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# EMERGENCE IN EXACT NATURAL SCIENCES

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## Abstract

The *context* of an operational description is given by the distinction between what we consider as relevant and what as irrelevant for a particular experiment or observation. A rigorous description of a context in terms of a mathematically formulated context-independent fundamental theory is possible by the restriction of the domain of the basic theory and the introduction of a new coarser topology. Such a new topology is never given by first principles, but depends in a crucial way on the abstractions made by the cognitive apparatus or the pattern recognition devices used by the experimentalist. A consistent mathematical formulation of a higher-level theory requires the closure of the restriction of the basic theory in the new contextual topology. The validity domain of the so constructed higher-level theory intersects nontrivially with the validity domain of the basic theory: neither domain is contained in the other. Therefore, higher-level theories cannot be totally ordered and theory reduction is not transitive. The *emergence* of qualitatively new properties is a necessary consequence of such a formulation of theory reduction (which does not correspond to the traditional one). Emergent properties are not manifest on the level of the basic theory, but they can be derived rigorously by imposing new, contextually selected topologies upon context-independent first principles.

Most intertheoretical relations are mathematically describable as *singular* asymptotic expansions which do not converge in the topology of the primary theory, or by choosing one of the infinitely many possible, physically inequivalent representations of the primary theory (Gelfand–Naimark–Segal-construction of algebraic quantum mechanics). As examples we discuss the emergence of shadows, inductors, capacitors and resistors from Maxwell's electrodynamics, the emergence of order parameters in statistical mechanics, the emergence of mass as a classical observable in Galilei-relativistic theories, the emergence of the shape of molecules in quantum mechanics, the emergence of temperature and other classical observables in algebraic quantum mechanics.

## Introduction

The problem of emergence in the exact natural sciences is related to the problem of reductionism. Philosophers of science usually consider reduction as a *logical relation* between theories, and they focus on the *deducibility* of one theory from another.[1] However, *there exists not a single physically well-founded and nontrivial example for theory reduction in the sense of* Hempel & Oppenheim (1948), Kemeny & Oppenheim (1956), Oppenheim & Putnam (1958), or Nagel (1961). The ideas discussed by these philosophers rest on much too simple a view about the structure of scientific theories describing the physical world. The standard examples for reduction and emergence discussed by the philosophers are usually taken from physical sciences, but they do not fulfill the legitimate requirements of conceptual and mathematical rigor of physics. The link between fundamental and higher-level theories is far more complex than presumed by most philosophers.

[1] For example, Nagel (1961), p. 352, defines: "A reduction is effected when the experimental laws of the secondary science (and if it has an adequate theory, its theory as well) are shown to be logical consequences of the theoretical assumptions (inclusive of the coordinating definitions) of the primary science." More reasonable variants require only that the higher-level theory can be *deduced* from the basic theory together with appropriate initial and boundary conditions, together with so-called "bridge laws" conditions. However, all these variants are not well-defined and of no use for a scientist.

*First example: The link between chemistry and physics is not a logical relation*

In their famous paper on reductionism, Kemeny and Oppenheim write: "… a great part of classical chemistry has been reduced to atomic physics."[2] No references for a proof are given. Evidently, the authors have no idea what they are claiming. For example: How are the non-linear differential equations of chemical kinetics derived from linear equations of motion of quantum mechanics? Or: What is the relation between chemically pure substances (like liquid water) and molecules (like $H_2O$, $(H_2O)_2$, $(H_2O)_3$, …)? Most certainly liquid water is not simply composed of molecules $H_2O$. The definition "substance x is chemically pure if and only if x is composed exclusively of either atoms or molecules of a single species or kind", as proposed by Mario Bunge[3] is not only factually untenable. It also represents a classical category mistake since it tries to explain the purity concept used for chemical substances by the purity concept relevant for a category of a different logical type.[4]

*Second example: The link between thermodynamics and mechanics is not a logical relation*

Kemeny and Oppenheim maintain: *"… the classical theory of heat has been reduced to statistical mechanics."*[5] Moreover, Ernest Nagel[6] claims that the concept of temperature can be reduced to statistical mechanics by a *mere definition*, namely by the mean kinetic energy $\frac{3}{2}kT$ of gas molecules. This example – cited again and again by philosophers[7] – shows an incredible ignorance of the most elementary concepts of physics. Clearly, temperature is *not* a mechanical or molecular concept.[8] It is defined via the fundamental zeroth principle of thermodynamics: *if two systems are both in equilibrium with a third system then they are in equilibrium with each other.* Systems fulfilling this highly nontrivial transitivity relation "have a property in common, namely the property of being in thermal equilibrium with each other. This property is called *temperature*."[9] It is plain that the zeroth principle cannot be derived from the principles of mechanics. Just as uninformed is the view of Lawrence Sklar that we have to "discard the thermodynamic notions of temperature and entropy." He claims: "The classical thermodynamics of Clausius, Kelvin and Carathéodory is simply an incorrect theory of the word. And not merely incorrect in a simple 'numerical' way, in which it is, in some sense, conceptually sound but just a little off in some predicted values. It is a 'conceptually incorrect' theory."[10]

These examples show that for a sensible discussion of the difficult problem of theory reduction we have to avoid handweaving arguments. So we require that a higher-level theory has to be deduced *rigorously and completely* from a basic theory, indicating within the adopted mathematical formalism *precisely all necessary auxiliary conditions*. Such a course of action is worth the effort since it turns out that *emergence and theory reduction are related.*

Even in the most recent and very careful exposition of the logical reduction of physical theories, Erhard Scheibe (1997) misses the opportunity to discuss the close relationship between reduction and emergence. For example, while his (unnecessarily simplified) discussion of the motion of electrons in the Coulomb field of nuclei with an almost fixed position is correct, it is restricted to the trivial comparison of the exact and approximative eigenvalues of the energy[11]. If theory reduction is related to a mature and fully mathematically formulated theory (in contrast to particular theoretical problems), then a mathematically rigorous discussion puts in evidence its singular character. Scheibe's example then turns out to be based on a singular asymptotic expansion which naturally leads to symmetry breaking and the emergence of qualitatively new properties. In molecular physics and chemistry such a procedure is well known under the name *Born–Oppenheimer picture*.[12] From a physical point of view the crucial point of a Born–Oppenheimer description is *not* a simplification of the mathematical problem, but the replacement of the basic theory by a related but qualitatively new one. Since nonconvergent singular asymptotic expansions usually yield very accurate numerical values, the fact that the new higher level theory has only approximate validity is often unimportant.

## Intrinsic and operational descriptions

According to George Spencer-Brown every description is based on the primitive act of a *distinction*.[13] A distinction splits the a universe of discourse into two parts and generates the fundamental duality of inside and outside.[14] A distinction allows an indication: the

[2] Kemeny & Oppenheim (1956), p. 7.

[3] Bunge (1985), p. 225.

[4] A correct discussion of the purity concept of chemical substances is anything but simple and hard to be found in in the literature. A most illuminating discussion is contained in the lectures on thermostatics by van der Waals (1927), §5, §92, §93.

[5] Kemeny & Oppenheim (1956), p. 7.

[6] Nagel (1961), chapt. 11.

[7] Compare for example Bunge (1973), pp. 182–183, or Scheibe (1988), p. 155.

[8] Compare also Feyerabend (1962).

[9] Guggenheim (1949), p. 6. See also Born (1921), §2.

[10] Sklar (1976). The quotations are from p. 30 and p. 16.

[11] Scheibe (1997), p. 215–218.

[12] Born & Oppenheimer (1927).

[13] Spencer-Brown (1969). Compare also Varela (1979), chapt. 11.

