and $\Lambda$ as follows:

$$e^{2\pi isB}\,T\,e^{-2\pi isB} \;=\; e^s\,T\,, \quad e^{2\pi isB}\Lambda\,e^{-2\pi isB} \;=\; e^{-s}\Lambda\,, \quad s \in \mathbb{R} \tag{3.25}$$

In an irreducible Hilbert-space representation on $\mathcal{N}_{\mathrm{acq}}$, the time reversal can be implemented by an *antilinear* and *antiunitary* operator $J$,

$$A \to JAJ^{-1} \quad \text{for all} \quad A \in \mathfrak{B}(\mathcal{N}_{\mathrm{acq}}) \quad . \tag{3.26}$$

The operator $J$ fulfills

$$J(a\Psi + b\Phi) \;=\; a^*J\Psi + b^*J\Phi \quad , \quad \langle J\Phi|J\Psi\rangle \;=\; \langle\Phi|\Psi\rangle^* = \langle\Psi|\Phi\rangle \tag{3.27}$$

for all $\Psi, \Phi \in \mathcal{N}_{\mathrm{acq}}$ and $a, b \in \mathbb{C}$.[28] The square of $J$ is a unitary operator which for spin-free systems has the property $J^2 = \mathbf{1}$. By definition, the time operator $T$ changes sign under time reversal, $\mathcal{T}(T) = JTJ^{-1} = -\,T$, so that the canonical commutation relations determine the following behavior of generators of the affine and the Heisenberg group under time reversal:

$$J\,TJ^{-1} = -\,T\ ,\ J\,e^{+2\pi i\lambda T}J^{-1} = e^{+2\pi i\lambda T}\ ,\ J\,U(\lambda)J^{-1} = U(\lambda) \quad , \tag{3.28}$$

$$J\,\Lambda J^{-1} = \ \Lambda \quad ,\ J\,e^{-2\pi i\tau\Lambda}J^{-1} = e^{+2\pi i\tau\Lambda}\ ,\ J\,V(\tau)J^{-1} = V(-\tau)\ , \tag{3.29}$$

$$J\,BJ^{-1} = -\,B\ ,\ J\,e^{+2\pi i\lambda B}J^{-1} = e^{+2\pi i\lambda B}\ ,\ J\,Y(s)J^{-1} = Y(s) \quad . \tag{3.30}$$

[28] Wigner (1932). Compare also Wigner (1959), chapter 26.

With the group multiplication laws (3.20) of the affine group and the relations (3.29), (3.30), and $J^2 = \mathbf{1}$, we find that the commutation relation (3.19) of the extended affine group is fulfilled by

$$G(s,\tau,j) := V(\tau)Y(s)\big\{\delta_{+1,j}\,\mathbf{1} + \delta_{-1,j}\,J\big\} \quad . \tag{3.31}$$

### *3.3.3 Lebesgue-Space Representation of the Time Group*

For an explicit evaluation of the time system it is convenient to realize the Hilbert space $\mathcal{N}_{\text{acq}}$ by the Lebesgue space $\boldsymbol{L}^2(\mathbb{R}, dx)$ of square-integrable complex-valued functions $x \mapsto \varPhi(x)$ on the real axis $\mathbb{R}$. We find:

$$\big\{V(\tau)\,\varPhi\big\}(x) = \varPhi(x-\tau) \quad , \quad x,\tau \in \mathbb{R} \quad , \tag{3.32}$$

$$\big\{Y(s)\,\varPhi\big\}(x) = e^{s/2}\,\varPhi(e^s x) \quad , \quad x,s \in \mathbb{R} \quad , \tag{3.33}$$

$$\big\{J\,\varPhi\big\}(x) = \varPhi(-x)^* \quad , \quad x \in \mathbb{R} \quad , \tag{3.34}$$

$$\big\{U(\lambda)\,\varPhi\big\}(x) = e^{2\pi i\lambda x}\,\varPhi(x) \quad , \quad x,\lambda \in \mathbb{R} \quad . \tag{3.35}$$

The generators of the unitary operators $V(\tau) = e^{-2\pi i\tau\varLambda}$, $Y(s) = e^{2\pi i s B}$, and $U(\lambda) = e^{2\pi i\lambda T}$ are unbounded selfadjoint operators which are well-defined on the Schwartz space $\mathcal{S}(\mathbb{R})$ of all rapidly decreasing complex-valued infinitely differentiable functions on $\mathbb{R}$. For every $\varPhi \in \mathcal{S}(\mathbb{R}) \subset \boldsymbol{L}^2(\mathbb{R}, dt)$ we find:

$$\big\{\varLambda\varPhi\big\}(x) = \frac{1}{2\pi i}\,\frac{\partial\,\varPhi(x)}{\partial x} \quad , \quad x \in \mathbb{R} \quad , \tag{3.36}$$

$$\big\{B\,\varPhi\big\}(x) = \frac{1}{4\pi i}\left\{\frac{\partial}{\partial x}\,x + x\,\frac{\partial}{\partial x}\right\}\varPhi(x) \quad , \quad x \in \mathbb{R} \quad , \tag{3.37}$$

$$\big\{T\,\varPhi\big\}(x) = x\,\varPhi(x) \quad , \quad x \in \mathbb{R} \quad . \tag{3.38}$$

## 3.4 Nonmaterial Processes

### *3.4.1 Innate and Novelty-Acquiring Processes*

Processes in the nonmaterial domain can be described by complex-valued functions $\tau \mapsto x(\tau) \in \mathcal{N}$. Every element $x$ of the Hilbert space $\mathcal{N}$ can be regarded as representing an equivalence class of complex-valued random variables with zero mean, $\mathcal{E}\{x\} = 0$, and finite variance $\mathcal{E}\{|x|^2\} < \infty$. The covariance is defined in terms of the inner product $\langle\cdot|\cdot\rangle$ of the Hilbert space $\mathcal{N}$ by $\mathcal{E}\{x^*y\} := \langle x|y\rangle$. A family of elements $x(\tau) \in \mathcal{N}$, where $\tau$ varies over the real axis $\mathbb{R}$, is called a zero-mean *second order stochastic process*. When $\tau$ varies over $\mathbb{R}$, the stochastic process can be considered as a *curve* in the Hilbert space $\mathcal{N}$. A zero-mean second order stochastic process $\{x(\tau)\,|\,\tau\in\mathbb{R}\}$ is said to be *weakly stationary* if its covariance function is invariant under the shift transformation, i.e. if for all $\tau,\tau',s\in\mathbb{R}$ we have $\mathcal{E}\{x(\tau+s)^*\,x(\tau'+s)\}=\mathcal{E}\{x(\tau)^*\,x(\tau')\}$.

The *history of the process* $\{x(\tau)\}$ is given by $\mathcal{N}(\infty):=\bigvee_\tau\mathcal{N}(-\infty,\tau)$, where the family of Hilbert spaces $\mathcal{N}(-\infty,\tau)$ is defined as the closed linear subspace of $\mathcal{N}$ generated by the family $\{x(\tau')\,|\,\tau'\leq\tau\}$,

$$\mathcal{N}(-\infty,\tau)\ :=\ \text{closed span}\,\{x(\tau')\,|\,-\infty\leq\tau'\leq\tau\}\quad, \tag{3.39}$$

The Hilbert space $\mathcal{N}(-\infty,\tau)$ is called *the past and present up to* $\tau$. Since the family $\{\mathcal{N}(-\infty,\tau)\,|\,\tau\in\mathbb{R}\}$ of Hilbert spaces $\mathcal{N}(-\infty,\tau)$ is monotonically increasing, the *remote past*

$$\mathcal{N}(-\infty)\ :=\ \textstyle\bigwedge_{\tau\leq 0}\mathcal{N}(-\infty,\tau) \tag{3.40}$$

is well defined. Two extreme cases may occur:

if $\mathcal{N}(-\infty)=\mathcal{N}$, then the process $\{x(\tau)\,|\,\tau\in\mathbb{R}\}$ is called *singular*, (3.41)
if $\mathcal{N}(-\infty)=\{0\}$, then the process $\{x(\tau)\,|\,\tau\in\mathbb{R}\}$ is called *regular*. (3.42)

Here $\{0\}$ is the null space consisting only of the random variable which is almost always equal to zero. Since the remote past of a singular process contains all information necessary for the whole development of the process, singular processes are also called *deterministic*. In our context they correspond to *innate processes*. A regular process is also called “completely nondeterministic” because its far future is essentially independent of the present, so that long-term predictions are not feasible.[29] In our context regular processes correspond to *novelty-acquiring processes*.

