EPR-like “funny business” in the theory of branching space-times

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Abstract

EPR-like phenomena are (presumably) indeterministic, but they furthermore suggest that our world involves seeming-strange “funny business.”† Without invoking any heavy mathematics, the theory of branching space-times offers two apparently quite different ways in which EPR-like funny business goes beyond simple indeterminism. (1) The first is a modal version of a Bell-like correlation: There exist two space-like separated indeterministic initial events whose families of outcomes are nevertheless modally correlated. That is, although the occurrence of each outcome of each of the two space-like separated initial events is separately possible, some joint occurrence of their outcomes (one from each) is impossible. (2) The second sounds like superluminal causation: A certain initial event can bear a cause-like relation to a certain *outcome event* without being in the causal past of that outcome. The two accounts of EPR-like funny business are proved equivalent, a result that supports the claim of each as useful to mark the line between mere indeterminism and EPR-like funny business.

†This is a “postprint” based on the version published in *Non-locality and modality*, eds. T. Placek and J. Butterfield, Kluwer Academic Publishers, Dordrecht, 2002, pp. 293–315. The archive at http://philsci-archive.pitt.edu contains two recent related articles by the author, “No-common-cause EPR-like funny business in branching space-times” (2002) and “A theory of causation: *causae causantes* (originating causes) as inus conditions in branching space-times” (2002).

1 Determinism, indeterminism, funny business

The language of branching space-times[1] as it is spelled out in Belnap 1992 is austere, concerning as it does only the causal ordering on possible point events of our world[2] and without any mention of probabilities.[3] By refraining from invoking the rich language needed for anything like a quantum-mechanical account, this very austerity appears to permit a sharp and simple delineation of three ways our world might be.

1. It might be strictly and universally deterministic: Given any initial event, its outcome is uniquely determined, which is to say, there is only one possible outcome.

2. It might be indeterministic, so that some initial events face multiple incompatible possibilities for their futures; but without any features that are EPR-like. (It was plausible to think of radium decay like that.)

3. It might be (and indeed seems to be) not only indeterministic, but exhibiting EPR-like "funny business," as I will say.[4]

The aim of this study is to add to our understanding of these distinctions, and especially to help with the much-too-fuzzy idea of "EPR-like funny business." Contri-

[1] Note the profound difference between "branching *space-times*" on the one hand and so-called "branching *time*" on the other. "Branching time," with "time" in the singular, is not, incidentally, good terminology, since there is no reason not to let "time" connote a linear order. The phrase is, however, fixed in a large literature, and to try to use another would be quixotic. On the other hand, "branching space-time," as in the title of BST-92, is little used, so that it seems reasonable to try to replace it, as I do here, with the less misleading plural phrase "branching space-times." This usage recognizes the intention that the branching be *between* space-times. Both branching time and branching space-times place a causal order on concrete "events," but in branching time, the terms of the causal order are giant Laplacean "simultaneity slices," not tiny little point events. Branching time can be used to represent a global version of indeterminism, but not—in the absence of additional vocabulary—seriously local indeterminism such as can be represented when one puts a causal order on point events. Chapters 6, 7, and 8 of Belnap, Perloff and Xu 2001 contain some foundational discussions pertaining to indeterminism as it is represented in "branching time" theory, including a sustained argument against employing the idea of an "actual future" in thinking about indeterminism, and a careful account of how one living in an indeterminist world must use the future tense. These discussions remain relevant when the switch is made to branching space-times.

[2] For instance, no language of "systems," "states," "particles," "laws," or "theories."

[3] Perhaps more thinkers than not claim to understand probability while finding *possibility* mysterious. You cannot, however, have probability without possibility. If you want to be technical, *possibility* lies in the so-called "probability space," an understanding of which seems essential to any adequate understanding of applied probability.

[4] Adding probabilistic language permits consideration of a concept of Bell-like funny business; but that is not our topic.

butions to this end can come from many quarters, historical, theoretical, experimental, metaphorical, etc.[5] My working hypothesis is that there can be complementary benefit by seeing what can be done within the "pre-physical" and even pre-metric language of branching space-times (BST-92[6]) whose only primitives are (1) the notion of point event and (2) a causal order on them in terms of which one can define ideas and relations that illuminate *both* indeterminism *and* space-time. The thought is that staying within the confines of what is there called "branching space-times" encourages one to try to make the three-fold distinction between determinism, indeterminism, and EPR-like funny business in a fashion that is sharp (absolutely rigorous), simple (only two uncomplicated primitives; no heavy mathematics), and intuitive (which is of course a subjective matter).

Trying, however, is not succeeding: I'll first go over some suggestions that are at best only partially successful. (There are obvious cases of such funny business that do not fall under those versions.) The aim of this study to give accounts that may or may not be "final," but are anyhow better than those that I initially survey. I give two that one may hope are satisfactory:

- Generalized primary space-like-related modal-correlation funny business
- Some-cause-like-locus-not-in-past funny business.

Each, which is rigorously defined in terms of the primitives of BST-92 theory, is intended to express a fundamental feature of one aspect (only) of quantum-mechanical wonderment.[7] These two ideas are proven to be in a certain sense equivalent, which I offer as evidence of their stability and suitability to their purpose. What purpose? I suggest that BST-92 theory, though "pre-physical"[8] rather than "physical," is a seriously helpful guide to broad-gauge physical understanding that goes beyond mere metaphor or arm-waving about the indeterminist and funny-business aspects of our world.

I begin with a hurried review of some fundamental definitions, postulates, and facts. Then I develop the two ideas of "funny business," and finally prove the supporting equivalence.

[5]In addition to offering his own approach, Placek 2000 provides valuable access to a rich variety of different such contributions.

[6]"BST-92" refers to the particular way in which the idea of branching space-times is worked out in Belnap 1992. The branching space-times theories of McCall 1994 and Placek 2000 share much of the underlying motivation of BST-92, but differ in key ways with respect to primitives, postulates, and definitions. Rakić 1997 puts the BST-92 version into a useful perspective.

[7]The language of BST-92 is not known to permit, for example, a representation of something as basic as the quantized nature of quantum-mechanical concepts; but since it does not pretend to do so, this is hardly a defect.

[8]A physicist after hearing a lecture once indicated impatience with the ideas of BST-92 by labeling them "common sense." Would that it were so.

2 Ancillary ideas of branching space-times

The BST-92 idealization of the causal structure of our world invokes just two primitives: the set of all possible point events (each taken to be as fully concrete as any point event in our actual past), which I call *Our World* (*OW* for short), and the causal order on them, $\leqslant$. Define $e_1 < e_2$ in the usual way by $e_1 \leqslant e_2$ & $e_1 \neq e_2$. Then $e_1 < e_2$ may be read indifferently either as "e_1 is in the causal past of e_2" or "e_2 is among the future of possibilities of e_1."

2.1 Postulates and basic definitions

All but one of the postulates of BST-92 are standard order-theoretic postulates. As can be seen from BST-92, however, each plays a definite role in giving an account of how histories branch from each other (indeterminism); none is given for its mere space-time content. That is why there are so few, as follows.

- *OW* is a nonempty set; its members are called *point events*, written e and sometimes p. $\leqslant$ is a partial order on *OW* (reflexive, transitive, antisymmetric). There are no maximal elements ($e_1 \in OW \rightarrow \exists e_2[e_1 < e_2 \,\&\, e_2 \in OW]$). $<$ is dense in *OW* ($e_1 < e_3 \rightarrow \exists e_2[e_1 < e_2 < e_3]$). A *chain* E is defined as a connected subset of *OW* ($e_1, e_2 \in E \rightarrow (e_1 \leqslant e_2$ or $e_2 \leqslant e_1)$.
- An *outcome chain*, written O, is a nonempty lower bounded chain, and each outcome chain O has an infimum, written $inf(O)$.
- A *history*, written h, is a subset of *OW* that is maximal with respect to containing an upper bound for each pair of its members.[9] Facts: Every point event is a member of (and every chain is a subset of) some history (by Zorn's lemma); each history is closed downward; and no history has a member that is maximal in that history.
- An *initial chain*, written I, is a nonempty and upper bounded chain, and each initial chain I has a supremum in each history h of which it is a subset, written $sup_h(I)$.
- Two histories h_1 and h_2 *separate* (or *divide* or *split*) at a point event e, written $h_1 \perp_e h_2$, $\leftrightarrow_{df}$ e is maximal in $h_1 \cap h_2$.

[9] Each history may usefully—but optionally—be pictured as a Minkowski space-time. It is to our purpose in clarifying indeterminism and funny business, however, that there be not nearly enough postulates to *force* that interpretation.

A point event e_1 prior to (or identical with) e belongs equally to both histories; relative to that earlier point event e_1, both future historical courses are equally possible. After e, however, no matter what happens next, at least one of h_1 and h_2 can only be termed "was possible but isn't anymore." Splitting at a point event is extended to sets of histories in a natural fashion, as follows. (Here and below we let $e_1 < O \leftrightarrow_{df} e_1 < e_2$ for all $e_2 \in O$, and we let $I < e$ and $I < O$ take on analogous universal meanings.)

- $h_1 \perp_e H \leftrightarrow_{df}$ the one point event e works for every member of H ($h_1 \perp_e h_2$ for every $h_2 \in H$). And $H_1 \perp_e H_2 \leftrightarrow_{df} h_1 \perp_e H_2$ for every $h_1 \in H_1$.

The idea of separating or splitting is required for the final postulate that I use in this study:[10]

- *The prior choice postulate*. $\forall O \forall h_1 \forall h_2[(O$ is an outcome chain and $O \subseteq (h_1 - h_2)) \rightarrow \exists e[e < O$ and $h_1 \perp_e h_2]]$. Choosing $O = e_0$ (that is, $O = \{e_0\}$) is an important special case.

The prior choice postulate says that if we are in the middle of some *outcome event* O that belongs to a history h_1, but we are not located within a specific complete history h_2, then there is *in the past* of O a definite locus e in *Our World* such that what happened at e kept h_1 possible at the expense of making h_2 no longer possible. If we take a "world line" back from where we are in O (that line will lie entirely within h_1), then e is exactly where our "world line" leaves h_2. Before e both h_1 and h_2 were possible courses of events for the future, but e, which is known to lie in the past of O, is the exact point event at which a "choice" was made that kept h_1 possible at the expense of rendering h_2 no longer possible.[11]

2.2 Definitions: Propositions and their truth, events and their occurrence

BST-92 permits a tightly organized and intuitively sensible theory of propositions to the effect that such and such an event "occurs."