[14] A review on inside/outside concepts in physics, philosophy and cognitive science can be found in Atmanspacher & Dalenoort (1994).

valuation and marking of the two distinct parts. Every distinction has a purpose but no a priori meaning: "There can be no distinction without motive, and there can be no motive unless contents are seen to differ in value."[15] In our context, we denote the distinct but interlocked parts as *object* and *environment*. Such a distinction creates a frame of reference, necessary for any kind of cognition and description. As Suzanne Langer observed "Our world 'divides into facts' because we so divide it."[16] For physical theories we may rephrase this statement as follows: *the distinction inside/outside is not covered by the most fundamental context-independent natural laws (first principles of physics).*

If we observe certain aspects of reality, we inevitably have to ignore certain other aspects of it so that every recognition of particular features has to be paid for by a loss of other features. Different points of view correspond to different abstractions. Concepts are introduced by neglecting elements that are considered as nonessential, and by accentuating features which are considered as essential for the chosen investigation. There exist mutually exclusive abstractions which provide alternative descriptions of the very same system. Such incompatible descriptions are not logically contradictory because they refer to different abstractions and pattern recognition methods.

The first principles of physics are intended to have universal validity and to give – as far as possible or reasonable – a *context-independent* description of the material world. Such an *intrinsic description* makes no reference to other physical systems. It is not operational since every operationally meaningful description we can give requires the abstractions from irrelevant features. By contrast, an *operational description* refers to empirical observations obtained by some pattern recognition methods which concentrate on those aspects we consider as relevant. If we isolate a phenomenon and assign individuality to it, we create an entity which we call a pattern. Put pointedly: "A pattern is something which somebody recognizes as a pattern."[17]

Recognition of patterns is an activity both in living and in technical systems. What is rejected as irrelevant is neither determined by subjective beliefs nor by physical laws. In living systems it is determined by the historical biological evolution, in technical systems by design principles. In both cases, patterns are created by rejecting *irrelevant* information. Since there exist no unprejudiced sense data or unbiased observations without data processing, *pattern recognition is not a purely logical process.* Observable patterns of the world do not exist in themselves. They come into being only through abstractions and contextual symmetry breakings. Yet, patterns do not arise like bolts from the blue. They reflect some structural features of the intrinsic reality. A phenomenological theory cannot be related directly to a fundamental theory since observable patterns depend on *both* the intrinsic properties of the world *and* the abstractions associated with the pattern recognition procedure. Besides of first principles, one has in addition use the very same abstractions as the pattern recognition methods of empirical science do.

This situation implies that the interrelations between intrinsic and operational descriptions are highly nontrivial. First principles are always exceedingly remote from our everyday experience – they cannot be deduced directly from experiments. The historical development of physics establishes that *symmetry* is an effective criterion for selecting first principles. On the other hand, all phenomenological theories are context-dependent, they cannot be deduced from context-independent first principles without taking into consideration the abstractions evoked by the pattern recognition devices necessary for the observation of phenomena. If one disregards this essential difference between context-independent fundamental laws and the context-dependent operational descriptions, one can hardly avoid Bas van Fraassen's conclusion that we should discard the idea that there are "laws of nature"[18], or Nancy Cartwright's verdict that "the fundamental laws of physics do not describe true facts about reality", and that "the fundamental laws patently do not get the facts right", but that "phenomenological laws are indeed true of the objects in reality".[19] The referents of fundamental laws and of phenomenological theories are categorically different: fundamental theories refer to the independent reality, while phenomenological theories refer to empirical reality.[20] Nonetheless, there exist most fertile relations between context-independent and context-dependent theories.

[15] Spencer-Brown (1969), p.1.
[16] Langer (1942), p.273.
[17] Giuliano (1967).

[18] Van Fraassen (1989).
[19] Cartwright (1983). The quotations are from p.54, p.3, and p.4.
[20] The concepts "independent reality" and "empirical reality" have been introduced Bernard d'Espagnat. Compare for example D'Espagnat (1995), chapters 14 and 15.

## On the topological characterization of a context

Provided we can describe precisely what we consider as relevant and what as irrelevant, what our deliberate lack of interest is, then we can *add* the very same abstractions to the fundamental first principles. In a mathematical description this additional information requires some concept of *nearness* which in turn leads to a *topological structure*.[21] Topology is the study of continuity within the framework of mathematics. Its objective is to investigate those properties of mathematical objects that are preserved under one-to-one bicontinuous transformations (so called homeomorphisms). Bicontinuous transformations map neighboring points into neighboring points, while distinct points are not crushed together.

Let $\varphi$ map a set $\boldsymbol{\Omega}$ into another set $\boldsymbol{\Omega}'$. Let $p$ be a point in $\boldsymbol{\Omega}$ and let $p' := \varphi(p)$ be the corresponding point in $\boldsymbol{\Omega}'$. Then the mapping $\varphi$ is said to be *continuous* if the points near to $p$ map into points near to $p'$. That is, a mapping is continuous if it preserves *nearness*. The notion of *nearness* is not intrinsic, it is defined by specifying a *topological structure* (often simply called a *topology*), defined by specifying the *neighborhood* of each point. In topology a point is said to be "near" to a set if the point is in the set or is a limit point of the set. In the latter case one says that the point belongs to the *closure* of this set. A topological space is a set with a topology imposed on it. More precisely, a topological space is a set $\boldsymbol{\Omega}$ (whose elements are called *points*) and a function (called *closure*) assigning to each set $\mathscr{X} \subset \boldsymbol{\Omega}$ a set $\overline{\mathscr{X}} \subset \boldsymbol{\Omega}$ satisfying the axioms $\mathscr{X} \subset \overline{\mathscr{X}}$, $\overline{\mathscr{X} \cup \mathscr{Y}} = \overline{\mathscr{X}} \cup \overline{\mathscr{Y}}$, $\overline{\varnothing} = \varnothing$, and $\overline{\overline{\mathscr{X}}} = \overline{\mathscr{X}}$. On a given set there can exist many distinct topologies, so that we may wish to compare them. To this end, we introduce a partial order into the class of all topologies on $\boldsymbol{\Omega}$. We say that a topology with the closure function $\mathscr{X} \to \overline{\mathscr{X}}$ is finer than that given by the closure function $\mathscr{X} \to \overline{\mathscr{X}}'$ if $\overline{\mathscr{X}} \subset \overline{\mathscr{X}}'$ for each $\mathscr{X} \subset \boldsymbol{\Omega}$. It is called coarser if $\overline{\mathscr{X}} \supset \overline{\mathscr{X}}'$ for each $\mathscr{X} \subset \boldsymbol{\Omega}$.[22] Since there are topologies which cannot be compared, the class of all topologies cannot be totally ordered.

In a mathematically formalized theory a context can be introduced by choosing a new coarser *contextual topology* which is compatible with the finer topology of the more fundamental theory. This new topology is never given a priori but depends in a crucial way on the abstractions made by the cognitive apparatus or the pattern recognition devices used by the experimentalist. The closure of such a higher-level description in the chosen contextual topology generates new context-dependent features which are not already present in the fundamental description. In this mathematically precise sense, one can speak of the *emergence of novelty* in descriptions of a higher level. The phenomenological theories corresponding to different branches of empirical science can be related to one and the same basic theory by introducing appropriate contextual topologies. The richness and variety of the phenomenological theories are due to the myriads of possibilities to choose coarser contextual topologies compatible with the topology of the underlying fundamental theory.

Given the contextual topology corresponding to a phenomenological description, the corresponding phenomenological theory can rigorously be derived from a more fundamental theory. Thereby, the specification of the context is at least as important as the first principles. If the derived theory would not be closed in the new contextual topology, the validity domain of the derived theory would be contained in the validity domain of the basic theory. To get a mathematically complete and consistent theory, one has to close the derived theory in the new topology. The result is that the validity domain of the derived theory and of the basic theory intersects nontrivially. Neither domain is contained in the other. Although higher-level theories can be derived rigorously from a more fundamental theory, they are not subtheories of the basic theory. Therefore, higher-level theories cannot be totally ordered, and theory reduction is not transitive.[23]

## Asymptotic descriptions

One of the most powerful tools to introduce new contextual descriptions is the use of singular asymptotic expansions.[24] An asymptotic expansion depicts

[21] Compare Herrlich (1974), Cameron, Hocking & Naimpally (1974).

[22] Compare Kuratowski (1966), §4 and §13.

