Symmetry, Structure and the Constitution of Objects[1]

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Introduction: From Epistemic to Ontic Structural Realism [2]

Structuralism has a long and honourable history in the philosophy of science, but interest in the programme has recently been re-awakened in the context of the realism-antirealism debate. Worrall, in particular, has presented an epistemological form of structuralism as a response to Laudan's 'Pessimistic Meta-Induction' (Worrall 1996). Put rather crudely and simply, the idea is that although the history of science is, to a significant extent, a history of changing ontologies - as one moves from the particle theory of light to the wave theory to Maxwell's theory and so on, for example - the same history suggests that important structural elements of theories are preserved through both 'normal' changes and, most importantly, revolutions. By 'ontology' here is meant the theoretical representation of scientific entities, such as light, electrons etc.; the relevant structures, on the other hand, are represented for Worrall by the appropriate mathematical equations - Snell's Laws are incorporated into Maxwell's Equations and so on. Thus whereas the ontological component of a theory may be subjected to a pessimistic meta-induction, as far as the structural component is concerned things look quite optimistic. This gives rise to a form of 'Structural Realism' (SR) which holds that one can, and should, adopt a realist attitude towards the well confirmed structural aspects of theories (see also Redhead 1995). As Ladyman has pointed out (1998), this should be regarded as an *epistemic* form of SR since it holds that all that we *know* are the structures, while the objects themselves remain epistemologically inaccessible. It is worth noting that in defending this position Worral draws on the history of structuralism in the form of those famous passages from *Science and Hypothesis* where Poincaré writes that theoretical terms '... are merely names of the images we substituted for the real objects which Nature will hide forever from our eyes. The true relations between these real objects are the only reality we can ever obtain.'(1905, p. 162). However, as Domski has emphasised, Poincaré may not be the most appropriate name to drop in order to give historical legitimacy to this version of SR, given his Kantian inclinations and rejection of truth as the aim of science (Domski preprint). Such inclinations appear again and again through the history of structuralism and the issue arises as to whether the structural realist can neatly peel them off from the rest of structuralist programme.

Leaving aside such historical issues for the moment, there are two philosophical concerns that arise in this context:

1. the idea of epistemologically inaccessible objects, hidden behind the structures as it were, may be thought to run counter to the scientific 'attitude' in general, or to hearken back to some form of pre-Scientific Revolution scholasticism (Psillos 1999).
2. the nature of these objects remains as problematic for the epistemic structural realist as it is for the 'standard' realist insofar as there exists a kind of metaphysical underdetermination whereby quantum mechanics supports both a metaphysics of individuals and a metaphysics of 'non-individuals' (French 1998; van Fraassen 1991).

Ladyman's 'ontic' form of SR (Ladyman 1998) can be seen as responding to both of these concerns by effectively eliminating the objects completely, leaving only the structures. Again, put simply, the idea is that it is not just that all that we *know* are the

[1]First draft, so its still rough and not so ready. Apology: this turned out more historical than I had anticipated. Acknowledgements: thanks to Jon Bain, Otávio Bueno, Anjan Chakravarrty, James Ladyman, Mary Domski, John Norton, Stathis Psillos and Peter Simons for useful conversations (useful to me, anyway).

[2]Further details can be found in Ladyman 1998 and French and Ladyman forthcoming.

structures but that all that there *is* are the structures. The elaboration and development of this position has raised a number of interesting issues, to do with the metaphysics of structure, the conceivability of structures without any underlying objects, the identity conditions for such structures and so on, some of which, at least, have been addressed elsewhere (Ladyman 1998; French 1999; French and Ladyman forthcoming). As with many developments in philosophy, one might feel a sense of deja vu as these or related issues can be discerned running like threads through the history of structuralism.

In this paper I want to focus on the general issue of the impact on structuralism of the quantum treatment of objects in terms of symmetry groups and, in particular, on the question as to how we might eliminate, or better, reconceptualise such objects in structural terms. With regard to the former, both Cassirer and Eddington not only explicitly and famously tied their structuralism to the development of group theory but also drew on the quantum treatment in order to further their structuralist aims and I want to sketch the relevant history here with an eye on what lessons might be drawn. With regard to the latter, Ladyman has explicitly cited Castellani's work on the group-theoretical constitution of quantum objects and I want to indicate both how such an approach needs to be understood if it is to mesh with ontic SR and how it might accommodate permutation symmetry through a consideration of Huggett's recent work.

Lets begin with a little of the history of structuralism.

Cassirer and Eddington on Structures, Symmetry and Subjectivity

In a recent contribution to the on-going revival of interest in Cassirer's work, Ihmig identifies the central theme running through Cassirer's writings in the philosophy of science as the analysis of the concept of object (Ihmig 1999). The fundamental perspective from which this analysis should proceed is epistemological:

'... epistemological reflection leads us everywhere to the insight that what the various sciences call the "object" is nothing in itself, fixed once for all, but that it is first determined by some standpoint of knowledge.' (Cassirer 1953, p. 356)

As is well known, Cassirer's interest in this issue can be traced back to his reflections on then nature of space and the influence of Klein's Erlanger programme. And the crucial insight offered by the latter concerned the introduction of the concept of group. What this yields, of course, is a structural conception of geometrical objects which shifts the focus from individual geometrical figures, grasped intuitively, to the relevant geometrical transformations and the associated laws. This shift is then manifested in Cassirer's neo-Kantian assertion of 'the priority of the concept of law over the concept of object.'

This assertion in turn forms an integral component of Cassirer's interpretation of the Kantian understanding of objectivity:

'For objectivity itself - following the critical analysis and interpretation of this concept - is only another label for the validity of certain connective relations that have to be ascertained separately and examined in terms of their structure. The tasks of the criticism of knowledge ("Erkenntniskritik") is to work backwards from the unity of the general object concept to the manifold of the necessary and sufficient conditions that constitute it. In this sense, that which knowledge calls its "object" breaks down into a web of relations that are held together in themselves through the highest rules and principles.' (Cassirer 1913, trans. in Ihmig *op. cit.*, p. 522)

These 'highest rules and principles' are the symmetry principles which represent that which is invariant in the web of relations itself. And these principles, in turn, are represented group-theoretically; thus the relevant group effectively lays down the general conditions in terms of which something can be viewed as an object.

Cassirer's 'application' of this framework to the foundations of relativity theory is well known (Ihmig *op. cit.*, pp. 524-528). According to Ihmig, what it does is restore the *unity* of the concept of object which is apparently undermined by the relativistic transformations. From the structuralist perspective, this unity is 'reinstated on a higher level.' (*ibid.*, p. 525) via the 'lawful unity' of inertial systems offered by the Lorentz transformations. The process of abstraction from a substantivalist conception of objects to a structuralist one is furthered by the General Theory of Relativity and what we are left with is an understanding of the objects of a theory as defined by those transformations which leave the relevant physical magnitudes invariant. Thus Cassirer saw General Relativity as the natural conclusion of the structuralist tendency.

Cassirer's understanding of the foundations of GR has been further pursued by Ryckman (1999), who points to the central importance of the principle of general covariance in this understanding. According to Ryckman, Cassirer viewed general covariance as a principle of objectivity which offers a 'deanthropomorphized' conception of a physical object. Furthermore, he (Ryckman) claims, this view of Cassirer's meshed with Einstein's own and underpinned the latter's objections to quantum mechanics through its implementation in the separability principle.

As the requirement that the laws of nature be formulated so that they remain valid in any frame of reference, general covariance '... is a further manifestation of the guiding methodological principle of "synthetic unity" necessary to the concept of the object of physical knowledge.' (*ibid.*, p. 604). Regarded as a synthetic requirement, general covariance comes to be seen as both a formal restriction and a heuristic guide for the discovery of general laws of nature (*ibid.*). Physical objectivity - apparently lost by space and time themselves - re-emerges in deanthropomorphised form in terms of the functional forms of connection and coexistence:

'With the demand that laws of nature be generally covariant, physics has completed the transposition of the substantial into the functional - it is no longer the existence of particular entities, definite permanencies propagating in space and time, that form "the ultimate stratum of objectivity" but rather "the invariance of relations between magnitudes".' (*ibid.*, p. 606, citing Cassirer 1957, p. 467).

There has been comparatively little discussion of Cassirer's analysis of the other major revolution of the twentieth century, namely quantum mechanics[3]. Ihmig states in a footnote that Cassirer assessed the results of this revolution in the same way as the above with regard to the objects of science (*op. cit.*, p. 515 fn. 1). However, there is the additional element of the loss of individuality of the particles themselves which was apparently implied by the new quantum physics[4].

The overall framework is the same, encompassing as it does a shift from things-as-substances to relations as the ground of objectivity in science; or as Cassirer put it, '[w]e are concerned not so much with the existence of things as with the objective validity of relations; and all our knowledge of atoms can be led back to, and depends on, this validity'(Cassirer 1936, p. 143). In classical mechanics objectivity rests on the spatio-temporal persistence of individual objects and here, '"[o]bjective" denotes a being which can be recognized as the same in spite of all changes in its individual determinations, and this recognition is possible only if we posit a spatial substratum.' (*ibid.*, p. 177). As Cassirer points out, 'The entire axiomatic system of classical mechanics is based on this presupposition.' (*ibid.*). As is well known, this presupposition features explicitly in Boltzmann's axioms for example and it forms the basis of the 'world-view' of classical (particle) physics in which we have individual objects possessing at all times well-defined properties and traversing well-defined spatio-temporal trajectories. It is this world-view that is apparently overturned by quantum mechanics (at least under the orthodox interpretation) and in the new situation in which we find ourselves, we cannot say that the particles unambiguously possess definite properties at all times, even beyond measurement interactions, or that they travel along well-defined trajectories. It is at this juncture that Cassirer asks a pair of crucial questions: '... what *are* these electrons whose path we can no longer follow? Is there any sense in ascribing to them a definite, strictly determined existence, which, however, is only incompletely accessible to us?' (*ibid.*, p. 178). In answering these questions, Cassirer makes the fundamental demand of the ontic form of structural realism, namely that we take the 'conditions of accessibility' as 'conditions of the objects of experience'. If we do that, then '... there will no longer exist an empirical object that in principle can be designated as utterly inaccessible; and there may be classes of presumed objects which we will have to exclude from the domain of empirical existence

[3]A sketch is given in Itzkoff (1997) pp. 83-98.
[4]What follows is taken from French and Ladyman forthcoming.

because it is shown that with the empirical and theoretical means of knowledge at our disposal, they are not accessible or determinable' (*ibid.*, p. 179). There are no epistemically inaccessible objects laying behind the structures which we can know.

What is an electron then? Not, Cassirer insists, an individual object (*ibid.*, p. 180) and he cites Born's comment (from 1926) that from the perspective of quantum statistics, the particles cannot be identified as individuals at all (*ibid.*, p. 184). Cassirer writes,

> 'The impossibility of delimiting different electrons from one another, and of ascribing to each of them an independent individuality, has been brought into clear light through the evolution of the modern quantum theory, and particularly through the considerations connected with the Pauli exclusion principle.' (*ibid.*, p. 184 fn. 17)[5]

Of course, this is to follow the 'received view' regarding the indistinguishability of quantum particles which draws the conclusion that they are non-individuals in some sense. Nevertheless, Cassirer takes it to further support the shift away from particles as substantival 'things'. If we want to continue to talk, in everyday language, about electrons as objects - because we lack the logico-linguistic resources to do otherwise - then we can do so 'only indirectly', '... not insofar as they themselves, as individuals, are given, but so far as they are describable as "points of intersection" of certain relations' (*ibid.*). And this relational conception of an object is taken straight from Kant himself: 'All we know in matter is merely relations ... but among these relations some are self-subsistent and permanent, and through these we are given a determinate object.' (Kant, quoted on p. 182) Charge, understood as an intrinsic or state-independent property of particles, is just such a 'self-subsistent and permanent relation' but as Cassirer points out, in an acute rebuttal of the assumption made by the 'standard' realist, ' ... the constancy of a certain relation is not at all sufficient for the inference of a constant carrier' (*ibid.*). The permanence of charge justifies our regarding the electron, say, as a 'determinate object', where the scare quotes indicate that the sense is that of an entity prior to reconceptualisation in structural terms, but it is does not justify what Cassirer calls the 'substantialization and hypostasis' of the electron in the sense of an entity which is not so reconceptualised.