Every nonmaterial process $\{x(\tau)\,|\,\tau\in\mathbb{R}\}$ can be uniquely represented as the orthogonal sum of a singular process $x^{\text{sing}}\in\mathcal{N}_{\text{inn}}$ and a regular process $x^{\text{reg}}\in\mathcal{N}_{\text{acq}}$.[30] Let $P_{-\infty}$ be the orthogonal projection operator from $\mathcal{N}$ onto the remote past $\mathcal{N}(-\infty)$. With $x^{\text{sing}}:=P_{-\infty}\,x$ and $x^{\text{reg}}:=(\mathbf{1}-P_{-\infty})\,x$ we can write

$$x\ =\ x^{\text{sing}}+x^{\text{reg}}\quad. \tag{3.43}$$

[29]The concept of regularity of one-dimensional stationary processes with discrete time has been introduced by Kolmogorov (1941). Kolmogorov’s analysis was based on the fundamental decomposition theorem by Wold (1938). In Wold’s thesis the two components were called singular and regular, the synonymous terms deterministic and nondeterministic were introduced by Doob (1944). In response to a question raised by Kolmogorov, Kreĭn (1945a,b) showed how to transfer Kolmogorov’s results for discrete time to continuous time by a simple transformation.

[30]This decomposition is due to Wold (1938) for the special case of discrete-time stationary processes, and to Hanner (1950) for the case of continuous-time processes.

This Wold decomposition implies that the Hilbert space $\mathcal{N}$ of the nonmaterial domain has the decomposition

$$\mathcal{N} \;:=\; \mathcal{N}_{\rm inn} \oplus \mathcal{N}_{\rm acq} \quad \text{with} \quad x^{\rm sing} \in \mathcal{N}_{\rm inn} \quad , \quad x^{\rm reg} \in \mathcal{N}_{\rm acq} \quad . \tag{3.44}$$

### *3.4.2 Nonmaterial K-Structures*

The development of a novelty-acquiring stationary process $\{x^{\rm reg}(\tau) \,|\, \tau \in \mathbb{R}\}$ with zero mean and finite variance is given by the time-translation group $\mathfrak{V}$

$$x^{\rm reg}(\tau) \;=\; V(\tau)\, x^{\rm reg}(0) \quad , \quad V(\tau) \in \mathfrak{V} \quad . \tag{3.45}$$

In the Hilbert space $\mathcal{N}_{\rm acq}$ we select the closed subspace $\mathcal{N}^+$ defined by

$$\mathcal{N}^+ \;:=\; \text{closed span}\, \{ x^{\,\rm reg}(\tau') \,|\, -\infty \le \tau' \le 0 \,\} \quad . \tag{3.46}$$

Then the strongly continuous one-parameter group $\mathfrak{V} := \{V(\tau) \,|\, \tau \in \mathbb{R}\}$ generates a *forward expanding Hilbert-space K-structure*[31] $\{\mathcal{N}_{\rm acq}, \mathcal{N}^+, \mathfrak{V}\}$, characterized by the conditions:

- $$\mathcal{N}^+ \subseteq V(\tau)\, \mathcal{N}^+ \quad \text{for all} \;\; \tau \ge 0 \quad , \tag{3.47}$$
- $$\textstyle\bigwedge_{\tau \le 0} V(\tau)\, \mathcal{N}^+ \;=\; \{0\} \quad , \tag{3.48}$$
- $$\textstyle\bigvee_{\tau \ge 0} V(\tau)\, \mathcal{N}^+ \;=\; \mathcal{N}_{\rm acq} \quad . \tag{3.49}$$

Here $\bigwedge$ denotes the intersection, $\bigvee$ the closure of the union. It follows that *a stationary nonmaterial process with zero mean and finite variance is novelty-acquiring (i.e. regular) if and only if* $\{\mathcal{N}, \mathcal{N}(-\infty, 0), \mathfrak{V}\}$ *is a forward expanding Hilbert-space K-structure.*

For every *Hilbert-space K-structure* $\{\mathcal{N}_{\rm acq}, \mathcal{N}^+, \mathfrak{V}\}$ there exists a unique strongly continuous one-parameter group $\mathfrak{U} := \{U(\lambda) \,|\, \lambda \in \mathbb{R}\}$ of unitary operators on $\mathcal{N}$, which satisfies Weyl's canonical commutation relations[32]

$$U(\lambda)\, V(\tau) \;=\; e^{2\pi i \lambda \tau}\, V(\tau)\, U(\lambda) \quad , \quad \lambda, \tau \in \mathbb{R} \quad . \tag{3.50}$$

### *3.4.3 Canonical Representations of Nonmaterial Processes*

Every stationary continuous regular process $\{x^{\rm reg}(\tau) \,|\, \tau \in \mathbb{R}\}$ with zero mean and finite variance has the following unique non-anticipative representation[33]

$$x(\tau) \;=\; \int_{-\infty}^{\tau} R(\tau - \tau')\, dw(\tau') \quad , \tag{3.51}$$

[31] Compare Lewis and Thomas (1974). For our purpose the *expanding* K-structure $\mathcal{N}^+ \subseteq V(\tau)\mathcal{N}^+$ is appropriate, while in the scattering theory by Lax and Phillips (1967) *contracting* K-structures $\mathcal{N}^+ \subseteq V(\tau)\mathcal{N}^+$ are used.

[32] Compare for example Cornfeld et al. (1982), p. 457.

[33] This result is due to Hanner (1950). For important generalizations compare Hida (1960), Cramér (1961a,b). In the context of statistical prediction theory Masani and Wiener (1959) called a non-anticipative one-sided moving-average representation an *innovation representation.* For details compare the monograph by Rozanov (1967).

where $\tau \mapsto R(\tau)$ is a non-random Borel measurable weighting function and $\tau \mapsto w(\tau)$ is a Wiener process with uncorrelated increments,

$$\mathcal{E}\{|dw(\tau)|^2\} = d\tau \quad , \quad \mathcal{E}\{dw(\tau)\, dw(\tau')\} = \delta(\tau - \tau')\, d\tau\, d\tau' \quad . \tag{3.52}$$

In engineering science the process $\tau \mapsto dw(\tau)/d\tau$ is called *white noise.*

A rigorous definition of white noise can be given in terms of Gel'fand triples.[34] On the Gel'fand triple $\mathcal{S}(\mathbb{R}) \subset \boldsymbol{L}^2(\mathbb{R}) \subset \mathcal{S}^*(\mathbb{R})$ white noise can be generated by an automorphic dynamical system. Here $\mathcal{S}(\mathbb{R})$ is the Schwartz space of real-valued rapidly decreasing infinitely often differentiable functions on $\mathbb{R}$, $\boldsymbol{L}^2(\mathbb{R})$ is the Hilbert space of Lebesgue square-integrable functions on $\mathbb{R}$, and $\mathcal{S}^*$ is the strong dual of $\mathcal{S}(\mathbb{R})$, that is, the Schwartz space of tempered distributions on $\mathbb{R}$. Let $\boldsymbol{\Sigma}(a,b)$ be the $\sigma$-field generated by white noise $\tau \mapsto dw(\tau)/d\tau$ in the interval $[a,b]$. It follows that *white noise generates the memoryless forward expanding Hilbert-space K-structure* $\big\{\boldsymbol{L}^2(\mathcal{S}^*, \boldsymbol{\Sigma}, \boldsymbol{\mu}), \boldsymbol{L}^2(\mathcal{S}^*, \boldsymbol{\Sigma}(-\infty, 0), \boldsymbol{\mu}), \{\mathfrak{V} \,|\, \tau \in \mathbb{R}\}\big\}$.[35]

The representation (3.51) can be interpreted as a non-anticipative input–output map producing the trajectories of a novelty-acquiring process in response to the application of a particular white noise trajectory at the input. However, in contrast to engineering systems, the "input" cannot be assigned arbitrarily from the outside. The fact that every novelty-acquiring nonmaterial process has a representation in terms of white noise suggests that the *memoryless* K-system generated by white noise can be considered as an *intrinsic structure element of the nonmaterial domain.* Every memory function $\tau \mapsto R(\tau)$ generates a particular nonmaterial process which may be used for descriptions of higher-level mental phenomena. The question whether such mental processes are correlated with material processes can be investigated only if more details about the structure of the nonmaterial domain are available.

