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Nonlinear Transverse Oscillations of a Geostrophic Front

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Abstract—A planar problem of nonlinear transverse oscillations of the surface (warm) front of a finite width is considered within the framework of a reduced-gravity model of the ocean. The source of oscillations is the departure of the front from its geostrophic equilibrium. When the current velocity is linear in the horizontal coordinate and the front's depth is quadratic in this coordinate, the problem is reduced to a system of four ordinary differential equations in time. As a result, the solution is obtained in a weakly nonlinear approximation and strongly nonlinear oscillations of the front are studied by numerically solving this system of equations by the Runge–Kutta method. The front's oscillations are always superinertial. Nonlinearity can lead to a decrease or increase in the oscillation frequency in comparison with the linear case. The oscillations are most intense when the current velocity is disturbed in the direction of the front's axis. A weakly nonlinear solution of the second order describes the oscillations very accurately even for initial velocity disturbances reaching 50% of its geostrophic value. An increase in the background-current shear leads to the damping of oscillations of the front's boundary. The amplitude of oscillations of the current velocity increases as the intensity of disturbances increases, and it is relatively small if background-current shears are small or large.

Frontal zones, which are characterized by rather large local gradients of the oceanic thermohydrophysical properties compared to their average values, rather frequently occur in the World Ocean and cover spatial scales from one meter to tens of kilometers [1]. The Gulf Stream and the Kuroshio Current, the zones of subtropical convergence, and the Antarctic circumpolar frontal zone are among planetary-scale frontal zones.

Such dynamic structures are subject to a significant time variation. Frontal zones change in their parameters and execute oscillations about a certain average position. Even within frontal zones, one can observe fast transformations of the structure, movements of frontal interfaces, and the meandering and formation of vortices. The significant influence of frontal zones on the dynamics and energetics of the ocean testifies to the necessity of analyzing both time variations in their characteristics and the physical mechanisms controlling these variations.

The frequency spectrum of time variations in oceanic hydrophysical fields is very wide [2]. A significant contribution to the variability of the ocean is made by quasi-inertial oscillations. As follows from current-velocity measurements, oscillations of the fields are superinertial in most cases, although the opposite situations can also occur [3]. Subinertial oscillations are frequently related to the transformation of long internal waves during their propagation in currents with a horizontal velocity shear [4].

This paper considers a mechanism of occurrence of superinertial oscillations in the ocean. This mechanism is related to nonlinear oscillations of the surface (warm) front about a geostrophic (stationary) state. The cause of the occurrence of oscillations is the initial deviation of hydrodynamic fields from geostrophic equilibrium. The initial disturbance of the medium is one of the possible sources of excitation of low-frequency oscillations, which is associated with the adaptation of the fields in the rotating ocean and atmosphere [5, 6]. The oscillations of frontal zones can also be initiated by atmospheric processes [7], by the scattering of externally incoming baroclinic waves on fronts [8], and by the instability of currents in the open ocean [9] or in the coastal zone [10].

The following analysis is based on a reduced-gravity model of the ocean. For hydrodynamic fields with a special spatial structure, planar transverse motions of a frontal zone are studied; i.e., its dynamics is considered in a vertical plane perpendicular to the front's axis. This problem is equivalent to the study of oscillations of an extended lens of cold water sliding over a sloping bottom [11] and is also applicable to the description of oscillations of a coastal jet current [12].

1. MATHEMATICAL FORMULATION OF THE PROBLEM

A finite-width frontal zone $x_1^* \le x^* \le x_2^*$ is considered. The geometry of its cross section is shown schematically in Fig. 1. Within the framework of the reduced-gravity model, the dynamics of this frontal zone is governed by the following system of equations [13]:

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} - fv^* = -g'\frac{\partial h^*}{\partial x^*},$$
$$\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} + fu^* = -g'\frac{\partial h^*}{\partial y^*}, \quad (1)$$
$$\frac{\partial h^*}{\partial t^*} + \frac{\partial(h^*u^*)}{\partial x^*} + \frac{\partial(h^*v^*)}{\partial y^*} = 0.$$