Charge, like the other intrinsic properties, features in the relevant laws of physics and according to Cassirer, what we have here is a reversal of the classical relationship between the concepts of object and law (*ibid.*, pp. 131-132): instead of beginning with a 'definitely determined entity' which possess certain properties and which then enters into definite relations with other entities, where these relations are expressed as laws of nature, what we now begin with are the laws which express the relations in terms of which the 'entities' are constituted. From the structuralist perspective, the entity '... constitutes no longer the self-evident starting point but the final goal and end of the considerations: the *terminus a quo* has become a *terminus ad quem*.' (*ibid.*, p. 131) Objectivity, therefore, is determinable through law, which is prior to it (*ibid.*, p. 176) and the boundaries of law mark the boundaries of objective knowledge (*ibid.*, p. 132).

As already indicated, Cassirer saw these developments in physics as confirming a neo-Kantian epistemology (Werkmeister 1949 p. 777) according to which the laws of physics - in particular those of quantum mechanics and relativity theory - provide the sole basis for our integration of experience. In this integration, a crucial role is played by the 'principle' of causality, regarded not as a proposition pertaining to events themselves, but, rather, '... a stipulation concerning the means through which things and events are constituted in experience.' (*ibid.*, p. 789). As such, the principle is not undermined by quantum mechanics; indeed, Cassirer insists, understood as a demand for strict functional dependence, the essence of causality remains untouched (*op. cit.*, p. 188). The significance of quantum physics for epistemology lies precisely with the above consideration regarding the nature of objects.

The retention of causality provides of course a further connection between Cassirer and Einstein. As I mentioned above, Ryckman also argues that general covariance underpins Einstein's criterion of observer independent objectivity in terms of his principle

[5]And here Cassirer follows Weyl in associating the Exclusion Principle with Leibniz's Principle of Identity of Indiscernibles.

of separability (Ryckman *op. cit.*). The connection is provided by Schlick who claimed that only general covariance can adequately satisfy the Maxwellian requirement that causal differences between two events should not depend upon the particular spatio-temporal locations of the events (Ryckman *ibid.*, p. 609). This further requires a way of distinguishing causal occurrences so that they may be regarded as similar but not identical and *this* is what the principle of separability allows. As is now well known, the latter is central to the EPR objection and by '... distinguishing physical systems by virtue of causal independence of measurement interactions, [it] serves as a principle of individuation in lieu of the usual identification of physical systems by reference to a fixed background of space and time ...' (*ibid.*, p. 615). The lesson drawn by Ryckman is that Einstein's criterion of 'observer objectivity' is not the expression of a 'simple minded realism', '... but rather a presupposition for the application of causal laws in the physical description of the world.' (*ibid.*, p. 616).

Howard understood separability both in spatio-temporal terms and as a sufficient condition for the individuality of physical systems. The failure of separability in quantum mechanics was then taken to imply a kind of non-individuality for quantum systems. Elsewhere, (French 1989), I've tried to resist this move, arguing that this presupposes that spatio-temporal location is the 'Principle of Individuality' and that on an alternative understanding of the latter, one could accommodate the failure of separability through the introduction of Teller's 'non-supervenient' relations holding between the particles[6]. Ryckman, however, takes Howard to have missed the point, since '... it is *not* possible to use the bare points of the manifold ... as a means of individuating separate physical systems' (*ibid.*, p. 617, fn. 51), because - and this is the 'central message' of general covariance - the bare manifold is not space-time. Thus the principle of separability is *not* to be understood as a form of spatio-temporal principle of individuality.

How is it to be understood then? And if, as Ryckman suggests, it does function as some kind of principle of individuation, how does this mesh with Cassirer's apparent realisation that quantum particles should not be regarded as individuals? What I would like to suggest (and this needs further elaboration, I know) is that it acts as a principle of 'pseudo-individuality' which allows us to distinguish systems - in a limited and localised way - in terms of their independent causal effects but does not give us licence to effectively import this principle beyond the observable effects and regard the systems as full-blown individual objects[7]. Citing Heisenberg, Cassirer writes, 'The process of observation cannot be simply objectified; its results cannot be turned immediately into real objects.' (1937, p. 142). The apparent failure of separability in EPR situations should then be read, not as a failure of the principle as a 'Principle of Pseudo-Individuality' but as a failure of the attempt to regard it as a Principle of (Full-Blown) Individuality and import it beyond the immediate measurement situation. In line with Ladyman's ontic structural realism, how this failure in turn should be understood is not in terms of the systems being non-individual objects, but in terms of their not being objects at all. Thus structuralism may offer a different ontological perspective on the implications of the Bell/EPR results.

Finally, it is interesting that both Ihmig and Ryckman mention Eddington in the context of Cassirer's structuralism (Ihmig *op. cit.*, p. 528; Ryckman op. it., p. 606); Ihmig in particular notes Eddington's emphasis on the importance of group theory in fleshing out the structural approach to knowledge. I'd like to turn to Eddington's form of structuralism now, as here the implications of quantum mechanics for the view of physical objects as individuals played an even more important role in its development.

In the preface to his later philosophical work, *The Philosophy of Physical Science* (Eddington 1939), Eddington remarks that in giving a name to his philosophy he hesitates between 'Selective Subjectivism' and 'Structuralism' (*ibid.*, p. viii)[8]. Both can be traced back

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[6]This re-emphasises the point that the physics alone will not break the above metaphysical underdetermination.

[7]This notion of a kind of 'pseudo-individuality' has been introduced by Toraldo di Francia and I shall return to it below.

[8]The presuppositions underlying Eddington's work have been nicely set out by Kilmister (1994) and I don't intend to question his analysis here; rather I simply want to highlight certain aspects to do with the constitution of objects and the quantum treatment of indistinguishable particles.

to his early reflections on the significance of relativity theory as presented in his *Mind* papers of 1920[9]. In the first (Eddington 1920a), Eddington rejects the standard approach of beginning with intervals as measured by clocks and rods and then obtaining the field equations, since the introduction of such clocks and rods before one has introduced the matter out of which they are supposedly composed would be 'inconvenient' when one is in the business of constructing the world in a 'strict analytical development' (*ibid.*, p. 152). We shall encounter this attitude again when we come to his view of particle indistinguishability. Instead, Eddington begins with point events, the aggregate of which constitute 'the World' (*ibid.*, p. 147) and which is postulated to be four-dimensional. Between any two neighbouring point events one can then define the interval, as a quantitative relation, and comparison of intervals leads to a 'rule of connexion' (*ibid.*, p. 148) which expresses a 'quality of the World' as measured by the usual coefficients $g_{\mu\nu}$. By an 'exceedingly complicated combination' of operations on the $g_{\mu\nu}$ one obtains the $G_{\mu\nu}$ (*ibid.*, pp. 149-150) and voilá, Eddington introduces the field equations for the case of empty space and for when matter is present[10]. These equations, he insists, should be read from left to right, not as *laws* of the World relating the continuum of points events and matter, since that leads to a kind of dualism (between the continuum of point events and matter) but as mathematical identifications denoting 'definite and absolute' conditions of the world (*ibid.*, p. 151) which give us the *perceptions* of emptiness and of matter respectively (Kilmister 1994, pp. 44-46)[11]. The field equation with non-vanishing stress-energy tensor describes how the theoretical quality represented by the left-hand side is 'appreciated' by the mind. Hence, 'Matter does not cause an unevenness in the gravitational field; the unevenness *is* matter.' (*ibid.*, p. 152)[12].

There are two important aspects to this, which relate to the structuralist and subjectivist components of Eddington's thought respectively. By matter as the putative cause of irregularities in the field, Eddington means *matter as substance* and thus this construction is seen as eliminating substance from our ontology in favour of relational structures. Secondly, 'matter', in this new sense, becomes dependent on the mind, since 'Matter is but one of a thousand relations between the constituents of the World, and it will be our task to show why one particular relation has a special value for the mind.' (*ibid.*, p. 153). In his later paper in the same volume (Eddington 1920b; contributed to the 1920 International Congress of Philosophy), Eddington draws an analogy with the construction of constellations out of the distribution of the stars:

'In a sense these patterns exist in the sky; but their recognition is subjective. So out of the primitive events which make up the external world, an infinite variety of "patterns" can be formed. There is one type of pattern which for some reason the mind loves to trace wherever it can; where it can trace it, the mind says, "Here is substance"; where it cannot, it says "How uninteresting! There is nothing in my line here". The mind is dealing with a real objective substratum; but the distinction of substance and emptiness is the mind's own contribution, depending on the kind of pattern it is interested in recognising.' (*ibid.*, p. 420).

Here we see how the structuralism and the selective subjectivism mesh.

Not all laws are subjective, however. What we have learned from relativity theory, according to Eddington, is that there is a certain quality which distinguishes substantial matter from mere emptiness. We have not yet discovered why the quality formerly known

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[9]Eddington apparently developed the idea that physics gives us knowledge of structure from reading Russell.

[10]In a footnote (*ibid.*, p. 150, fn. 1) Eddington writes the these tensors 'occupy a position intermediate between intrinsic qualities of the World, and qualities which involve space and time haphazardly.' (*ibid.*). Both the vanishing of a tensor and the equality of two tensors are intrinsic, absolute relations.

[11]Here Eddington talks as if the equations were correspondence rules, which allow us to 'identify the symbols of theory with things familiar to experience...' (*ibid.*, p. 151).

[12]In the second paper, it is interesting that Eddington cites Schlick as extending Minkowski's famous phrase about space and time sinking to 'mere shadows' to things themselves: 'The combination or oneness of space, time and things is alone reality; each by itself is an abstraction' (quoted in Eddington 1928b, p. 419). We recall Domski's suggestion that the structural realist might be better off looking to Schlick rather than Poincaré.

as matter comes in lumps; hence the 'law of atomicity' may be a law of the World itself. Accommodating the latter requires another analysis which must start from a different basis:

'... starting from the postulate that the mind can appreciate only relations, the theory we have described is, or is intended to be, the most general possible theory of the way in which relations can combine to form permanent substance; and accordingly the laws of physics which result depend solely on this postulate as to the mind. Whatever the constitution of the external world, we can pick out a four-dimensional aggregate of entities which we may take to be our point -events since these have been left undefined. But if we attempt to push the analysis behind the point-events, we are, I think, bound to particularise the structure.' (1920a, p. 157).

It is this 'particularising' of the structure that is described by quantum mechanics, of course, and here Eddington is quite explicit that in order to understand how it is that the same quality which is chosen by the mind as substantial matter is singled out by Nature for the property of atomicity, we must understand how relativity theory and quantum physics can be related. This, of course, is the aim of the programme pursued in his later work.