## 3.5 Breaking the Time-Reversal Symmetry

### *3.5.1 Time Symmetries*

First principles are always characterized by a high degree of symmetry. The reason why we prefer a symmetric time concept in fundamental *ontic* descriptions has been stated by Poincaré:[36]

> "Time must be defined in such a way that the
> statements of the natural laws be as simple as possible."

Starting with a description of a timeless universe of discourse, we introduced the concept of time via an intrinsic one-parameter group of automorphisms. This approach yields two basic time symmetries:

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[34] Compare for example Gel'fand and Vilenkin (1964), chapter III; or Hida (1980), section 3.4.

[35] Compare also Hida (1980), p. 146.

[36] Translated from Poincaré (1899), p. 6.

1. The fundamental dynamical equations are invariant under translations $t \to t' = t + a$, $a \in \mathbb{R}$. This invariance is called *time-translation symmetry*.

2. The fundamental dynamical equations are invariant under an involution which changes the time parameter $t$ to $-t$.[37] This invariance is called *time-reversal symmetry*.

For time-reversal invariant systems there is a distinction between past and future, but no distinction between cause and effect. The success of phenomenological nonanticipative physical descriptions suggests that in *epistemic* descriptions the time-reversal symmetry is always broken. Likewise, conscious perception, cognition and communication presupposes the usual forward direction of time, distinguishing memory of the past from anticipation of the future.

### *3.5.2 Why Do All Processes Show the Same Arrow of Time?*

The phenomenon of symmetry breaking is well-understood in modern physical theories. Nevertheless, an important problem remains unsolved. The time-reversal symmetry is represented by a group of order two. If the time-reversal symmetry is broken one gets *two* representations. One of them satisfies the generally accepted rules of retarded causality, the other one satisfies the strange rules of advanced causality. In our description of the nonmaterial domain the canonical forward representation (3.51)

$$x(\tau) \;=\; \int_{-\infty}^{\tau} R(\tau - \tau')\, dw(\tau')$$

manifests the *breaking of the time-reversal symmetry* of the underlying unitary time-evolution group $\mathfrak{V} := \{V(\tau) \mid \tau \in \mathbb{R}\}$ of the forward expanding Hilbert-space K-structure $\{\mathcal{N}_{\text{acq}}, \mathcal{N}^+, \mathfrak{V}\}$ and the associated novelty-acquiring processes in the nonmaterial domain. A backward expanding Hilbert-space K-structure would lead to nonmaterial processes characterized by an anticipating canonical backward representation of the type

$$\int_{\tau}^{\infty} R(\tau - \tau')\, dw(\tau') \quad .$$

The usual choice of retarded causality in the material domain cannot be explained by a statistical mechanical formulation of the "second

[37] The time-reversal transformation is an involution, i.e. an operation whose square is the identity. The involution associated with time-reversal does not only change the direction of time but also associated quantities like the velocity, the momentum, the angular momentum, the electric current and the magnetic field. In elementary particle physics, the invariant involution $PCT$ associated with time reversal $T$ ($T^2 = 1$) also involves space reflection $P$ ($P^2 = 1$) and charge conjugation $C$ ($C^2 = 1$) ($PCT$-theorem).

law" without an *a priori* postulate imposing an asymmetric evolution toward increasing time. Thus, the decision which of the two possibilities is appropriate is not derivable from the first principles of physics.

While in classical physics a breaking of the time-reversal symmetry permits that two noninteracting subsystems have different directions of the time arrow, this is in general impossible in quantum theory. The fact that even noninteracting subsystems of a quantum system are in general entangled implies that in quantum theory the time-reversal operation is *global*. The reason is that for quantum systems the time-reversal map is positive but *not completely positive*. Therefore it is impossible to define a *local* time-reversal operation for one subsystem only.

This can be explained in detail in the C*-algebraic description of physical systems, where the time reversal is represented by an involutary anti-automorphism $\mathcal{T} : \mathfrak{A} \to \mathfrak{A}$ of the underlying C*-algebra $\mathfrak{A}$,

$$\mathcal{T}(A^*) = \mathcal{T}(A)^* \ , \ \mathcal{T}(AB) = \mathcal{T}(B)\mathcal{T}(A) \ , \ \mathcal{T}\{\mathcal{T}(A)\} = A \ , \ A, B \in \mathfrak{A} \ .$$

The time-reversal map $\mathcal{T}$ is positive, but for quantum systems (where $\mathfrak{A}$ is noncommutative) the map $\mathcal{T}$ is *not completely positive.*[38] That is, if $(\mathfrak{A}_1, \mathcal{T}_1)$ and $(\mathfrak{A}_2, \mathcal{T}_2)$ describe two noninteracting quantum systems, then the *local* map $\mathcal{T}_1 \otimes \mathbf{1}_2$ and the *local* map $\mathbf{1}_1 \otimes \mathcal{T}_2$ on the minimal C*-tensor product $\mathfrak{A}_1 \otimes \mathfrak{A}_2$ are not positive, hence they are not time-reversal maps. The time-reversal map for the composite quantum system is given by the *global* positive map

$$(\mathcal{T}_1 \otimes \mathbf{1}_2)(\mathbf{1}_1 \otimes \mathcal{T}_2) \ = \ \mathcal{T}_1 \otimes \mathcal{T}_2 \quad .$$

This implies that, even if the two quantum systems do *not interact*, the time-reversal for the first quantum system is *not* given by $\mathcal{T}_1 \otimes \mathbf{1}_2$. *The map $\mathcal{T}_1 \otimes \mathbf{1}_2$ represents the time-reversal operator for the first subsystem if and only if the two systems are not entangled.* In an entangled system with broken time-reversal symmetry the direction of the arrow of time has to point to the same direction for all (even noninteracting) subsystems.

## 4. Mind–Matter Entanglement

### 4.1 An Intrinsic Symmetry of the Timeless Unus Mundus

We write the generator $G$ of the underlying symmetry group of the timeless universe of discourse in the form

$$G \ = \ \Lambda \otimes \mathbf{1} + \mathbf{1} \otimes H/h \quad . \tag{4.1}$$

[38] A linear map $\mathcal{T} : \mathfrak{A} \to \mathfrak{A}$ is said to be *completely positive* if the linear map $\mathcal{T} \otimes \mathbf{1}_n : \mathfrak{A} \otimes \mathfrak{B}(\mathbb{C}^n) \to \mathfrak{A} \otimes \mathfrak{B}(\mathbb{C}^n)$ is positive for all $n \geq 1$, where $B(\mathbb{C}^n)$ is the C*-algebra of all complex $n \times n$-matrices and $\mathbf{1}_n$ is the identity transformation of $B(\mathbb{C}^n)$ onto itself.

where $H$ is the Hamiltonian of the material domain and $h$ is Planck's constant $2\pi\hbar$. We assume that Planck's constant refers only to matter and radiation, so it is not included in the generator $\Lambda$ of the nonmaterial domain. The selfadjoint operator $G$ generates a one-parameter group $\{\alpha_\tau \mid \tau \in \mathbb{R}\}$ of automorphisms $\alpha_\tau$ of the algebra $\mathfrak{B}(\mathcal{N}\otimes\mathcal{M})$ of all bounded operators acting on the Hilbert space $\mathcal{N}\otimes\mathcal{M}$ by

$$\alpha_\tau(X) = e^{2\pi i\tau G} X\, e^{-2\pi i\tau G} \ , \quad X \in \mathfrak{B}(\mathcal{N}\otimes\mathcal{M}) \quad , \quad \tau \in \mathbb{R} \quad . \tag{4.2}$$