Here, x^* and y^* are the coordinates along the horizontal axes directed perpendicularly to and aligned with the front's axis, respectively; t^* is the time; (u^*, v^*) and h^* are the projections of the horizontal current velocity within the frontal zone and its depth, respectively; $f > 0$ is the Coriolis parameter; $g' = g(1 - \rho_1/\rho_2)$ is the reduced acceleration of gravity; g is the acceleration of gravity; and ρ_1 and ρ_2 are the densities of the fluids in the upper (warm) and lower layers of the ocean, respectively ($\rho_1 < \rho_2$). Within the framework of this approximate prognostic model, the lower layer of the ocean is at rest, although the interface between the layers can move. The model allows one to determine changes in the total depth of the front depending on x^* and the time; however, the absolute displacements of the lower and upper boundaries of the frontal zone remain undetermined.

In the following analysis, we restrict ourselves to planar free motions of the front. In this case, $\partial/\partial y^* \equiv 0$ in (1). In terms of dimensionless variables, for which the sign * is omitted, the motion of the fluid in the cross section of the front with a mobile boundary is governed by the following system of three equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - v = -\frac{\partial h}{\partial x},$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + u = 0, \quad (2)$$
$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} = 0,$$

where the front's depth h^* is normalized by the maximum thickness h^+ in a stationary state; u^* and v^* are normalized by the velocity $V = \sqrt{g'h^+}$; and (x^*, y^*) and t^* are normalized by V/f and $1/f$, respectively. The dimensionless depth h is positive on the interval $x_1(t) < x < x_2(t)$, corresponding to the frontal zone, and $h(x_{1,2}) = 0$. Let u, v, and h depend only on the normal (to the front) horizontal coordinate x and time t. We note that system (2) coincides formally with the system describing the one-dimensional dynamics of a lens of heavy (cold) water situated on the bottom of the basin [11].

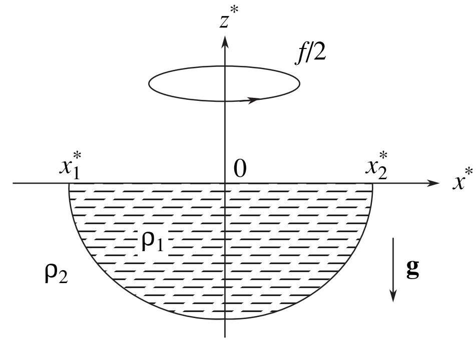

Fig. 1. Diagram of the cross section of the frontal zone.

System (2) should be subject to the initial conditions

$$u = u_0(x), \quad v = v_0(x), \quad h = h_0(x) \quad (t = 0), \quad (3)$$

where u_0, v_0, and h_0 are known functions specified on the interval $x_1(0) \le x \le x_2(0)$.

For an arbitrary finite-width distribution of the front's depth $h = h^g(x)$, nonlinear system of equations (2) admits the stationary alternating geostrophic current

$$u^g = 0, \quad v^g = \frac{\partial h^g}{\partial x} \quad (4)$$

along the front. In the following, we will consider the nonlinear oscillations of the front that are caused by the initial deviations of the fields from the geostrophic current given by (4).

2. FRONT WITH A SPECIAL STRUCTURE

Let us consider the motion of the front under the assumption that the velocity projections are linear functions and the distribution of the front's depth is a quadratic function of the coordinate x, i.e.,

$$u = A(t)x, \quad v = B(t)x, \quad h = C(t)x^2 + D(t). \quad (5)$$

The substitution of (5) into (2) and (3) leads to the following Cauchy problem for the nonlinear system of ordinary differential equations for the unknown coefficients appearing in (5):

$$\dot{A} = B - 2C - A^2, \quad (6)$$
$$\dot{B} = -A - AB, \quad (7)$$
$$\dot{C} = -3AC, \quad (8)$$
$$\dot{D} = -AD, \quad (9)$$
$$A(0) = A_0, \quad B(0) = B_0, \quad C(0) = C_0, \quad D(0) = D_0. \quad (10)$$

Thus, specifying the structure of the fields in form (5) makes it possible to replace the initial model given in (2) and (3) by system of four ordinary differential equations (6)–(9). This allows one to significantly simplify the study of the nonlinear oscillations of this front without amplitude limitations. Historically, such an approach was initially employed in analyzing tides in a parabolic basin [14]. The simplification of the initial problem by specifying the velocity field and the vortex thickness in the form of a linear and a quadratic function of spatial coordinates, respectively, was effectively used to study nonlinear oscillations of elliptic vortices in a two-layer ocean [15–17].