[10] It must be added that in connection with a theory of how probabilities work in branching space-times, M. Wiener discovered the necessity of an additional postulate to the effect that given two initial chains and two histories, the order of the respective suprema is preserved as the histories are varied. The consequences of this postulate are not used here.

[11] Choice? All there is to say in the language of BST-92 is that up to and including e both histories remained possible futures, but immediately after e (and indeed forever after e) that was no longer true. Perhaps this is also a place to remark that my use of tensed constructions is wholly unobjectionable, but makes sense only if one pictures oneself somewhere within *Our World*, for example, somewhere in O. One can of course retail the same causal facts tenselessly. See Chapters 6 and 8 of Belnap *et al*. 2001 for a careful working out of how tenses must be used in the context of indeterminism.

- In the spirit of possible-worlds theory, a *proposition* is a set of histories H; and H is said to be *true in* each of its members.

- $H_{(e)} = \{h\colon e \in h\}$ (the proposition that e *occurs*); $H_{[I]} = \{h\colon I \subseteq h\}$ (the proposition that I, *considered as an initial event*, *occurs* in the sense that it finishes or (fully) passes away). $H_{\langle O \rangle} = \{h\colon O \cap h \neq \varnothing\}$ (the proposition that O, *considered as an* outcome event, *occurs* in the sense that it begins or comes to be).[12]

BST-92 permits consistency concepts that are both rigorous and natural. The reason for this is its controlled treatment of the relation between events and propositions—consistency is *prima facie* understandable only for propositions, but BST-92 can then piggy-back consistency for events on propositional consistency.

- H is *consistent* (*inconsistent*) $\leftrightarrow_{df}$ H is nonempty (empty). H_1 is *consistent* (*inconsistent*) *with* H_2 $\leftrightarrow_{df}$ $H_1 \cap H_2$ is consistent (inconsistent). More generally, propositions forming a set $\mathbf{H}$ are *jointly consistent* $\leftrightarrow_{df}$ $\bigcap \mathbf{H} \neq \varnothing$.

Consistency (inconsistency) in application to point events or sets of point events is always mediated via appropriate sets of histories standing for occurrence propositions. The postulates readily guarantee that I and O are each consistent (every nonempty chain can be extended to a history so that $H_{[I]}$ and $H_{\langle O \rangle}$ are certain to be nonempty), so until Sections 4.2 and 4.4 invoke more complex notions of initial and outcome respectively, we only need to attend to *joint* consistency of events:

- Two or more events each considered in a certain way (i.e., each considered either as initial or as outcome) are *(jointly) consistent* $\leftrightarrow_{df}$ their occurrence-propositions are so. Important example: e_1 is consistent with e_2 iff $H_{(e_1)} \cap H_{(e_2)} \neq \varnothing$; which is to say, iff $\exists h[e_1 \in h \;\&\; e_2 \in h]$.

2.3 Definitions and facts: transitions and spreads

In order to put the matter of funny business in the clearest possible light, I first develop the notions of "transition" and "spread," which come from Szabo and

[12] There are good things to mean by "proposition" other than the given timeless sense, and other good ways to use "occurs." The current definitions are offered nonexclusively, as useful in their own right. I note that if one wishes to indulge in a tensed usage of "occur" that is both carefully controlled by theory and also easy to understand, it would appear to be better to insert a past tense: a certain initial or *outcome event* "*occurred*"; or, if reference to the future is wanted, "will *have occurred*." (Note 1 above cites a careful discussion of how the future tense must be understood in the context of indeterminism.)

Belnap 1996. This development is alas a bit tedious, but it is hard to be clear about the ideas without it.

- A *chain transition* is defined as an ordered pair of an initial chain I (or a single point event e) and an outcome chain O, where $I < O$. I write $I \rightarrowtail O$. Such a transition in *Our World* is intended in the spirit of Russell's "at-at" account of motion. A good deal of the "mystery" of indeterminism can be attributed to the neglect of the theory of transitions.[13] See Belnap 1999 and Xu 1997 for further discussions.

- A *chain spread* is defined as an ordered pair of an initial chain I and a set Ω of outcome chains such that (1) the pairing of I with each $O \in \Omega$ is a chain transition, and (2) exactly one member of Ω occurs in any history in which I occurs. I write $I \rightarrowtail \Omega$. A spread is our theoretical stand-in for any local indeterministic situation such as a quantum-mechanical "measurement" or a choice by an investigator.

Szabo and Belnap 1996 considered transitions $I_1 \rightarrowtail O$ such that the outcome chain O begins in the perhaps distant future of the finishing of the initial chain I_1. From the present perspective, the critical feature of such a transition is that there is time-like room between I_1 and O. Such room permits the possibility that there be an "influence" from an outside initial I_2 represented by the fact that a "world line" can run from I_2 up to O. I bring this up only to contrast it with the present targets, which are certain *immediate* transitions and spreads.

- $I \rightarrowtail O$ is an *immediate chain transition* $\leftrightarrow_{df}$ $I \rightarrowtail O$ is a chain transition such that there is no point event e such that $I < e$ and $e < O$.

- $I \rightarrowtail \Omega$ is an *immediate chain spread* $\leftrightarrow_{df}$ $I \rightarrowtail \Omega$ is a chain spread such that $I \rightarrowtail O$ is an immediate chain transition for each $O \in \Omega$.

Because of denseness, given any immediate chain transition $I \rightarrowtail O$, *either* I has a last member e and O is downward nonterminating with $inf(O) = e$, *or* O has a first member p and I is upward nonterminating with $sup_h(I) = p$ for some $h \in H_{[I]}$. For all our particular analytic purposes, we can think of the former as having the form $e \rightarrowtail O$ (with $inf(O) = e$) and the latter as having the form $I \rightarrowtail p$ (with $sup_h(I) = p$ for appropriate h).[14]

[13]For instance, "when" is a transition? It has, in Whitehead's phrase, no "simple location," and to speak as if it did may (but of course need not) invite confusion.

[14]That is, I feel free here and elsewhere to neglect the distinction between a unit set and its member.

Both are important, but they are profoundly different in their causal properties. A transition of the type $e \rightarrowtail O$ is a simple and apt candidate for a *causa causans* or "originating cause" for the following reason. Consider $e \rightarrowtail O$. Suppose you think that some initial I in the causal past of O that is separate and independent of e can be the initial of a spread that "influences" the occurrence of O. But then, because $inf(O) = e$, any "line of influence" from I to O will need to pass through e, and so not after all be separate and independent. Given that $e \rightarrowtail O$ is *immediate*, there is no "room for outside influences from the past."[15] In other words, if you are looking for a place in *Our World* such that what happens there is relevant to the transition from e to O, you cannot find any such place in the past of O except for e itself.

In contrast, in spite of the immediacy of $I_1 \rightarrowtail p$, some initial I_2 may well "influence" the occurrence of p: There is still room for a "line of influence" or a "world line" to run up from I_2 up to p without passing through I_1. It is for this reason that transitions $I_1 \rightarrowtail p$ are best taken as more similar in their causal character to non-immediate transitions than they are to transitions of the form $e \rightarrowtail O$. It is *only* in the latter case that all routes from initials in the proper past of O up to O are forced to pass through e.

I call a transition of type $e \rightarrowtail O$ "primary" in order to emphasize its theoretical role as a *causa causans*.

- $e \rightarrowtail O$ is a *primary chain transition* $\leftrightarrow_{df}$ $e \rightarrowtail O$ is an immediate chain transition whose initial is a single point event.[16]

- Fact: $e \rightarrowtail O$ is a primary chain transition iff $e < O$ and $inf(O) = e$.

- $e \rightarrowtail \Omega$ is a *primary chain spread* $\leftrightarrow_{df}$ $e \rightarrowtail \Omega$ is an immediate chain spread whose initial is a single point event.

- Fact: If $e \rightarrowtail \Omega$ is a primary chain spread, then for each $h \in H_{(e)}$ there is exactly one $O \in \Omega$ such that $h \in H_{\langle O \rangle}$. In other words, $\{H_{\langle O \rangle}\colon O \in \Omega\}$ is a partition of $H_{(e)}$. In fact, $\{H_{\langle O \rangle}\colon O \in \Omega\}$ is uniquely determined as Π_e, defined below.

It is often awkward or circuitous to speak of immediate outcomes of e as chains, for outcomes naturally lead a double life as events and as propositions. It is intuitive to say that when $e \rightarrowtail O$ is an immediate transition, so is $e \rightarrowtail H_{\langle O \rangle}$. But what is "immediate" about a *propositional* outcome (set of histories)? Of course we wish $H_{\langle O \rangle}$ to be one, but what is the species? BST-92 suggests the following.

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[15]This reasoning is given with shudder-quotes in the absence of a proper theory of "influence."

[16]A primary chain transition $e \rightarrowtail O$ can be vacuous: $H_{(e)} = H_{\langle O \rangle}$. Even though in that case $e \rightarrowtail O$ is hardly of much use as a *causa causans*, this won't get in our way.

Consider any collection of histories all containing e. If any two of them h_1 and h_2 contain a point event that is in the proper future of e, then any split between them must occur after e. At e the histories h_1 and h_2 are "undivided," and must remain so for some stretch after e. Therefore, since they do not divide until later, so as far as *immediate* outcomes of e go, h_1 and h_2 must stay together as part of the same immediate outcome.

This line of thought (which the article BST-92 suggests as the heart of its account) leads to the following definitions and facts.

- h_1 and h_2 *are undivided at* e, written $h_1 \equiv_e h_2$, $\leftrightarrow_{df}$ $e \in h_1 \cap h_2$ but e is not maximal in $h_1 \cap h_2$.

  Being undivided at e, $h_1 \equiv_e h_2$, and being separated at e, $h_1 \perp_e h_2$, both imply that e belongs to $h_1 \cap h_2$, but given that presupposition alone, being-undivided-at and being-separated-at must have opposite truth values.

- Fact: The relation $h_1 \equiv_e h_2$ of undividedness is, given the postulates of BST-92, transitive: the "transitivity of undividedness." See BST-92 for a proof, which turns out to require nearly all of the postulates. Undividedness at e is therefore an equivalence relation on $H_{(e)}$ since it is easily seen to be reflexive on $H_{(e)}$ (proof in BST-92) and symmetric (no proof needed).

Because it is an equivalence relation, undividedness at e gives us a smooth theory of primary propositional outcomes, transitions, and spreads.

- H is a *primary propositional outcome* of e $\leftrightarrow_{df}$ $\exists h_1[h_1 \in H$ and $\forall h_2[h_1 \equiv_e h_2 \leftrightarrow h_2 \in H]]$.