It is important to note that Eddington's structuralism is limited both globally and locally. It is limited globally in that structuralism is appropriate only for metrical (or as he later calls it 'symbolic') knowledge (see Eddington 1928, p. 321), such as we obtain through physics, and not non-metrical (or 'intimate') knowledge (*ibid.*, p. 322), which would include biology as well as theology[13]. This is not the place to discuss Eddington's religious beliefs but it is interesting to note that Dingle, in his critique (Dingle 1954), characterises the difference between the metrical and non-metrical in terms of that between *structure* and *nature*; thus non-metrical knowledge is knowledge of the nature of things. If Dingle is correct, the Eddington would count as an epistemic structuralist (of a rather peculiar, religious stripe, perhaps).I shall return to this point shortly.

His structuralism is also limited locally in the way already indicated, namely with regard to the lack of a good theory of matter itself. As he wrote, 'The possibility of the existence of an electron in space is a remarkable phenomenon which we do not yet understand. The details of its structure must be determined by some unknown set of equations ...' (1928, p. 153). In this regard, substance makes a reappearance:

'The fundamental basis of all things must presumably have *structure* and *substance*. We cannot describe substance; we can only give a name to it. Any attempt to do more than give a name leads at once to an attribution of structure. But structure can be described to some extent; and when reduced to ultimate terms it appears to resolve itself into a complex of relations.' (*ibid.*, p. 224)

As Kilmister notes, Eddington concludes, again, that atomicity may be a reflection of non-subjective laws of the World and hence represents non-structural substance.

By the time of his 1927 Gifford lectures, published the following year as *The Nature of the Physical World* (Eddington 1928), Eddington was able to say rather more about quantum physics than his earlier rather brief and simplistic remarks concerning the quantum of action[14]. Nevertheless, he was not able to say enough to be able to incorporate atomicity within his 'world building' (*ibid.*, Ch. XI). This is presented even more clearly

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[13]Certain colleagues have expressed the criticism that structural realism is too closely tied to the physical sciences, since it is here that we have the appropriate mathematical structures. The response is to simply note that we can expand out notion of structure to include the non-mathematical and represent this set-theoretically (see for example French and Ladyman forthcoming).

[14]It is in this work that we find the much quoted remark, 'It would probably be wiser to nail up over the door of the new quantum theory a notice, "Structural alterations in progress-No admittance except on business" , and particularly to warn the doorkeeper to keep out prying philosophers.' (1928, p. 211). The context is Eddington's own 'rough impressions' of Schrödinger's theory (he also briefly discusses Dirac's as well as matrix mechanics), about which he concludes that although from the point of view of applications it has become indispensable, 'I do not see the least likelihood that his [Schrödinger's} ideas will survive long in their present form.' (*ibid.*, p. 211). It is also in the Introduction to this book that we find the infamous discussion of the 'two tables' . Here Eddington refers to substance as 'smoke' and an 'illusion' (p. xii, p. 280 and p. 318) but it is an illusion which we cannot fully exorcise.

than in the 1920 paper but with regard to the structural 'building material', Eddington quite clearly does not take the relata as the basic building blocks:

'The relations unite the relata; the relata are the meeting points of the relations. The one is unthinkable apart from the other. I do not think that a more general starting-point of structure could be conceived.' (*ibid.*, pp. 230-231).

It is here that we see elements of Eddington's 'numerological' tendencies, as he expressed the hope that from a 'structural interlocking' of relations, one might derive the desired physical properties of the world.

In particular, by applying symmetry constraints to the structure, Eddington claimed that we could construct geometry and mechanics, on the one hand, and electromagnetism on the other[15]. Again, the relevant laws are described as 'mathematical identities', whose violation is 'unthinkable'[16]. And again, the construction obtained is too coarse to accommodate the microscopic structure of the world. Here Eddington acknowledges that he 'scarcely knows' what to think. Perhaps the laws of quantum physics will also come to be seen as mathematical identities, arising 'only in the presentation of the world to us'; or perhaps they will be acknowledged as genuine 'laws of control' of an external world. According to Kilmister, the summary given in this work of the state of play in quantum mechanics around 1926-7 essentially cemented into place Eddington's understanding of the theory, with the exception of the Dirac equation to be mentioned below (as we shall see, this is not quite the case).

Here too the selective subjectivism comes into play. Why are the properties of the building we obtain ordered the way they are? The answer is that the theoretical world building must converge to the mental world building which gives us familiar experiences:

'The Hamiltonian derivative has just that kind of quality which makes it stand out in our minds as an active agent against a passive extension of space and time; and Hamiltonian differentiation is virtually the symbol for creation of an active world out of the formless background. Not once in the dim past, but continuously by conscious mind is the miracle of the Creation wrought.' (*ibid.*, p. 241)

In particular, these familiar experiences are subject to the mind's demand for permanence and it is this which underpins the conservation laws and also gives rise to the illusion of substantiality. Again, Eddington makes an allusion to picking out constellations from the stars, so that this world building actually amounts to '... a selection from the patterns that weave themselves' (*ibid.*).

As far as Eddington was concerned, the most suitable representation of the world structure was through the tensor calculus. Indeed, he wrote that,

'I do not think it is too extravagant to claim that the method of the tensor calculus, which presents all physical equations in a form independent of the choice of measure-code, is the only possible means of studying the conditions of the world which are at the basis of physical phenomena.' (1923, p. 49)

It is not surprising, then, that Dirac's equation had a dramatic impact, expressed as it was in terms of spinors (see Kilmister *op. cit.*, Ch. 5). It prompted Eddington to elaborate and investigate a new set of algebraic structures, described by what he called the 'wave-tensor' calculus, which, he believed, would provide the bridge between relativity and quantum theory. Furthermore, he maintained, the laws constituting this bridge have the above form of mathematical identities and thus the construction of the bridge proceeds on the same analytic basis. In particular, and famously, manipulation of this wave-tensor calculus appeared to give the values of certain fundamental physical constants, such as the fine

[15]In the 1920 paper Eddington cites Weyl's gauge theoretic attempt to extend Einstein's theory to cover electromagnetism (1920a, p. 156) and he persisted in regarding this as an 'essential part of the relativistic conception' (1939, p. 28).

[16]Which is not to say that we are infallible. Eddington acknowledges that it may turn out that what we are accustomed to measure with our instruments is not actually the thing conserved in the relevant law (*ibid.*, p. 239).

structure constant[17]. As far as Eddington's contemporaries were concerned, this was nothing more than a form of numerology which transformed into necessities numbers which were only contingent[18]. Dingle's criticism is representative: even supposing that Eddington's mathematics is correct, it does not follow that his conclusions are strictly 'epistemological' since they depend on the choice of certain postulates and this choice is ultimately guided, at least in part, by experience (Dingle *op. cit.*, pp. 55-57)[19].

There is the further question whether his mathematics is correct and answering this is partly the aim of Kilmister's project (Kilmister *op. cit.*)[20]. It is also to show that Eddington's manipulations, although apparently bizarre and poorly motivated, are perfectly plausible from the perspective of his own philosophy. I'm not going to go through Kilmister's courageous reconstruction here; all I want to do is emphasise the crucial role played by considerations of particle indistinguishability in quantum mechanics.

The issue is that of constructing a bridge between quantum mechanics and electromagnetism and, as the ratio between the Compton wavelength and classical electron radius, the fine structure constant was seen by Eddington as the capstone of the bridge (just as the speed of light was for the unification of electricity and magnetism; see Eddington 1936, p. 4). Originally, measurements of the reciprocal of this constant (and it is the reciprocal which Eddington persists in calling the fine structure constant) gave a value of 136. Beginning with four algebraic elements, related to Dirac's operators and decomposed into 3 + 1, Eddington generated a complex algebraic structure, applied to the case of two particles not just one as for Dirac. This was for two reasons: first, in the treatment of the hydrogen atom the electron and proton should be placed on an equal footing; secondly, according to the principle of relativity, if the electron was the 'object' particle, a 'comparison' particle also had to be introduced, representing, in idealised form, the environment (Kilmister *op. cit.*, p. 112).

As Kilmister points out, the whole project gains a certain plausibility if it is viewed from a structuralist perspective: if physics is primarily the investigation of structures, then the most appropriate tool for this investigation will be forms of mathematics in which structure is paramount (*ibid.*, p. 118)[21]. Kilmister somewhat downplays Eddington's selective subjectivism here (and emphasises his form of operationalism according to which the origin of a law is revealed by the 'unravelling' of the series of operations resulting in the relevant physical quantity[22]) but I think he would admit that it is crucial in justifying the very basis of Eddington's algebraic manipulations. This is precisely the purpose of Eddington's 1928 discussion mentioned above. Finally, as Kilmister notes, although he uses the phrase 'algebraic structure' to describe the mathematics employed, what Eddington was primarily concerned with was group theory (Kilmister *ibid.*, p. 118 and 119, fn. 1). With regard to the last point, we have already touched on Eddington's enthusiasm for tensors and, according to Kilmister, his conviction '... that here there is everything needed to describe the 'condition of the world' simply rested on the prolific character of this generation of [group-theoretic] representations ...' (*ibid.*, p. 72). Eddington was also

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[17]An attempt at a summary in the context of a popular exposition of his philosophy can be found in his (1939, pp. 162-168).

[18]Pauli described it as 'romantic poetry, not physics' (Kilmister *op. cit.*, p. 116).

[19]One such specific postulate that Dingle mentions is that the world can be described in terms of identical particles (*op. cit.*, p. 56).

[20]In a critical review of Eddington's *Philosophy of Physical Science*, Broad wrote 'It is greatly to be wished that some competent mathematical physicist, with a critical rather than a creative intellect, should undertake an "Examination of Eddington's Mathematics" comparable to my *Examination of McTaggart's Philosophy*. It is possible that, although McTaggart was ploughed, Eddington might pass with honours.' (Broad 1940, p. 312 - 'Sir Arthur Eddington's *Philosophy of Physical Science*', Philosophy, 15 pp. 301-312). I do not mean to suggest either that Kilmister is not possessed of a 'creative intellect' or that his analysis demonstrates that Eddington does indeed pass with honours!

[21]Of course the structuralist should not be found guilty by association here! That structuralism in part motivated Eddington's numerology should not be used to form the basis of some kind of modus tollens. Indeed, since the numerology rests on selective subjectivism as well, the structuralist's attempt to detach the latter should block this derogatory inference.

[22]Again, Eddington himself is careful to distinguish his approach from that of the logical positivists (1939, p. 189).

explicit in his insistence that the structure of the world is of a kind defined and investigated by group theory (see Eddington 1936, Ch. XII and 1939, Ch. IX). However, it appears that by the late 1920s/early 1930s, this group-theoretic structuralism was also motivated by the implications of quantum mechanics for particle indistinguishability. These played a crucial role in the rescue of his structuralist derivation of the fine structure constant when experimental results revised the value of the latter from 136 to 137 (Eddington 1936).

Eddington was well aware of the philosophical implications of the new quantum statistics and understood the non-classical indistinguishability of the particles to be the logically prior notion. However, his understanding went beyond that of other physicists in shaping his notion of 'interchange' (Kilmister *ibid.*, pp. 130-132). Following a geometric analogy with rotation, Eddington considered the conditions under which an interchange of particles made no difference from an algebraic perspective. Such conditions provide a representation of indistinguishability which he can then effectively feed into his algebraic programme and by a great detail of jiggery-pokery obtain the desired result. The details are once again given by Kilmister (*ibid.*, Chs. 8 and 9) but there are two curious, not to say bizarre, features of Eddington's use of indistinguishability which throw further light on his form of structuralism. The first concerns a technical issue: Eddington not only had to account for the revised value of the fine structure constant, but also had to accommodate the fact that as applied to the hydrogen atom, Dirac's equation gave an extra term 1/137r, where the 1/r acts like the Coulomb potential. Eddington's response was to argue that the Coulomb force could be identified with Fermi-Dirac exclusion and hence was a consequence of particle indistinguishability for fermions. The second feature is that, with regard to the hydrogen atom, Eddington regarded the proton as *indistinguishable* from the electron (Kilmister's gloss on this is uncharacteristically not as helpful as one would wish).