It is necessary to specify limitations on the coefficients appearing in the expression for the distribution of depth, namely, $C(t) < 0$ and $D(t) > 0$. This provides a nonzero depth and a finite width of the front at any moment of time t. Let us show that, if $C_0 < 0$ and $D_0 > 0$, the inequalities $C < 0$ and $D > 0$ are satisfied at any t. Indeed, we find from Eqs. (8) and (9) that

$$C(t) = C_0 e^{-3I(t)}, \quad D(t) = D_0 e^{-I(t)},$$

$$I = \int_0^t A(\xi)d\xi.$$

Thus, the functions C and D do not vanish and, therefore, retain the sign at all $t \geq 0$. For nonlinear oscillations of elliptic vortices in the ocean, this property of fields was established in [15].

From (4) and (5), expressions for geostrophic fields are obtained:

$$u^g = 0, \quad v^g = -2\gamma x = \partial h^g/\partial x, \quad h^g = 1 - \gamma x^2. \tag{11}$$

Here, $\gamma > 0$ is an arbitrary constant which characterizes the horizontal shear of the background current and is related to the physical parameters of the frontal zone by the formula $\gamma = g'h^+/(fW)^2$, where W is the half-width of the frontal zone. For this case, the coefficients assume the values

$$A = 0, \quad B = -2\gamma, \quad C = -\gamma, \quad D = 1. \tag{12}$$

These values correspond to the equilibrium position of system of equations (6)–(9).

In order to describe front oscillations about the state of geostrophic equilibrium (11), it is convenient to introduce disturbances in coefficients (12) by the following formulas:

$$A = a(t), \; B = -2\gamma + b(t), \; C = -\gamma + c(t), \; D = 1 + d(t). \tag{13}$$

The substitution of (13) into (6)–(10) leads to the new problem

$$\dot{a} = b - 2c - a^2, \tag{14}$$

$$\dot{b} = (2\gamma - 1)a - ab, \tag{15}$$

$$\dot{c} = 3\gamma a - 3ac, \tag{16}$$

$$\dot{d} = -a - ad, \tag{17}$$

$$a(0) = a_0, \quad b(0) = b_0, \quad c(0) = c_0, \quad d(0) = d_0. \tag{18}$$

This is the main problem in the following analysis. The limitations $c_0 < \gamma$ and $d_0 > -1$ provide the fulfillment of the conditions $C_0 < 0$ and $D_0 > 0$. This ensures the existence of a finite-width frontal zone at all $t \geq 0$.

We note that the general solution of system of ordinary differential equations (6)–(9), as well as that of the problem given by (14)–(18), can be obtained analytically in an implicit integral form. A procedure allowing this solution to be obtain in the context of a planar nonlinear problem is described in [11, 18] as applied to oscillations of a near-bottom lens of cold water and transverse barotropic seiches in a channel of parabolic cross section. This solution establishes the existence of periodic oscillations of the front at arbitrary amplitudes of the initial disturbance and makes it possible to analyze the dependence of the period of strongly nonlinear oscillations of fields on both the parameters of the background current and the intensity of its initial disturbances [11]. However, the use of an implicit solution for the quantitative analysis and study of physical regularities of field oscillations in the frontal zone is significantly less effective in comparison with a direct numerical integration of the Cauchy problem given by (14)–(18).

