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Vertical coherence of short-periodic current variations*

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Abstract—A study is described which attempts to obtain information about the vertical correlation of ocean currents at frequencies higher than inertial. Current velocity and temperature data for sensor separations of 4–12 m were taken with a mooring at ' Site D '. The coherence and phase spectra for velocity component pairs reveals that motions are rotational at low frequencies. A cut-off frequency exists above which coherence drops to low values. The limiting frequency coincides with the minimum Väisälä frequency of the total water column. These cross-spectral properties support the assumption that the motion in this frequency range is governed by internal wave dynamics. The coherence and phase spectra of temperature pairs indicate that a field of temperature structure is superimposed on the mean field which is weakly correlated to the field of motion.

INTRODUCTION

DATA from current meters moored in the deep ocean lead to the conclusion that currents outside the wind wave regime can usually be described by a typical shape of the energy spectrum (WEBSTER, 1968; FOFONOFF, 1969). The maximum of the spectral curve for horizontal currents is found at very low frequencies, and peaks occur at the tidal and/or the inertial frequencies. The spectral energy decreases from this frequency range towards the Väisälä frequency almost according to a power law. Observations indicate that motions are usually strongly correlated in space at frequencies much lower than the inertial frequency. However, there apparently exists a weakcor relation in space at frequencies near the inertial frequency and in the range of decreasing spectral energy above this frequency. A quantitative investigation of the spatial correlation of inertial-period motions was performed by WEBSTER (1968). The following study attempts to obtain information about the vertical correlation of ocean currents at frequencies higher than inertial.

APPROACH

A vertical array of Richardson-type current meters was moored near ' Site D ' at $\phi = 39°21'00''$, $\lambda = 70°03'40''$W in the Western North Atlantic Ocean. The mooring configuration is shown on Fig. 1. Four meters recording currents and temperatures were placed at approximately 100 m depth on a subsurface mooring. To obtain simultaneous information about vertical displacements of the sensors by mooring motion, a pressure recorder was mounted in the array. An acoustic beacon and an acoustic release were included for locating and recovering the instruments. To obtain a back-up system in case of release malfunction, a small surface float was

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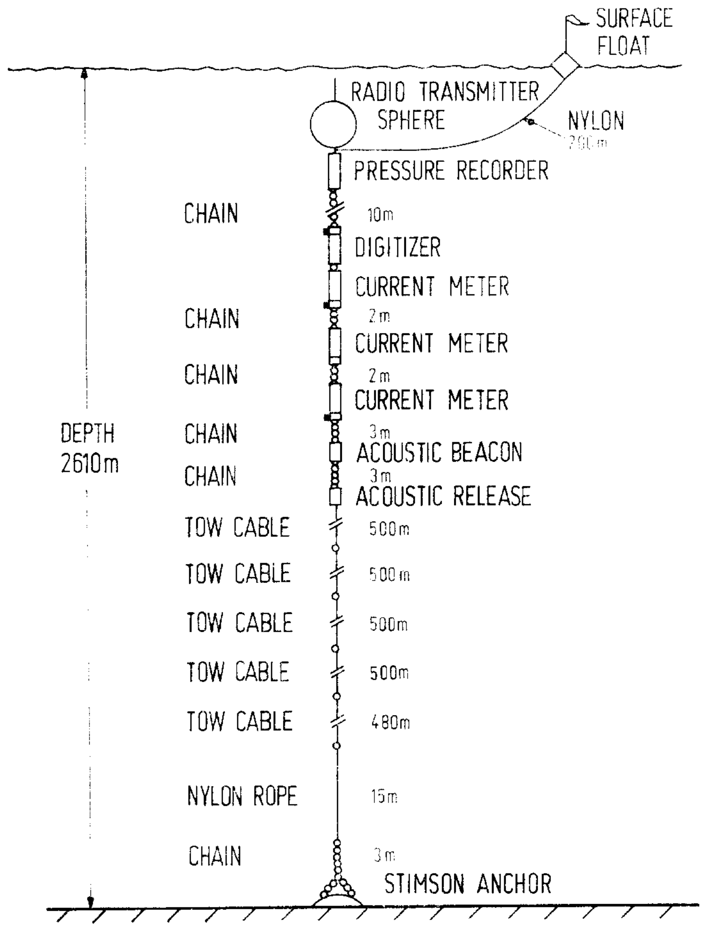

Fig. 1. Mooring configuration.

attached to the subsurface float. This back-up system actually had to be used for recovery.

The meter array is shown in more detail in Fig. 2. The digitizer unit included current speed and direction sensors (C1) and was connected to three temperature sensors (T1 to T3) by electrical cables. The film recording meters carried current sensors only (C2 to C4). The depths of the individual sensors are shown in Fig. 2, indicating the correlation distances of 4, 8 and 12 m. The digitizer unit recorded with a sampling rate of 2·5 sec at intervals of 2·5 min, the film current meters were running in the continuous mode with a sampling rate of 5 sec. The mooring was set on October 3, 1967 and recovered on October 10, 1967.