In order to get a grip on these claims, we need to start with a principle which Eddington himself identifies as *the* fundamental epistemological principle of this 1936 work, the 'Principle of the Blank Sheet'. The basic idea is that, in order to get the analysis going, we must first formulate some kind of background in terms of which physical phenomena can then be distinguished:

'To develop a theory of the characteristics which can be distinguished and of the measurement of the distinction, we require a blank sheet to write on - not a sheet already scribbled over with vaguely recognised distinctions.' (Eddington 1936, p. 32).

Thus we begin with intrinsically indistinguishable particles *and* space-time frames (*ibid.*, p. 33 and p. 56) in order that the relevant physical differences are introduced openly rather than smuggled in via the initial assumptions[23].

Such differences include mass and charge, of course, neither of which are regarded by Eddington as intrinsic properties of the particles. Rest mass is nothing more than the energy of the particle in an assembly of particles in statistical equilibrium (1936, p. 268 and p. 262). Similarly charge has its origins in the permutation of indistinguishable particles (1929 paper; 1936, p. 283). This is what Eddington claims to have demonstrated through the identification of the Coulomb force with the results of the Exclusion Principle. The heuristic origins of this identification are not entirely clear and I shall simply note that the basis of Eddington's demonstration is permutation invariance. As already indicated, Eddington saw this as a 'new kind of relativity transformation' (*ibid.*, p. 283) in which the interchanges of indistinguishable particles is represented by a rotation of the system in configuration space (this foreshadows more recent configuration space approaches to particle indistinguishability)[24]. This allows the permutation to be represented as a continuous transformation, as probability is gradually transferred from one 'identification' to the opposite one (*ibid.*, p. 284). The interchange energy is then the momentum conjugate to the permutation co-ordinate and it is this that Eddington claims is equal to the

[23]Thus one of the first distinctions to be scribbled on the sheet is that of the symmetry properties of his algebraic elements (*ibid.*, p. 40).

[24]There is a further anticipation of more generalised forms of statistics corresponding to higher order symmetries (*ibid.*, p. 290). Of course, for Eddington these correspond to 'non-minimum' charges.

observable value of the Coulomb energy. Furthermore, and crucially, it is the addition of this permutation co-ordinate which requires the extension of his original 136-dimension phase space by a further dimension to provide the algebraic foundation of the (new value for) the fine structure constant (*ibid.*, p. 286).

Thus from the point of view of Eddington's analysis, protons and electrons begin life, as it were, as completely indistinguishable units, to which various attributes are added as the analysis proceeds:

'The Principle of the Blank Sheet requires that at the start we should recognise no intrinsic distribution between the particles which we contemplate, in order that we may trace to their very source the origin of those distinctions which we recognise in practical observation. The fundamental dynamics is the dynamics of indistinguishable particles; the dynamics of distinguishable particles is a practical application to be used when we do not wish to analyse the phenomena so deeply.' (*ibid.*, p. 287)

Thus the mass cannot be used to distinguish a proton from an electron because it is represented within quantum mechanics by an appropriate operator and this cannot be applied to a given particle until we have first determined how that particle should be identified at different times. To use mass as a criterion to distinguish particles presupposes that they have already been distinguished (*ibid.*)[25]. More generally, the identification of particles is always relative since a change in attributes such as position or colour (Eddington gives several illustrative examples involving different coloured balls throughout the discussion) can always be effected by a shift in reference frame. Even the permutation co-ordinate is not absolute in terms of observational meaning and all we can do is adopt some conventional criterion for determining the constancy of the co-ordinate and measure changes in it relative to this standard (*ibid.*, p. 291)[26].

The introduction of observation here and in the quote above suggests a role for Eddington's selective subjectivism in his philosophical attitude towards indistinguishability. As he puts it, there is nothing 'mystical' about the effects of indistinguishability (*ibid.*, p. 285), in the sense that they arise from some ontological peculiarity of the particles, such as 'non-individuality', say. A being 'more gifted than ourselves' could identify individual particles and apply the ordinary equations relevant for distinguishable particles, but, crucially, the results obtained would be of no interest for, or use to, us because we have no access to the relevant observational data (*ibid.*). We are unable to identify particles at different spatio-temporal locations and thus for Eddington quantum indistinguishability is ultimately observational in origin. This is a familiar position among physicists, but Eddington understands it within the framework of his epistemology according to which this observational limitation is subjective. It may be asked, why should the statistical behaviour of particles be affected by our inability to distinguish them? As he says, this would be absurd or incredible, '... unless we bear in mind the subjectivity of the world described by physics and of all that it is said to contain.' (1939, p. 37). The question would be a legitimate one to ask with regard to wholly objective particles displaying wholly objective behaviour but '... our generalisations about their behaviour ... describe properties imposed by our procedure of observation ...' (*ibid.*) Indistinguishability for Eddington is thus a form of epistemological principle; one can imagine it being tested but the test would be perfunctory 'like the experimental verification of propositions of Euclid' (*ibid.* - significant analogy?).

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[25]We recall his earlier attitude towards clocks and rods in relativity theory noted above.

[26]See also the discussion in his final work (1946, pp. 50-51), where he writes that 'The supposed non-interchangeability of the proton and electron is based on the mistaken assumption that we begin with *free information* as to which of them is at x_1.' (*ibid.*, p. 51). This, he claims, also applied in classical mechanics, giving, as an example, a double star whose components are so similar that a telescope cannot distinguish them (interestingly, a similar example has been given more recently by Dalla Chiara and Toraldo di Francia in their discussion of particle indistinguishability and proper names). He continues: 'The 'indistinguishability' of particles is best understood if we think of them as carriers. It does not apply to the contents of the carriers, and it is to be noted that the contents include the mass and sign of charge as well as less permanent characteristics.' (*ibid.*). This could be confusing since the notion of a 'carrier' meshes well with that of substance which Eddington wants to reject, of course; cf. Cassirer's comments on 'carriers'.

There is a further distinction to be made. If the reduction of the Coulomb force to Fermi-Dirac statistics is extended to cover all interaction forces, as Eddington believed it could be (1939, p. 128), then interaction has a subjective origin due to indistinguishability. There is a further subjectivity attached to the 'ultimate particles' which is strictly independent of the former[27]. That these ultimate particles are 'identical structural units' arises as a 'specialisation' of the concept of analysis, viewed as an ingrained form or 'frame' of thought (*ibid.*, p. 122). Apparently intrinsic attributes can be resolved into relational ones, so that 'All the variety in the world, all that is observable, comes from the variety of relations between entities.' (*ibid.*) but the entities themselves are precisely alike. And this is not because the objective universe is built of such units but rather that our knowledge is 'impressed' by a fundamental form of thought (*ibid.*, p. 123 and p. 125). Thus, the laws of atomicity, which Eddington had earlier speculated might be objective, are brought within the subjectivist fold, thanks, at least in part, to indistinguishability and permutation invariance.

From this perspective, such a unit cannot be taken as separate or disassociated from the system of analysis of which it is a part. Taken as it stands, this is quite general but it becomes more precise when it is expressed mathematically, so that the relevant frame of thought is transformed into a mathematical frame[28]. And the appropriate mathematics, of course, is group theory (*ibid.*, pp. 137-140)[29]. Here it is interesting that Eddington doesn't tie the introduction of group theory explicitly to the presence of his identical structural units. It comes in as a way of expressing the relationships between relations and the important point is that whatever the nature of the entities, the use of group theory allows us to abstract away the 'pattern' or structure of relations between the entities[30]. Knowledge of structure, therefore, is communicable whereas other forms of knowledge (my knowledge of what something tastes like, for example) are not[31]. Hence, it is through structure that we can have inter-subjective knowledge and Eddington proposes group theory as the answer of modern physics to the old philosophical question, 'what sort of thing is it that I know?':

'What sort of thing is it that I know? The answer is structure. To be quite precise, it is structure of the kind defined and investigated in the mathematical theory of groups.' (*ibid.*, p. 147)

The philosopher's perplexity arises from the assumption that knowledge of the external world must be based on sensations, which are mental and hence the problem arises as to how the mental can give us knowledge of the non-mental. The assumption is incorrect, however, as a single sensation tells us nothing about the physical world. The logical starting point of physical knowledge is '... knowledge of *the group-structure of a set of sensations* in a consciousness.' (*ibid.*, p. 148; Eddington's emphasis)[32]. These fragments of structure are then collected together, represented through the fundamental forms of

[27]The distinction between 'knowable' and 'objective' systems - criticised by Dingle (*op. cit.*) - plays a role here (1939, p. 128).

[28]In an important sense what Eddington is tackling here is the problem of the applicability of mathematics: 'The question to be discussed in this chapter is, At what point does the mathematician contrive to get a grip on material which intrinsically does not seem particularly fitted for his manipulations?' (*ibid.*, p. 137).

[29]The above point that the unit cannot be disassociated from the structure then receives a mathematical gloss: 'As a structural concept the part is a symbol having no properties except as a constituent of the group-structure of a set of parts.' (*ibid.*, p. 145).

[30]And this is how he solves the applicability problem: 'In this way mathematics gets a footing in knowledge which intrinsically is not of a kind suggesting mathematical conceptions.' (*ibid.*, p. 141).

[31]The mathematics is essential because that is the only way in which the assertions of physical knowledge can be restricted to structural knowledge. But this very mathematics then prevents access to non-structural knowledge of what underlies the structure: 'Every path to knowledge of what lies beneath the structure is then blocked by an impenetrable mathematical symbol.' (*ibid.*, 142). This is strongly reminiscent of Husserl's view, presented in *The Crisis of European Sciences and Transcendental Phenomenology*, that mathematised nature, although responsible for so many scientific discoveries, also conceals the 'meaning-fundament' (in the life-world) of the theories discovered by it (Husserl 1970).

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thought and completed by inference to unobservable structures to give the 'structure [formerly] known as the physical universe.' Thus,

'Physical science consists of purely structural knowledge, so that we know only the structure of the universe which it describes. This is not a conjecture as to the nature of physical knowledge; it is precisely what physical knowledge as formulated in present-day theory states itself to be. In fundamental investigations the conception of group-structure appears quite explicitly as the starting point; and nowhere in the subsequent development do we admit material not derived from group-structure.' (*ibid.*, pp. 142-143).

Note that group theory enters both at the bottom level, as it were, in representing the structure of sensations, and at the top-most theoretical level, in representing the structure of the ultimate theoretical elements[33]. That the sensations themselves are non-structural, and our knowledge of them is by direct awareness, might seem to resurrect a form a dualism but Eddington insists that this is a 'logical confusion' according to which we cannot give meaning to the notion of dualism without making certain presuppositions which undermine that very dualism (*ibid.*, pp. 150-151). The idea is this: if we conceptually distinguish that part of the world of which we have direct awareness, namely that part which has to do with our sensations, from that part of which we have structural knowledge, then structurally the latter is no different from the former. Yet in order to give meaning to the dualism we would have to suppose that we have some non-structural knowledge of that part by which we could assess its difference from the sensational part. But that is to suppose that we could have direct awareness of the structural part which would show that it is non-sensational. But that is impossible, for if we had direct awareness of it, then it would be sensational; hence the very possibility of dualism is undermined[34].