[23] The traditional view claims the contrary. For example Steven Weinberg (1988) takes "for granted a kind of transitivity. If a large number of experimental facts a, b, c, … are explained by a set of theories X, Y, … , and then these theories are in turn explained by a more satisfying theory Z, then I would say that the facts a, b, c, … are explained by theory Z even though we already understood them in terms of the previous theories X, Y, … ." For a more detailed discussion of the nontransitivity of general theory reductions, compare Primas (1977).

[24] A similar point of view has been discussed by Michael Berry (1995).

the behavior of a function near the boundary of its domain of definition. For example, we may be interested in the behavior of an analytic function $z \mapsto f(z)$ as $z \to \infty$. If $f$ has a removable singularity or a pole, this behavior is trivial. Nontrivial asymptotic expansions deal with functions that have either an essential singularity at infinity or are defined merely in some angular region extending to infinity. A *divergent* series $f_0 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3} + \cdots$ is said to be a asymptotic expansion of a function $z \mapsto f(z)$ if for every $n = 0, 1, 2, \cdots$

$$\lim_{z \to \infty} \{ z^n [ f(z) - f_0 - f_1 z^{-1} - f_2 z^{-2} - \cdots - f_n z^{-n} ] \} = 0$$

for $n$ fixed, even though

$$\lim_{n \to \infty} \{ z^n [ f(z) - f_0 - f_1 z^{-1} - f_2 z^{-2} - \cdots - f_n z^{-n} ] \} = \infty$$

for $z$ fixed. When this is the case, we can make

$$\left| z^n [ f(z) - f_0 - f_1 z^{-1} - f_2 z^{-2} - \cdots - f_n z^{-n} ] \right|$$

arbitrarily small by taking $|z|$ sufficiently large. One then writes $f(z) \sim \sum_{n=0}^{\infty} f_n z^{-n}$ and says that the series is a *singular asymptotic expansion* of the function $z \mapsto f(z)$ for $z \to \infty$ in the sense of Poincaré.[25]

Many intertheoretical relations can be mathematically described by asymptotic expansions. *Singular* asymptotic expansions are never uniformly convergent in the intrinsic topology of the basic theory. This nonuniformity is not a disaster but an indication that the limiting case represents a caricature, suppressing irrelevant details and enhancing contextually relevant features. The discontinuous change in the limit leads to a discontinuous change in the semantics and therewith to a description in a new language in terms of emergent properties. In the same sense as a photograph can never replace a brilliant caricature, an asymptotic description can – for the intended purpose – be more adequate than the exact description.

A singular asymptotic expansion depends on a parameter (or several parameters) so that the description behaves nonuniformly as the parameter tends towards some limiting value. For a consistent new description one seeks a uniformly valid reformulation. Such a regularization of a singular asymptotic description often requires a rescaling of the time variable. A powerful analytic tool to achieve this goal is the so-called multiple-time-scale perturbation theory.[26] The essence of this method is the rescaling of the relevant observables together with the time variable. If the singular asymptotic expansion can be formulated with respect to a dimensionless positive parameter $\varepsilon$ ( $\varepsilon = z^{-1} \downarrow 0$), one can replace the original time variable $t$ by a "fast time" $\tau_0 := t$, a "slow time" $\tau_1 := \varepsilon t$, a still "slower time" $\tau_2 := \varepsilon^2 t$, and so on. The time dependence of a physical quantity $X$ can then be written in the rescaled form as

$$X(t) = X_0(\tau_0, \tau_1, \tau_2, \cdots) + \varepsilon X_1(\tau_0, \tau_1, \tau_2, \cdots) + \varepsilon^2 X_2(\tau_0, \tau_1, \tau_2, \cdots) + \cdots ,$$

where $\tau_0, \tau_1, \tau_2, \ldots$ are considered as *independent* variables so that the time derivative of $X(t)$ is given by

$$\frac{\partial X(t)}{\partial t} = \frac{\partial X_0}{\partial \tau_0} + \varepsilon \left\{ \frac{\partial X_0}{\partial \tau_1} + \frac{\partial X_1}{\partial \tau_0} \right\} + \varepsilon^2 \left\{ \frac{\partial X_0}{\partial \tau_2} + \frac{\partial X_1}{\partial \tau_1} + \frac{\partial X}{\partial \tau_0} \right\} + \cdots .$$

The requirement that the multiple-time expansion is uniformly valid for all times (no "secular terms" in the language of astronomers) leads to a hierarchy of equations of motion for the rescaled quantities $X_0$, $X_1$, $X_2$, … and therewith to a hierarchical structure characterized by different characteristic time scales. Often, the separation of time scales is of fundamental importance to understand a complex system. However, it would not be correct to say that the system *is* structured hierarchically. It is the new viewpoint with its associated abstractions which generates the higher levels of a hierarchical description. *The task of higher-level descriptions is not to approximate the fundamental theory but to represent new patterns of reality.*

## Atomism is not the proper starting point for theory reduction

Present-day natural science is still strongly influenced by atomism and tries to analyze the material reality in terms of some *elementary* building blocks. For example, discussing the relationship between biology and the physical sciences, Franciso Ayala maintains that a reductionist defends the position "that organisms are ultimately made up of the same atoms that make up inorganic matter, and of nothing else."[27] However, the key ideas of reductionism and atomism should not be muddled. Reductive atomism is the dated belief that ultimately everything comes down to the mechanics of independently existing atoms. Typical for this view is the following statement by Emil du Bois-Reymond of 1872:

> "Denken wir uns alle Veränderungen in der Körperwelt in Bewegungen von Atomen aufgelöst, die durch deren konstante Zentral-

[25] Poincaré (1886).

[26] Compare for example Nayfeh (1973), chapter 6.

[27] Ayala (1983), p. 526.

kräfte bewirkt werden, so wäre das Weltall naturwissenschaftlich erkannt."[28]

The historical idea that the material world is already structured by some kind of interacting "atoms" is in sharp contradiction to basic insights suggested by quantum mechanics. According to quantum theory the material world is a whole, *a whole which is not made out of independently existing parts*. As a rule, separated subsystems of a quantum system do not exist.[29] Matter as described by first principles is not a substance but the carrier of patterns. Quarks, photons, electrons, atoms, or molecules are not building blocks, they are contextual objects without an independent existence. Questions such as "Do organisms exhibit properties other than those of their constituent atoms and molecules?"[30] are ill-posed since most certainly organisms are *not made out of atoms and molecules*, even if it should turn out that all properties of organisms can be explained in terms of quantum mechanics. The assertions

- "something *consists of* elementary systems",
- "something *can be decomposed* into elementary systems",
- "something *can be described* in terms of elementary systems",

are not equivalent. According to quantum mechanics the first statement is almost always false, the second one is often true, and the third one may be sensible. Under appropriate conditions a material whole can be *described* in terms of parts but there is a great variety of feasible non-isomorphic decompositions of the whole into parts. Non-isomorphic decomposition lead to different descriptions, reflecting different perspectives. That is, in quantum theory we are naturally led to a *multitude* of inequivalent descriptions. They are not only admissible, but they are equally entitled and necessary. The proper choice is not given a priori but depends on the chosen context.

Elementary systems (usually, but misleadingly called "elementary particles") are merely auxiliary group-theoretically indecomposable structures without any direct ontological meaning. They are just conceptual carriers of ergodic representations of the appropriate kinematical symmetry group (like the group SU(3), the Poincaré group, or the Galilei group). In the jargon of quantum mechanics such group-theoretically defined elementary systems are called *bare systems*, they are idealized as objects without an environment – a quite outrageous and incongruous theoretical construct. At least such bare systems have no operational meaning. A bare elementary system carrying a mass and an electrical charge (like a bare electron) inevitably interacts with the gravitational and the electromagnetic field generated by itself.