To obtain information about hydrographic conditions in the area during the time of the current measurements, hydrographic stations and repeated expendable bathythermograph lowerings were carried out shortly after launching on October 3, 1967 and before recovering the mooring on October 10, 1967.

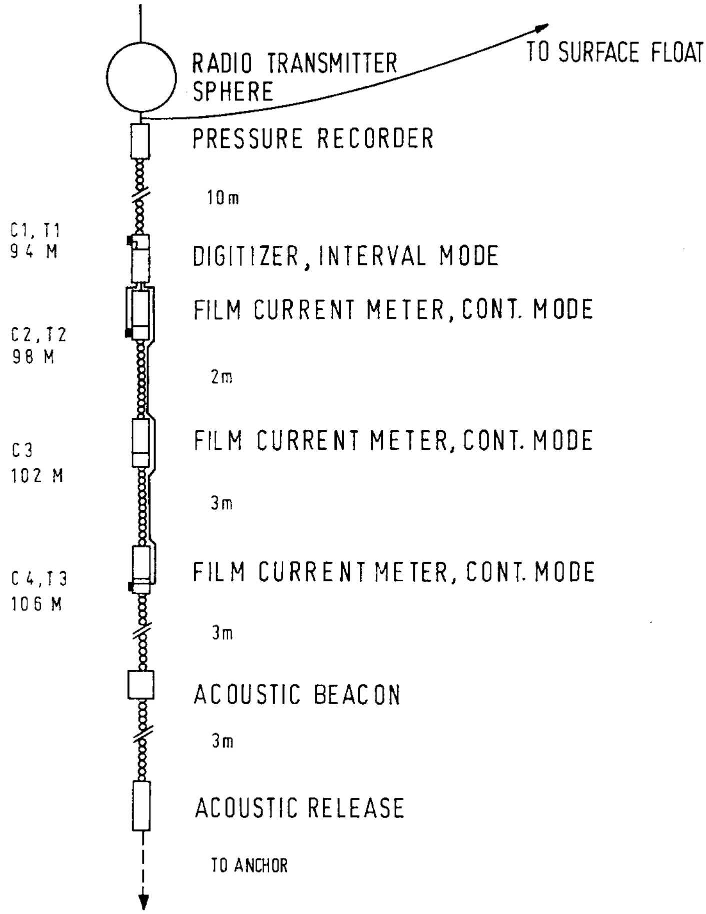

Fig. 2. Configuration of meter array.

DATA PROCESSING

The digitizer magnetic tape record proved to be of low quality. Speed and direction data from this meter contained too much instrumental noise to be used for this study. Because of the slow time response of the temperature probes, uncontaminated temperature data from this meter varied only slightly in each burst of data points per interval. Usable temperature data could therefore be obtained by simply selecting one apparently good temperature value from each burst.

The data from the three film recording current meters were of good quality.* However, no speed data were obtained for the lowermost meter because the rotor had been locked by a rod that had been bent during the launching operations. The most serious problem of the data processing turned out to be the time base correction of these three records. The sampling rates determining the sampling intervals were controlled by electronic circuits which were considerably sensitive to fluctuations of

*A constant off-set in the direction of approximately 14° appeared at C2. Although the reason of this possibly instrumental effect could not be detected, the deviation was removed from the data.

ambient temperatures. An exact correspondence of time bases, however, is of fundamental importance for any study of correlation.

Fortunately, a procedure could be found by which the time bases could be corrected. The compass data revealed that the whole instrument array rotated uniformly for certain periods. A total of 10 such events could be used to obtain time reference points. Each record was divided into 9 sections given by the 10 reference points, and appropriate sampling intervals were selected for the individual sections to match the time-bases of the three records. The data were then vector-averaged over 1-min intervals to obtain equally spaced time series.

The pressure record on strip chart was read at hourly values. The expendable bathythermograph traces on strip chart were read at vertical intervals of 2 m.

PRESENTATION OF RESULTS

Two methods were used to study the spatial correlation of the data. For periods longer than half a day, it seemed appropriate to make a qualitative comparison of the progressive vector diagrams because of the limited duration of the measurements. The results for C2 and C3 are shown in Fig. 3. It can be seen that current variations