This issue, of the relationship between the structural and non-structural components of our knowledge, is obviously a fundamental one. Thus the ordinary 'frames of thought' which, as indicated above, are transformed by mathematics feature non-structural, 'general' concepts, from which structural concepts are obtained by eliminating everything which is not essential to the role the concept plays in a group-structure. If the structural concept becomes a mere element, whose properties are those of a mathematical symbol, then a general concept '... is our conception of what the symbol represents in our ordinary non-mathematical form of thought.' (*ibid.*, p. 144). However, with the exception of those general concepts concerning things of which we are directly aware, such concepts - albeit ingrained as ordinary forms of thought- may be no more than forms of 'self-deception' which persuade us '... we have an apprehension of something which we cannot apprehend.' (*ibid.*, p. 144). I mention this because it can be made to relate to the motivations for ontic structural realism mentioned above: we have a general concept of an object as an individual, which is so ingrained as a form of thought that we export it from the classical to quantum realm and are persuaded that we have an apprehension of that which we cannot apprehend. All that we can apprehend, following Eddington, is the

[33]It is interesting to note that although Born argued vehemently against Eddington's over-emphasis on theory (he considered the latter's ideas to be a 'considerable danger to the sound development of science'; 1943, p. 2) he drew on the analysis of sense impressions offered by Gestalt psychology to argue that ' ... the 'shapes' of physical things are the invariants of the equations' (*ibid.*, p. 12) and that these have the same kind of objective reality as any shape of more familiar things (see also Born 1956, p.163). Cassirer also approached the results of Gestalt psychology from a group-theoretical perspective in his (1944).

[34]As Eddington acknowledges, that structuralism 'abolishes' dualism had been previously noted by Russell, for example, and at this point in the text Eddington recalls a passage from *Introduction to Mathematical Philosophy* which concludes that the difference between structural and non-structural knowledge '... must lie in just that essence of individuality which always eludes words and baffles description, but which for that very reason is irrelevant to science.' The difference between Russell's position in 1919 and Eddington's in 1939, or so Eddington claims, is that in Russell's case structuralist knowledge was regarded as '... the kernel of truth which would outlast the changing theories which enhulled it' (Eddington *op. cit.*., p. 152), whereas by 1939 it was no longer hidden but had been dug out from physical theories through the techniques of group theory (see the brief discussion of Braithwaite's objection below).

relevant group-theoretic structure, of course, as it is represented in terms of symmetric and anti-symmetric state functions[35].

Bizarrely perhaps, Eddington applies his distinction between 'general' and 'structural' concepts to the issue of what we mean by the term 'exist'. He rejects '... any metaphysical concept of "real existence"' (*ibid.*, p. 162) and introduces in its place a 'structural concept' of existence according to which it only makes sense to ask if a given entity exists in the given structure or not. Since there are only two possibilities, existence and non-existence (of course), '*The structural concept of existence is represented by an idempotent symbol*.' (1939, p. 162; Eddington's emphasis). Embodying the simplest possible structure, an entity represented by such a symbol must have no parts and the entity in physics corresponding to this element of analysis is, of course, the elementary particle. Now an interesting question arises: from the perspective of individuality and indistinguishability how are these particles to be regarded? As Eddington notes (1946, p. 131), this mathematical representation in terms of idempotent quantities encourages a treatment of the particles as 'pseudo-individuals':

> 'It will not be surprising if in our gropings into the structure of things a legend of individuality has attached itself to the carrier of an idempotent variate. In statistically grounded theory it is the closest counterpart of the obsolete classical particle. We now know that matter cannot be analysed into elements having the individual distinctness that classical particles were supposed to have; but in the carriers of idempotent variates we reach elements which, though not less statistical than other carriers, do not betray their statistical character in the ordinary calculations of dynamics.' (*ibid.*)[36]

According to Eddington, the association of this notion with idempotency has 'profound implications' for the logical structure of physical science, involving, as it does, the transfer of a metaphysics appropriate to the macroscopic realm, to the microscopic. What we observe are macroscopic (Eddington uses the term 'molar') phenomena and underlying this realm '... we are accustomed to picture a microscopic world populated by individuals ... and it is further supposed that protons and electrons are such individuals.' (*ibid.*). Such a picture encourages the view that the process of analysis has a terminus (in the individuals) but if the only kind of individuality is actually this pseudo- form conferred by idempotency, then there is no reason to suppose that the process will ever have to stop for *metaphysical* reasons. We may decide to stop once we have achieved our analytical aims but that is another matter entirely. Furthermore, once we realise that the analysis of macroscopic objects into microscopic carriers has a goal that is mathematically, rather than objectively, defined in this way, the numerological efforts of Eddington's programme may not seem so implausible (or so he hopes; *ibid.*, p. 132)[37]. From this perspective, then, 'the elementary particle is a product of analysis of ... group structure.' (*ibid.*, p. 164).

This programme was subjected to vigourous criticism by Braithwaite in his 1941 review of Philosophy of Physical Science (Braithwaite 1941). What's interesting here is to note the similarity between Braithwaite's objections to Eddington's structuralism and Psillos' more recent concerns, as cited above, particularly with regard to the structure-object distinction. Thus Braithwaite rejects as invalid what he sees as Eddington's dichotomy between structure and the incommunicable 'Erkenntnis' of the content of

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[35]Another example Eddington gives is that of space and his discussion is nicely suggestive of recent structural realist accounts.

[36]With regard to the aspect of 'carriers' here, we recall the comments above.

[37]Here he appears to be referring to the work given in the final chapter, probably written on the last day of his working life, in which he calculates the 'cosmical number' of protons and electrons in the universe (1946, pp. 265-283). Issues of identity intrude again, when he writes, 'It may be asked, why should we trouble to distinguish ... particles ... from one another, seeing that it is a fundamental tenet of quantum theory that elementary particles have no distinctive identity? But, as we pointed out earlier, it is the conception that matters - not whether the particles are distinguishable, but whether they are conceived as distinct. Clearly, the physicist conceives electrons as distinct, otherwise he would not talk about exchanging them; if they are continually exchanging identity, they must be conceived as having an identity to exchange. We are not concerned with any metaphysical conception of identity, Whatever it is that is exchanged - whether it is called 'identity' or merely a 'suffix' - has to have a structural equivalent.' (1946, p. 273).

experience (1940, p. 462). Focusing on the claimed group-theoretic nature of the structure, Braithwaite insists that such groups are defined only with respect to given 'modes of combination', so that the group structrure is in fact less abstract than Eddington supposes. He writes,

'To say that two sets of things have the same group-structure would be to say noting of interest unless the modes of combination of both the groups had been specified. The fact that structure depends upon content is one reason why the structure-content dichotomy of knowledge is untenable.' (ibid., p. 463)

In other words, the group-structure is only given once the relevant transformations have been specified (i.e. whether we're talking about rotations or permutations, for example), but to do this is to supply content and so we no longer have pure structure. This appears to be analogous to Psillos' argument which, the latter claims, leads to the collapse of epistemic structural realism (op. cit.).

Even more interesting, in this latter context, is that in a footnote to the above passage, Braithwaite refers to Newman's famous result that for any collection of objects of a given cardinality, the claim that there exists a particular structure, expressed in terms of the relevant relations, defined over this set, can be trivially satisfied (see, for example, the discussions in Demopolous and Friedman (1995) and Psillos (op. cit., pp. 63-65)). Thus he writes,

'... his [Newman's] strictures are applicable to Eddington's group-structure. If Newman's conclusive criticism had received proper attention from philosophers, less nonsense would have been written during the last twelve years on the epistemological virtue of pure structure.' (Braithwaite op. cit., p. 463).

Eddinsgton's reply is revealing. With regard to Braithwaite's claim that a group is only defined with respect to a particular 'mode of combination', Eddington points out that what Braithwaite appears to have in mind here are group representations, whereas in order to represent the 'pattern of interweaving', he has been talking about *abstract* groups. And it is precisely the abstract aspect that renders the concept of a group so useful in the philosophy of physics. From this perspective we lose the distinction between the nature of the element and the nature of the combining relaiton which makes it an element of the group: '*The element is what it is because of its relation to the group structure*.' (Eddington 1941, p. 269; his emphasis). Since the elements of the abstract group are operators, the combining relation emphasised by Braithwaite, is taken up into the manner in which the elements operate. Eddington continues,

'... I must insist that I am rescuing out of the mathematical formalism what it is for physical purposes the most essential feature of the group conception of structure, namely, that primarily the elements of a group (or ring or algebra) are defined solely by their role in that group (or ring or algebra). Therefore when Braithwaite argues that it is possible to regard the elements of a group in such a way that they are not elements of a group, I answer that there is *no* other way of regarding them. Unless we import qualities not inherent in them by definition (by adopting a special realisation or representation of the group [and as he notes he means this in the non-technical sense] there is nothing to lay hold of that *could* be regarded from another point of view.' (*ibid.*, p. 269).

There are a couple of things to note about this. First of all, it is important to bear in mind that the elements of the algebra should not be identified with the elements of the group, of course (**more here?**). But then the worry is that by shifting from the group to the associated algebra, Eddington might have evaded the issue somewhat. In stating that an element 'is what it is' by virtue of its relation to the group structure, Eddington is not quite offering a version of the ontic form of structural realism which sees particles as being what they are by virtue of their relation to the overall structure, since the 'element' for Eddington, here, is an operation, like rotation (see below). Secondly, however, there is a feature of this form of structural realism present in Eddington's remarks that there is no non-group-theoretic *content* to 'lay hold of'. In this case, Eddington agrees that there is no structure-content dichotomy, not because structure depends on content but rather because it is content - as represented in this case by Braithwaite's 'combining relations' - that depends on the structure. This dialectic is nicely mirrored in the present day differences between

Psillos, in responding to Worrall by insisting that the latter's distinction between structure and content is untenable in the scientific context, and Ladyman in arguing that the structure-content distinction collapses because all the physical 'content' can be cashed out in structural terms.

Referring to Newman's criticism of Russell Eddington argues that,

> 'Russell, in his pioneer development of structuralism, did not get so far as the concept of group-structure. He had glimpsed the idea of a purely abstract structure; but since he did not concern himself with the technical problem of describing it, he had no defence against Newman's criticisms. Russell's vague conception of structure was a pattern of entities, or at most a pattern of relations; but the elements of group theory make it clear that pure structure is only reached by considering a pattern of interweaving, i.e. a pattern of interrelatedness of relations [and here Eddington refers to *The Philosophy of Physical Science*, pp. 137-140].' (Eddington 1941, p. 278).

As an example of what he means by this 'pattern of interrelatedness of relations', Eddington presents the algebra of operators representing rotations acting on rotations, for which the 'pattern of interrelatedness' is manifested in the associated multiplication table. The information encoded in such a table is by no means trivial and hence Eddington feels able to conclude that there is no foundation to Braithwaite's contention that the Newman objection applies to the structure as described by this multiplication table; indeed, he accuses Braithwaite of not having grasped 'the main idea' of structuralism.