For $A \in \mathfrak{B}(\mathcal{N})$ and $B \in \mathfrak{B}(\mathcal{M})$ we get

$$\alpha_\tau\big(A\otimes B\big) = \big(e^{2\pi i\tau\Lambda} A\, e^{-2\pi i\tau\Lambda}\big) \otimes \big(e^{i\tau H/\hbar} B\, e^{-i\tau H/\hbar}\big) \quad . \tag{4.3}$$

### 4.2 Maximally Entangled Quantum States

A state which cannot be represented as a product state with respect to the tensorization $\mathcal{N}\otimes\mathcal{M}$ is called an *entangled state* (Schrödinger 1935). We describe the state of the timeless universe of discourse by a continuous superposition of the product state vectors $e^{-2\pi i\tau G}\,\Phi\otimes\Psi = (e^{-2\pi i\tau\Lambda}\,\Phi)\otimes(e^{-i\tau H/\hbar}\,\Psi)$,

$$\Xi = \int_{\mathbb{R}} g(\tau)\,\big(e^{-2\pi i\tau\Lambda}\,\Phi\big)\otimes\big(e^{-i\tau H/\hbar}\,\Psi\big)\,d\tau \quad , \tag{4.4}$$

where $\tau \mapsto g(\tau)$ is some complex-valued weight function on $\mathbb{R}$. The reference vector $\Psi \in \mathcal{M}$ specifies the possible states of the material system. It corresponds to the initial state vector of the material system in traditional quantum theoretical descriptions with an external parameter time.

The Leibnizian requirement of a *perfect* pre-established harmony between the mental time and the material time demands a *maximally* entangled state vector $\Xi$. A time-entangled state is said to be maximally entangled if $|g(\tau)| = 1$ for all $\tau \in \mathbb{R}$. The further prerequisite that the description of the timeless universe of discourse is invariant under the action of the unitary group $\{e^{-2\pi i\tau G} \mid \tau \in \mathbb{R}\}$ implies that $e^{-2\pi i\tau G}\,\Xi = \Xi$, so that $g \equiv 1$.

### 4.3 Covariant Reference States

We demand that the reference state vector $\Phi$ of the time system transforms *covariantly* under the time-translation group. This is fulfilled by any coherent state vector $\Phi = |\lambda, t\,\rangle \in \mathcal{N}$ related to the Heisenberg group:

$$e^{-2\pi i\tau\Lambda}\,|\lambda, t\rangle = e^{-\pi i\tau\lambda}\,|\lambda, t+\tau\rangle \quad . \tag{4.5}$$

In terms of the temporal Weyl system (3.10) a coherent state vector is defined by

$$|\lambda, t\,\rangle := W(\lambda, t)|0, 0\,\rangle \quad , \quad \lambda, t \in \mathbb{R} \quad , \tag{4.6}$$

where the reference state vector $|0,0\rangle \in \mathcal{N}$ is defined by the state-generating function

$$\begin{aligned} \langle 0,0|W(\lambda,t)|0,0\rangle &= \exp\Big\{-2\pi^2\sigma_T^2\lambda^2 - 2\pi^2\sigma_\Lambda^2 t^2\Big\} \ , \\ \sigma_T\sigma_\Lambda &= 1/(4\pi) \ , \end{aligned} \tag{4.7}$$

The mean values $\lambda, t \in \mathbb{R}$ and the standard deviations $\sigma_T > 0$, $\sigma_\Lambda > 0$ with respect to the coherent states are given by

$$\lambda \ := \ \langle\lambda,t\,|\,\Lambda\,|\,\lambda,t\rangle \quad , \quad t \ := \ \langle\lambda,t\,|\,T\,|\,\lambda,t\rangle \quad , \tag{4.8}$$

$$\sigma_\Lambda^2 \ := \ \langle\lambda,t\,|\,(\Lambda-\lambda)^2\,|\,\lambda,t\rangle \quad , \quad \sigma_T^2 \ := \ \langle\lambda,t\,|\,(T-t)^2\,|\,\lambda,t\rangle \quad . \tag{4.9}$$

In the multiplication representation of the time operator, the coherent state vector $|\lambda,t\,\rangle$ in the Lebesgue space $\boldsymbol{L}^2(\mathbb{R},dx)$ is given by

$$|\lambda,t\,\rangle(x) \ = \ \left(\frac{1}{2\pi\sigma_T^2}\right)^{1/4} e^{2\pi i\lambda(x-t/2)\,-\,\frac{1}{4}(x-t)^2/\sigma_T^2} \tag{4.10}$$

Without loss of generality, we can choose the vacuum vector $|0,0\rangle$ of the Weyl system $\{W(\lambda,t)\,|\,\lambda,t\in\mathbb{R}\}$ as the reference vector $\Phi$ of the time system,

$$\Phi \ := \ |0,0\rangle \quad \text{with} \ \langle\Phi\,|\,T\,|\,\Phi\rangle \ = \ 0 \ , \ \langle\Phi\,|\,\Lambda\,|\,\Phi\rangle \ = \ 0 \quad . \tag{4.11}$$

The covariant transformation of this reference vector under the action of the group $\{e^{-2\pi it\Lambda}\,|\,t\in\mathbb{R}\}$ is given by $e^{-2\pi it\Lambda}\,|0,0\rangle = |0,t\rangle$.

***Conclusion:*** *The global description of the timeless universe of discourse is represented in terms of the maximally time-entangled generalized vector*

$$\Xi \ = \ \int_{\mathbb{R}} |0,t\rangle \otimes e^{-itH/\hbar}\,\Psi\,dt \quad , \tag{4.12}$$

*invariant under the action of the intrinsic unitary group* $\{e^{-i\tau G}\,|\,\tau\in\mathbb{R}\}$.

## 5. The Emergence of Time

### 5.1 Quantum Correlations and Relative States

The timeless description in terms of the global generalized state vector $\Xi$ gives rise to many relational descriptions. In spite of the fact that the system as a whole is in a stationary state, dynamical aspects of the material subsystem are still present in form of *quantum correlations.* A long time ago Schrödinger (1931, p. 243) proposed to replace the Platonic concept of time by correlations. From an epistemic point of view, Rovelli pointed out that[39]

> "physics is the theory of the relative information that systems have about each other. ... Quantum mechanics can therefore be viewed as a theory about the states of systems and values of physical quantities relative to other systems."

Starting with a timeless description, all results of traditional quantum mechanics can be recovered in terms of the *correlations* and *conditional expectations* between the nonmaterial time system and a material object system.[40] In the superposition $\varXi = \int_{\mathbb{R}} dt\, |0,t\rangle \otimes e^{-itK/\hbar}\varPsi$ the component $e^{-itK/\hbar}\varPsi$ is considered as the *relative state vector* of the material domain, given that the state of the nonmaterial time system is described by the state vector $|0,t\rangle$.

## 5.2 Every Probability is a Conditional Probability

Normalizable maximally entangled quantum state vectors exist only in finite-dimensional Hilbert spaces. In a probabilistic interpretation they correspond to *uniform* probabilities which exist only for compact probability spaces. Fortunately, a relational probabilistic interpretation requires only conditional probabilities whose definition does neither depend on bounded measures nor on absolute probabilities.

In Kolmogorov's mathematical foundation of probability theory conditional probabilities are defined in terms of absolute probabilities. Yet, the basic notion of probability theory is not an absolute probability but the notion of the *conditional probability* $P(A|B)$ of $A$ under the condition $B$. An axiomatic theory using the concept of conditional probability as the primary concept has been developed by Rényi.[41] This approach avoids the use of absolute a priori probabilities, nevertheless every result of Kolmogorov's probability theory can be translated into a theory based on conditional probabilities (cf. Rényi 1970b, chapter 2). Moreover, it also permits the use of unbounded measures. Rényi proposed the following definition for *conditional probability spaces*.