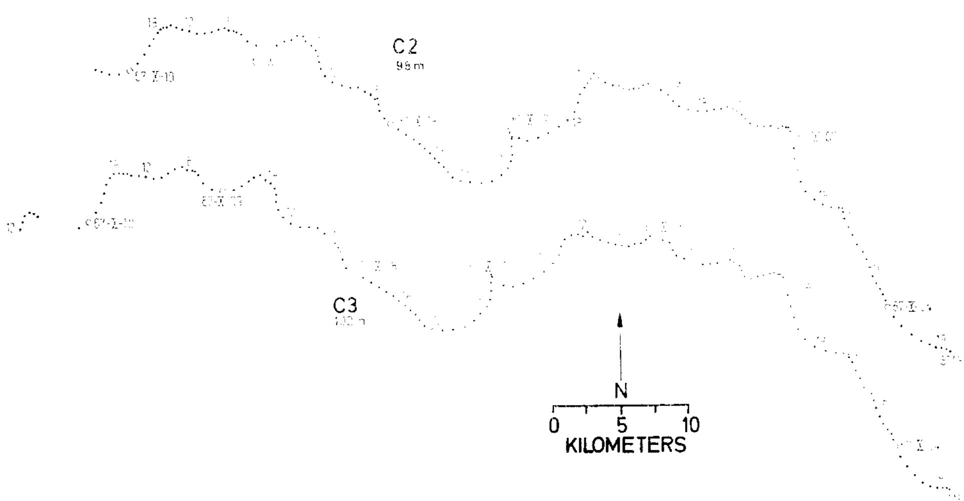

Fig. 3. Progressive vector diagrams with hourly means for the sensors C2 and C3. Numbers indicate date and time.

with a time-scale of a few days are approximately the same in both records. Even the semi-diurnal tidal motions appearing in the diagrams form a very similar pattern in both series. This comparison indicates a very good correlation in amplitude and phase for long-term variations over a vertical distance of 4 m. It should be mentioned here that the pressure recorder above the current meters displayed mooring motions with a semi-diurnal period and an approximate vertical amplitude of up to 10 m. No significant pressure variations occurred at shorter periods. The observed variations are due to the varying drag generated by the super-position of the semi-diurnal tidal motion and a stronger mean current. The progressive vector diagrams therefore probably indicate a correlation of long-term current variations over a vertical distance exceeding the sensor separation of 4 m.

For shorter periods, a quantitative cross-spectral analysis of the data could be carried out (see GRANGER and HATANAKA, 1964; WEBSTER, 1968). A measure of the correlation between two components u_i and u_j of the current vector time series is then given by the normalized cross-spectrum:

$$C_{ij}(\omega) = \left[\frac{P_{ij}^2(\omega) + Q_{ij}^2(\omega)}{P_{ii}(\omega) \cdot P_{jj}(\omega)}\right]^{\frac{1}{2}}. \tag{1}$$

C_{ij} is called the coherence, P_{ii} and P_{jj} are the autospectra, P_{ij} is the co-spectrum, and Q_{ij} is the quadrature spectrum of the two series. A corresponding frequency function for the phase angle is obtained from:

$$\phi_{ij}(\omega) = \tan^{-1}\left[\frac{Q_{ij}(\omega)}{P_{ij}(\omega)}\right]. \tag{2}$$

Taking the east and north components of the two vector series $\vec{u}_i$ and $\vec{u}_j$, coherence and phase can be expressed in the following matrix:

$$\begin{bmatrix} C_{E_iE_i} & C_{E_iN_i} & C_{E_iE_j} & C_{E_iN_j} \\ \phi_{E_iN_i} & C_{N_iN_i} & C_{N_iE_j} & C_{N_iN_j} \\ \phi_{E_iE_j} & \phi_{N_iE_j} & C_{E_jE_j} & C_{E_jN_j} \\ \phi_{E_iN_j} & \phi_{N_iN_j} & \phi_{E_jN_j} & C_{N_jN_j} \end{bmatrix}.$$

The diagonal terms are always equal to 1. The off-diagonal terms are a measure of the amplitude and phase relations between the indicated time series. If the phase angle increases from north to east, the normalized cross-spectral matrix for a uniform rotational motion turning to the right is given by:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ +90^\circ & 1 & 1 & 1 \\ 0^\circ & -90^\circ & 1 & 1 \\ +90^\circ & 0 & +90^\circ & 1 \end{bmatrix}.$$

The off-diagonal terms of the cross-spectral matrix for series C2 and C3 as a function of frequency are shown in Figs. 4 and 5. The coherence spectra of all terms are above the 95% confidence level for zero coherence at low frequencies. The spectra for $E_2 E_3$ and $N_2 N_3$ display a very high coherence of 0·95–0·7 up to a frequency ω_0 where the coherence starts to drop rapidly to low values. This cut-off frequency corresponds to a period of approximately 100 min. If the autospectrum of the shear vector series for C2 and C3 is compared with the auto-spectra of the individual vector series, a corresponding result is obtained: The shear spectrum is much smaller at low frequencies and somewhat larger at high frequencies, with a cross-over point at ω_0, indicating a loss of spatial correlation for frequencies above ω_0.