A further indication of what Eddington had in mind is given in his example of spin, where the information encoded, as above, in the relevant structure gives all the information we can get (*ibid.*, p. 279). What's particularly interesting here is the way in which Eddington deploys a certain structuralist strategy which amounts to assuming certain non-structural elements in order to be able to articulate the structure in the first place, only to discard these elements once the structure has been constructed (see French 1999; French and Ladyman forthcoming; the strategy can be traced back to Poincaré's approach to geometry, where he refers to objects as a kind of 'crutch' which eventually is thrown away). Thus he notes that the components of spin can be specified in a set of mutually orthogonal planes and also that this represents non-trivial knowledge. Now of course, the Newman-Braithwaite objection would be that such knowledge is non-structural because we are acquainted with such orthogonal planes in the 'external' world. The way round it is to consider the set of operations represented by rotations through 90^o in each of the planes. This yields a group-multiplication table which Eddington takes to define the relevant structure and now '[w]e need ... trouble no further about the planes;' (*op. cit.*, p. 279). We initially associated the components of spin with the planes but we could equally as well have associated them with unit rotations in the plane, so that initial association was just a kind of heuristic move which takes us to the group-multiplication table which in turn represents what is important, namely the structure. The information encoded in the latter is definitely non-trivial, since it conflicts with other statements, some plausible, but the apparent non-structural knowledge acquired by our acquaintance with the planes is in fact 'non-existent'. Thus the appearance of a non-structural component is illusory, deriving from the heuristic role played by certain objects[38].

There is much more to say of course, but I want to conclude this historical section, finally, by reflecting on the forms of structuralism proposed by Cassirer and Eddington[39]. In both cases, they appear to offer a strong dose of Kantian epistemology with their structuralism. Of course, we have to be a little careful with the labels here. Eddington himself wrote, 'We do not accept the Kantian label; but, as a matter of acknowledgment, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern development of physics.' (1939, pp. 188-189). And as Ryckman notes, by historicizing the inquiry into the conditions of the possibility of knowledge, Cassirer moves away from Kant in conceiving of synthesis as a

[38]There are further interesting aspects of this debate, concerning Eddington's attempted derivation of his 'E'-algebra from his 'structural concept' of existence and also his view of relations, but I will leave these for another opportunity.

[39]Of course, Cassirer and Eddington disagreed over the impact of quantum mechanics on the principle of causality (see, for example, Cassirer *op. cit.*, pp. 60-61 fn. 4).

methodological requirement. Nevertheless, his account of objectivity and the constitution of objects possesses clear idealist characteristics. As indicated in French and Ladyman (forthcoming), the modern structural realist might want to avail herself of the structural analysis of objects, whilst articulating an alternative account of physical law, for example. Of course, as far as Cassirer is concerned, this would be to impale oneself on one horn of the 'old dilemma' of phenomenalism vs. naive realism which his structural understanding of objectivity is intended to avoid (*op. cit.*, pp. 142ff).

Turning to Eddington, in one sense, his view is clearly epistemological: the world is not entirely structural. However, all our physical knowledge is knowledge only of structure and so we cannot have physical knowledge of these non-structural aspects. It is this perhaps that non-structuralists find so repugnant; in Eddington we see the Kantian noumena acquire a mystical resonance. But as we have seen, it is not quite that simple. The particles of physics are not to be found in the non-structural world; as far as Eddington is concerned, such a suggestion would be completely absurd. Thus the standard realist's cry that this epistemic structuralism leaves something beyond the reach of physics, belabouring as she is under the misapprehension that this something *should* be within the reach of physics, would be regarded with something approaching derision. There was never any possibility that this aspect could be grasped by physics because it could only be so if it were structural in the first place. Thus we cannot set Eddington beside Worrall in holding that when it comes to the particle of physics say, what we know are the structural relations they enter into but their 'natures' lie hidden. From the synthetic point of view, of course, we start with individual particles and combine them to form perceptible objects (Eddington 1939, p. 220) but from the analytic perspective of Eddington's programme, it is the relation (between phenomena) which comes first and the elementary particle emerges as the product of analysis of the group structure. This aspect, of course, is closer to ontic structuralism but for Eddington to be placed next to Ladyman we would have to dispense with the selective subjectivism. Kilmister seems to think we can understand Eddington's structuralism without this but I'm not convinced[40]. If we *could* cleanly excise it, however, we would lose the distinction between the structural and non-structural, or physical and 'external' worlds, leaving a form of ontic structuralism.

The Group-Theoretic Conception of Objects

It is odd that neither Cassirer nor Eddington appear to refer to the introduction of group theory into quantum mechanics by Weyl and Wigner. As Weyl noted, this hinged on the identification of two different kinds of symmetries: spatio-temporal symmetries, such as the rotational symmetry associated with the representation of the atomic nucleus as a fixed centre of force; and permutation symmetry, or invariance under a particle permutation. Given Cassirer's understanding of spatio-temporal symmetries, and their incorporation into his epistemology, we might speculate that he saw little that was particularly new in these developments, from the structuralist perspective. Eddington, on the other hand, did make an explicit attempt to accommodate permutation invariance within his account, but he gave it a particular geometric twist as I indicated above. Weyl, however, clearly recognised the import of this new symmetry and not only explored its physical consequences in *The Theory of Groups and Quantum Mechanics* but later, in *The Philosophy of Mathematics and Natural Sciences*, considered in more detail its philosophical implications. As I have noted elsewhere, that Weyl was aware that here was a new aspect of symmetry in the world goes some way to responding to Donini's perplexity over why, in *The Theory of Groups* ..., he appeared to have forgotten all his work on ‘relativity and invariance matters’ (Donini 1987, p. 109). It’s not a question of forgetting the latter but of focusing on a new fundamental symmetry.

Mackey characterises the formal moves in terms of two, intertwined sets of developments: what can be called the 'Weyl programme', which saw group theory as a way of bringing order to the collection of principles and ad hoc rules that constituted quantum

[40]On this point, Broad also claimed that the two can be separated: 'I do not think there is much connection between the "selective subjectivism" and the "structuralism" of Eddington's theory. Of course both of them may be true. But the structuralism might be true and important, so far as I can see, even if the selective subjectivism were false or greatly exaggerated.' (*op. cit*., p. 312)

mechanics in the late 1920s, and setting the foundations of the theory on a secure basis; and the 'Wigner programme' which saw group theory as a way of bypassing the computational intractability associated with tackling the dynamics head on. Of course, both Weyl and Wigner made important contributions to each. Wigner, in particular, further extended the reach of group theory within physics by applying it both to the nucleus[41] (see French 2000) and elementary particles. This latter extension was presented by Wigner at the 1935 'Pittsburgh Symposium on Group Theory and Quantum Mechanics' (Wigner 1935)[42] where he notes the 'unique correspondence' between possible Lorentz invariant equations of quantum mechanics and the representations of the inhomogeneous Lorentz group. Such a representation, ' ... though not sufficient to replace the quantum mechanical equations entirely, can replace them to a large extent.' (Wigner 1939, p. 151) It can give the change through time of a physical quantity corresponding to a particular operator, but not the relationships holding between operators at a given time. The issue then is to determine the irreducible representations of this group and these are established in Wigner's famous 1939 paper[43].

It is this work of Wigner's - specifically, the association of 'elementary physical systems' with irreducible representations of the inhomogeneous Lorentz group - which is drawn upon by Castellani in her analysis of the group theoretic constitution of objects.

From Objectivity to Object

According to Weyl, 'objectivity means invariance with respect to the group of automorphisms [of space-time]' (Weyl, 1952). On the basis of this statement, Castellani presents an 'objectivity condition' (for the physical description of world), namely invariance with respect to the space-time symmetry group. The issue then is to move from objectivity to objects:

> 'What is of interest, from the point of view of the object question, is how this objectivity condition for the laws of physics can be used with regard to the determination of 'objects' within a given physical domain.' (Castellani 1993, p. 108)

The basis for such a move is precisely Wigner's association of an 'elementary system' with an irreducible representation of the space-time symmetry group, such that the set of states of the system constitutes a representation space for the irreducible representation. For quantum systems, the appropriate representation space will be the Hilbert space, of course. The labels of the irreducible representations are thus associated with values of the invariant properties characterising the systems. Further details are given in Castellani 1993 and 1998 **- more here?**

Two issues then arise. First of all, what this group-theoretic construction yields are classes or *kinds* of particles, not distinct objects (Castellani 1993, p. 109; 1998, p. 183-184). As Castellani puts it,

> 'The invariant properties which are ascribed to a 'particle-object' on the basis of group-theoretical considerations - as, for example, definite properties of mass and spin are ascribed to a (quantum) particle which is associated with an irreducible representation of the Poincaré group - are necessary for determining that given particle (an electron couldn;t be an *electron* without given properties of mass and spin), but they are not sufficient for distinguishing it from other similar particles. In addition to these 'necessary'

[41]It is in this context that Eddington does refer to Wigner, although the group-theoretical basis of the latter's 'theory of the nucleus' does not rate a mention (Eddington 1946, p. 205).

[42]Recalling Eddington's reaction to the Dirac equation, this symposium also featured a paper by Breit on group theoretic applications to Dirac's theory.

[43]It is interesting that the abstract of the 1935 presentation indicates that a detailed discussion of this work was supposed to appear in a joint paper with Dirac 'who first perceived this problem.' I don't know if such a joint project was ever begun. In the 1939 paper, Wigner again acknowledges Dirac, stating that the topic of the paper was suggested by him as early as 1928 and that even then, Dirac realised the connection between representations and the equations of quantum mechanics (1939, p. 156). The paper is presented as the outgrowth of 'many fruitful conversations', especially during 1934/5. Dirac also published his own work in this area, presenting more elegant derivations of Majorana's results on the classification of representations of the Lorentz group. As Wigner notes, his results provide aposteriori justifications of the work of Dirac and Majorana.

properties (sometimes called 'essential' properties), one does need further specifications in order to constitute a particle as an individual object.' (1993, p. 109)

Secondly, in addition to spatio-temporal symmetries, there is the permutation symmetry which needs to be accommodated within this approach. Let us consider each of these issues in turn.

With regard to the first issue, we need something else to give ‘individual objects’; Castellani identifies this something else as 'imprimitivity'. This was originally introduced by Mackey in the 1950s (see Mackey, 1978), although it is implicit in Wigner's 1939 work, and it has been notably applied to the definition of physical particles by Piron (1976). The basic idea is to use the notion of a 'system of imprimitivity' associated with a symmetry group in order to determine 'individuating' observable quantities such as position and momentum and thus move from kinds to individual objects by supplementing the above group-theoretic account.

Putting things somewhat crudely, we obtain an imprimitivity system in the following way: we associate with a system, in addition to the group G, a configuration space S (strictly a Borel space) on which G acts. A projection valued measure is then defined on S (where a projection valued measure is a mapping from a Borel subset of S to the relevant projection operator) and if the projection valued measure satisfies a certain identity ($U_x^{-1}P_EU_x = P_{Ex^{-1}}$; where P_E is a projection operator and U is a unitary representation) then the projection valued measure constitutes a ‘system of imprimitivity’ for U based on S. The importance of the system of imprimitivity associated with U is that it determines the structure of U as an induced representation (Mackey *op. cit.* p. 71; Varadarajan 1970/1968 Ch. 9). In particular, if S is transitive and L is a unitary representation of a closed subgroup of G, then the equivalence class of L is uniquely determined by the pair U,P, where P is a system of imprimitivity for U and the commuting algebra for L is isomorphic to the subalgebra consisting of all bounded linear operators that commute with all P_E (Mackey *op. cit.*, pp. 71-72). This amounts to a statement of the 'imprimitivity theorem' (**more here???**) which has a number of important applications.

The virtues of imprimitivity have been extolled by Varadarajan, who writes that

‘The approach through systems of imprimitivity enables one to view in a unified context many apparently separate parts of quantum mechanics - such as the commutation rules, the equivalence of wave and matrix mechanics, the correspondence principle, and so on. The same treatment leads moreover in a natural fashion to the notion of spin ...’ (Varadarajan 1970, p. viii).