Let $\boldsymbol{\varOmega}$ be an arbitrary set, $\boldsymbol{\varSigma}$ a $\sigma$-algebra of subsets of $\boldsymbol{\varOmega}$, and $\boldsymbol{\varSigma}_B$ a nonempty subset of $\boldsymbol{\varSigma}$. A function $P(\mathcal{A}|\mathcal{B})$ of two set-valued variables $\mathcal{A} \in \boldsymbol{\varSigma}$ and $\mathcal{B} \in \boldsymbol{\varSigma}_B$ is called a conditional probability if and only if it satisfies for every $\mathcal{A} \in \boldsymbol{\varSigma}$ and all $\mathcal{B}, \mathcal{C}, \mathcal{B} \cap \mathcal{C} \in \boldsymbol{\varSigma}_B$ the axioms:

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[39]Rovelli (1996). From a different viewpoint, Mermin (1998a,b) also takes the notion of *correlation* as one of the primitive concepts for an interpretation of quantum mechanics. In the present discussion, motivation and details are different.

[40]Compare also Page and Wootters (1983), Wootters (1984), Englert (1990), where the notion of time is replaced by the notion of correlations between subsystems.

[41]Rényi (1955). Compare also Rényi (1970a), chapter II, §11, as well as Rényi (1970b), section 2.2.

1. $P(\mathcal{A}|\mathcal{B}) \geq 0$, $P(\mathcal{B}|\mathcal{B}) = 1$,
2. For each $\mathcal{B}$, $P(\cdot|\mathcal{B})$ is a measure,
3. $P(\mathcal{A}|\mathcal{B} \cap C)P(\mathcal{B}|\mathcal{C}) = P(\mathcal{A} \cap \mathcal{B}|\mathcal{C})$

If $\boldsymbol{\nu}$ is a (not necessarily bounded) measure on $\boldsymbol{\Sigma}$ and if $\boldsymbol{\nu}(\mathcal{B}) > 0$, then $P(\mathcal{A}|\mathcal{B}) = \boldsymbol{\nu}(\mathcal{A} \cap \mathcal{B})/\boldsymbol{\nu}(\mathcal{B})$ is a conditional probability in the sense of Rényi. Every real- or complex-valued integrable function $\boldsymbol{\omega} \mapsto x(\boldsymbol{\omega})$ on the space $(\boldsymbol{\Omega}, \boldsymbol{\Sigma}, \boldsymbol{\nu})$ is also integrable with respect to the measure $\mathcal{A} \mapsto P(\mathcal{A}|\mathcal{B})$. The corresponding integral $\mathcal{E}\{x|\,\mathcal{B}\} = \int_{\boldsymbol{\Omega}} x(\boldsymbol{\omega}) P(d\boldsymbol{\omega}|\mathcal{B})$ is called the *conditional expectation of the random variable* $x$.

### 5.3 Conditional Expectation of a Material Observable

We consider the description of the timeless universe of discourse in terms of the Hilbert space $\mathcal{N} \otimes \mathcal{M}$. In the time system we choose the spectral measure

$$E_T(\mathcal{T}) \;:=\; \int_{\mathcal{T}} E_\tau \, d\tau \tag{5.1}$$

of the time operator (3.5) as reference. In the material system we select the spectral measure $E_A$ of a selfadjoint operator $A \in \mathfrak{B}(\mathcal{M})$. Then the expression

$$P(\mathcal{M}|\mathcal{T}) \;:=\; \frac{\big\langle \Xi \big| E_T(\mathcal{T}) \otimes E_A(\mathcal{M})\, \Xi \big\rangle}{\big\langle \Xi \big| E_T(\mathcal{T}) \otimes \mathbf{1}\, \Xi \big\rangle} \quad , \tag{5.2}$$

describing the probability of the material event $\mathcal{M}$, given the temporal event $\mathcal{T}$, satisfies the defining properties for a *conditional probability* in the sense of Rényi. The quantum correlations between the time system and the material system are given by the *conditional expectation* $\mathcal{E}\{A\,|\,\mathcal{T}\}$ of the material observable $A = \int_{\mathbb{R}} \alpha \, E_A(d\alpha)$, given the temporal event $\mathcal{T}$,

$$\mathcal{E}\{A\,|\,\mathcal{T}\} \;:=\; \frac{\big\langle \Xi \big| E_T(\mathcal{T}) \otimes A\, \Xi \big\rangle}{\big\langle \Xi \big| E_T(\mathcal{T}) \otimes \mathbf{1}\, \Xi \big\rangle} \quad . \tag{5.3}$$

Using the time-independent vector

$$\Xi \;=\; \int_{\mathbb{R}} \Phi_t \otimes \Psi_t \, dt \quad \text{with} \quad \Phi_t \;:=\; e^{-2\pi i t \Lambda}\,|0,0\rangle \;=\; |0,t\rangle \quad ,$$

which describes the state of the timeless universe of discourse, we get

$$\begin{aligned}\big\langle \Xi \big| E_T(\mathcal{T}) \otimes A\, \Xi \big\rangle &= \int_{\mathbb{R}} du \int_{\mathbb{R}} dv \, \big\langle \Phi_u \otimes \Psi_u \big|\, E_T(\mathcal{T}) \otimes A \,\big| \Phi_v \otimes \Psi_v \big\rangle \\ &= \int_{\mathbb{R}} du \int_{\mathbb{R}} dv \, \big\langle \Phi_u \big|\, E_T(\mathcal{T}) \,\big| \Phi_v \big\rangle_{\mathcal{N}} \big\langle \Psi_u \big|\, A \,\big| \Psi_v \big\rangle_{\mathcal{M}} \quad .\end{aligned}$$

We introduce new variables $r$ and $s$ by $u = r - s$ and $v = r + s$, so that

$$\begin{aligned}&\big\langle \Xi | E_T(\mathfrak{T}) \otimes A\, \Xi \big\rangle \\ &= 2 \int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \big\langle \Phi_{r-s} \big| \, E_T(\mathfrak{T}) \, \big| \, \Phi_{r+s} \big\rangle_{\mathcal{N}} \big\langle \Psi_{r-s} \big| \, A \, \big| \Psi_{r+s} \big\rangle_{\mathcal{M}} \ .\end{aligned}$$

Using equation (4.10), we find

$$\Phi_t(x) \ = \ |0, t\,\rangle(x) \ = \ \left( \frac{1}{2\pi\sigma_T^2} \right)^{1/4} e^{-\,\frac{1}{4}(x-t)^2/\sigma_T^2} \quad , \tag{5.4}$$

so that

$$\begin{aligned}\big\langle \Phi_{r-s} \big| \, E_T(\mathfrak{T}) \, \big| \, \Phi_{r+s} \big\rangle_{\mathcal{N}} \ &= \ \big\langle 0, r-s \big| \, E_T(\mathfrak{T}) \, \big| 0, r+s \big\rangle_{\mathcal{N}} \\ &= \ e^{-\,\frac{1}{2}s^2/\sigma_T^2} \left( \frac{1}{2\pi\sigma_T^2} \right)^{1/2} \int_{\mathfrak{T}} dx \, e^{-\,\frac{1}{2}(x-t)^2/\sigma_T^2} \quad .\end{aligned}$$