- $\Pi_e =_{df}$ the set of primary propositional outcomes of e. Π_e is a partition of $H_{(e)}$.

- $\Pi_e\langle h\rangle$ (for $e \in h$) $=_{df}$ the member of Π_e to which h belongs. $\Pi_e\langle h\rangle$ (for $e \in h$) is a primary propositional outcome of e, and every primary propositional outcome of e can be expressed in the form $\Pi_e\langle h\rangle$ for some $h \in H_{(e)}$.

- $e \rightarrowtail H$ is a *primary propositional transition* $\leftrightarrow_{df}$ H is a primary propositional outcome of e.

- $e \rightarrowtail \mathbf{H}$ is a *primary propositional spread* $\leftrightarrow_{df}$ $\mathbf{H}$ is the set of all primary propositional outcomes of e; which is to say, iff $\mathbf{H} = \Pi_e$.

- So $e \rightarrowtail \Pi_e$ is a primary propositional spread. The "spread" terminology is justified since exactly one member of Π_e is true in each history in which e occurs.[17]

Finally, having introduced two kinds of primary transitions and spreads (chain and propositional), one may observe that there is a smooth passage in each direction.

- If $e \rightarrowtail O$ is a primary chain transition, then $H_{\langle O\rangle}$ is a primary propositional outcome of e, so that $e \rightarrowtail H_{\langle O\rangle}$ is a primary propositional transition.
- Conversely, if H is a primary propositional outcome of e, so that $e \rightarrowtail H$ is a primary propositional transition, then there is an O such that $H = H_{\langle O\rangle}$ and $e \rightarrowtail O$ is a primary chain transition.
- Adding a little more detail, given $e \in h$, there is always an outcome chain O such that $e < O$ and $O \subseteq h$ and $inf(O) = e$. In other words, if $e \in h$, there is a primary chain transition $e \rightarrowtail O$ such that $O \subseteq h$. Furthermore, when the latter three conditions hold, $H_{\langle O\rangle} = \Pi_e\langle h\rangle$.
- If $e \rightarrowtail \Omega$ is a primary chain spread then $\Pi_e = \{H_{\langle O\rangle} \colon O \in \Omega\}$.
- Conversely, given a primary propositional spread $e \rightarrowtail \Pi_e$, there is a set Ω of outcome chains such that $e \rightarrowtail \Pi_e = e \rightarrowtail \{H_{\langle O\rangle} \colon O \in \Omega\}$.

These natural moves between the chain and propositional ideas of primary transition and spread, having been concentrated here, will be made without comment.

The beginning of this section introduced general ideas of *chain* transitions and spreads, but as yet we haven't made room for the general ideas of *propositional* transitions and spreads.

- $I \rightarrowtail H$ is a *chain-proposition transition* $\leftrightarrow_{df}$ $H \subseteq H_{[I]}$. When the subset relation is proper, the transition is *contingent*.
- $I \rightarrowtail \mathbf{H}$ is a *chain-proposition spread* $\leftrightarrow_{df}$ $\mathbf{H}$ is a partition of $H_{[I]}$.

These last ideas are perhaps too general to be of much service, and they are used here just a little. Observe that even this very abstract idea of "spread" insists on some sort of location in branching space-times via the location of I.[18]

[17] Since the matter is not as memorable as one might have hoped, I call attention to the fact that with respect to a certain history, our jargon allows that an event *occurs* or not, whereas a proposition is *true* or not.

[18] There is room left for "proposition-proposition" transitions and spreads, but I know of no particular application for them.

3 Generalized primary space-like-related modal-correlation funny business

What is a "modal correlation," and when does it constitute EPR-like "funny business"? If you have two spreads each with its own set of possible outcomes, then "correlation" means that knowing what happens at one of them gives you some information about what happens at the other.[19] In a "modal correlation," the information is in terms of consistency and inconsistency.

We can get the right definition of "modal correlation" by paying attention to the most general case. Suppose we have two chain-proposition spreads $I_1 \rightarrowtail \mathbf{H}_1$ and $I_2 \rightarrowtail \mathbf{H}_2$.

- $I_1 \rightarrowtail \mathbf{H}_1$ and $I_2 \rightarrowtail \mathbf{H}_2$ are *modally correlated* $\leftrightarrow_{df}$ $H_1 \cap H_2 = \varnothing$ for some $H_1 \in \mathbf{H}_1$ and $H_2 \in \mathbf{H}_2$.[20]
- If we specify both the two spreads and an inconsistent pair of outcomes (one from each), we say that we have a *modal correlation*.

It is built into the notation (and the "given" clauses) that the outcome H_1 of the first transition and the outcome H_2 of the second are each possible. Each is individually possible. The question is, can both H_1 and H_2 be true in some one history? Modal correlation says No. Absence of modal correlation says Yes. (Permit me to emphasize that this analysis absolutely requires that *what* is correlated are spreads; no vague notion of "variable" will serve.)

3.1 Simplest kind of space-like-related modal-correlation funny business

Whether modal or probabilistic, correlation between spreads $I_1 \rightarrowtail \mathbf{H}_1$ and $I_2 \rightarrowtail \mathbf{H}_2$ is, as such, uninteresting; it happens too often. It seems part of the literature concerning various quantum-mechanical "entanglement" phenomena such as EPR that a key feature of the interesting (surprising?) cases is that the correlation occurs between space-like related measurements. If one for the moment idealizes a measurement as a *primary* spread, a suggestive kind of *a priori* deduction of the relevance of space-like relatedness becomes possible. To see this, let us first give the definition of "space-like related" from BST-92.

- Two point events e_1 and e_2 are *space-like related*, written $e_1 \ \mathrm{SLR}\ e_2$, $\leftrightarrow_{df}$ they are (1) consistent, (2) $e_1 \neq e_2$, and (3) both $e_1 \not< e_2$ and $e_1 \not< e_2$.

[19] The epistemic language is of course only for expository convenience.

[20] Modal correlation is a stronger property than probabilistic correlation. Contrariwise, absence of modal correlation means much less than absence of probabilistic correlation.

This definition of "space-like related" is the same as that of Minkowski space-time, with one addition required by BST-92: There must be at least one history that contains both point events. They cannot be inconsistent.

It is "scientifically natural" to find interest in a case of two primary spreads at space-like remove that are nonetheless correlated, and the upcoming "deduction" does not touch on the sufficiency of being space-like related for being interesting. Rather, I show that if the initials of two primary spreads are *not* space-like related, then there is no interest in their modal correlation.

Consider then two primary spreads $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$. There are three ways (1)–(3) in which their initials can fail to be space-like related. In each of these cases, I indicate why the question of modal correlation is obviously uninteresting.

1. If the initials e_1 and e_2 are inconsistent, there is inevitable and indeed rampant modal correlation, since every member of every $H_1 \in \Pi_{e_1}$ must contain e_1, whereas in virtue of the inconsistency of e_1 and e_2, no member of any $H_2 \in \Pi_{e_2}$ can contain e_1. So in this case, modal correlation is trivially inescapable. Since the existence of inconsistent pairs of point events is a consequence of the merest hint of even straightforward no-funny-business indeterminism (that is, it follows from the bare existence of more than one history), such modal correlations do not by themselves warrant our interest.

2. When $e_1 = e_2$, then intuitively we might not speak of "correlation." But it is illuminating to see that in BST-92, if $e_1 = e_2$ then we can give reasons of a sort: There are two equally uninteresting cases. Case *(a)*. If Π_{e_1} (which is the same as Π_{e_2}) has more than one member, say $H_1 \neq H_2$, then of course you are going to find a modal correlation. Since $e_1 \rightarrowtail \Pi_{e_1} = e_2 \rightarrowtail \Pi_{e_2}$ is a spread, its distinct outcome propositions have empty intersection: $H_1 \cap H_2 = \varnothing$, and modal correlation cannot be avoided. Case *(b)*. On the other hand, if Π_{e_1} is trivial (has just one member, namely, $H_{(e_1)}$), then the absence of modal correlation is vacuous and of equal lack of interest.

3. If e_1 is in the causal past of e_2, then according to BST-92 theory,[21] the very occurrence of e_2 is consistent with one and only one primary outcome of e_1 (member of Π_{e_1}). Two cases. Case *(a)*. Perhaps e_1 has $H_{(e_1)}$ as its single vacuous primary outcome. This is evidently a case of uninteresting absence of modal correlation. Case *(b)*. e_1 has more than one primary outcome. We know that e_2 is consistent with only one of them, so that each of the other outcomes of e_1 is inconsistent with *any and every* outcome of e_2; which is equally uninteresting. So if $e_1 < e_2$, "modal correlation" is in either case

[21] The transitivity of undividedness is involved.

uninteresting. And, of course, the case is the same if e_2 lies in the causal past of e_1.

What happens when one eliminates these three uninteresting cases? In BST-92 that is *exactly* to say that e_1 and e_2 are "space-like related": Modal correlation between primary spreads $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$ is interesting only if e_1 is space-like related to e_2. In my view such correlations are interesting in exactly the way that EPR-like phenomena are, which is why I call such modal correlations a kind of "funny business," an opinion built into the wording of the following *definiendum*.

- Two primary spreads $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$ are *space-like-related modally correlated* $\leftrightarrow_{df}$ e_1 SLR e_2 and for some h_1 and h_2 such that $e_1 \in h_1$ and $e_2 \in h_2$, $\Pi_{e_1}\langle h_1 \rangle \cap \Pi_{e_2}\langle h_2 \rangle = \varnothing$
- Two primary spreads $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$ taken together with two outcome-determining histories h_1 and h_2 constitute a case of *space-like-related modal-correlation funny business of the simplest kind* $\leftrightarrow_{df}$
  - e_1 SLR e_2 and
  - $e_1 \in h_1$ and $e_2 \in h_2$ and
  - $\Pi_{e_1}\langle h_1 \rangle \cap \Pi_{e_2}\langle h_2 \rangle = \varnothing$.[22]

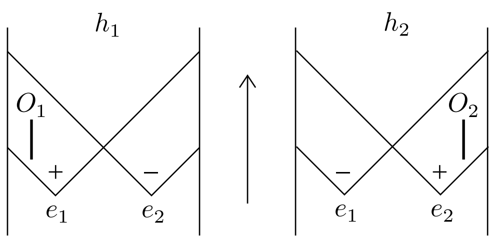

Figure 1: BST-92 picture of Einstein-Rosen-Podolski.