In particular, if S denotes physical space (3 dimensional, Euclidean, affine), and G is now the Euclidean group of all rigid motions of space, then the position of a particle, regarded as an ‘S valued observable’, can be described by a projection valued measure defined on S. The relevant projection operator is then the self-adjoint operator corresponding to the real-valued observable which has the value 1 when the particle is ‘in’ Borel sub-set/at a given position and 0 when it is not. If we impose the requirement that the description of the system be covariant with respect to G, then the projection operator must satisfy the identity which renders the projection valued measure a system of imprimitivity. Introducing momentum observables and applying certain group-theoretic results, one can then obtain the usual commutation relations, not by analogy with the Poisson brackets of classical mechanics but as a consequence of Euclidean invariance (Mackey Ch. 18; Varadarajan Ch. 11)[44].

Furthermore, one can show that every irreducible representation of the commutation rules is equivalent to the Schrödinger representation. The apparently special choices in the latter for representing position and momentum observables are in fact the most general ones possible subject to the commutation rules, if we assume irreducibility.

[44] There is the possibility of further underdetermination here: ‘Given any quantum system with a complex Hilbert space defining the logic, we may obtain another whose logic is defined by a real Hilbert space by simply composing the given one with a new independent system whose logic is the set of all subspaces of a real two-dimensional Hilbert space.’ (p. 197). According to Mackey, the ambiguity can be analyzed and ‘to some extent removed’ by the application of group theoretic notions.

On this basis, it is claimed, we can prove the isomorphism of Schrödinger wave mechanics (based on the Schrödinger representation) and matrix mechanics (based on the commutation rules only) (Varadarajan *op. cit.* p. 151). And the results just keep on coming: if the relevant configuration space is affine, we get the Born interpretation of $|\psi|^2$ and an 'illustration' of complementarity in the sense that one can show that no single state exists in which both position and momentum can be localised sharply (Varadarajan *ibid.*, pp. 154-155)

As far as the current discussion is concerned, the important point arising from all this is that, 'All we need to discuss physical events are position observables and a dynamic group.' (Mackey *op. cit.* p. 195). In particular, through the imposition of a condition of covariance for observables, imprimitivity allows us to accommodate, in group theoretic terms, the spatio-temporal location of particles (Piron 1976, pp. 93-95). According to Castellani, this restores the notion of an object and thus we get the group theoretic characterisation (or for her, *constitution*) of not only kinds but individual objects:

'The aim is to arrive at a definition of a particle by determining "individuating" observable quantities (such as the position and momentum) with the help of the imprimitivity systems.' (Castellani 1998, p. 190)

Now this is not what the structuralist wants[45]!

So let us consider the philosophical implications of the above formal moves in a little more detail. First of all, these moves have not reintroduced 'substance' of course (we recall Cassirer's point about the constancy of the relation not implying the constancy of the carrier). However, what we appear to have arrived at, via this long detour through group theory, is nothing less than the good ol' 'bundle theory' of individual objects, according to which such objects are regarded as nothing more than a 'bundle' of properties, with spatio-temporal location typically privileged as that property which confers individuality, distinguishability and (classically at least) reidentifiability. As Castellani makes clear in her discussion of imprimitivity,

'The view at issue here is that according to which individuality is conferred upon an object by some of its properties and, in particular, by space-time properties.' (1998, p. 193 fn. 21)

Now the bundle theory, as usually understood, requires some form of the Principle of Identity of Indiscernibles in order to effectively guarantee individuation and as we all know, the status of this Principle is problematic in quantum mechanics (for a recent discussion see Massimi forthcoming). I'm not going to get into that discussion again, except to note the following: on the one hand, from the perspective of the configuration space approach a form of the Identity of Indiscernibles is manifested by the removal of coincidence points in the relevant configuration space. This, in turn, is effectively written into the guidance equations of Bohm theory (Brown et. al. 1999) and thus the latter can be understood as embodying a metaphysics of individual particles[46]. On the other, if one were to accept the well known arguments that the Identity of Indiscernibles is at best inapplicable, at worst violated in quantum physics, one would have an ontology of bundles which aren't tied together. This amounts to an ontology of non-individual objects described by something like qua/quasi-set theory.

Our long discussion of group theory and systems of imprimitivity seems to have led us right back to where we started, namely the underdetermination between individuality and non-individuality. Is there a way the structural realist can accommodate the central insight of Mackey's comment above without being committed to objects that are either individuals or non-individuals? A possible response is to understand imprimitivity as giving a group-theoretic grasp on the position of a 'particle' (perhaps understood as Bell's 'beable') but to insist that this does not yield objecthood (beables don't give objects). In other words, we can buy into the whole group-theoretic analysis/reduction of 'objects' but

[45]A point that has also been made by Chakravarrty.
[46]A rather peculiar metaphysics, granted, in which certain fundamental properties, which are regarded classically as 'intrinsic', are 'shared', in some sense, between the particle and the quantum potential.

simply resist the exportation of position, say, beyond the temporally limited domain of the immediately observable and into the realm of quantum objects as a whole[47]. The question now is, what kind of ontological picture does this give?

First of all, position can be regarded as yielding, not individuality per se, but only a kind of ‘pseudo-individuality’, as noted above, or what Toraldo di Francia refers to as 'mock individuality’ in the sense that one can pretend the particles are individual objects at the point of measurement, as it were, but only temporarily (Toraldo di Francia 1985; Dalla Chiara and Toraldo di Francia 1993). It is significant that this notion is articulated in the context of what can be taken as a form of structuralism[48], according to which particles are regarded as 'nomological' objects in the sense that '... physical objects are today *knots of properties*, prescribed by physical laws' (1978, p. 63). It is in this context that Dalla Chiara and Toraldo di Francia develop their view of quantum particles as 'anonymous' in the sense that proper names cannot be attached to them, although here too there is a tension between this and the underlying structuralism. However, the important point is that pseudo-individuality allows us to refer to 'objects', without compromising our structuralism[49]:

'This is why an engineer, when discussing a drawing, can *temporarily* make an exception to the anonymity principle and say: "Electron a, issued from point S, will hit the screen at P, while electron b, issued from T, will land at Q."' (1985, p. 209; Dalla Chiara and Toraldo di Francia *op. cit.*, p. 266)

Furthermore, a more congenial metaphysical framework for this structuralist view of objects might be found in trope theory. The basic idea here is that a 'trope' is a particular instance of a property, such as Bush's inarticulateness (is that a word , never mind a trope?!) and the proclaimed advantage is that, with both particulars and properties constructed out of, or reduced to, tropes, we get a parsimonious one category ontology (see Bacon 1997)[50]. Trope bundle theory represents an object as, surprise, surprise, a bundle of such tropes, rather than universal properties per se, but, again, some principle is required to tie the bundle together. In this case, Identity of Indiscernibles, as standardly formulated, would be inappropriate, and typically the second-level relation of compresence is invoked. Mertz (1996) has criticised this on the grounds that compresence simply cannot provide the level of unity required to bundle the tropes together into an object:

'Tropes have no 'attaching' aspect; their unity or 'togetherness' in constituting a thick particular must be accounted for by positing some uniquely ordained relation of 'compresence', which, to perform its ontic duty, must be predicative - that is, actually relating. Yet, as required of all relations in trope theory ... compresence reduces to monadic properties of the subject tropes so related and hence can be but further tropes, which, by the theory, are nonpredicative. Consequently, thick particulars dissolve back into disjoint atomistic tropes, and the goal in positing a compresence relation is defeated.' (*ibid.*, pp. 27-28).

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[47]Since I've mentioned Bell's beables, it is worth noting that in the context of Bohm theory one can dispense with trajectories. In his comparison/synthesis of Everett and Bohm (‘Measurement Theory of Everett/de Broglie’s Pilot Wave’) Bell takes the lesson from Everett to be that ‘... if we do not like these trajectories we can simply leave them out. We could just as well redistribute the configuration (x_1, x_2, ...) are random (with weight $|\psi|^2$) from one instant to the next. For we have no access to the past, but only to memories, and these memories are just part of the instantaneous configuration of the world.’ (Speakable and Unspeakable ..., p. 98).

[48] Thus he refers to the process of 'objectuation' by which the mind 'decomposes' the world into objects (1978, p. 58; see also Toraldo di Francia 1981, p. 220 - *The Investigation of the Physical World*, CUP) Crucially, '... objectuation is strictly connected with, or consists of, the mind's ability to distinguish *this* and *other*' (*ibid.*). Further discussion can be found in French and Krause forthcoming (book).

[49]With regard to the issue of how one can have structures (as relational) without objects (the relata), one can understand this pseudo-individuality as allowing us to introduce a notion of object, as a descriptive convenience, on the basis of which we can employ group theory, but which can subsequently be discarded once we have a grip on the relevant structures as described by the mathematics (French 1999; French and Ladyman forthcoming).

[50]Tropes also do useful service in acting as truth-makers for non-existential propositions about particulars.

Simons' approach avoids this form of criticism by replacing compresence with a form of Husserlian 'foundation relation':

'An electron must have a certain mass, charge and spin, and in addition is variably endowed with a position relative to other things and with a velocity and acceleration in particular directions at any time. When individual tropes require other individual tropes we say they are *rigidly dependent* or *founded* on these. When founding is mutual then a group of tropes must either all exist or none do. The mass, charge and spin of an electron must coexist, they require each other and form a bundle. A bundle consisting of all the tropes mutually founding one another directly or indirectly we may call a *nucleus*.' (Simons 2000, p.148; see also Simons 1994).

Tropes may also require other tropes as members of a kind and in such cases, instead of 'founding' we have 'generic dependence', with the tropes generically required forming a 'halo'.

Further reasons for rejecting compresence are derived from examples in physics. Thus, Simons argues, compresence is neither necessary nor sufficient for tropes to form a bundle:

'... it is not necessary because a trope bundle may be widely distributed, as in particle pair formation where paired tropes constituting electromagnetic polarisation or spin may be vastly separated yet mutually dependent. It is not sufficient because more than one trope bundle can be compresent as when two or more electrons occupy the same shell of an atom.' (*ibid.*, p. 148).

In other words, compresence cannot do the job because of quantum non-locality and indistinguishability! Replacing compresence by the above notion of rigid dependence or founding is supposed to accommodate examples like these. Of course, compresence does not disappear from the picture entirely, as it may still be concomitant. If the trope bundle theory is sufficiently 'flexible', then perhaps it can cover both pseudo-individuals and structures: a pseudo-individual is a bundle of compresent tropes, whereas a structure, or 'kind-structure', in the above sense, is a bundle of tropes which are not compresent[51]. This is not to say that the physics somehow *requires* trope theory; it may be that some form of bundle theory of individuality can do the same job, but at least we don't have substance and we don't have the Identity of Indiscernibles[52].

What about the second issue? How are we to understand the action of the permutation group? In the final section I'd like to explore this question in the light of two options, attached to each of the 'horns' of the metaphysical underdetermination with which we started.

Option 1: the particles are regarded as individual objects and permutation symmetry is either regarded as a property of the particles themselves, or, more plausibly but still problematically, as an emergent property of the assembly;
Option 2: permutation symmetry reflects the metaphysically weird nature of quantum objects as non-individuals.

And I shall conclude that Huggett's attempt to avoid either option leads us back to structural realism.