With this we get

$$\big\langle \Xi | E_T(\mathfrak{T}) \otimes A\, \Xi \big\rangle = 2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} ds \, \big\langle \Psi_{-s} \big| \, \widetilde{A}_r \, \big| \Psi_s \big\rangle_{\mathcal{M}} \, e^{-\,\frac{1}{2}s^2/\sigma_T^2} \int_{\mathfrak{T}} \boldsymbol{\mu}_r(dx) \quad ,$$

where $\widetilde{A}_r$ is defined by

$$\widetilde{A}_r \ := \ e^{irH/\hbar} A \, e^{-irH/\hbar} \quad , \tag{5.5}$$

and $\boldsymbol{\mu}_r(dx)$ is the Gaussian measure

$$\boldsymbol{\mu}_r(dx) \ := \ \frac{e^{-\,\frac{1}{2}(r-x)^2/\sigma_T^2}}{\sqrt{2\pi\sigma_T^2}} \, dx \tag{5.6}$$

with mean $r \in \mathbb{R}$ and variance $\sigma_T > 0$. The conditional expectation $\mathcal{E}\{A \,|\, \mathfrak{T}\}$ of a material observable $A$, given the temporal event $\mathfrak{T}$, is then given by

$$\mathcal{E}\{A \,|\, \mathfrak{T}\} \ = \ \frac{\int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \big\langle \Psi_{-s} \big| \, \widetilde{A}_r \, \big| \Psi_s \big\rangle_{\mathcal{M}} \, e^{-\,\frac{1}{2}s^2/\sigma_T^2} \int_{\mathfrak{T}} dx \, \boldsymbol{\mu}_r(dx)}{\int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \big\langle \Psi_{-s} \big| \Psi_s \big\rangle_{\mathcal{M}} \, e^{-\,\frac{1}{2}s^2/\sigma_T^2} \int_{\mathfrak{T}} dx \, \boldsymbol{\mu}_r(dx)} \quad . \tag{5.7}$$

If $\mathfrak{T}$ is the interval $t - \varepsilon \le x \le t + \varepsilon$, the limit $\varepsilon \to 0$ provides the *conditional expectation $\mathcal{E}\{A \,|\, t\}$ of the material observable $A$ given that the nonmaterial time operator $T$ has the mean value $t$*,[42]

$$\mathcal{E}\{A \,|\, t\} \ = \ \frac{\int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \big\langle \Psi_{-s} \big| \, \widetilde{A}_r \, \big| \Psi_s \big\rangle_{\mathcal{M}} \, e^{-\,\frac{1}{2}(r-t)^2/\sigma_T^2 - \,\frac{1}{2}s^2/\sigma_T^2}}{\int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \big\langle \Psi_{-s} \big| \Psi_s \big\rangle_{\mathcal{M}} \, e^{-\,\frac{1}{2}(r-t)^2/\sigma_T^2 - \,\frac{1}{2}s^2/\sigma_T^2}} \quad . \tag{5.8}$$

[42]Compare Feller (1966), sections III.2, V.9 and V.10.

Using the spectral resolution of the selfadjoint Hamitonian $H$ we can easily evaluate $\langle \Psi_{r-s} | A | \Psi_{r+s} \rangle_{\mathcal{M}}$ as a function of $r$ and $s$, so that we get

$$\int_{\mathbb{R}} dr \int_{\mathbb{R}} ds \, \langle \Psi_{r-s} | A | \Psi_{r+s} \rangle_{\mathcal{M}} \, e^{-\frac{1}{2}(r-t)^2/\sigma_T^2 - \frac{1}{2}s^2/\sigma_T^2} = \langle \Psi | \, e^{-\sigma_T^2 H^2/\hbar^2} \widetilde{A}_r \, e^{-\sigma_T^2 H^2/\hbar^2} \, | \Psi \rangle_{\mathcal{M}} \quad .$$

***Conclusion:*** *The conditional expectation of a material observable $A$, given that the time operator $T$ has the mean value $t$, equals*

$$\mathcal{E}\{A \,|\, t\} = \langle \widehat{\Psi}_t | \, A \, | \widehat{\Psi}_t \rangle_{\mathcal{M}} \quad . \tag{5.9}$$

*The dynamics of the material system is determined by the usual Schrödinger-type equation for the effective state vector $\widehat{\Psi}_t \in \mathcal{N}$*

$$i\hbar \frac{\partial \widehat{\Psi}_t}{\partial t} = H \widehat{\Psi}_t \quad , \quad \widehat{\Psi}_t := e^{-itH/\hbar} \widehat{\Psi}_0 \quad . \tag{5.10}$$

*However, in contrast to traditional quantum mechanics, the initial state vector $\widehat{\Psi}_0$ is not given by $\Psi$, but by*

$$\widehat{\Psi}_0 = e^{-\sigma_T^2 H^2/\hbar^2} \Psi \big/ \| e^{-\sigma_T^2 H^2/\hbar^2} \Psi \| \quad , \tag{5.11}$$

*where the operator $e^{-\sigma_T^2 H^2/\hbar^2}$ describes the effects of the Gaussian time distribution.*

## 5.4 Asymptotically Cartesian Description

The singular case $\sigma_T = 0$ implies a dispersionfree parameter-time. In this exceptional case the algebra generated by the nonmaterial time operator $T$ lies in the center of the algebra of all observables of the unus mundus so that quantum correlations between the nonmaterial and the material system do not exist. This situation corresponds to the so-called *Cartesian separation* of the holistic reality into non-entangled mental and material domains (Primas 1993) which is used in the traditional description of matter in terms of quantum mechanics.

Since in our approach the quantum correlations between the nonmaterial and the material domain are crucial, a Cartesian description is possible only in an asymptotic sense. An expansion in powers of $\sigma_T^2$ around the singular point $\sigma_T = 0$

$$\widehat{\Psi}_0 = \left\{ 1 - \frac{\sigma_T^2}{\hbar^2} H^2 + O(\sigma_T^4) \right\} \Psi \quad ,$$

corresponds to an *asymptotically Cartesian description*. If the variance $\sigma_T^2$ of the time operator $T$ is sufficiently small, the correlations between the nonmaterial and the material domain are small and one finds

$$\left\|\Psi_t - \widehat{\Psi}_t\right\|^2 \;=\; \sigma_T^4 \left\langle \Psi \middle| \left(H^2 - \left\langle \Psi \middle| H^2\, \Psi \right\rangle\right)^2 \Psi \right\rangle \;+\; O(\sigma_T^6) \quad , \tag{5.12}$$

so that in the limit $\sigma_T^4 \langle (H^2 - \langle H^2\rangle)^2\rangle \to 0$ the expectation value $\langle \widehat{\Psi}_t |\, A\, | \widehat{\Psi}_t\rangle_{\mathcal{M}}$ of the timeless formulation approaches the expectation value $\langle \Psi_t |\, A\, |\Psi_t\rangle_{\mathcal{M}}$ of the traditional parameter-time formulation. For the special case of two-level systems the Hamiltonian $H^2$ can be taken as a multiple of the identity, so that we get for all values of $\sigma_T$ the exceptional result $\mathcal{E}\{A \,|\, t\} = \langle \Psi_t |\, A\, |\Psi_t\rangle_{\mathcal{M}}$.

## 6. Tentative Conclusions

Clearly, the proposed ideas are of fragmentary and speculative character so that this essay should be considered as an exercise, whose aim is not to solve any concrete problem but to discuss new ways of thinking about the mind-matter problem. Postulating a holistic conception concerning mind and matter and using the non-Boolean logical framework of quantum theory, we describe mind and matter as complementary aspects of one holistic reality (unus mundus) in such a way that in the limit of a Cartesian separation no results of the traditional quantum theory of matter and radiation are violated. The Leibnizian notion of a noncausal synchronization of tensed nonmaterial and tenseless material aspects of the unus mundus can be implemented by representing the timeless state of the unus mundus as a *maximally entangled state* with respect to a contextual tensorization into a tensed and a tenseless domain. The order parameter characterizing this continuous-parameter entanglement corresponds to Leibniz's "order of non-contemporaneous things". The tensed domain is the carrier of nonmaterial mental phenomena, while the tenseless domain is the carrier of nonmental material phenomena.

Since tensed time is not associated with any material system, all tensed time phenomena are related to the nonmaterial domain. It can be introduced in terms of a Kolmogorov structure, representing a nonmaterial memory which defines chronological order via the growth of the set of mental events. This Kolmogorov structure is associated with a nonclassical *time operator* which induces in the material domain a time variable with a Gaussian distribution. Only in the Cartesian limit this time variable degenerates into a dispersion-free parameter.

Even if the nonmaterial time system does not interact with the material system, it is nevertheless in a nonclassical way strictly correlated with the material part. Due to the perfect noncausal quantum correlations the dynamical aspects of traditional quantum mechanics of matter can be described in terms of the *correlations* and *conditional expectations* between the nonmaterial time system and the material object system. In the Cartesian limit of vanishing correlations between the nonmaterial and the material domain, the usual equations of motion are recovered with an emergent parameter-time as independent variable.