The state representing an electron *as actually observed in the laboratory* is called a *dressed electron*, it has a very complicated structure. Without an appropriate concept of an environmental background the concept of an individual quantum object makes no operational sense. Heuristically, a dressed electron can be thought of as consisting of the bare electron, interacting with its own radiation field by emitting and reabsorbing virtual photons. The presence of a virtual cloud does not only modify the properties of the bare elementary system and its bare environment, but also its dynamics. A dressed object is not only adapted to its environment. Its individuality *emerges* as by the interplay with the environment. This applies to electrons as well as other objects such as atoms and molecules.

## First example: Shadows, inductors, and capacitors in Maxwell's electrodynamics

Singular asymptotic expansions are always mathematically very delicate, but if executed properly they may lead to amazing insights which are outside the scope of ad hoc approximations. The necessary standard of mathematical rigor has been set by Kurt O. Friedrichs (1955) with his general analysis of asymptotic phenomena in mathematical physics.

When a light wave passes an object, a typical discontinuity – called the *shadow* – can be observed. However, in Maxwell's electrodynamics – the fundamental theory for the propagation of light – shadows do not exist. Maxwell's electrodynamics is governed by partial differential equations which have only *continuous* solutions. The discontinuities associated with shadows appear only in geometric optics, the limiting case of vanishing wavelength $\lambda$, $\lambda \to 0$. An appropriate asymptotic expansion of the Max-

[28] Du Bois–Reymond (1872).

[29] From an operational viewpoint, two physical objects are considered as *separated* if and only if an experiment performed on one of the systems does not change the state of the other systems. It is a mathematical property of classical mechanics that the individual states of the subsystems of a physical system determine the individual state of the whole system. This property is called *separability*. A physical system is called *holistic* if it does *not* possess the property of separability. Quantum mechanics is the first mathematically formalized holistic theory we know of.

[30] Ayala (1983), p. 525.

well equation with the expansion parameter $\varepsilon = \lambda$ is discontinuous at $\varepsilon = 0$, leading to the pattern "shadow". The boundary of the shadow is an emergent pattern of this singular asymptotic description. In the fundamental Maxwell theory there is just a continuous transition from light to darkness which takes place across a very narrow strip along the shadow boundary of geometric optics.

It would be misleading to consider geometric optics just as an approximation to Maxwell's electrodynamics. These two theories are related but *qualitatively* different. The task of geometric optics is not to approximate Maxwell's electrodynamics but to describe optical phenomena in terms of the higher-level language of geometry. In many respects geometric optics is much more powerful than the more fundamental electrodynamics of Maxwell. Traditional geometric optics is the standard tool for the construction of optical instruments. It is described by the leading term in the asymptotic short-wavelength expansion. Diffraction effects are not included in geometric optics, but they are taken care of in the higher order terms of the asymptotic expansion. The asymptotic description by geometric optics is not just a convenient tool in the mathematical analysis of light. Friedrichs writes:

> "On the one hand, discontinuities appear to play a secondary role, namely when they are considered as approximate descriptions of continuous phenomena involving quick transitions. On the other hand, discontinuities play a primary role. For, the experimental description of nature and the theoretical description based on it involves objects with more or less sharp outlines. Therefore, nature could not be described in this way if natural objects did not possess sharp outlines, i.e. discontinuities. In other words, the quantities employed to describe nature could not even be defined if discontinuities did not occur. In this sense, discontinuities appear to play a primary role. It may be debated whether or not this situation involves a vicious or a nonvicious circle. In any case, one may say that asymptotic description is not just a matter of imperfection, but is an essential element in the mathematical description of nature."[31]

While geometric optics is the first term of a high-frequency asymptotic expansion of Maxwell's equations around the singular point $\nu = \infty$, $\nu := c/\lambda$, electrical network theory arises as a low-frequency asymptotic expansion around the singular point $\nu = 0$. The concept of a voltage difference as used in electrical circuit theory is defined in terms of field quantities if and only if the field is electrostatic, otherwise the line integral of the electric field intensity is dependent on the path of integration. Such electric fields, characterized by a vanishing energy loss by radiation, are called quasi-static. They are described by neglecting of the displacement current $\partial \boldsymbol{D}/\partial t$ in the Maxwell equation $\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D}/\partial t$ for the magnetic induction field vector $\boldsymbol{H}$. For electrical networks the vector current density $\boldsymbol{J}$ is a function of the electric field vector $\boldsymbol{E}$, $\boldsymbol{J} = \boldsymbol{J}(\boldsymbol{E})$. Many media are isotropic, homogeneous and linear so that throughout a wide range of conditions Ohm's law may be assumed, $\boldsymbol{J} = \sigma \boldsymbol{E}$, where the scalar $\sigma$ is the conductivity of the medium. If a typical length $\ell$ of the system is small compared to the wavelength $c/\nu$, the propagation delay $\ell/c$ is an irrelevant factor. Under these conditions Kirchhoff's laws for electrical networks can be derived from Maxwell's equations by going to the limit of infinitely thin conductors (wires). It is not trivial, but possible to find in a mathematically rigorous way a singular asymptotic expansion of Maxwell's electrodynamics with the expansion parameter $\varepsilon = \nu$, whose dominant term is characterized by the equation of motion of electrical circuit theory.[32] The network elements are characterized by the differential equations $v_L = L\, d j_L/dt$, $j_C = C\, dv_C/dt$, $v_R = R\, j_R$, where $v_L$, $v_C$, $v_R$ are the voltages across the network elements, and $j_L$, $j_C$, $j_L$ the currents through the network elements. The constant $L$ is called the self inductance of the emergent circuit element "inductor", the constant $C$ is called the capacitance of the emergent circuit element "capacitor", while the constant $R$ is called the resistance of the emergent circuit element "resistor".

## Second example: Phase transitions and order parameters

It is well known that a theoretical description of phase transitions is possible in terms of statistical mechanics. In the basic microscopic description the interactions are translation and rotation invariant and given by gauge fields. The correct description of phase transitions requires the spontaneous breakdowns of some of the mentioned symmetries. For example, ferromagnetism involves the spontaneous breakdown of

[31] Friedrichs (1955), pp. 495–496.

[32] Compare for example Magid (1972), chapter 9; Carey & O'Brien (1986).

the rotation symmetry, crystallization realizes the spontaneous breakdown of the translation and rotation symmetry, superfluidity is related to the breakdown of the special Galilei symmetry, while superconductivity is connected with the spontaneously broken gauge symmetry. However, it can be proved that both in classical and in quantum mechanics phase transitions and spontaneously broken symmetries exist only for systems having infinitely many degrees of freedom. It has been argued that actual physical systems have only finitely many degrees of freedom so that one may ask why in a theoretical description we should use infinite systems.

An analogous question could be posed in electronics: why do we not restrict ourselves to networks with finitely many resistors, capacitors and inductors, but discuss transmission lines in terms of transfer functions with line singularities? Or even in arithmetic: why do we not restrict ourselves to a finite number system? The answer is: *In many respects the infinite is simpler than the very large but finite*. Usually the mathematical discussion of physical phenomena is based on the *uncountably infinite* set of real numbers. For all practical purposes such an analysis gives results which are physically equivalent to a computer calculation based on a large but *finite* number system. However, a transfinite analysis is much simpler. For example, a circle is much simpler to be characterized than an approximating polygon with very many edges.

If one tries to describe macroscopic systems in terms of Hamiltonian mechanics of a microscopic system with very many mechanical degrees of freedom, one is confronted with a severe conceptual difficulty. It turns out that the exact description of systems with myriads of degrees of freedom is *not robust*. That is, if we start with a finite system with $N$ degrees of freedom and make $N$ larger and larger, some expectation values become extremely sensitive with respect to small changes of the model or to very small external perturbations. If the mathematical model for the system is not robust, it deteriorates *qualitatively* for small deviations from a chosen model. Therefore the environment of a macroscopic system can never be left out of consideration, and one has to consider open systems.