The Ontological Status of Permutation Symmetry[53]
Let me begin with the first option. If permutation symmetry is regarded as an intrinsic property and intrinsic properties are understood as delineating natural kinds, then electrons,

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[51]How we 'get' from the structures to the pseudo-individuals is part and parcel of the measurement problem.
[52]Trope theory may also be congenial to structuralism insofar as some trope theorists emphasise and defend the irreducible nature of relations (see, for example, Mertz 1996). Of course, they don't go so far as to allow relations in the absence of relata (they're not that mad) but it is, perhaps, not such a huge step to go from the view of relations as ontoglial to the view of them as ontological.
[53]What follows is an expanded version of the final section of (French forthcoming).

say, obeying different statistics - which is a possibility raised by violations of the spin-statistics theorem - would constitute different such kinds by virtue of possessing a different 'kind' of permutation symmetry. The particles of each such kind would still be indistinguishable, of course, and in a way that generates problems for the identity of indiscernibles and hence the bundle theory of individuality. And if we were to reject the latter and adopt some form of Transcendental Individuality, then this option of understanding permutation symmetry as an intrinsic property is still problematic, as Hilborn and Yuca have recently spelled out (preprint).

First of all, the only thing that would distinguish these electrons of different kinds would be their permutation symmetry; in all other respects they are indistinguishable. If two or more such electrons were to come together in an atomic system, the question arises as to which statistics would dominate (Hilborn and Yuca *ibid.*, p. 41). Perhaps we would need some form of 'meta-quantum statistics' to answer it! More fundamentally, as Hilborn and Yuca emphasise, we don't *measure* the permutation symmetry of individual particles; more than one particle is needed. Thus, as a property in this intrinsic sense, the permutation symmetry of the individual particle can be regarded as 'empirically superfluous'. For these sorts of reasons, Hilborn and Yuca reject this option. Instead they prefer a 'holistic' perspective which can accommodate the 'emergent' quality. Permutation symmetry 'emerges' at the collective level as a property, not of the particles themselves, but of the quantum state.

They argue that the possibilities of non-standard statistics and violations of the spin-statistics theorem in general can be accommodated quite naturally within this framework:

'This holistic point of view is both more faithful to the possibilities of physics (including possible violations of the spin-statistics connection) and a stronger philosophical stance. It also has the merits of simplicity and efficiency. On this account, permutation symmetry is a property of the collective state of the identical particles, not an intrinsic property to be associated with each particle.' (*op. cit.*, p. 44).

However, some care needs to be taken concerning what this holistic point of view amounts to. Hilborn and Yuca understand Redhead and Teller to be arguing in its favour, particularly with regard to '[t]he fact that holism places no epistemological limitations on the observer' (*ibid.*, p. 46). Redhead and Teller themselves understand the metaphysics of this holism in terms of non-individual quanta; on such an account, permutation symmetry is a manifestation of the metaphysically peculiar nature of the quanta. However, this account of permutation symmetry as a property of the state can also be accommodated within the alternative metaphysics of particles as individuals. In this case, as I have indicated elsewhere, the state space breaks up into sub-spaces of different symmetry, with transitions between such sub-spaces suitably prohibited. And permutation symmetry is then a kind of initial condition representing a further structural characteristic of the state space. Thus we seem to face our metaphysical underdetermination once again.

Huggett's recent analysis of permutation invariance (1999) can be seen as an attempt to establish an alternative to both the particles-as-individuals and particles-as-non-individuals packages, where the former takes permutation invariance to be a kind of mysterious 'brute fact' of the universe and the latter takes it to be associated with the non-individuality of quanta. The central motif of Huggett's approach is that permutation invariance should be regarded straightforwardly as a symmetry on a par with rotational symmetry, for example, and hence it is symmetry considerations, rather than either 'brute fact' or metaphysics, which explains quantum statistics. Of course, what is meant by 'a symmetry' needs to be spelled out and Huggett takes us through three such explications:

First of all, permutations are covariant, in the sense - as Huggett takes it - that the permutation group has a unitary representation in the state space. However, the explanation of (non-relativistic) spin requires not just that the rotation group has a unitary representation but also, of course, that the state vectors lie within multiplets of distinct intrinsic angular momenta, 0, 1/2, 1, and so on (Huggett *ibid.*, p. 337). In other words, the representations must also be irreducible. This gives a stronger notion of symmetry, which Huggett calls 'elementary state covariance': a symmetry group is said to be elementary state covariant if and only if the particle state vectors transform according to the unitary representation of the group (*ibid.*, p. 338). The (philosophical) point then is that we now have an account of the relationship between quantum statistics and permutations which, Huggett claims, is identical to that which is given for spin and rotations in non-relativistic quantum physics: if permutations are included in the full group of symmetries and it is postulated that this group is elementary state covariant, then only those many-particle states are allowed which are appropriately symmetrized or para-symmetric. The advantages of this are three-fold: i) it provides an explanation of the state space restrictions in terms of symmetry '... without the unnecessary extra logical strength of further (possibly questionable) assumptions' (*ibid.*, p. 339); ii) it provides an understanding of symmetrization within the many-particle tensor product formalism without having to invoke the Fock space that the non-individual quanta interpretation hinges upon; and, crucially, iii) it provides a unified treatment of quantum statistics and spin in terms of a fundamental symmetry principle (*ibid.*, pp. 339-340)[54].

Now, of course, as Huggett acknowledges, permutation symmetry is very different from rotational symmetry: a quantum system is not just covariant but invariant, in the sense that permutations are not just indistinguishable to similarly permuted observers but to *all* observers. Nevertheless, he argues, permutation invariance is implied by a further symmetry principle which space-time symmetries also obey, together with the formal structure of the permutation group. This further principle is what he calls 'global Hamiltonian symmetry' which implies that the relevant symmetry operator commutes with the relevant Hamiltonian. What we take the relevant Hamiltonian to cover is crucial here because, again as Huggett acknowledges, the principle would appear to be violated in the case where, for example, we have a noncentral potential term in the Hamiltonian of an atomic system, but, he insists, the symmetry is restored if we consider the 'full' Hamiltonian of system plus field, which does commute with the operators of the rotation group. As he points out (*ibid.*, p. 345), if observers are taken to be systems too, this symmetry principle is equivalent to covariance for space-time symmetries.

With regard to the permutation group, of course, permutations of a sub-system are permutations of the whole system and the above 'global Hamiltonian symmetry' very straightforwardly implies permutation invariance, without any additional assumptions concerning the structure of state space (*ibid.*, pp. 344-345). Hence, Huggett concludes,

> '... we should view permutations in a similar light to rotations: we should not take [permutation invariance] as a fundamental symmetry principle in order to explain quantum statistics. Instead we should recognize that it is a particular consequence of global Hamiltonian symmetry given the group structure of the permutations. Further, if we accept the similarity of permutation and rotation symmetry, it becomes natural to see quantum statistics as a natural result of the role symmetries play in nature, *via* [elementary state covariance]' (*ibid.*, p. 346).

Thus quantum statistics comes to be explained in terms of a fundamental symmetry principle. This is an attractive proposal which meshes nicely with the history of this subject. Nevertheless, one might feel that the proposed explanation contains a crucial lacuna. Consider: as Huggett acknowledges, permutation symmetry *is* different from space-time symmetries and permutation invariance only follows from his general symmetry principle given the particular structure of the permutation group. This generates the obvious question: why should the group structure be this way and not like that of the rotation group? Or, better perhaps, since the question could be answered by simply insisting 'that's the way the maths is', why should this particular piece of maths be applicable?[55] One obvious answer is to say that it reflects the nature of the objects themselves, as non-individual quanta. In other words, the explanatory gap gets filled by metaphysics and we fall back to the particles-as-non-individuals view. Alternatively, we might insist that it has nothing to do with the objects themselves, which can still be regarded as individuals but is a reflection of some kind of initial condition for our universe. Of course, Huggett would be unhappy with either option and might insist that it is enough to note the general symmetry principle plus the structure of the permutation group - the explanatory buck stops there. And indeed it has to stop somewhere, so the issue comes down to that of what counts as an appropriate terminus. At this point we might recall some

[54]And as Huggett notes, unification is deemed to be of central importance in certain philosophical accounts of explanation.

[55]There is another, related question, which is '*how* does this mathematics get applied/' The answer to this question involves spelling out the sorts of moves - particularly idealisations and approximations - made by the likes of Wigner, Weyl and von Neumann, for example.

more history: Newton famously refused to elaborate on the metaphysics of gravity and insisted that he could explain the phenomena using his law of universal gravitation and that was enough. Leibniz, caught up in his own baroque form of metaphysics, equally famously objected that this left gravity as an 'occult' force. We'll leave it to the student of counterfactual history to wonder whether Leibniz would have been satisfied with Einstein's attempt to bring gravity back into the light!

What was it that Leibniz found unsatisfactory with Newton's account and does it have anything to do with the uneasiness that one might feel at Huggett's attempt at an explanatory terminus? Perhaps it has to do with the feeling that for an explanation to be satisfactory, it has to incorporate some aspect of how the world is, from a realist perspective, or how the world could be, from that of today's anti-realist. Let's consider the analogy with spin: here one might object that the rotation group doesn't actually *explain* spin in the sense of accounting for its existence. Spin is a property that is attributed to objects, originally for experimental reasons, and its nature, possible values etc. came to be *described* group theoretically in the well known way: for massive particles the possible representations of the rotation group, as the relevant little group of the Poincaré group, yield the allowed values of spin, and this is the case in both relativistic and non-relativistic QM (the latter being the case with which Huggett is concerned), whereas for massless particles one needs to introduce parity and the explanation of the two spin states for such particles is relativistic. But the point is that what one is explaining here is not the existence of spin, as an intrinsic property, but its structure, the values it can take in particular cases and so on.

In the case of quantum statistics, on the other hand, it is the existence of these statistics themselves, as expressed via permutation invariance, that we are trying to explain and we might feel justified to ask, with regard to this explanation, what is it about the world that gives rise to this phenomenon? If the spin-statistics theorem could be proven, we could follow the reductionist route and the explanation would terminate in spin, understood as an intrinsic property of things. (Of course, if the theorem could be proven, one might feel tempted to push the reduction the other way and have spin emerging as a result of the statistics, understood in one of the ways to be canvassed below.) Or we could suggest that the statistics is a holistic or emergent property of particle collectives, as indicated by Hilborn and Yuca above, but then we have to come up with an appropriate metaphysics of emergence[56]. Or we could insist that it has to do with the peculiar metaphysical nature of the particles themselves, as non-individual objects. Or we could say that the objects are not peculiar at all, metaphysically speaking, and that permutation invariance is a reflection of certain initial or boundary conditions that pertain in our universe. All these options provide a metaphysical component and simply saying that permutation invariance is nothing more than a result of a general symmetry principle together with the structure of the permutation group seems metaphysically and hence explanatorily deficient.

If we're going to take up Huggett's option and shy away from considerations of the metaphysical nature of the particles as *objects*, then we're going to need an appropriate metaphysics of symmetry and it is precisely this that structuralism aims to provide. This amounts to a broadening of the group-theoretical approach to elementary particles by incorporating permutation invariance, understood not in terms of an intrinsic property of the particles but, just as these properties themselves, as an aspect of the 'world-structure', if you like. In other words, the world is ultimately and metaphysically *structural* in nature and permutation invariance is simply one manifestation of this structure.

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[56]Humphreys has attempted to develop a framework for representing emergence - albeit in a somewhat different context - which introduces the notion of 'fusion' according to which the causal effects of the fused whole cannot be reduced to or represented by the causal effects of the component parts (Humphreys 1997). Interestingly, he offers the example of entangled states in QM but it is not immediately apparent whether this framework can be extended to cover statistics.

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