At present, we have insufficient empirical information for a more exhaustive mathematical description of the nonmaterial domain. Even so, many problems which have been considered as difficult or mysterious in traditional philosophical discussions appear transparent in the proposed quantum theoretical framework. As examples we may mention:

- In the proposed non-Boolean description, the concepts time, mind and matter are not given *a priori*. They refer to a symmetry-broken description of the unus mundus.
- Time emerges as a universal ordering principle by breaking the timeless holistic reality into a tensed nonmaterial domain and a tenseless material domain.
- The concept of "nowness" and the direction of time originate in the nonmaterial domain.
- The tensed time of the nonmaterial domain is synchronized with the tenseless time of fundamental physics by quantum correlations.
- The arrow of time is globally unique both in the nonmaterial and in the material domain.

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## References

Augustine A. (1994): *The Confessions of St. Augustine.* Translated and edited by Albert Cook Outler, Dover, New York.

Barbour J. (1999): *The End of Time.* Weidenfeld and Nicolson, London.

Boltzmann L. (1877): Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung respektive den Sätzen über das Wärmegleichgewicht. *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften, Wien* **76**, 373–435.

Carnap R. (1963): Intellectual Autobiography. In *The Philosophy of Rudolf Carnap*, ed. by P.A. Schilpp, Open Court, La Salle, pp. 1–84.

Carrier M. (2003): How to tell causes from effects: Kant's causal theory of time and modern approaches. *Studies in History and Philosophy of Science* **34**, 59–71.

Cornfeld I.P., Fomin S.V., and Sinai Ya.G. (1982): *Ergodic Theory.* Springer, New York.

Cramér H. (1961a): On some classes of non-stationary stochastic processes. In *Proceedings of the Fourth Berkeley Symposium on Statistics and Applied Probability. Volume II: Contributions to Probability Theory*, ed. by J. Neyman, University of California Press, Berkeley, pp. 57–78.

Cramér H. (1961b): On the structure of purely non-deterministic stochastic processes. *Arkiv för Matematik* **4**, 249–266.

Davies P.C.W. (1974): *The Physics of Time Asymmetry.* Surrey University Press, London.

Denbigh K.G. (1970): In defence of the direction of time. *Studium Generale* **23**, 234–244.

Denbigh K.G. (1978): The objectivity, or otherwise, of the present. In *The Study of Time*, ed. by J.T. Fraser, N. Lawrence, and D. Park, Springer, New York, pp. 307–332.

Denbigh K.G. (1981): *Three Concepts of Time.* Springer, Berlin.

Dirac P.A.M. (1927): Relativity quantum mechanics with an application to Compton scattering. *Proceedings of the Royal Society (London)* **A 111**, 405–423.

Doob J.L. (1944): The elementary Gaussian processes. *Annals of Mathematical Statistics* **15**, 229–282.

Eddington A.S. (1928): *The Nature of the Physical World.* Cambridge University Press, Cambridge.

Einstein A. (1916): *Über die spezielle und die allgemeine Relativitätstheorie.* Vieweg, Braunschweig.

Einstein A. and Besso M. (1979): *Correspondance 1903–1955.* Traduction, notes et introduction de Pierre Speziali. Hermann, Paris.

Englert F. (1990): From quantum correlation to time and entropy. *Physicalia Magazine* **12**, Supplement 2, 75–90.

Feller W. (1966): *An Introduction to Probability Theory and its Applications, Volume II.* Wiley, New York.

Flesia C. and Piron C. (1984): La théorie de la diffusion de Lax et Phillips dans le cas quantique. *Helvetica Physica Acta* **57**, 697–703.

Franck G. (2000): Time, actuality, novelty and history. In *Life and Motion of Socio-Economic Units*, ed. by A. Frank, J. Raper, and J.-P. Cheylan, Taylor & Francis, London, pp. 111–134.

Franz, M.L. von (1966): Time and synchronicity in analytic psychology. In *The Voices of Time*, ed. by J.T. Frazer, Braziller, New York, pp. 218–232.

Frege G. (1979): *Posthumous Writings*, ed. by M. Beaney, F. Kambartel and F. Kaulbach, Blackwell, Oxford.

Freud S. (1940a): *Gesammelte Werke. Dreizehnter Band. Jenseits des Lustprinzips. Massenpsychologie und Ich-Analyse. Das Ich und das Es.* Imago Publishing, London.

Freud S. (1940b): *Gesammelte Werke. Fünfzehnter Band. Neue Folge der Vorlesungen zur Einführung in die Psychoanalyse.* Imago Publishing, London.

Gasking D. (1955): Causation and recipes. *Mind* **64**, 479–487.

Gel'fand I.M. and Vilenkin N.Ja. (1964): *Generalized Functions Volume 4. Applications of Harmonic Analysis.* Academic Press, New York.

Gold T. (1967): *The Nature of Time.* Cornell University Press, Ithaca.

Grünbaum A. (1967): The anisotropy of time. In *The Nature of Time*, ed. by T. Gold, Cornell University Press, Ithaca, pp. 149–186.

Grünbaum A. (1971): The status of temporal becoming. In *Time in Science and Philosophy*, ed. by J. Zeman, Elsevier, Amsterdam, pp. 67–87.

Grünbaum A. (1973): *Philosophical Problems of Space and Time.* Second enlarged edition, Reidel, Dordrecht.

Gödel K. (1949): A remark about the relationship between relativity theory and idealistic philosophy. In *Albert Einstein: Philosopher–Scientist*, ed. by P.A. Schilpp, Open Court, Evanston, pp. 557–562.

Gödel K. (1995): *Kurt Gödel. Collected Works. Volume III. Unpublished Essays and Lectures*, ed. by S. Feferman, Oxford University Press, Oxford.

Gustafson K. and Misra B. (1976): Canonical commutation relations of quantum mechanics and stochastic regularity. *Letters in Mathematical Physics* **1**, 275–280.

Halliwell J.J., Pérez-Mercander J., and Zurek W.H. (1994): *Physical Origins of Time Asymmetry.* Cambridge University Press, Cambridge.

Hanner O. (1950): Deterministic and non-deterministic stationary random processes. *Arkiv för Matematik* **1**, 161–177.

Hausman D. (1998): *Causal Asymmetries.* Cambridge University Press, Cambridge.

Hawking S.W. (1988): *A Brief History of Time.* Bantam Press, London.

Hida T. (1960): Canonical representations of Gaussian processes and their applications. *Memoirs of the College of Science, University of Kyoto* **A 33**, 109–155.

Hida T. (1980): *Brownian Motion.* Springer, New York.

Hollinger H.B. and Zenzen M.J. (1985): *The Nature of Irreversibility.* Reidel, Dordrecht.

Horwich P. (1987): *Asymmetries in Time.* MIT Press, Cambridge Ma.

Horwitz L.P. and Piron C. (1993): The unstable system and irreversible motion in quantum theory. *Helvetica Physica Acta* **66**, 693–711.

Isham C.J. (1994): Canonical quantum gravity and the question of time. In *Canonical Gravity: From Classical to Quantum*, ed. by J. Ehlers and H. Friedrich, Springer, Berlin, pp. 150–169.

Jung C.G. (1958): *The Collected Works of C.G. Jung. Volume 11. Psychology and Religion: West and East.* Princeton University Press, Princeton.

Jung C.G. (1969): *The Collected Works of C.G. Jung. Volume 8. The Structure and the Dynamics of the Psyche.* Second edition, Princeton University Press, Princeton.

Jung C.G. (1970): *The Collected Works of C.G. Jung. Volume 14. Mysterium Conjunctionis.* Second Edition, Princeton University Press, Princeton.

Jung C.G. (1975): *The Collected Works of C.G. Jung. Volume 18. The Symbolic Life.* Princeton University Press, Princeton.

Kallianpur G. and Mandrekar V. (1965): Multiplicity and representation theory of purely non-deterministic stochastic processes. *Theory of Probability and its Applications* **10**, 553–581.