3. WEAKLY NONLINEAR FRONT OSCILLATIONS

Let us consider the motions of a front caused by small but finite disturbances of geostrophic current (11). We specify these disturbances as initial current velocities. In this case, one can set $a_0 = \varepsilon$, $b_0 = \mu$, and $c_0 = d_0 = 0$, assuming that the parameters ε and μ are small.

The following identities follow from Eqs. (15) and (16):

$$b(t) = \mu J + (1 - 2\gamma)[J(t) - 1], \quad c(t) = \gamma[1 - J^3(t)],$$

where $J = \exp[-\int_0^t a(\xi)d\xi]$. Substituting these identities into Eq. (14) and taking into account the independence of Eqs. (14)–(16) and Eq. (17), we can replace the problem given by (14)–(18) by the equivalent problem

$$\begin{aligned} \dot{a} &= \omega^2\delta + \mu + \mu\delta - a^2 + 6\gamma\delta^2 + 2\gamma\delta^3, \\ \dot{\delta} &= -a - a\delta, \\ a(0) &= \varepsilon, \quad \delta(0) = 0, \end{aligned} \tag{19}$$

where the new dependent variable $\delta(t) = J(t) - 1$ is introduced.

The solution of (19) in the case of a weak nonlinearity can be obtained using a perturbation method with respect to the two small parameters ε and μ [19]. We restrict ourselves to the consideration of nonlinear effects in the lowest-order approximation.

Let us introduce a new independent variable τ through the relation

$$t = \tau(1 + \alpha_{10}\varepsilon + \alpha_{01}\mu + \alpha_{20}\varepsilon^2 + \alpha_{11}\varepsilon\mu + \alpha_{02}\mu^2 + \ldots), \tag{20}$$

where the coefficients α_{jk} are to be determined. The functions $a(t)$ and $\delta(t)$ are also sought as power series in ε and μ with coefficients depending on the time τ:

$$\begin{aligned} a &= \varepsilon a_{10}(\tau) + \mu a_{01}(\tau) + \varepsilon^2 a_{20}(\tau) \\ &\quad + \varepsilon\mu a_{11}(\tau) + \mu^2 a_{02}(\tau) + \ldots, \\ \delta &= \varepsilon\delta_{10}(\tau) + \mu\delta_{01}(\tau) + \varepsilon^2\delta_{20}(\tau) \\ &\quad + \varepsilon\mu\delta_{11}(\tau) + \mu^2\delta_{02}(\tau) + \ldots. \end{aligned} \tag{21}$$

The substitution of (20) and (21) into (19) leads to the sequence of Cauchy problems for finding a_{jk} and δ_{jk}. The constants α_{jk} are determined from the conditions of eliminating secular terms in the solutions of the corresponding approximations. In particular, it is found that $\alpha_{10} = \alpha_{11} = 0$.

The solution of the second order in the small parameters ε and μ can be written as

$$\begin{aligned} a \approx{}& \varepsilon\cos\Omega t + \mu\frac{\sin\Omega t}{\omega} \\ &+ \frac{\varepsilon^2}{2\omega^3}[8\gamma\sin\Omega t - (1+8\gamma)\sin 2\Omega t] \\ &+ \frac{\varepsilon\mu}{\omega^4}(1+8\gamma)(\cos\Omega t - \cos 2\Omega t) \\ &- \frac{\mu^2}{2\omega^5}[2(1+2\gamma)\sin\Omega t - (1+8\gamma)\sin 2\Omega t], \end{aligned} \tag{22}$$

$$\Omega \approx \omega\left[1 + \frac{6\gamma}{\omega^4}\mu - \frac{3\gamma(1-\gamma)}{\omega^6}\varepsilon^2 - \frac{15\gamma(1-3\gamma)}{\omega^8}\mu^2\right]^{-1}, \tag{23}$$

$$\omega = \sqrt{1+4\gamma}. \tag{24}$$

From formulas (22) and (23), it follows that, in the linear approximation, front oscillations are harmonic oscillations with superinertial frequency (24). The increase in the frequency with respect to the inertial frequency is due to the horizontal shear of the background current. If the initial disturbance of the geostrophic current is specified in the direction transverse to the front ($\varepsilon \neq 0$ and $\mu = 0$), nonlinearity brings about ($\gamma < 1$) an additional quadratic (in ε and, thus, independent of the sign of $\varepsilon = a_0$) increase in the carrier frequency of oscillations and the generation of multiple harmonics.