In the spectra for $E_2 N_2$, $E_3 N_3$, $E_2 N_3$ and $N_2 E_3$ a level of 0·7–0·8 is found at

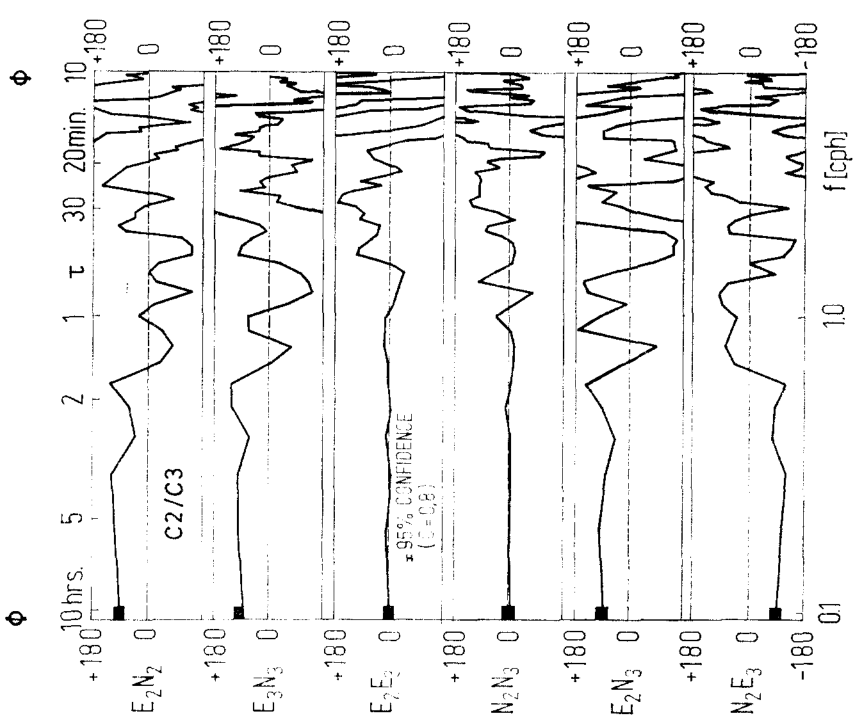

Fig. 5. Phase spectra for east (E) and north (N) components of the data obtained by sensors C2 and C3.

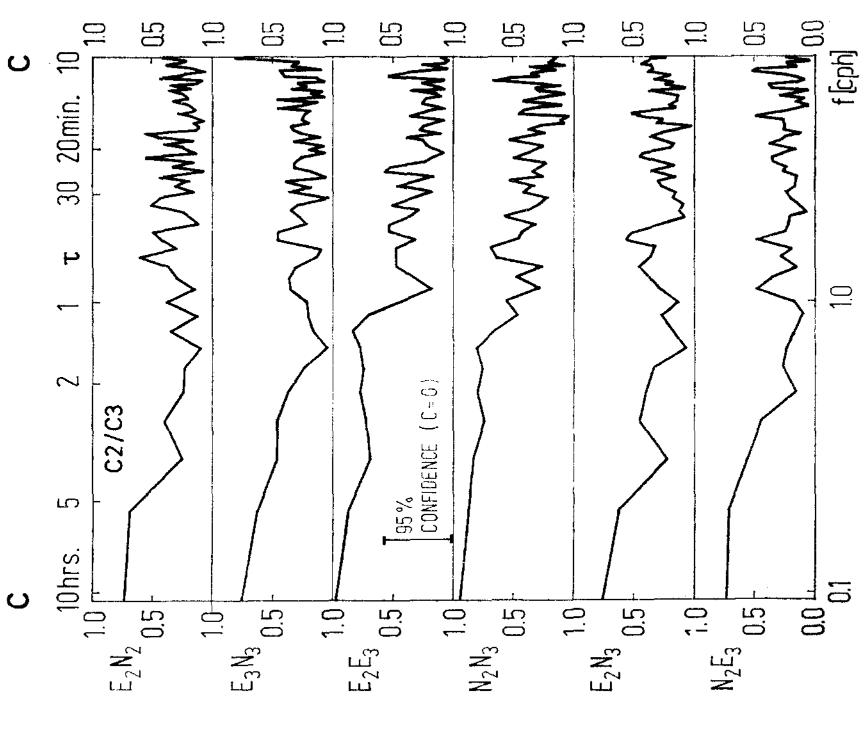

Fig. 4. Coherence spectra for east (E) and north (N) components of the data obtained by sensors C2 and C3.

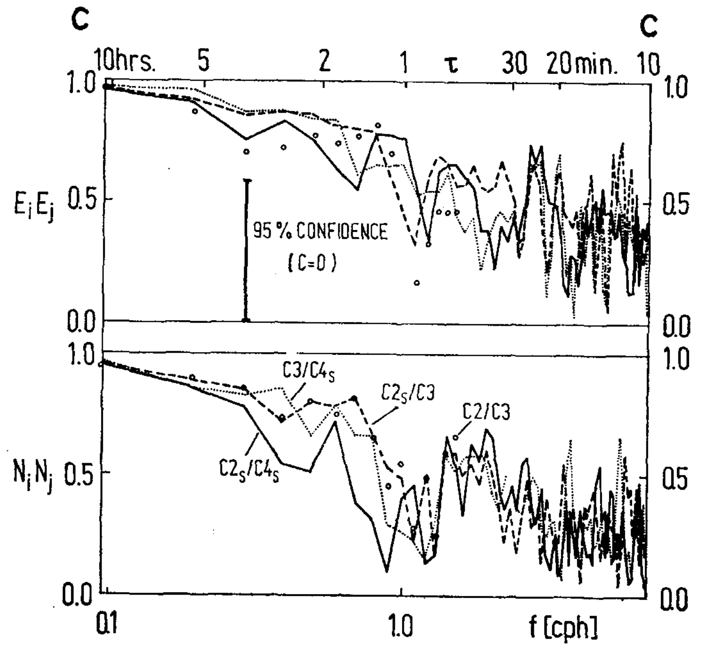

Fig. 6. Coherence spectra for east (E) and north (N) components of the data obtained by sensors C2, C3 and C4, where speeds of C3 are substituted in all series. Circles indicate the spectra of Fig. 4.