[22]Two comments. (1) The phrase "outcome-determining" is redundant. It is intended to highlight the role of the histories in defining a particular modal correlation; namely, the histories serve only to determine which two outcomes are inconsistent. (2) The passage from "inconsistent" to "not space-like related" is valid only for the present case, where the initials are single point events, but fails when later we consider initials that are sets of point events. Then two initials in the more general sense can be (according to the definitions we give) both space-like related and inconsistent.

Simplest cases of SLR modal-correlation funny business are EPR-like. The BST-92 picture of EPR is given in Figure 1.[23] EPR is overkill for SLR modal correlation in two ways. (1) Not only are e_1 and e_2 consistent, but they occur in exactly the same histories. (2) There is more than one joint outcome that is impossible: Both $\Pi_{e_1}\langle h_1\rangle \cap \Pi_{e_2}\langle h_2\rangle = \varnothing$ and $\Pi_{e_1}\langle h_2\rangle \cap \Pi_{e_2}\langle h_1\rangle = \varnothing$. Informally we can say this by the following: If you are sitting immediately after e_1, then you are sure to know *exactly* what you will find out has happened at e_2, namely, the opposite sign. To see how EPR itself goes beyond the weakest sort of SLR modal correlation, look at Figure 2. Here, sitting just after e_1, if you see that a "$-$" has happened, then you know which outcome of e_2 you will eventually find in your past; but if you see that a "$+$" has happened, then you know nothing (yet) about which outcome of e_2 you will eventually find in your past. That is because there is only a *single* modal correlation.[24] I still suggest that this is "essentially" the same, however, as the EPR case. What is "essential" is that there be at least one modal correlation (impossible joint outcome).

If two primary transitions $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$ are SLR modally correlated according to definition, it follows that at least one of Π_{e_1} and Π_{e_2} is nontrivial, which is to say, its initial does not predetermine an outcome.[25] One of $e_1 \rightarrowtail \Pi_{e_1}$ and $e_2 \rightarrowtail \Pi_{e_2}$ might by the definition be a trivial spread, but let us put this aside: Assume that both are nontrivial. Then the situation is this: Although the primary spreads originating in e_1 and e_2 are space-like separated, and although in each case there is no predetermination of which outcome comes to be, it is nevertheless

[23] BST-92 is about many space-times branching. To picture this is not so easy, and to explain the pictures is even more difficult. The convention is that a figure represents each history in *Our World* with a separate two-dimensional Minkowski diagram. Identically labeled point events are identical, each typically belonging to more than one history. If the regions immediately after a labeled point event are marked the same in two histories (for example, in Figure 2, "$+$" after e_1 in each of h_1 and h_2) then those histories are undivided there, and contain exactly the same points there. On the other hand, if in two histories the regions immediately after one and the same point event are marked differently (for example, in Figure 2, after e_1 there is "$+$" in h_1 and "$-$" in h_3), the convention is that the labeled point event is a "choice point": The two histories split at that point event, so that after the choice point, no point events—not even those on light-like paths emanating from the choice point—are common to the two histories. A further "negative" convention is that using the same mark for the immediate future of *distinct* labeled point events (for example, in Figure 2, "$+$" after both e_1 and e_2 in h_1) has *no* strictly causal meaning. When worrying about indeterminism and funny business according to BST-92, "similarity" doesn't count.

[24] It seems to me certain that anyone who is sufficiently familiar with quantum mechanics should be able to rig up an experimental situation having the causal shape described in Figure 2; that is part of what I intend by suggesting that BST-92 can serve as a sort of guide to finding one's way around amid scary physics. I myself cannot give you a concrete example, however, because I fail the test of familiarity.

[25] If two primary transitions are each trivial, they have one and only one joint outcome, which, if they are space-like-related, must certainly be consistent and hence not a modal correlation.

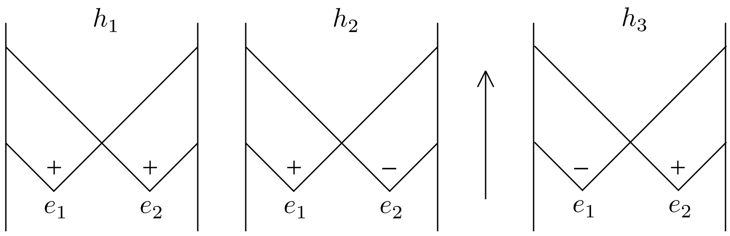

Figure 2: Space-like-related modal correlation of the simplest kind.

guaranteed in advance that a certain combination of outcomes will not happen.[26] This is a type of EPR-like "funny business" abstractly described. I intend this as kind of a claim: If you yourself come across a concrete pair of measurement-like situations for which this idealization seems apt, then you yourself will find that you have found something that belongs in the EPR family with regard to its causal structure.

One should reject the converse: There are kinds of funny business that are not space-like-related modal correlations of the simplest kind. Each of Figure 3 and Figure 4 is an illustration of funny business that does not, however, exhibit SLR modal correlation of the simplest kind. Look at Figure 3. There is no SLR modal correlation of the simplest kind. If you want a modal-correlation account of this funny business that is *like* the simplest kind in its concentration on single-point-event initials, you have to divide into *three* space-like separated initials rather than just two. You will find that each of these three initials has its own outcomes, but that there is a combination of one outcome from each that is impossible (namely, the $+++$ combination).

3.2 Primary space-like related modal correlation generalized

There is, however, a way to keep our primary SLR modal-correlation account binary and still have it apply to this case, and this is the way that we take.[27] Let one initial consist of two point events instead of just one; namely, let the initials be defined as sets of point events that are consistent in the way that an initial should be:

[26] In pictures this looks like "missing histories." Thinking in this way with your right brain is fine as long as it does not lead you to decide that funny business is abnormal.

[27] Szabo and Belnap 1996 works with SLR correlations that are neither binary nor primary. Note that here I am not trying to use "intuition" in order to show up Figure 3 as exhibiting funny business; I have already done that. Now the task is to see how to modify a *binary* account in order to apply to this case.

- $\mathbf{I}$ *is an initial event* $\leftrightarrow_{df}$ $\mathbf{I} \neq \varnothing$ and $\exists h[\mathbf{I} \subseteq h]$.[28]

Let $\mathbf{I}$ range over arbitrary initial events: $\mathbf{I}$ is always a nonempty set of point events that is consistent in the sense that they occur together in at least one history. Let $\mathbf{I}_1$ = $\{e_1\}$ in Figure 3, and let $\mathbf{I}_2$ = $\{e_2, e_3\}$. Evidently $\Pi_{\mathbf{I}_1}\langle h_2\rangle$ = $\Pi_{e_1}\langle h_2\rangle$ = $\{h_2, h_3, h_4\}$, by our prior definitions. We need, however, to figure out what to mean by $\Pi_{\mathbf{I}}$ when the initial $\mathbf{I}$ is more than a single point event. A tactic that plays out well in the present context is to define a "generalized primary propositional outcome" of an initial event via quantified undividedness.[29]

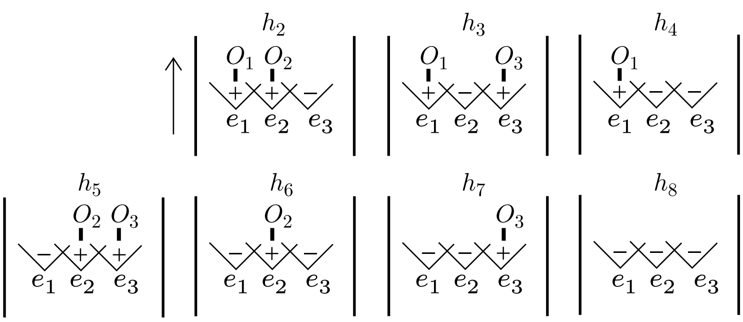

Figure 3: A piece of funny business with seven histories.

- For any initial event $\mathbf{I}$, h_1 *is undivided from* h_2 *at* $\mathbf{I}$, written $h_1 \equiv_{\mathbf{I}} h_2$, $\leftrightarrow_{df}$ $h_1 \equiv_e h_2$ for every $e \in \mathbf{I}$; the relation is evidently an equivalence relation on $H_{[\mathbf{I}]}$.
- H is an *generalized primary propositional outcome of* $\mathbf{I}$ $\leftrightarrow_{df}$ for some $h_1 \in H_{[\mathbf{I}]}$, $H = \{h_2 \colon h_1 \equiv_{\mathbf{I}} h_2\}$.
- $\Pi_{\mathbf{I}}$ $=_{df}$ the set of generalized primary propositional outcomes of $\mathbf{I}$; $\Pi_{\mathbf{I}}$ is a partition of $H_{[\mathbf{I}]}$.
- For $\mathbf{I} \subseteq h_1$, $\Pi_{\mathbf{I}}\langle h_1\rangle =_{df} \{h_2 \colon h_1 \equiv_{\mathbf{I}} h_2\}$, and is the generalized primary propositional outcome of $\mathbf{I}$ to which h_1 belongs.
- Where $\mathbf{I}$ is an initial, $\mathbf{I} \rightarrowtail H$ is a *generalized primary propositional transition* $\leftrightarrow_{df}$ $H \in \Pi_{\mathbf{I}}$; and a *generalized primary propositional spread* is defined as a spread having the form $\mathbf{I} \rightarrowtail \Pi_{\mathbf{I}}$.

[28]We defined an initial *chain* as requiring an upper bound, which is indeed a necessary condition of having a supremum. In the present investigation, however, the upper-bound requirement has seemed unnecessary in connection with using the more general notion of an initial *event*.

[29]The double-orthocomplement idea as employed in Placek 2000 (p. 143) should not be neglected.

- Let "GP" stand for "generalized primary."

With these definitions, $\Pi_{\mathbf{I}_2}\langle h_5\rangle = \{h_5\}$ in Figure 3 is a GP propositional outcome of $\mathbf{I}$; and evidently we have a modal correlation:

$$\Pi_{\mathbf{I}_1}\langle h_2\rangle \cap \Pi_{\mathbf{I}_2}\langle h_5\rangle = \varnothing.$$

Further, and this is an essential observation, each member of $\mathbf{I}_1$ is space-like related to each member of $\mathbf{I}_2$, so that we have not just any modal correlation; we have a *space-like-related* modal correlation. Let us convert this observation to a definition.

- $\mathbf{I}_1$ SLR $\mathbf{I}_2 \leftrightarrow_{df} e_1$ SLR e_2 for every $e_1 \in \mathbf{I}_1$ and $e_2 \in \mathbf{I}_2$.