The inclusion of the interactions with the environment can have dramatic qualitative effects such as symmetry breakdown and the emergence of qualitatively new properties. However, these interactions and the state of the environment are never precisely known. Fortunately, perfectly detailed information is unnecessary to get a robust description. Nonrobust models can be investigated with Bogoliubov's method of quasi-averages.[33] One considers an expectation value $\langle A\rangle_{N,\varepsilon V}$ of an observable $A$ of a system with $N$ degrees of freedom and the Hamiltonian $H+\varepsilon V$. Here $H$ is the Hamiltonian of the object under study and $\varepsilon V$ ($\varepsilon>0$) is an arbitrary external perturbation. The physically correct limit for a very large number of degrees of freedom is then taken to be $\lim_{\varepsilon\to 0}\lim_{N\to\infty}\langle A\rangle_{N,\varepsilon V}$. Usually this limit depends in a critical way on the choice of the perturbation $V$, and it is different from the unphysical limit $\lim_{N\to\infty}\lim_{\varepsilon\to 0}\langle A\rangle_{N,\varepsilon V}$. Different perturbations lead to different limits, but none of them are a priori distinguished. Depending on the choice of the arbitrarily small perturbation one gets as a rule infinitely many physically inequivalent representations with the associated qualitatively new features. Since robustness is a continuity property, the proper choice of one of the many inequivalent representations is tantamount to a selection of a contextual topology. The introduction of a new contextual topology regularizes the Hamiltonian model and generates a new level of description.

For infinite systems phase transitions are perfectly sharp. Such an idealized description fits the experimental data well but should be considered just as the first term of an asymptotic description. For example, ferromagnets show a second-order phase transition which is idealized as a singularity at the critical Curie temperature $T_{\text{Curie}}$. In this description, for $T<T_{\text{Curie}}$ there is a spontaneous magnetization which decreases with increasing temperature and vanishes at the Curie temperature $T_{\text{Curie}}$ Moreover, the magnetization has a precisely defined distinguished *direction* so that one speaks of a *spontaneous symmetry breaking.* In a more realistic description one has to turn to a thermodynamic theory of large but finite systems which can be expressed by an asymptotic expansion of the thermodynamic functions. For example, the Gibbs free energy $G$ may have an asymptotic expansion of the form[34]

$$G(P,T,N) = g(P,T)\,N + a(P,T)\,N^{3/2} + b(P,T)\ln N + \dots\ ,$$

where $N$ is the number of molecules. The first term is the free energy in the limit $N\to\infty$ in the proper contextual topology. The second term may be a surface energy. The additional terms of the order $N^{3/2}$

[33] Bogoliubov (1970).
[34] Hill (1963).

or $\ln N$ will be noticeable at phase transitions and are the reason why phase transitions are not completely sharp but blurred by fluctuations. Such soft phase transitions are linked to approximate order parameters. Although for finite systems the traditional thermodynamic concepts of "phase transition", "order parameter" and "symmetry breakdown" become somewhat fuzzy, they are qualitatively significant for large enough $N$. Thus, by idealizations we can establish qualitatively new phenomena (e.g. phase transitions) and qualitatively new properties (e.g. order parameters) which are appropriate for the description of empirical phenomena. That is, in the *appropriate contextual topology* one can approximate an infinite system *arbitrarily closely* by a finite one.

## Third example: The mass in Galilei-relativistic theories

The transition from the more fundamental Lorentz-relativistic quantum mechanics to Galilei-relativistic quantum mechanics is governed by the contraction of the Lorentz group to the Galilei group – a highly singular limit.[35] While the Lorentz group is semisimple, the Galilei group is not but has a more complicated mathematical structure. The emergent quantity associated with this contraction is the *mass* in the sense of a *classical observable* (which commutes with all other observables and can therefore be treated as a real parameter). In traditional quantum mechanics this result is known under the name univalence superselection rule or Bargmann's superselection rule which guarantees the strict conservation of mass in Galilei-relativistic quantum theory.[36] A systematic asymptotic expansion of Lorentz-relativistic quantum mechanics in terms of powers of the inverse velocity of light allows the inclusion of Lorentz-relativistic effects into the Galilei-relativistic picture.[37]

## Fourth example: The shape of molecules and the molecular hierarchy

All molecular chemistry is based on the concept of molecular structure. For example, every chemist attributes to a benzene molecule in the ground state a shape which is characterized by a planar nuclear frame with the symmetry group $D_{6h}$. This pattern does not exist in a full quantum-theoretical description since electrons and nuclei are entangled by Einstein–Podolsky–Rosen correlations. The concept of molecular structure does not follow from first principles – all molecules with the same empirical formula have the same Schrödinger equation, so that, at this level, the shape of a molecule as the main feature of molecular chemistry is simply not in evidence. In a quantum theoretical description the molecular shape emerges by abstracting from the actually existing Einstein–Podolsky–Rosen correlations between the electrons and the nuclei. Historically, the structure concept has been introduced into quantum chemistry by the so-called Born-Oppenheimer approximation.[38] But this terminology is misleading since the main issue is not an approximation, but the breaking of a holistic symmetry. A more proper appreciation of the Born–Oppenheimer-description stresses its singular nature: it is an expansion about the singular point of infinite nuclear masses. An asymptotic expansion can be formulated in terms of the ratio $\varepsilon = (m/M)^{1/4}$, where $m$ is the mass of an electron and $M$ is a mean nuclear mass of the molecular system. In the limiting case $\varepsilon = 0$ the holistic correlations between nuclei and electrons are suppressed so the description of a molecule reduces to the description of the motion of electrons in the electric field of a *classical* nuclear framework. In this description the molecular structure is a property described by an emergent classical observable.[39] The singular limiting case $\varepsilon = 0$ leads to a discontinuous change in the description and is the starting point for an asymptotic expansion in terms of the emergent property at higher levels of description.

A mathematically rigorous formulation requires the use of the method of multiple scales, a rescaling of the nuclear masses and the nuclear momentum operators.[40] In the asymptotic expansion into powers of the small parameter $\varepsilon$ the terms proportional to $\varepsilon$ and $\varepsilon^3$ vanish. The term proportional to $\varepsilon^2$ describes the oscillation of the nuclear frame quantum-mechanically. The term proportional to $\varepsilon^4$ describes the ro-

[35] Compare Inönü & Wigner (1953), Saletan (1961).

[36] Bargmann (1954). Compare also the lucid appendix 7 "*If Galileo Had Known Quantum Mechanics*" in Kaempffer (1965).

[37] Compare Itoh (1965), deVries & Jonker (1968).

[38] Born & Oppenheimer (1927). For a modern version which includes the induced non-Abelian Berry-type gauge fields, compare the synoptic presentations by Bohm (1993), chapter XXIII, and by Littlejohn & Reinsch (1997).

[39] Compare Primas (1981), section 6.4.

[40] Compare Hagedorn (1980), Hagedorn (1986), Hagedorn (1988), Kargol (1994).

tation of the nuclear frame quantum-mechanically.[41] The asymptotic separation of time scales is the basis of molecular spectroscopy. The electronic spectra (typically in the visible and ultraviolet region) are discussed on the time scale $\tau_0 := t$, the vibration spectra (typically in the infrared region) are governed by the slower time scale $\tau_2 := \varepsilon^2 t$, while the rotation spectra (typically in the radio frequency region) have to be discussed on the even slower time scale $\tau_4 := \varepsilon^4 t$. This asymptotic expansion leads to a hierarchic description of molecular dynamics.

In such dynamical hierarchies a level of description is classified as a higher level if its reaction time is much longer than that of a lower level. Moreover, adjacent hierarchic levels are connected by a feedback loop such that the lower level is subordinated by an authority relation to the next higher level which constrains the behavior of the lower level in the sense of some coherent activity. No hierarchic level can be considered as an independent entity on its own right; it exists only in virtue of the lower levels and has to be described as a nonautonomous open system. If in a dynamical hierarchic system a higher level acts slowly enough, the subordinate lower levels can follow so that we have a dynamically stable hierarchy. However, a forced sudden change on a higher level usually leads to a loss of hierarchic control and to structural instabilities.