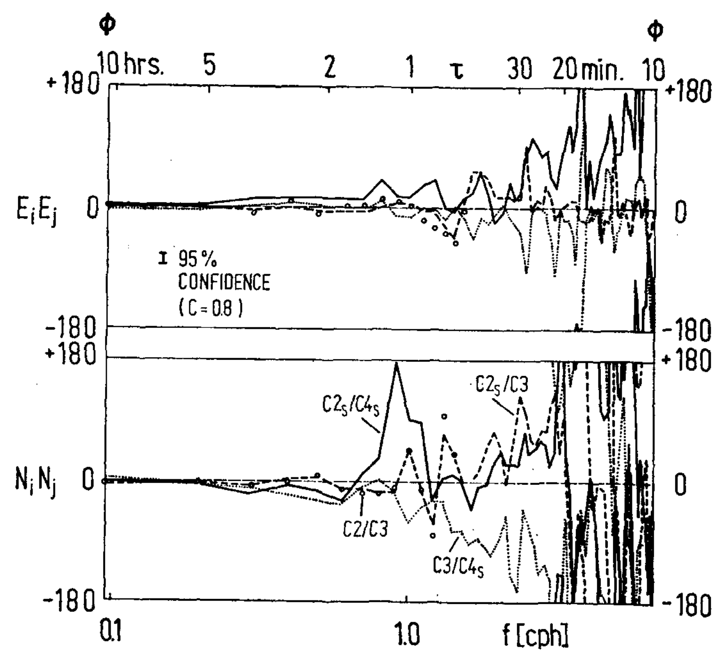

Fig. 7. Phase spectra for east (E) and north (N) components of the data obtained by sensors C2, C3 and C4 where speeds of C3 are substituted in all series. Circles indicate the spectra of Fig. 5.

low frequencies, and the coherence drops to low levels at a frequency corresponding to approximately 5 hr.

At low frequencies the phase spectra have values very near the ones which can be expected for rotational motion. The phase angles given in the corresponding matrix above are indicated by marks on the left side. It can be stated from these spectra that the motion was very near to rotational down to periods shorter than 5 hr.

An attempt was made to obtain similar information for a greater sensor separation. Unfortunately, no speed data were available for C4. Therefore, speed data from C3 were substituted in series C4. To check whether such a procedure will change the results described above, the speeds of C3 were also substituted in C2. Terms computed with substituted speed data are marked by index *s*.

Spectra of coherence and phase were computed from these series. The results

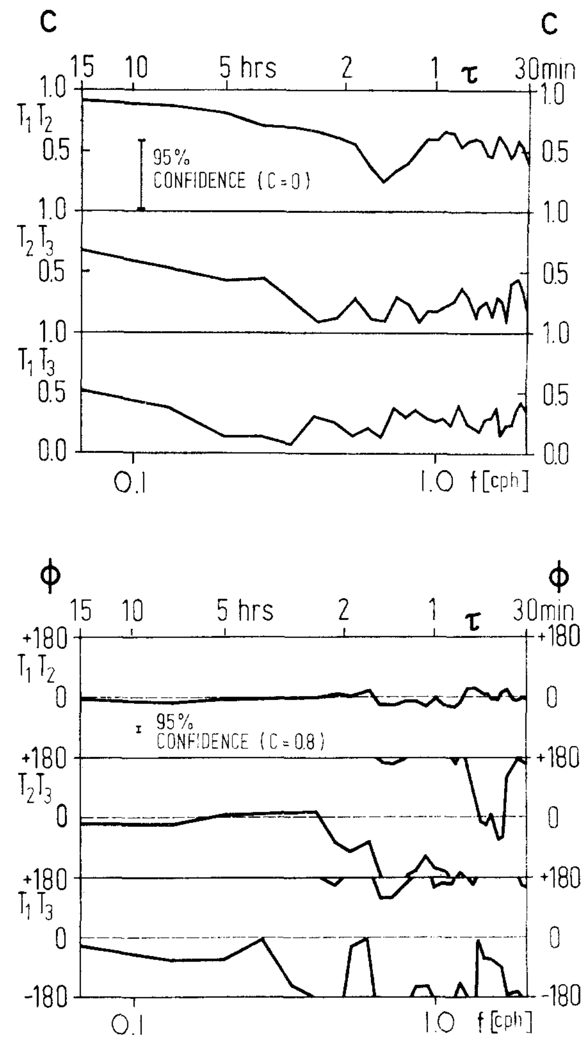

Fig. 8. Coherence spectra and phase spectra for the temperature data obtained by sensors T_1, T_2 and T_3.

for $E_i E_j$ and $N_i N_j$ are given in Figs. 6 and 7. First, a comparison of the spectra for $E_{2s} E_3$, $N_{2s} N_3$ with those for $E_2 E_3$, $N_2 N_3$ leads to the conclusion that the cross-spectra remain essentially the same at low frequencies up to the cut-off values after the speed substitution has been done. No significant systematic variation of the cut-off frequency can be detected with increasing current sensor separation for the distances from 4 to 8 m.