That is, to say of two initial events that they are space-like related is to say that each member of one is space-like related to each member of the other.[30]

Now that all concepts are sharply defined, we can enlarge the account of SLR modal correlation:

- Two GP propositional spreads $\mathbf{I}_1 \rightarrowtail \Pi_{\mathbf{I}_1}$ and $\mathbf{I}_2 \rightarrowtail \Pi_{\mathbf{I}_2}$ are *space-like-related modally correlated* $\leftrightarrow_{df}$ $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and for some h_1 and h_2 such that $\mathbf{I}_1 \subseteq h_1$ and $\mathbf{I}_2 \subseteq h_2$, $\Pi_{\mathbf{I}_1}\langle h_1\rangle \cap \Pi_{\mathbf{I}_2}\langle h_2\rangle = \varnothing$.

It is easy to see that the funny business illustrated in Figure 3 falls under this account, so that it no longer stands as a counterexample to the thesis that all situations exhibiting EPR-like funny business fall under the rubric of "SLR modal correlation" as defined in the language of BST-92. SLR modal correlation of the simplest kind may not be enough, but it is worth conjecturing that the more generalized notion is indeed "enough." I express this conjecture by means of the following definition.

- Two GP propositional spreads $\mathbf{I}_1 \rightarrowtail \Pi_{\mathbf{I}_1}$ and $\mathbf{I}_2 \rightarrowtail \Pi_{\mathbf{I}_2}$ together with two outcome-determining histories h_1 and h_2 such that $\mathbf{I}_1 \subseteq h_1$ and $\mathbf{I}_2 \subseteq h_2$ constitute a case of *generalized primary space-like-related modal-correlation funny business* $\leftrightarrow_{df}$ $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and $\Pi_{\mathbf{I}_1}\langle h_1\rangle \cap \Pi_{\mathbf{I}_2}\langle h_2\rangle = \varnothing$.[31]

We can test this conjecture just a little by considering another diagram, namely Figure 4. You can check each pair of space-like related point events with more than

[30] I note with some surprise that one should not strengthen this definition to say that $\mathbf{I}_1$ and $\mathbf{I}_2$ are consistent (i.e. to say that some history contains them both). The reason is that any case in which both $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and $H_{[\mathbf{I}_1]} \cap H_{[\mathbf{I}_2]} = \varnothing$ is a case of funny business. There are two arguments for this: (1) There are examples, which I omit. (2) In such a case, by the theorem to be proved, you will also find a case of some-cause-like-locus-not-in-past funny business.

[31] The adjective "primary" is important. When outcomes are distant from initials, then SLR modal correlation is not enough for funny business, since the correlation can be due to perfectly "ordinary" circumstances such as a "common cause." In the primary case, however, there is, as we have suggested in Section 2.3, no "room" for additional causal "influences" from the past.

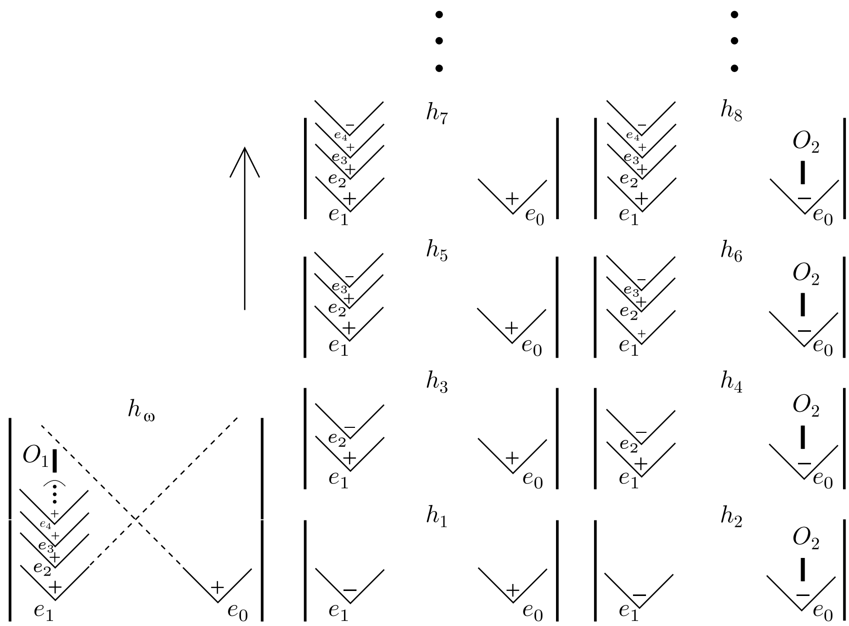

Figure 4: A piece of funny business that does not exhibit SLR modal correlation of the simplest kind.

one primary outcome (i.e., the labeled point events) to see that every combination of outcomes, one from each, is possible. To appreciate that Figure 4 nevertheless pictures some funny business, think of the entire infinite chain on the left as one initial event $\mathbf{I}_1$, and let e_0 be the other. These two initials look space-like related, and indeed are so in our defined sense: Each member of $\mathbf{I}_1$ is space-like related to e_0. What smacks of funny business in Figure 4 is that if you know that each member of $\mathbf{I}_1$ "chose" plus rather than minus, then you know that e_0 did not "choose" minus. Symmetrically, if you know that e_0 "chose" minus, then you know that not every member of $\mathbf{I}_1$ "chose" plus; and all of this in spite of the space-like relatedness of e_0 and each member of $\mathbf{I}_1$. This indeed has the flavor of EPR.[32]

It is effortless to see that the GP concepts apply to this case. Figure 4 shows that $\mathbf{I}_1$ has but a single GP outcome, $\{h_\omega\}$; and it shows that this GP outcome of $\mathbf{I}_1$ is inconsistent with the outcome $\Pi_{e_0}\langle h_2\rangle$ of e_0. Since we verified space-like

[32] I do not know whether the infinite sequence of choice points defining $\mathbf{I}_1$ does or does not make "real scientific sense." It seems to me certain, however, that the ability of BST-92 theory to treat of such cases cannot reasonably be counted against it.

relatedness of $\mathbf{I}_1$ and e_0, we have a *bona fide* case of GP SLR modal-correlation funny business described in strict BST-92 terms.

Rather than further retail testing of the conjecture that GP SLR modal-correlation funny business catches all EPR-like phenomena to the extent that the language of BST-92 makes that possible, I turn to a second approach to funny business. The proof that the two approaches come to the same thing is offered as evidence of their individual stability.

4 Some-cause-like-locus-not-in-past funny business

The second effort at extracting a notion of funny business tries to find inspiration in the (I should think incontestable) fact that when we look for a token-causal explanation of why things are one way rather than another, we always look to the past. Something like this thought lies behind much discussion of Reichenbach's "common cause" principle as for example in Szabo and Belnap 1996 or Placek 2000, and also behind the prior choice principle of BST-92 cited in Section 2.1.

4.1 Cause-like locus (simplest kind)

My approach to the second notion of funny business begins with a contrastive observation: Modal correlation invokes the idea of *undividedness* between histories at a point event, whereas the prior choice principle relies for its statement on the idea of *separation*. Separation is as important as undividedness, maybe more so. I suggest that separation is cause-like. Start with the prior choice postulate, which says that if some history is inconsistent with the beginning-to-be of O (i.e., if $h \cap O = \varnothing$), then there is bound to be a point event e *in the past of* O such that $h \perp_e H_{\langle O \rangle}$. Such an e is the initial of a *causa causans*, a primary or originating transition that objectively explains or accounts for the fact that being in the *outcome event* O is possible only at the expense of not being in the history h. The transition from the event e to the primary propositional outcome of e that is determined by O (provably exactly one, call it $\Pi_e\langle O\rangle$, will be consistent with O[33]) may properly be taken as a kind of partial "cause" of O. That transition $e \rightarrowtail \Pi_e\langle O\rangle$ keeps O possible, while itself being a *contingent* matter, since if the transition from O had gone towards h, O would thereafter have been impossible.

Let us therefore permit ourselves to say that e itself is a "cause-like locus" for O. That means: e is the initial of a nontrivial primary propositional spread, at least one outcome of which renders O impossible and at least one of which leaves O possible—at least in the immediate future of e. One may define this cause-like

[33] In fact when e is in the causal past of O, it is provable that $H_{\langle O \rangle} \subseteq \Pi_e\langle O\rangle$.

concept without coming out with a theory (or even a syntax) of exactly what causes what.

- e is a *cause-like locus of the simplest kind for* O *with respect to* $h \leftrightarrow_{df} h \perp_e H_{\langle O \rangle}$.

Why do I say "cause-like" instead of "causal"? There are three reasons. The first is merely to avoid premature commitment to a theory of *causality*.[34] The second and more important reason depends on the observation that this notion of "cause-like locus" *does not include a statement that* e *lies in the past of* O. We know of course by the prior choice postulate that *at least one* cause-like locus for O lies in its past. The point is that the prior choice postulate says that something cause-like lies in the past of O, but it is no part of the definition of *what* is said to lie in the past (namely, a cause-like locus) that in fact it does so. Because of this analytical separation, it makes sense to inquire of a particular cause-like locus whether or not it lies in the past. Of course our instincts almost drive us to declare that whatever is a cause *must* occur in the past; but the analytical separation enshrined in the phrase "cause-*like*" allows us to appreciate the third reason, which is critical: Neither the definition of cause-like locus nor the prior choice postulate commit us to the theory that *all* cause-like loci for O lie in the past of O. And in fact in numerous EPR-like situations, although (according to the present suggested analysis) invariably *some* cause-like locus does lie in the past, some cause-like locus for O fails to lie in its past. This makes quick intuitive statements liable to be misleading. For example, it would seem as if the meaning of the prior choice principle of BST-92 is that "causes lie in the past," and the principle is defended intuitively by the observation that when looking for a token-explanation of why things are the way they are rather than otherwise, we always look to the past. It is therefore arresting to learn that the principle only says that there is always *at least one* cause-like locus in the past; it does not say that *all* of them lie there.

This suggests that the existence of a cause-like locus that fails to reside in the past is itself a plausible account of what is strange about EPR-like phenomena. If there is a place (in *Our World*) where a transitional determination is made as to whether the beginning of some *outcome event* remains possible on the one hand or is made impossible on the other, one would expect that place to be in the past.[35] So:

[34] Research proposal for a happy syntax of *causation*: Construe "causes"—and perhaps "*effects*" as well—as transitions in *Our World*. The role of transitions in understanding causation is emphasized in Xu 1997.

[35] The sense of "expect" here is tied to a particular mental set; perhaps it is akin to the special sense in which one expects that there be no "irrational" or "imaginary" numbers.