Kanthack l. and Wegener U. (1976): Zum Zusammenhang zwischen Projektionsoperatoren und Eigenschaften. *Zeitschrift für allgemeine Wissenschaftstheorie* **7**, 249–257.

Kolmogorov A.N. (1941): Stationary sequences in Hilbert space [in Russian]. *Bjulletin Moskovskogo Gosudarstvenogo Universiteta Matematika* **2**, No. 6, 1–40. (English translation in *Linear Least-Squares Estimation. Benchmark Papers in Electrical Engineering and Computer Science, Vol. 17*, ed. by T. Kailath, Dowden, Huchinson & Ross, Stroudsburg 1977, pp. 66–89).

Kreĭn M.G. (1945a): On a generalization of some investigations of G. Szegö, W.M. Smirnov, and A.N. Kolmogorov. *Doklady Akademii Nauk SSSR* **46**, 91–94 [in Russian].

Kreĭn M.G. (1945b): On a problem of extrapolation of A.N. Kolmogorov. *Doklady Akademii Nauk SSSR* **46**, 306–309 [in Russian].

Lax P.D. and Phillips R.S. (1967): *Scattering Theory.* Academic Press, New York.

Lebowitz J.L. (1993a): Macroscopic laws, microscopic dynamics, time's arrow and Boltzmann's entropy. *Physica* **A 194**, 1–27.

Lebowitz J.L. (1993b): Boltzmann's entropy and time's arrow. *Physics Today* **46**, No. 9, 32–38.

Lewis J.T. and Thomas L.C. (1974): A characterization of regular solutions of a linear stochastic differential equation. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* **30**, 45–55.

Masani P. and Wiener N. (1959): On bivariate stationary processes and the factorization of matrix-value functions. *Theory of Probability and Its Applications* **4**, 322–331.

McTaggart J.E. (1908): The Unreality of Time. *Mind* **17**, 456–473.

Meier C.A. (1950): *Zeitgemässe Probleme der Traumforschung.* Kultur- und Staatswissenschaftliche Schriften der ETH Zürich Nr. 75.

Meier C.A. (1975): Psychosomatik in Jungscher Sicht. In *Experiment und Symbol*, ed. by C.A. Meier, Walter, Olten, pp. 138–156.

Meier C.A. (1988): Science and synchronicity. *Psychological Perspectives* **19**, 320–324.

Mellor D.H. (1998): *Real Time II.* Routledge, London.

Menzies P. and Price H. (1993): Causation as a secondary quality. *British Journal for the Philosophy of Science* **44**, 187–203.

Mermin D.M. (1998a): The Ithaca interpretation of quantum mechanics. *Pramana* **51**, 549–565.

Mermin D.M. (1998b): What is quantum mechanics trying to tell us? *American Journal of Physics* **66**, 753–767.

Minkowski H. (1923): Space and time. In *The Principle of Relativity*, ed. by W. Perrett and G.B. Jeffery, Methuen, London, pp. 73–91.

Newton I. (1960): *The Mathematical Principles of Natural Philosophy.* Translated into English by Andrew Motte in 1729, edited by Florian Cajori. University of California Press, Berkeley.

Page D.N. and Wootters W.K. (1983): Evolution without evolution: Dynamics described by stationary observables. *Physical Review D* **27**, 2885–2892.

Pauli W. (1933): *Die allgemeinen Prinzipien der Wellenmechanik.* In *Handbuch der Physik*, ed. by H. Geiger and K. Scheel, 2. Auflage, Band 24, 1. Teil, Springer, Berlin, pp. 83–272.

Pauli W. (1994): *Writings on Physics and Philosophy*, ed. by C.P. Enz and K. von Meyenn. Springer, Berlin.

Poincaré H. (1899): La mesure du temps. *Révue de Métaphysique et de Morales* **6**, 1–13.

Price H. (1992): Agency and causal asymmetry. *Mind* **101**, 501–520.

Price H. (1996): *Time's Arrow and Archimedes' Point.* Oxford University Press, Oxford.

Primas H. (1993): The Cartesian cut, the Heisenberg cut, and disentangled observers. In *Symposia on the Foundations of Modern Physics 1992. The Copenhagen Interpretation and Wolfgang Pauli*, ed. by K.V. Laurikainen and C. Montonen, World Scientific, Singapore, pp. 245–269.

Raggio G.A. (1988): A remark on Bell's inequality and decomposable normal states. *Letters in Mathematical Physics* **15**, 27–29.

Reichenbach H. (1956): *The Direction of Time.* University of California Press, Berkeley.

Reichenbach H. (1958): *The Philosophy of Space and Time.* Dover, New York.

Ritz W. and Einstein A. (1909): Zum gegenwärtigen Stand des Strahlungsproblems. *Physikalische Zeitschrift* **10**, 323–324.

Rényi A. (1955): A new axiomatic theory of probability. *Acta Mathematica Academia Scientiarum Hungaricae* **6**, 285–335.

Rényi A. (1970a): *Probability Theory.* North-Holland, Amsterdam.

Rényi A. (1970b): *Foundations of Probability.* Holden-Day, San Francisco.

Rovelli C. (1996): Relational quantum mechanics. *International Journal of Theoretical Physics* **35**, 1637–1678.

Rozanov Yu.A. (1967): *Innovation Processes.* Wiley, New York.

Russell B. (1913): On the notion of cause. *Proceedings of the Aristotelian Society* **13**, 1–26.

Savitt S.F. (1995): *Time's Arrows Today.* Cambridge University Press, Cambridge.

Scheibe E. (1973): *The Logical Analysis of Quantum Mechanics.* Pergamon Press, Oxford.

Schrödinger E. (1931): Spezielle Relativitätstheorie und Quantenmechanik. *Sitzungsberichte der Preussischen Akadademie der Wissenschaften, physikalisch-mathematische Klasse* **1931**, 238–247.

Schrödinger E. (1935): Discussion of probability relations between separated systems. *Proceedings of the Cambridge Philosophical Society* **31**, 555–563.

Strauss Y., Horwitz L.P., and Eisenberg E. (2000): Representation of quantum mechanical resonances in the Lax–Phillips Hilbert space. *Journal of Mathematical Physics* **41**, 8050–8071.

Suppes P. (1970): *A Probabilistic Theory of Causation.* North Holland, Amsterdam.

Takesaki M. (1979): *Theory of Operator Algebras I.* Springer, New York.

Tjøstheim D. (1975): Multiplicity theory for multivariate wide sense stationary generalized processes. *Journal of Multivariate Analysis* **5**, 314–321.

Tjøstheim D. (1976a): Spectral generating operators for non-stationary processes. *Advances in Applied Probability* **8**, 831–846.

Tjøstheim D. (1976b): A commutation relation for wide sense stationary processes. *SIAM Journal of Applied Mathematics* **30**, 115–122.

Weyl H. (1922): *Space–Time–Matter.* Methuen, London.

Weyl H. (1927): Quantenmechanik und Gruppentheorie. *Zeitschrift für Physik* **46**, 1–46.

Weyl H. (1928): *Gruppentheorie und Quantenmechanik.* Hirzel, Leipzig.

Weyl H. (1949): *Philosophy of Mathematics and Natural Science.* Princeton University Press, Princeton.

Wiener P.P. (1951): *Leibniz Selections.* Charles Scribners's Sons, New York.

Wigner E.P. (1932): Über die Operation der Zeitumkehr in der Quantenmechanik. *Nachrichten von der Gesellschaft der Wissenschaften Göttingen, Mathematisch Physikalische Klasse* **31**, 546–559.

Wigner E.P. (1959): *Group Theory.* Academic Press, New York.

Wold H. (1938): *A Study in the Analysis of Stationary Times Series.* Almquist and Wiksell, Stockholm.

Wootters W.K. (1984): "Time" replaced by quantum correlations. *International Journal of Theoretical Physics* **23**, 701–711.

Zeh H.D. (1999): *The Physical Basis of the Direction of Time.* 3rd. ed., Springer, Berlin.

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