A similar cross-spectral analysis can be carried out for the temperature data T1, T2 and T3. The coherence and phase spectra for these three time series are shown in Fig. 8, indicating the correlation over 4, 8 and 12 m sensor separation. A similar cut-off frequency corresponding to a period of approximately 100 min seems to exist for $T_1 T_2$, but the coherence level is lower than for the current series $E_i E_j$ and $N_i N_j$. The temperature variations are in phase up to this frequency. In the case of temperatures, however, there exists a systematic shift of the coherence curve to lower frequencies for increasing sensor separations. This shift of the cut-off frequency towards lower values for increasing distances is also indicated in the phase spectra.

DISCUSSION OF RESULTS

The measurements show that there exists a distinct cut-off frequency ω_0 for the coherence of corresponding current vector components. Above this frequency the coherence drops to very low values. It seems difficult to imagine that such an abrupt change could be caused by noise generated by increasing turbulence. However, a comparison between the minimum Väisälä frequency N_{min} as given in Fig. 9 and the cut-off frequency leads to the conclusion that

$$\omega_0 \sim N_{min}.$$

A possible explanation of the drop in coherence can then be found by assuming that in the frequency range under discussion the motion can be described by a superposition of internal wave modes. If the modes have no fixed phase relation to each other and occur and disappear in time scales which are small with respect to the observation time, coherence is lost for a large spread δk of vertical wavenumbers k. Observations of inertial-period motions indicate that the time scale of such motions will often be of the order of a few periods only (WEBSTER, 1968), and similar intermittent processes for shorter periods can be expected.

For a certain order of internal wave modes, the vertical wavenumber is larger for improper internal waves with frequencies above N_{min} (KRAUSS, 1966; PHILLIPS, 1966) than for ordinary internal waves below N_{min}. According to MUNK and PHILLIPS (1968) the vertical coherence scale δz is given by

$$\delta k \,.\, \delta z = 0\,(1) \tag{3}$$

A change of the spread δk of vertical wave numbers at N_{min} will therefore lead to a change of the coherence scale δz.

If one considers the motion with frequencies $\omega \ll N$ (N = Väisälä frequency) in an incompressible ocean with constant depth, the amplitude W of vertical velocity can be approximately described by:

$$\frac{d^2 W}{dz^2} + k^2 W = 0 \qquad \text{with} \qquad k(z) = r \frac{\pi N(z)}{h \overline{N}} \tag{4}$$