- A history h, an outcome chain O, and a point event e constitute a case of *some-cause-like-locus-not-in-past funny business of the simplest kind* $\leftrightarrow_{df}$ $h \perp_e H_{\langle O \rangle}$, but $e \not< O$.

The account needs generalizing, and in two directions. The "cause-like locus" needs generalizing from a single point event e to the more complex idea of an initial event **I** (a set of point events) and the "*outcome event*" in question also needs to be generalized to take account of outcomes of a sort too complex to be represented by a single outcome chain. Each of these generalizations will bring some subtleties in their respective trains.

4.2 From e to I as cause-like locus

The need to generalize the "causal locus" from point event e to initial event **I** we can see from either of two previous examples. Consider that in Figure 4 h_ω is inconsistent with O_2. In this diagram there is no some-cause-like-locus-not-in-past funny business of the simplest kind; that is, in essence, because no single point event except e_0 serves to separate h_ω from $H_{\langle O_2 \rangle}$, and e_0 is after all in the past of O_2. But generalize. Let $\mathbf{I} = \{e_1, e_2, \ldots\}$. Observe that you need all of (and only) **I** to separate h_ω and $H_{\langle O_2 \rangle}$: $\forall h[h \in H_{\langle O_2 \rangle} \rightarrow \exists e[e \in \mathbf{I}$ & $h \perp_e h_\omega]]$. You do not need e_0: For every history in which O_2 occurs, there is a "splitter" in **I** that splits that history from h_ω. What's *funny* is that although **I** can count as an adequate "cause-like locus" for O_2, no part of it falls in the causal past of O_2. So noticing this fact is one way of seeing something funny about the causal structure exhibited in Figure 4. As for Figure 3, in order to separate h_5 from $H_{\langle O_1 \rangle}$, you do not need e_1; the set $\mathbf{I} = \{e_2, e_3\}$ also suffices as cause-like locus, even though no part of **I** lies in the causal past of O_1.

These examples suggest that the idea of separation, which starts with histories splitting at point events ($h_1 \perp_e h_2$), needs to be generalized, and I have already done so in one way, having defined $h_1 \perp_e H$ by universal quantification: $h_1 \perp_e H$ $\leftrightarrow_{df}$ $h_1 \perp_e h_2$ for every $h_2 \in H$. A further generalization is, however, essential. One wishes to be able to say that h is separated from H at a *set* of point events **I**, meaning that which member of **I** might be needed for separation might depend on which member of H is to be separated from h. This idea needs to be existential:

- h_1 *is separated from* H *at* **I**, written $h_1 \perp_{\mathbf{I}} H$, $\leftrightarrow_{df}$ $\forall h_2[h_2 \in H \rightarrow \exists e[e \in \mathbf{I}$ and $h_1 \perp_e h_2]]$

That is, h_1 is separated from H at **I** iff for each history $h_2 \in H$ there is a point event in **I** that splits h_2 from h_1. Note the alternation of the quantifiers.

4.3 Relevance of a cause-like locus

Before one can base on these thoughts a rigorously defined more general notion of cause-like locus and a more general notion of some-cause-like-locus-not-in-past funny business, one needs to face up to a consideration that comes into play only after separation-at-e is generalized to separation-at-$\mathbf{I}$. My prose description of this kind of funny business implicitly relied, in a certain way, on the fact that you need *only* $\mathbf{I}$ to separate h_ω from $H_{\langle O_2 \rangle}$. In what way? If $\mathbf{I}$ is too large, it may well contain loose pieces. The nub is that since $h \perp_{\mathbf{I}} H_{\langle O \rangle}$ is existential on $\mathbf{I}$, separation is preserved even if a number of irrelevant "junk" point events are added to $\mathbf{I}$. These "extra" points—the ones that do no work in the separation—could be anywhere without affecting the truth of $h \perp_{\mathbf{I}} H_{\langle O \rangle}$. Therefore, one would certainly not have a reason to issue a judgment of "funny business" just because some of these "extra" point events failed to lie in the past of O. So at this point a refinement is needed. We wish to be able to say that each member of $\mathbf{I}$ is relevant to separating h from $H_{\langle O \rangle}$. That's not the same as "essential": I just mean that $\mathbf{I}$ is "entirely relevant" in the sense that each piece of $\mathbf{I}$ can be used in the separation. I mean that $\forall e[e \in \mathbf{I} \rightarrow \exists h_1[h_1 \in H_{\langle O \rangle}$ and $h \perp_e h_1]]$.[36]

These considerations lead us to enter the following important definitions, albeit definitions that are not yet in final form.

- *h is relevantly separated from H at* $\mathbf{I}$, written $h \perp\!\!\!\perp_{\mathbf{I}} H$, $\leftrightarrow_{df}$ $h \perp_{\mathbf{I}} H$ and $\forall e[e \in \mathbf{I} \rightarrow \exists h_1[h_1 \in H$ and $h \perp_e h_1]]$.
- $\mathbf{I}$ is a *cause-like locus for O with respect to h* $\leftrightarrow_{df}$ $h \perp\!\!\!\perp_{\mathbf{I}} H_{\langle O \rangle}$.
- An initial event $\mathbf{I}$ and a history h and an outcome chain O constitute a case of *some-cause-like-locus-not-in-past funny business (simple kind)* $\leftrightarrow_{df}$ $\mathbf{I}$ is a cause-like locus for O with respect to h, but no member of $\mathbf{I}$ lies in the causal past of O.

From the some-cause-like-locus-not-in-past point of view, we can find funny business in the example of Figure 3. Set $\mathbf{I} = \{e_2, e_3\}$. Then $\mathbf{I}$ is a cause-like locus for O_1 with respect to h_5 ($h_5 \perp\!\!\!\perp_{\mathbf{I}} H_{\langle O_1 \rangle}$), but none of $\mathbf{I}$ lies in the past of O_1. So again, and to this limited extent, the stability of the definition is confirmed.

4.4 From O to $\mathbf{O}$ as an outcome event

In order to complete generalizing some-cause-like-locus-not-in-past funny business, one needs to see that representing outcome events as outcome chains O is too

[36] Such a set is not necessarily "minimal" with respect to separating h from $H_{\langle O \rangle}$; it could be that one of its proper subsets also suffices. It just turns out that relevance matters whereas minimality does not matter.

special. Just as I generalized from single-point-event initials e to consistent sets-of-point-events initials $\mathbf{I}$, so I now generalize from single-outcome-chain outcome events O to sets-of-outcome-chains outcome events $\mathbf{O}$. Then it will be possible to prove the equivalence of the statements that there is funny business in the GP SLR modal-correlation sense with funny business in the some-cause-like-locus-not-in-past.

For a simple case in which the more generalized notion of outcome is needed, consider once more Figure 3. The outcome event $\mathbf{O} = \{O_2, O_3\}$ occurs if and only if *both* O_2 and O_3 occur, which is to say, in h_5 and in no other history. That is, using $H_{\langle \mathbf{O} \rangle}$ for the occurrence proposition for $\mathbf{O}$, $H_{\langle \mathbf{O} \rangle} = H_{\langle O_2 \rangle} \cap H_{\langle O_3 \rangle} = \{h_5\}$. Therefore, $\mathbf{O}$ does not occur in h_2. Certainly e_3 is a perfectly good cause-like locus for $\mathbf{O}$ remaining possible at the expense of h_2 ($h_2 \perp_{e_3} H_{\langle \mathbf{O} \rangle}$), and it is heartwarming to observe that e_3 lies in the past of a member of $\mathbf{O}$; namely, $e_3 < O_3$; nothing funny about that. The funny business lies in the fact that e_1 is also a cause-like locus for $\mathbf{O}$ with respect to h_2 ($h_2 \perp_{e_1} H_{\langle \mathbf{O} \rangle}$), even though e_1 does not lie in the past of any member of $\mathbf{O}$ and so cannot provide a source of "influence" that travels along a "world line" up from e_1 to a member of $\mathbf{O}$. Funny business indeed. Note that O_2 alone will not show up the funny business because O_2 occurs in h_2; nor will attending to O_3 alone help. The reason is that although O_3 does not occur in h_2, neither e_1 nor e_2 nor even $\mathbf{I} = \{e_1, e_2\}$ serves as a cause-like locus for O_3 with respect to h_2, since $\mathbf{I}$ thus defined does not serve to separate h_2 from (every history in) $H_{\langle O_3 \rangle}$. You need the joint occurrence of O_2 and O_3 in order to exhibit some-cause-like-locus-not-in-past funny business.

4.5 Final account of some-cause-like-locus-not-in-past funny business

I extract rigorous definitions from these deliberations as follows.

- $\mathbf{O}$ is an outcome event $\leftrightarrow_{df}$ $\mathbf{O}$ is a nonempty set of outcome chains that is consistent in the sense that $\bigcap \{H_{\langle O \rangle} : O \in \mathbf{O}\} \neq \varnothing$.[37]

  So an *outcome event* can have disjoint pieces, and to say that it occurs is to say that each piece begins to be.[38]

- $H_{\langle \mathbf{O} \rangle}$ is defined as $\bigcap \{H_{\langle O \rangle} : O \in \mathbf{O}\}$.

- $\mathbf{I}$ *is a cause-like locus for* $\mathbf{O}$ *with respect to* h $\leftrightarrow_{df}$ $h \perp_{\mathbf{I}} H_{\langle \mathbf{O} \rangle}$.

[37] This definition seems helpful, but there is no reason to believe that it is exclusively so. See note 29.

[38] The idea of representing certain events as sets of sets is due to von Kutschera 1993, an idea that is also made use of by Xu 1997. It is obvious that the source of the idea lies in theoretical considerations rather than "common sense"; those considerations seem to me to be powerful.

- To say that an initial event $\mathbf{I}$ "does not lie in the past of" an *outcome event* $\mathbf{O}$ is to say something doubly universal: for every $e \in \mathbf{I}$ and every $O \in \mathbf{O}$, $e \not< O$ (no member of the initial event lies in the past of any member of the *outcome event*).

- An initial event $\mathbf{I}$ and a history h and an *outcome event* $\mathbf{O}$ constitute a case of *some-cause-like-locus-not-in-past funny business* $\leftrightarrow_{df}$ $\mathbf{I}$ is a cause-like locus for $\mathbf{O}$ with respect to h, but no member of $\mathbf{I}$ lies in the causal past of any member of $\mathbf{O}$.

This is our "final" account of some-cause-like-locus-not-in-past funny business. It might be adequate to all cases of EPR-like funny business to the extent that they can be represented in the language of BST-92. Evidence in this direction is provided by the fact, to be proved shortly, that there is a case of some-cause-like-locus-not-in-past funny business if and only if there is a case of generalized primary space-like-related modal-correlation funny business.