A "more exact" description without such a dynamical hierarchy is, of course, possible but it would leave many interesting phenomena out of evidence and would be of no use to a chemist. The separation of time scales and the distinction between short-term and long-term memories is crucial for our attempts to understanding the material world in simple terms. Every hierarchic level has its own expressive natural language which should not be eliminated in favor of a universal language. The description of a higher level with a language appropriate for a lower level may be feasible but is highly impracticable since such a description would be very complex and almost incomprehensible.

## Fifth example: Classical observables in quantum theory

A fairly context-independent general formulation of the first principles of physics can be expressed algebraically. The corresponding formalism of *algebraic quantum mechanics* is nothing but a mathematically correct codification of the original ideas of quantum theory. It is a general representation theory of the basic kinematical symmetry group and the associated canonical commutation relations. Given the appropriate context, this general formulation allows a mathematically rigorous derivation of most physical phenomenological theories such as traditional quantum mechanics, quantum field theory, classical point mechanics, classical continuum mechanics, or statistical mechanics.

In algebraic quantum mechanics one can formulate the first principles (which are related to fundamental symmetries) individually and non-probabilistically in terms of an abstract C*-algebra $\mathcal{A}$, called the *algebra of intrinsic observables*.[42] A C*-algebra is a topological algebra with the extraordinary property that its topology (the so-called *norm topology*) is determined algebraically. Hence the topology of a C*-algebra is intrinsic and does not depend on any context. The *intrinsic properties* are represented by the selfadjoint elements of the C*-algebra $\mathcal{A}$. They describe what is physically real independently of any observation.

Considering only first principles of quantum mechanics it is impossible to get any operational description of the material world. For example, fundamental symmetries are never directly accessible by experiments, they can only retrospectively inferred by contingent symmetry breakings. This fundamental insight has been recognized clearly by Pierre Curie (1894): "C'est la dissymétrie qui crée le phénomène." Every operational description requires a metatheoretical "Archimedean point" which provides us with a distinction separating an observed object from its environment and from observing tools. Without a division of the world into a part "which sees" and a part "which is seen" experimental science is not conceivable. This division has to be introduced by the *context* of inquiry, that is by *our* decision of what we consider as relevant and what as irrelevant. In algebraic quantum mechanics a particular context can be introduced by imposing a new, contextually selected topology upon the state space of the C*-algebra of intrinsic observables with an intrinsic topology.

[41] Compare Primas & Müller-Herold (1984), chapter 4.

[42] A *-algebra is a collection of mathematical objects $A, B, \cdots$ that can be combined linearly, multiplied in a bilinear and associative way, and mapped by the conjugate linear *-operation $A \to A^*$ which satisfies $A^{**} = A$ and $(AB)^* = B^* A^*$. If a *-algebra $\mathcal{A}$ is endowed with a Banach-space norm $\|\cdot\|$ with the properties $\|AB\| \leq \|A\|\|B\|$ and $\|A^* A\| = \|A\|^2$, then $\mathcal{A}$ is called a C*-algebra.

A most interesting feature of algebraic quantum mechanics is that it provides the mathematical tools for the construction of an "Archimedean point". A new *coarser,* contextually selected topology can be introduced by picking out a particular *reference state*, given by a positive linear state functional $\rho$ on the context-independent abstract C*-algebra $\mathcal{A}$ of intrinsic observables. The so-called GNS-construction (according to Gelfand, Naimark and Segal) then allows the construction of a context-dependent Hilbert space $\mathcal{H}_\rho$ and an associated faithful representation $\pi_\rho(\mathcal{A})$ of the C*-algebra $\mathcal{A}$ acting on $\mathcal{H}_\rho$.[43] The closure of $\pi_\rho(\mathcal{A})$ in the weak topology of the algebra $\mathcal{B}(\mathcal{H}_\rho)$ of all bounded operators acting on $\mathcal{H}_\rho$ is a context-dependent W*-algebra $\mathcal{M}_\rho$, $\pi_\rho(\mathcal{A}) \subset \mathcal{M}_\rho \subset \mathcal{B}(\mathcal{H}_\rho)$, called the *algebra of contextual observables.* Every W*-algebra is a C*-algebra, but not every C*-algebra is closed in the coarser W*-topology. The new contextual topology on $\mathcal{A}$ which is induced by the reference state functional $\rho$ corresponds to the weak operator topology on $\pi_\rho(\mathcal{A}) \subset \mathcal{B}(\mathcal{H}_\rho)$. It reflects the continuity requirement necessary for a continuous representation of the contingent initial conditions. That is, not all states on the C*-algebra $\mathcal{A}$ of intrinsic observables are admissible states for a contextual description in terms of the W*-algebra $\mathcal{M}_\rho$ of contextual observables. A contextual W*-algebra $\mathcal{M}_\rho$ is strictly larger than the faithful representation $\pi_\rho(\mathcal{A})$ of the C*-algebra $\mathcal{A}$ of intrinsic observables. That is, all intrinsic observables appear also as contextual observables, but in addition there are new observables which are not intrinsic. The observables in the W*-algebra $\mathcal{M}_\rho$ of contextual observables, but not in the faithful representation $\pi_\rho(\mathcal{A})$ of the C*-algebra $\mathcal{A}$ of intrinsic observables, are called *emergent observables*. They represent properties which are novel in the sense that they are absent in the more fundamental context-independent C*-algebraic description. The emergence of novelty in contextual descriptions is a compelling consequence of algebraic quantum theory.

The adopted ontology for the basic C*-algebraic theory induces an operationally meaningful interpretation in terms of statistical states which refer to our knowledge of the state as it appears in the context-independent description. Statistical states are represented by the *normal* positive linear functionals on the W*-algebra $\mathcal{M}_\rho$ of contextual observables.[44] Note that in a particular representation $\pi_\rho$ only a small portion of the states on the algebra $\mathcal{A}$ of intrinsic observables corresponds to operationally accessible $\sigma$-additive statistical states (i.e. to elements of the predual $(\mathcal{M}_\rho)_*$). The contextually selected topology is characterized by the fact that in the representation $\pi_\rho$ the reference state $\rho$ is the restriction of an operationally accessible statistical state.

With the only exception of von Neumann's codification of traditional quantum mechanics (where the basic C*-algebra is the algebra of compact operators), there are always infinitely many physically inequivalent W*-representations of the underlying basic C*-algebra of intrinsic observables. Different inequivalent representations represent physically inequivalent contextual descriptions of one and the same basic system. They can be distinguished by *classical observables* which commute with all observables. Even if the algebra $\mathcal{A}$ of intrinsic observables (hence also $\pi_\rho(\mathcal{A})$) has no center, the contextually constructed algebra $\mathcal{M}_\rho$ usually has a large center $\mathcal{Z}_\rho(\mathcal{M}_\rho)$,

$$\mathcal{Z}_\rho(\mathcal{M}_\rho) :=$$
$$\{Z \,|\, Z \in \mathcal{M}_\rho,\, Z\,M = M\,Z \ \text{ for every } \ M \in \mathcal{M}_\rho\} \quad ,$$

describing the *classical features* of the contextual quantum system. The nontrivial selfadjoint operators of the center $\mathcal{Z}_\rho(\mathcal{M}_\rho)$ are called *classical observables. Most classical observables are emergent;* they are elements of $\mathcal{Z}_\rho(\mathcal{M}_\rho)$ but not elements of $\pi_\rho(\mathcal{A})$. Such emergent classical observables (like temperature or order parameters describing phase transitions) are neither contained in the basic C*-algebra $\mathcal{A}$ nor are they functions of the intrinsic observables. They are generated by the basic C*-algebra $\mathcal{A}$ of intrinsic observables together with a context which selects a particular representation.

In the algebraic approach emergent properties are not postulated but *derived* from contingent conditions which are necessary to describe physical systems besides the natural law. However, since there are uncountably many inequivalent possibilities to intro-

[43] For all mathematical questions we refer to Takesaki (1979), chapter I, section 9.