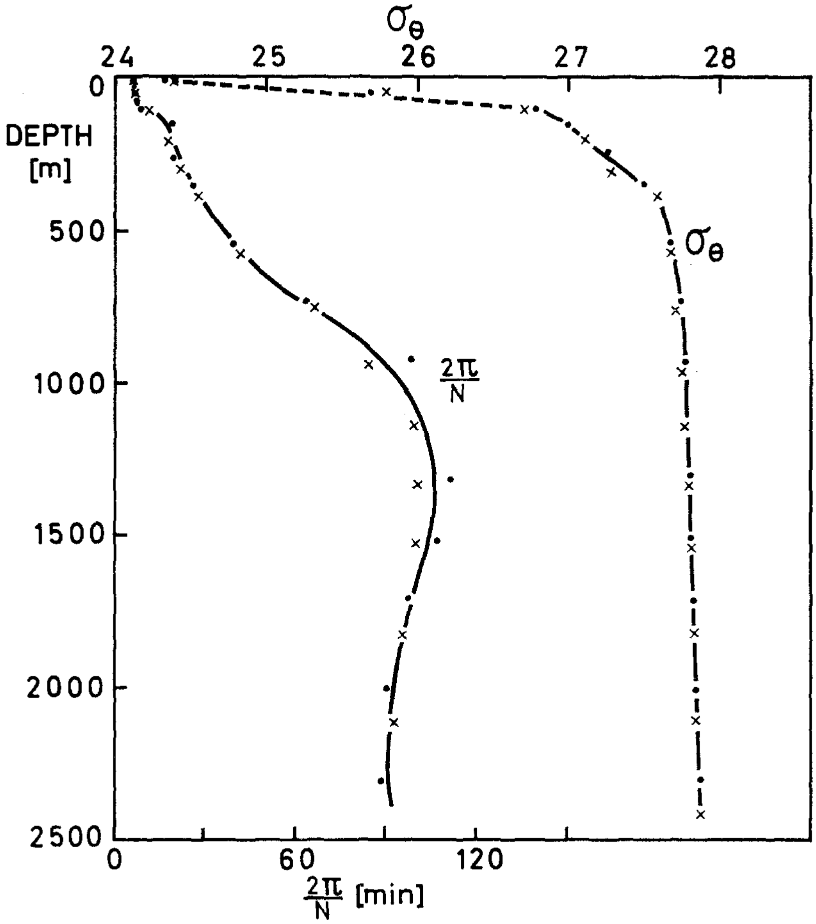

Fig. 9. Potential density (represented by σ_θ) and Väisälä period ($2\pi/N$ with N = Väisälä frequency) at Site D obtained from hydrocast data at the beginning (dots) and end (crosses) of the experiment.

[k = vertical wavenumber, r = number of nodal levels in W, $N(z)$ = local Väisälä frequency, $\bar{N}$ = mean Väisälä frequency, h = total vertical scale, z = downward co-ordinate].

From (3) and (4) one obtains the following relation between the coherence scale and the spread of order of modes:

$$\delta r = 0\,(1)\,\frac{h\bar{N}}{\pi\,N(z)}\,\frac{1}{\delta z}\,. \tag{5}$$

From the described coherence spectra the following information was revealed:

$$\delta z < 4\text{ m} \quad \text{for } \omega > \omega_0 \tag{6}$$

$$\delta z > 8\text{ m} \quad \text{for } \omega < \omega_0. \tag{7}$$

Ordinary internal wave modes can only exist for:

$$\omega < N_{min} \quad \text{with} \quad 0 \leqslant z \leqslant H \sim 2600\text{ m} \quad \text{(bottom depth).} \tag{8}$$

The range of improper internal wave modes is given by:

$$\mathrm{N}_{\min} < \omega < N(z) \quad \text{with} \quad 0 \leqslant z \leqslant z_{(\omega=N(z))} \leqslant z_{(\omega=\mathrm{N}_{\min})} \sim 1000\,\mathrm{m}. \tag{9}$$

The distribution of modes with frequencies just below and above $\mathrm{N}_{\min}$ will be similar to the case illustrated by Fig. 10. If one assumes that the spread δr of order of modes is approximately the same in both cases, the change of the total vertical scale h of modes from H to $z_{(\omega=\mathrm{N}_{\min})}$ will lead to a sudden change in coherence scale according to (5) as observed by the measurements.

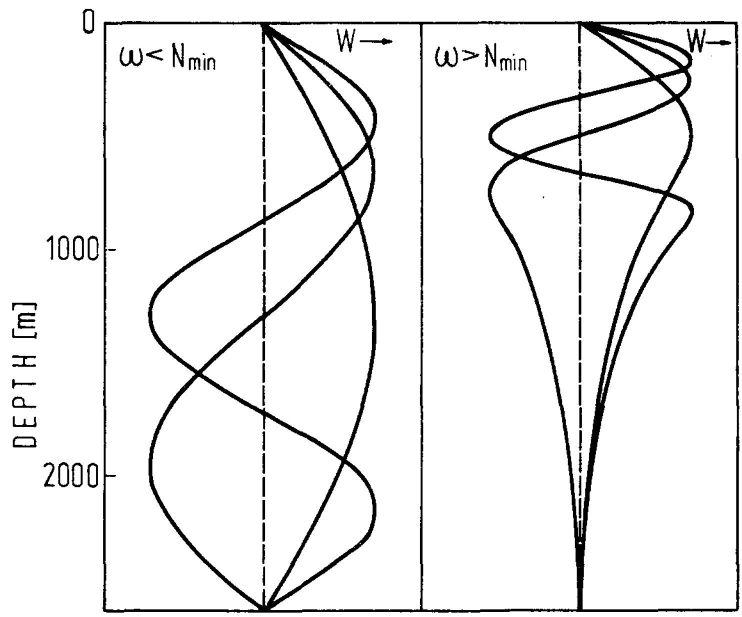

Fig. 10. Typical high-order modes for a frequency below ($\omega < \mathrm{N}_{\min}$) and above ($\omega > \mathrm{N}_{\min}$) the minimum Väisälä frequency.

It is necessary to check whether such an argument leads to reasonable numbers in δr and δz. Using $N(z)/\bar{N} \sim 3$ one obtains for $\omega > \mathrm{N}_{\min}$ from (5), (6) and (9):

$$\delta r > 0\,(1)\,.\,25. \tag{10}$$

If δr is of the same order $0\,(1)\,.\,25$ just above and below $\mathrm{N}_{\min}$, one obtains for $\omega < \mathrm{N}_{\min}$ from (5) and (8):

$$\delta z = 0\,(1)\,.\,10\,\mathrm{m}. \tag{11}$$

This is in agreement with (7).

The spread δr of orders of modes is somewhat higher than usually anticipated for lower frequencies. The vertical coherence scale δz for $\omega < \mathrm{N}_{\min}$ is of the order of the vertical coherence scale of inertial motion given by WEBSTER's (1968) results.