5 Equivalence of the two ideas of funny business

Let us state again what counts as a case of funny business in each of the two senses.

5-1 DEFINITION. *(Two ideas of funny business)*

- Two GP propositional spreads $\mathbf{I}_1 \rightarrowtail \Pi_{\mathbf{I}_1}$ and $\mathbf{I}_2 \rightarrowtail \Pi_{\mathbf{I}_2}$ together with two outcome-determining histories h_1 and h_2 such that $\mathbf{I}_1 \subseteq h_1$ and $\mathbf{I}_2 \subseteq h_2$ constitute a case of *generalized primary space-like-related modal-correlation funny business* $\leftrightarrow_{df}$ $\mathbf{I}_1$ SLR $\mathbf{I}_2$ and $\Pi_{\mathbf{I}_1}\langle h_1\rangle \cap \Pi_{\mathbf{I}_2}\langle h_2\rangle = \emptyset$.

- An initial event $\mathbf{I}$ and a history h and an *outcome event* $\mathbf{O}$ constitute a case of *some-cause-like-locus-not-in-past funny business* $\leftrightarrow_{df}$ $\mathbf{I}$ is a cause-like locus for $\mathbf{O}$ with respect to h, but no member of $\mathbf{I}$ lies in the causal past of any member of $\mathbf{O}$.

We aim to show agreement between the notion of generalized primary SLR modal-correlation funny business and the some-cause-like-locus-not-in-past accounts of funny business in the following sense.

5-2 THEOREM. *(Equivalence of two ideas of funny business)* There is a case of generalized primary space-like-related modal-correlation funny business iff there is a case of some-cause-like-locus-not-in-past funny business.

This is a consequence of the two conditionals Lemma Lemma 5-3 and Lemma Lemma 5-4. Here is the first half.

5-3 LEMMA. *(Left to right)* Suppose that $\mathbf{I}_{\mathrm{L}} \rightarrowtail \Pi_{\mathbf{I}_{\mathrm{L}}}$ and $\mathbf{I}_{\mathrm{R}} \rightarrowtail \Pi_{\mathbf{I}_{\mathrm{R}}}$ together with outcome-determining histories h_{L} and h_{R} constitute a case of GP SLR modal correlation funny business.[39] Then we can find a case of some-cause-like-locus-not-in-past funny business.

PROOF. The hypothesis gives us that although $\mathbf{I}_{\mathrm{L}} \subseteq h_{\mathrm{L}}$ and $\mathbf{I}_{\mathrm{R}} \subseteq h_{\mathrm{R}}$, so that $\Pi_{\mathbf{I}_{\mathrm{L}}}\langle h_{\mathrm{L}}\rangle$ and $\Pi_{\mathbf{I}_{\mathrm{R}}}\langle h_{\mathrm{R}}\rangle$ are each GP propositional outcomes of their respective initial events, and although also $\mathbf{I}_{\mathrm{L}}$ SLR $\mathbf{I}_{\mathrm{R}}$, nevertheless $\Pi_{\mathbf{I}_{\mathrm{L}}}\langle h_{\mathrm{L}}\rangle \cap \Pi_{\mathbf{I}_{\mathrm{R}}}\langle h_{\mathrm{R}}\rangle = \varnothing$. We aim to prove the existence of a case of some-cause-like-locus-not-in-past funny business. We can use h_{L} as-is; so we want to use $\mathbf{I}_{\mathrm{L}}$, $\mathbf{I}_{\mathrm{R}}$, and h_{R} somehow in order to find an $\mathbf{I}_{\mathrm{L}'}$ and an $\mathbf{O}_{\mathrm{R}}$ such that

$$h_{\mathrm{L}} \perp_{\mathbf{I}_{\mathrm{L}'}} H_{\langle \mathbf{O}_{\mathrm{R}}\rangle}, \text{ and } \forall e \forall O[(e \in \mathbf{I}_{\mathrm{L}'} \text{ and } O \in \mathbf{O}_{\mathrm{R}}) \to e \not< O].$$

The construction of $\mathbf{O}_{\mathrm{R}}$ is straightforward: For each $e_{\mathrm{R}} \in \mathbf{I}_{\mathrm{R}}$, choose O_{R} such that $h_{\mathrm{R}} \cap O_{\mathrm{R}} \neq \varnothing$ and $e_{\mathrm{R}} < O_{\mathrm{R}}$ and $\mathit{inf}(O_{\mathrm{R}}) = e_{\mathrm{R}}$, and let $\mathbf{O}_{\mathrm{R}}$ contain exactly these outcome chains. It is a well-known fact (see Section 2.3) that such a choice guarantees that $H_{\langle O_{\mathrm{R}}\rangle} = \Pi_{e_{\mathrm{R}}}\langle h_{\mathrm{R}}\rangle$.

The definition of $\mathbf{I}_{\mathrm{L}'}$ involves a more complicated system of choices. Let $h_{\mathrm{R}'} \in \Pi_{\mathbf{I}_{\mathrm{R}}}\langle h_{\mathrm{R}}\rangle$. We need to choose a point event to be put into $\mathbf{I}_{\mathrm{L}'}$ that will separate h_{L} from $h_{\mathrm{R}'}$.

Suppose that $\mathbf{I}_{\mathrm{L}} \subseteq h_{\mathrm{R}'}$. $h_{\mathrm{L}} \equiv_{\mathbf{I}_{\mathrm{L}}} h_{\mathrm{R}'}$ would contradict that no history belongs to both $\Pi_{\mathbf{I}_{\mathrm{L}}}\langle h_{\mathrm{L}}\rangle$ and $\Pi_{\mathbf{I}_{\mathrm{R}}}\langle h_{\mathrm{R}}\rangle$ (recalling that the latter is $H_{\langle \mathbf{O}_{\mathrm{R}}\rangle}$), so there must be an offender in $\mathbf{I}_{\mathrm{L}}$ at which undividedness fails. Choose one of these $e_{\mathrm{L}'}$, and note that, since $e_{\mathrm{L}'} \in (h_{\mathrm{L}} \cap h_{\mathrm{R}'})$, not only does undividedness fail, but $h_{\mathrm{L}} \perp_{e_{\mathrm{L}'}} h_{\mathrm{R}'}$. Put $e_{\mathrm{L}'}$ into $\mathbf{I}_{\mathrm{L}'}$. That $\mathbf{I}_{\mathrm{L}}$ SLR $\mathbf{I}_{\mathrm{R}}$ guarantees that $e_{\mathrm{L}'}$ won't be in the past of any member O_{R} of $\mathbf{O}_{\mathrm{R}}$: If $e_{\mathrm{L}'} < O_{\mathrm{R}}$ then $e_{\mathrm{L}'} \leqslant \mathit{inf}(O_{\mathrm{R}}) = e_{\mathrm{R}} \in \mathbf{I}_{\mathrm{R}}$, contradicting the space-like relatedness of $\mathbf{I}_{\mathrm{L}}$ and $\mathbf{I}_{\mathrm{R}}$.

Suppose, however, that $\mathbf{I}_{\mathrm{L}} \not\subseteq h_{\mathrm{R}'}$; that is, suppose that for some e_{L}, $e_{\mathrm{L}} \in (\mathbf{I}_{\mathrm{L}} - h_{\mathrm{R}'})$, hence $e_{\mathrm{L}} \in h_{\mathrm{L}} - h_{\mathrm{R}'}$. Then by prior choice there is bound to be an $e_{\mathrm{L}'}$ such that $e_{\mathrm{L}'} < e_{\mathrm{L}}$ and $h_{\mathrm{L}} \perp_{e_{\mathrm{L}'}} h_{\mathrm{R}'}$. Put this $e_{\mathrm{L}'}$ into $\mathbf{I}_{\mathrm{L}'}$. We show that $e_{\mathrm{L}'} \not< O_{\mathrm{R}}$ (all $O_{\mathrm{R}} \in \mathbf{O}_{\mathrm{R}}$) by a *reductio*; so suppose $e_{\mathrm{L}'} < O_{\mathrm{R}}$. Setting $e_{\mathrm{R}} = \mathit{inf}(O_{\mathrm{R}})$, it would follow that $e_{\mathrm{L}'} \leqslant e_{\mathrm{R}}$. The identity possibility is ruled out by $e_{\mathrm{L}'} < e_{\mathrm{L}}$ together with the space-like relatedness of $\mathbf{I}_{\mathrm{L}}$ and $\mathbf{I}_{\mathrm{R}}$, so (under the hypothesis for *reductio*) $e_{\mathrm{L}'} < e_{\mathrm{R}}$. Now find a history h_0 that witnesses the space-like relation between e_{L} and e_{R}. Argue that $h_{\mathrm{L}} \equiv_{e_{\mathrm{L}'}} h_0$ (since $e_{\mathrm{L}'} < e_{\mathrm{L}}$ and $e_{\mathrm{L}} \in h_{\mathrm{L}} \cap h_0$) and $h_0 \equiv_{e_{\mathrm{L}'}} h_{\mathrm{R}'}$ (since $e_{\mathrm{L}'} < e_{\mathrm{R}}$ and e_{R}

[39] I am picturing the affair in two dimensions, using "L" informally to point to the left wing and "R" to the right.

$\in h_0 \cap h_{\mathrm{R}'}$, the latter because $e_{\mathrm{R}} \in \mathbf{I}_{\mathrm{R}} \subseteq h_{\mathrm{R}'}$), so that $h_{\mathrm{L}} \equiv_{e_{\mathrm{L}'}} h_{\mathrm{R}'}$ by transitivity of undividedness. Which contradicts $h_{\mathrm{L}} \perp_{e_{\mathrm{L}'}} h_{\mathrm{R}'}$ and finishes the *reductio*.

The so-defined $\mathbf{I}_{\mathrm{L}'}$ evidently splits h_{L} from every required history ($h_{\mathrm{L}} \perp_{\mathbf{I}_{\mathrm{L}'}} H_{\langle \mathbf{O}_{\mathrm{R}} \rangle}$), and the "relevance" condition is also met, because we didn't put anything into $\mathbf{I}_{\mathrm{L}'}$ unless it was good for something ($h_{\mathrm{L}} \underline{\perp}_{\mathbf{I}_{\mathrm{L}'}} H_{\langle \mathbf{O}_{\mathrm{R}} \rangle}$). With equal evidence none of $\mathbf{I}_{\mathrm{L}'}$ is less than any member of $\mathbf{O}_{\mathrm{R}}$. End proof: Each case of GP SLR modal-correlation funny business implies a case of some-cause-like-locus-not-in-past funny business. □

5-4 LEMMA. *(Right to left)* Suppose that there is a case of some-cause-like-locus-not-in-past funny business. Then there is a case of generalized primary space-like-related modal-correlation funny business.