[44] The $\sigma$-weak topology of a W*-algebra is of crucial importance for the representation of *statistical states*. In analogy to the concept of additivity of measure in classical probability theory, a state functional $\rho \in \mathcal{M}^*$ is said to be completely additive if it satisfies $\rho(\vee F_n) = \Sigma_n \rho(F_n)$ for every set $\{F_n\}$ of pairwise orthogonal projections $F_n \in \mathcal{P}(\mathcal{M})$, $F_n F_m = 0$ for $n \neq m$. In measure theory, the additivity of a measure implies Lebesgue's monotone convergence theorem. In analogy, a state $\rho$ is said to be *normal* when $\rho(M_n) \uparrow \rho(M)$ for each monotonically increasing net $\{M_n\}$ of operators $M_n$ in $\mathcal{M}$ with least upper bound $M$.

duce a consistent coarser contextual topology, the traditional view that emergent properties cannot be predicted makes sense: only when we already know the phenomenon we can guess the appropriate contextual topology necessary to derive the corresponding phenomenological theory from first principles.

## Sixth example: Temperature is a classical observable in quantum statistical mechanics

In phenomenological thermodynamics the concept of "temperature" is introduced as an equivalence relation via the zeroth principle of thermodynamics: *if two systems are both in equilibrium with a third system then they are in equilibrium with each other*. Since the zeroth law is not a first principle of mechanics, temperature is not represented by an intrinsic observable. However, it is possible to select a reference state functional on the context-independent abstract C*-algebra $\mathcal{A}$ of intrinsic observables such that in the associated W*-GNS-representation the zeroth principle of phenomenological thermodynamics is fulfilled.

For thermal systems the appropriate reference states are represented by the so-called KMS-state functionals, named after the physicists R. Kubo, P. C. Martin and J. Schwinger.[45] They are characterized by strong relaxation and stability properties with respect to small local perturbations.[46] A GNS-construction with a family of KMS-states as distinguished reference states on a C*-algebra $B \in \mathcal{Z}_{\mathrm{KMS}}(\mathcal{M}_{\mathrm{KMS}})$ without intrinsic classical observables generates a W*-algebra $\mathcal{M}_{\mathrm{KMS}}$ with a center $\mathcal{Z}_{\mathrm{KMS}}(\mathcal{M}_{\mathrm{KMS}})$ which contains a classical observable $B \in \mathcal{M}_{\mathrm{KMS}}$.[47] Real-valued parameters in quantum-theoretical descriptions are always values of a classical observable. Here, the reciprocal absolute temperature $\beta$ is a spectral value of the classical variable $B$. This classical temperature observable is emergent since it belongs to the algebra $\mathcal{M}_{\mathrm{KMS}}$ of contextual observables but not to the algebra $\mathcal{A}$ of intrinsic observables, $B \in \mathcal{M}_{\mathrm{KMS}}$, $B \notin \mathcal{A}$. Moreover, while all mechanical descriptions are given by so-called type I W*-algebras, the contextual W*-algebra $\mathcal{M}_{\mathrm{KMS}}$ appropriate for the description of thermal systems is of type III. This fact shows explicitly that a direct mechanistic description is not possible. Depending on the choice of the family of contextual reference states for the GNS-construction we can get mutually exclusive descriptions of the material reality, either in terms of electrons, atoms and molecules, or in terms of thermodynamics.

## Seventh example: In quantum theory facts are generated in an asymptotic long-time limit

According to the modern point of view, quantum mechanics is a fundamental theory while classical mechanics is a higher level contextual theory of limited validity. According to this view, a quantum system (depending on Planck's constant $\hbar$) is called *classical* if its algebra of observables is commutative. All classical quantum systems emerge by breaking the holistic Einstein–Podolsky–Rosen symmetry. The still widely held view that classical mechanics is the limiting case for vanishing Planck's constant $\hbar$ is untenable. It is easy to prove rigorously by counter-examples that *there is no universal classical limit of quantum mechanics*. That is, the fictitious limit $\hbar \to 0$ does not exist in the norm topology. Nonetheless it is possible that for distinguished small families of quantum states certain quantum systems may behave exactly as classical systems. In this case, we speak of *classical quantum systems*, whose behavior, of course, depends on the physical value of Planck's constant $\hbar$. Whether a quantum system behaves classically or not, is *not* an intrinsic but a *contextual* property of the system. Depending on the context, it is possible to associate various *inequivalent* classical quantum systems to one and the same quantum system.

A family of quantum states is called classical if this family does not contain any coherent superposition of its elements. Nontrivial observables which commute with all observables are called *classical observables.* A related notion is the concept of *disjointness*. Two pure states are called disjoint if there exists a classical observable such that the expectation

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[45] Denoting the reciprocal inverse temperature by $\beta$, a $\beta$-KMS state $\rho_\beta$ of a dynamical system with the C*-algebra $\mathcal{A}$ and the dynamics $\{\alpha_t \mid t \in \mathbb{R}\}$ is characterized by a function $z \mapsto F_\beta(z)$ of a complex variable $z$ on the closed strip $\{z \in \mathbb{C} \mid 0 \le \Im\mathrm{m}(z) \le \beta\}$ such that $z \mapsto F_\beta(z)$ is bounded and continuous on the closed strip, holomorphic in the interior of the strip, and satisfies for all $A, B \in \mathcal{A}$ and all $t \in \mathbb{R}$ the two boundary conditions $F_\beta(t) = \rho_\beta\{A\alpha_t(B)\}$ and $F_\beta(t + i\beta) = \rho_\beta\{\alpha_t(B)\,A\}$. For more details, compare for example Bratteli & Robinson (1981), section 5.3.1.

[46] Compare Haag, Kastler & Trych-Pohlmeyer (1974).

[47] This result follows from a theorem by Masamichi Takesaki (1970).

values with respect to these states are different. That is, mutually disjoint states can be distinguished and classified in a classical manner. Classically indecomposable states are called *factor states*. More precisely, a factor state is characterized by the fact that it is dispersion-free with respect to every classical observable. Every quantum state can be decomposed *uniquely* into a sum or integral of mutually disjoint factor states.[48] This so-called *central decomposition* represents the finest unique decomposition of a nonpure state into a *classical mixture.* A classical mixture allows an *ignorance interpretation* with respect to the central decomposition of a nonpure quantum state into factor states. A factor state is in general nonpure and allows infinitely many physically different pure states. Therefore, an ignorance interpretation of a factor state is not possible. Since in traditional quantum mechanics all states are factor states, an ignorance interpretation is inadmissible in traditional quantum mechanics. *The only permissible ignorance interpretation of nonpure quantum states refers to the central decomposition into a classical mixture of mutually disjoint factors states.*

Processes which produce *facts* are ubiquitous as objectively occurring natural processes as well as measuring processes in the laboratory. A fact is an event in the *past* which is created by a dynamical process. Any reasonable statement about such a fact has to be either true or false, even if we do not know it. This condition requires a domain of discourse which has a classical *Boolean* description. That is, *the nonexistence of coherent superpositions of facts* is a characteristic property for facts. Therefore, *in quantum theory facts are emergent quantities which are generated by irreversible processes with asymptotically disjoint final states.*[49] The notorious "measurement problem of quantum mechanics" is related to a so-called insolubility theorem which in its most general modern version says that *an automorphic time evolution on any C*-algebra cannot generate disjoint states*.[50] Even in the very general framework of modern algebraic quantum theory an automorphic dynamics cannot describe a measurement process which generates classically different final states in finite time. However, there exist automorphic time evolutions which transform an arbitrary initial factor state asymptotically into a classical mixture of disjoint final factor states.