It remains to be clarified why it is possible to detect the same cut-off frequency $\mathrm{N}_{\min}$ in the temperature spectra for a very short vertical distance, but a shift of the

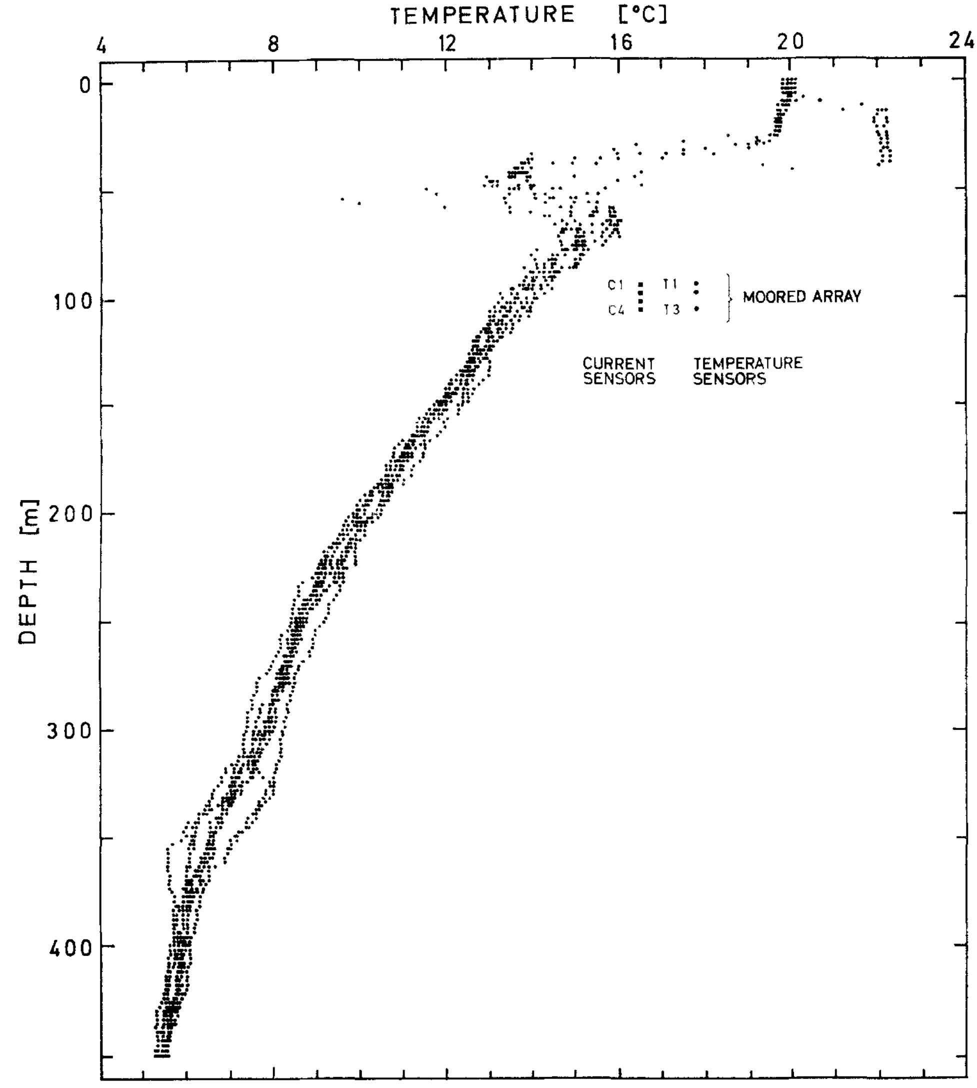

Fig. 11. Temperature vs. depth data at Site D taken during the experiment by repeated expendable bathythermograph lowerings.

coherence curve towards lower frequencies for increasing sensor separation. The same type of normalized cross-spectrum should be expected for temperature and current series if the temperature variations were only caused by the internal wave modes discussed above. Apparently, this is not the case. In Fig. 11, the temperature data are plotted which were obtained from repeated expendable bathythermograph lowerings. They indicate the existence of a rapidly changing temperature finestructure. If such a field of temperature, uncorrelated or weakly correlated with the internal wave field and with varying vertical scales, is superimposed to the mean field, coherence will drop for increasing sensor separation as observed by the measurements.

CONCLUSIONS

The results presented lead to the conclusion that the vertical coherence for corresponding current velocity components dropped rapidly at the minimum Väisälä frequency of the total water column. It is suggested that this fact may be explained by a superposition of internal wave modes with no fixed phase relation in the range from tidal to local Väisälä frequencies. Such a drop in coherence can then be expected if the spread of order of modes is approximately the same below and above this limiting frequency.

Simultaneous temperature measurements reveal that the coherence levels are lower for temperature variations than for current variations. Simultaneous measurements of vertical temperature profiles indicate that a field of temperature fine-structure with varying vertical scales is superimposed on the mean temperature field causing a loss of coherence for increasing vertical distances.

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