PROOF. Suppose that $\mathbf{I}_{\mathrm{L}}$, h_{L}, and $\mathbf{O}_{\mathrm{R}}$ constitute a case of some-cause-like-locus-not-in-past funny business:[40]

(a) $h_{\mathrm{L}} \perp_{\mathbf{I}_{\mathrm{L}}} H_{\langle \mathbf{O}_{\mathrm{R}} \rangle}$,

(b) $\forall e[e \in \mathbf{I}_{\mathrm{L}} \to \exists h_{\mathrm{R}}[h_{\mathrm{R}} \in H_{\langle \mathbf{O}_{\mathrm{R}} \rangle}$ & $h_{\mathrm{L}} \perp_e h_{\mathrm{R}}]]$, and

(c) $\forall e \forall O[(e \in \mathbf{I}_{\mathrm{L}}$ and $O \in \mathbf{O}_{\mathrm{R}}) \to e \not< O]$.

Choose h_{R} such that *(d)* $h_{\mathrm{R}} \in H_{\langle \mathbf{O}_{\mathrm{R}} \rangle}$. Use the axiom of choice to provide three choice functions. The first, Ξ_1, is defined on all sets of histories, and is such that $\Xi_1(H) \in H$ (unless $H = \varnothing$). The second, Ξ_2, is defined on all sets $\mathbf{O}$ of outcome chains, and is such that $\Xi_2(\mathbf{O}) \in \mathbf{O}$ (unless $\mathbf{O} = \varnothing$). The third, Ξ_3, is defined on all subsets of *OW*, and is such that $\Xi_3(E) \in E$ (unless $E = \varnothing$). We proceed to define ordinal-length sequences of initial events, histories, outcome chains, and point events in such a way as to guarantee that eventually we will reach a stage exhibiting a GP SLR modal correlation:[41]

(e) $\mathbf{I}_\alpha = \{e_\gamma \colon \gamma < \alpha\}$.

(f) $h_0 = h_{\mathrm{L}}$. For $0 < \alpha$, there are two possibilities. If $\Pi_{\mathbf{I}_{\mathrm{L}}} \langle h_{\mathrm{L}} \rangle \cap \Pi_{\mathbf{I}_\alpha} \langle h_{\mathrm{R}} \rangle \neq \varnothing$, let $h_\alpha = \Xi_1(\Pi_{\mathbf{I}_{\mathrm{L}}} \langle h_{\mathrm{L}} \rangle \cap \Pi_{\mathbf{I}_\alpha} \langle h_{\mathrm{R}} \rangle)$. Otherwise, let $h_\alpha = h_{\mathrm{L}}$.

(g) $O_\alpha = \Xi_2(\{O \colon O \in \mathbf{O}_{\mathrm{R}}$ and $h_\alpha \cap O = \varnothing)\}$.

(h) $e_\alpha = \Xi_3(\{e \colon e < O_\alpha$ and $h_\alpha \perp_e H_{\langle O_\alpha \rangle}\})$.

[40] I am using "R" as associated with the outcome, imagined on the right, and "L" as associated with the initial, imagined on the left, that is not in its past.

[41] The circularity of *(e)–(h)* is only apparent: e_α depends on O_α and h_α; O_α depends on h_α; h_α depends on $\mathbf{I}_\alpha$; and $\mathbf{I}_\alpha$ depends on various e_γ, but only for $\gamma < \alpha$.

We begin by observing that under the hypotheses *(a)–(h)*, the following hold for any ordinal α.

1. $h_\alpha \in \Pi_{\mathbf{I}_\mathrm{L}} \langle h_\mathrm{L} \rangle$.
2. $h_\alpha \perp_{\mathbf{I}_\mathrm{L}} H_{\langle \mathbf{O}_\mathrm{R} \rangle}$.
3. $O_\alpha \in \mathbf{O}_\mathrm{R}$ and $h_\alpha \cap O_\alpha = \varnothing$.
4. $e_\alpha < O_\alpha$.
5. $h_\alpha \perp_{e_\alpha} H_{\langle O_\alpha \rangle}$.
6. $h_\mathrm{R} \in H_{\langle O_\alpha \rangle}$.
7. $h_\alpha \perp_{e_\alpha} h_\mathrm{R}$.
8. $\mathbf{I}_\mathrm{L} \subseteq h_\mathrm{L}$ and $\mathbf{I}_\alpha \subseteq h_\mathrm{R}$.
9. If $0 < \alpha$ then $\mathbf{I}_\mathrm{L}$ SLR $\mathbf{I}_\alpha$.
10. If $0 < \alpha$ and $\mathbf{I}_\mathrm{L} \rightarrowtail \Pi_{\mathbf{I}_\mathrm{L}}$ and $\mathbf{I}_\alpha \rightarrowtail \Pi_{\mathbf{I}_\alpha}$ together with outcome-determining histories h_L and h_R do not constitute a case of GP SLR modal correlation, then $h_\alpha \in \Pi_{\mathbf{I}_\alpha} \langle h_\mathrm{R} \rangle$.

Items (1)–(10) are straightforward as follows. (1) holds directly by *(f)*. Next use (1) together with *(a)* and the transitivity of undividedness to infer (2). This implies that $h_\alpha \cap O = \varnothing$ for some $O \in \mathbf{O}_\mathrm{R}$, which in turn with *(g)* implies (3). The prior choice postulate then implies that $\{e\colon e < O_\alpha \text{ and } h_\alpha \perp_e H_{\langle O_\alpha \rangle}\} \neq \varnothing$, so that (4) and (5) come by the definition *(h)* of e_α. The first part of (3) together with *(d)* implies (6), which implies (7) when used with (5).

The first part of (8) is a consequence of *(b)*, whereas the second part follows from (4) and (3) and *(d)* (histories are closed downward).

For (9), choose $e_\mathrm{L} \in \mathbf{I}_\mathrm{L}$ and $e_\gamma \in \mathbf{I}_\alpha$, and show that e_L and e_γ are space-like related. By definition that requires first that they be distinct, which follows from *(c)* and (3) and (4). Space-like relatedness also requires that e_L and e_γ are consistent (belong in common to at least one history) and are not causally ordered. It will suffice to find two histories that split at each of e_L and e_γ, for points at which two histories split must belong to those histories (so that those points must be consistent), and the points must also be maximal in the intersection of the two histories (so that neither can lie in the causal past of the other).

We proceed as follows. By *(b)*, choose $h_{\mathrm{R}'} \in H_{\langle \mathbf{O}_\mathrm{R} \rangle}$ (and hence *(i)* $h_{\mathrm{R}'} \in H_{\langle O_\gamma \rangle}$) such that *(j)* $h_\mathrm{L} \perp_{e_\mathrm{L}} h_{\mathrm{R}'}$, and observe that (1) implies that $h_\gamma \in \Pi_{e_\mathrm{L}} \langle h_\mathrm{L} \rangle$. Applying the transitivity of undividedness to this and *(j)* gives $h_\gamma \perp_{e_{\mathrm{L}}} h_{\mathrm{R}'}$, which is half of what we want (h_γ and $h_{\mathrm{R}'}$ will be the required histories). Combining (5) with *(i)* implies $h_\gamma \perp_{e_\gamma} h_{\mathrm{R}'}$, which is the other half, and completes the proof that every member of $\mathbf{I}_{\mathrm{L}}$ is space-like related to every member of $\mathbf{I}_\alpha$.

Turning now to (10), assume $0<\alpha$, and that $\mathbf{I}_{\mathrm{L}} \rightarrowtail \Pi_{\mathbf{I}_{\mathrm{L}}}$ and $\mathbf{I}_\alpha \rightarrowtail \Pi_{\mathbf{I}_\alpha}$ together with outcome-determining histories h_{L} and h_{R} do not constitute a case of GP SLR modal correlation. Observe that by (8) and (9) we have two individually consistent space-like related initial events $\mathbf{I}_{\mathrm{L}}$ and $\mathbf{I}_\alpha$ such that $\mathbf{I}_{\mathrm{L}} \subseteq h_{\mathrm{L}}$ and $\mathbf{I}_\alpha \subseteq h_{\mathrm{R}}$, so that the condition of no GP SLR modal-correlation funny business ensures the nonemptiness of the set from which *(f)* makes Ξ_1 pick the history h_α, which must therefore belong to $\Pi_{\mathbf{I}_\alpha}\langle h_{\mathrm{R}}\rangle$, as required by (10).

Finally, take advantage of the fact that *Our World* is a set; therefore, since the ordinals outrun any given set, the ordinal-length sequence of e_α must contain repetitions. Let β be the smallest ordinal such that $e_\beta = e_\gamma$ for some $\gamma<\beta$; hence $e_\beta \in \mathbf{I}_\beta$. Observe that for all α, (7) and (10) taken together imply that $e_\alpha \notin \mathbf{I}_\alpha$, provided that $\mathbf{I}_{\mathrm{L}} \rightarrowtail \Pi_{\mathbf{I}_{\mathrm{L}}}$ and $\mathbf{I}_\alpha \rightarrowtail \Pi_{\mathbf{I}_\alpha}$ together with outcome-determining histories h_{L} and h_{R} do not constitute a case of GP SLR modal correlation. Since $e_\beta \in \mathbf{I}_\beta$, it must be that $\mathbf{I}_{\mathrm{L}} \rightarrowtail \Pi_{\mathbf{I}_{\mathrm{L}}}$ and $\mathbf{I}_\beta \rightarrowtail \Pi_{\mathbf{I}_\beta}$ together with outcome-determining histories h_{L} and h_{R} constitute a case of GP SLR modal correlation. This completes the proof that some-cause-like-locus-not-in-past funny business implies the existence of generalized primary space-like-related modal-correlation funny business. $\square$

6 Conclusion

The equivalence of the two notions of funny business supports the underlying conjecture of this essay, namely, that the theory of BST-92 plus a no-funny-business postulate gives us a good idealization of a world that is as indeterministic as you like, and even filled with Bell-like probabilistic correlations, but nevertheless innocent of EPR-like funny business. The conjecture is that the defined ideas of EPR-like funny business cut at a joint.[42]

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[42]Thanks to A. Gupta and B. Gyenis, T. Müller, and S. Wölfl for specific help, and to T. Placek for many valuable interchanges.

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