It has been objected that processes with asymptotically disjoint final states require an infinite measurement time.[51] This is a misunderstanding: every measurement in engineering physics is asymptotic. To illustrate this situation we consider a customary statistical decision procedure. In experimental science all observations have to be considered to be subject to random variations. The result of a statistical experiment is characterized by a probability measure on some measurable space $(\boldsymbol{\Omega}, \boldsymbol{\Sigma})$. Consider a statistical test for deciding whether the state of the apparatus is given either by the probability measure $\boldsymbol{\mu}'$ or by the probability measure $\boldsymbol{\mu}''$. Independently of how we perform this test, the *minimal error probability* is given by

$$e(\boldsymbol{\mu}', \boldsymbol{\mu}'') :=$$

$$\inf_{\mathcal{B} \in \boldsymbol{\Sigma}} \{\boldsymbol{\mu}'(\mathcal{B}) - \boldsymbol{\mu}''(\boldsymbol{\Omega} - \mathcal{B})\} = 1 - \| \boldsymbol{\mu}' - \boldsymbol{\mu}'' \|,$$

where $\| \boldsymbol{\mu}' - \boldsymbol{\mu}'' \|$ is Kolmogorov's variation distance between $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}''$, $0 \leq \| \boldsymbol{\mu}' - \boldsymbol{\mu}'' \| \leq 1$.[52] If the minimal error probability vanishes, a perfect decision can be made with probability one. Such statistical tests are called *singular*. *In engineering science, models which lead to singular decisions problems are considered as ill-posed and unacceptable*.[53] If the essential degrees of freedom of the measuring instrument are described by a commutative C*-algebra, a classical quantum state induces a GNS-representation as a Lebesgue space $L^{\infty}(\boldsymbol{\Omega}, \boldsymbol{\Sigma}, \boldsymbol{\mu})$ where the probability measure $\boldsymbol{\mu}$ represents a classical quantum state. If $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}''$ represent disjoint classical quantum states, then the two probability measures are singular with respect to each other, $\| \boldsymbol{\mu}' - \boldsymbol{\mu}'' \| = 1$.[54] That is, disjoint states lead to singular decision problems which engineers reject as unrealistic idealizations.

Since the quantum states of the apparatus can be classically described in terms of probability measures, the well-established distance measure between probability distributions can be used to introduce a physically meaningful *measure of the approximate disjointness of two classical quantum states*. It is given by the minimal error probability of the associated statistical decision problem. In models describing the measurement process statistically with asymptotically disjoint final states the effective

[48] For details, compare for example chapter IV.6 in Takesaki (1979).
[49] Compare also Primas (1997).
[50] Hepp (1972), lemma 2, p.246.
[51] For example by Landsman (1995), p.55.
[52] Rényi (1966), Rényi (1967).
[53] Root (1963); Root (1964); Root (1968).
[54] Compare for example theorem 2 on p.82 and theorem 4 on p.86 in Nelson (1969).

measuring time can be arbitrarily short. For example, if $\boldsymbol{\mu}'(t)$ and $\boldsymbol{\mu}''(t)$ describe two classical quantum states which become asymptotically disjoint, $\lim_{t\to\infty}\|\boldsymbol{\mu}'(t)-\boldsymbol{\mu}''(t)\|=1$, then the error probability for a decision at time $t$ as to which of the two possible final states $\boldsymbol{\mu}'(\infty)$ and $\boldsymbol{\mu}''(\infty)$ will asymptotically be realized, is given by $1-\|\boldsymbol{\mu}'(t)-\boldsymbol{\mu}''(t)\|$. If the threshold level for the error probability is given by $\varepsilon \ll 1$, then the effective measuring time $\tau_\varepsilon$ is given by $\varepsilon = 1-\|\boldsymbol{\mu}'(\tau_\varepsilon)-\boldsymbol{\mu}''(\tau_\varepsilon)\|$. In the mathematical model for this process the parameters in the Hamiltonian can be chosen so that for any fixed threshold level $\varepsilon > 0$ the effective measuring time $\tau_\varepsilon$ can be made arbitrarily small.

## Contextual ontologies

The problem of the *ontology of a scientific theory* refers to the problem of the existence of the postulated entities. An ontic interpretation refers to a theory about things "as they really are", independently of any observational or descriptive context. It is not assumed that an ontic interpretation refers to an ultimate fundamental theory of matter, but it is to be understood "relative to one scheme for describing and explaining physical phenomena."[55] On the other hand, epistemic interpretations refer to our knowledge of observable patterns or modes of reactions of systems.[56] Bernard d'Espagnat defended convincingly the inevitability of distinguishing between an independent reality (i.e. independent of our existence) and the various aspects of empirical reality.[57]

Fundamental theories are characterized by their independence of particular contexts and are distinguished by their conceptual simplicity. They are supposed to refer to a hypothetical *context- and mind-independent reality*. The requested context-independence of a fundamental theory precludes its direct application to concrete problems. This circumstance corresponds to the fact that the independent reality cannot be observed directly by our five main senses or by our instruments. Yet, as argued by d'Espagnat, "the great mathematical laws of physics may let us catch some glimpses on the true structure of mind-independent reality".[58] *Experimentally inaccessible theories are not meaningless but can be used to generate context-dependent operational theories.* A fundamental theory of independent reality can be operationalized by specifying a *context* which describes our cognitive apparatus or the pattern recognition devices used by the experimentalist. Only by referring to a specified context one can distinguish between relevant and irrelevant features. All the richness and variety of empirical reality manifest themselves only by abstractions resulting from pattern recognition devices which determines what is relevant and what is not. In a mathematically formulated fundamental theory, a distinction between relevant and irrelevant features can be accomplished by the introduction of a new *contextual topology* which is compatible with the intrinsic topology of the underlying fundamental theory.

In the first place, such derived theories refer to our knowledge of the properties or modes of reactions of contextual objects as we perceive them. Since every epistemic description contains a non-removable reference to the observing tools, the referent of such derived theories is the empirical realty. If the underlying fundamental theory has an ontic interpretation, the epistemic formulation of a derived context-dependent theory gives rise to a well-defined interpretation: *an epistemic state of a derived theory refers to our partial knowledge of the ontic state of the fundamental theory*. It is true that the results arising from empirical investigations depend on the chosen categorization and the conceptual framework, but a context-dependent theory derived from a fundamental theory in an ontic interpretation still reflects *some aspects* of the independent reality.

This situation allows us to introduce a consistent *contextual ontology* for derived theories: if we agree on the context, we are entitled to speak about contextual objects *as if* they were actually existing. For example, independent of a context it makes no sense to speak of the moon, of an ammonia molecule, of a hydrogen atom, or of an electron. But, in everyday life we take notions like "planets", "atoms", "molecules" literally and ontologize them. There are no objections to do so provided we keep the contextuality of this ontologization in our mind. It would not be reasonable (and would contradict common sense and scientific practice) to assume that there is only *one* ontological level, and it would be preposterous if we were forced to change our beliefs about the ontological status of the common-sense world.

A contextual ontology does not refer to an independent reality but to *emergent properties* arising from latent features of the independent reality. Such

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[55] Putnam (1982), p. 4.

[56] Ontic and epistemic formulations have been introduced by Scheibe (1964), Scheibe (1973).

[57] D'Espagnat (1995), chapters 14 and 15.

[58] D'Espagnat (1998), p. 15.

hidden structures become manifest only by choosing a topology capable to distinguish the relevant and irrelevant features. It cannot be a question whether a particular ontologization is "more true" or "more real" than another. No single operational description is uniquely legitimate, and none is sufficient; all of them together are necessary.[59] For example, when a scientist commits himself to the ontological assumptions implicit at the molecular level, his vision of the world would be severely limited if he would consider, say, the thermodynamic level as derived from and secondary with respect to the "more real" molecular phenomena. Of course, if we approach matter from a molecular point of view, we will get molecular answers, and our molecular view will be confirmed. However, questions of a different kind which cannot be answered within the molecular language may be important as well. Only if we maintain multiple sets of contextual ontologies, we can tolerate the coexistence of complementary views in our experience of reality. While an independent reality itself is directly inaccessible, the numerous inequivalent contextual descriptions allow us to get deeper insight into the structure of independent reality.

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[59] Discussing the structure of music, Guerino Mazzola (1986) defends a similar thesis on the basis of the Yoneda-lemma of category theory: "A structure is determined as soon as you know how it looks from the 'perspective' of every other structure. Musicians feel this by saying: 'A work of music is known by playing all possible interpretations'." Compare also Mazzola (1985), p.88.

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