A different situation occurs if the initial disturbance of the geostrophic current is specified along the axis of the front. This disturbance corresponds to $\varepsilon = 0$ and $\mu \neq 0$. As previously, nonlinearity leads to the generation of multiple harmonics. According to (23), in the lowest-order approximation, the disturbance of the background current along the front results in the distortion of the frequency of linear oscillations proportional to μ. Therefore, such disturbances can be responsible for both an increase ($b_0 < 0$) and a decrease ($b_0 > 0$) in the oscillation frequency in comparison with its value in the linear case.

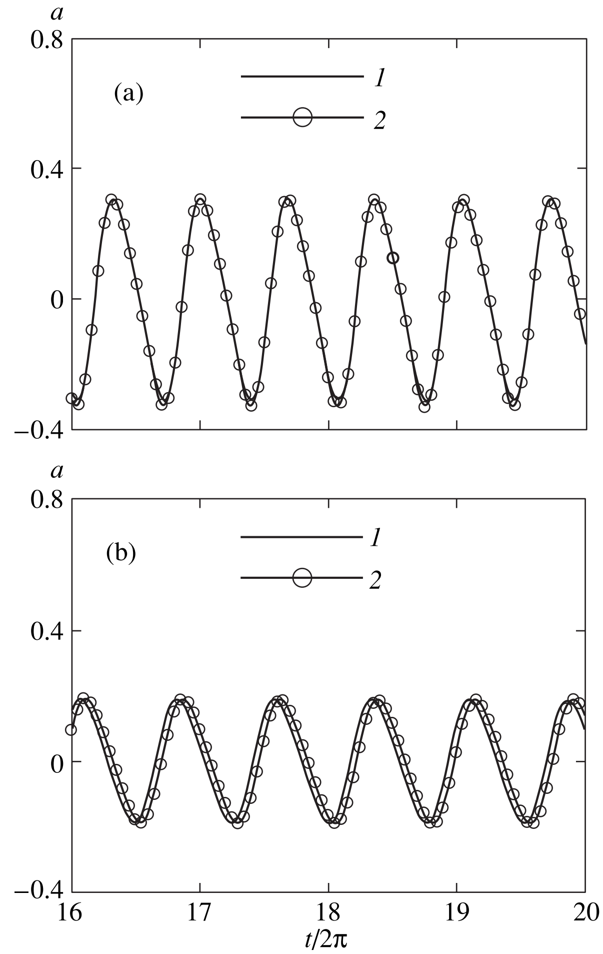

Fig. 2. (*1*) Exact and (*2*) asymptotic solutions of problem (14)–(18) for different initial disturbances of the geostrophic current at $\gamma = 0.2834$: (a) initial disturbance in the form of a velocity transverse to the front, $a_0 = 0.3$, $b_0 = c_0 = d_0 = 0$, and (b) initial disturbance in the form of a velocity along the front, $b_0 = 0.3$, $a_0 = c_0 = d_0 = 0$. The time on the abscissa is specified in inertial periods, $t/(2\pi)$.

A situation similar to the last situation also appears under initial disturbances in the depth of the frontal zone, when $a_0 = b_0 = 0$, $0 < |c_0| \ll 1$, and d_0 is chosen from the condition that the cross-section area is equal to that for the undisturbed current described by (11).

Formulas (22) and (23) are characterized by errors of the third order in the small parameters ε and μ, i.e., of about $O(\varepsilon^3)$ if it is assumed that $\varepsilon \sim \mu$. A numerical solution of the problem given by (14)–(18) makes it possible to assess the closeness between the asymptotic and exact solutions for any values of these parameters. Such a comparison is given in Fig. 2, which shows the time variations of the numerically calculated and asymptotic values of the coefficient $a(t)$ for two types of

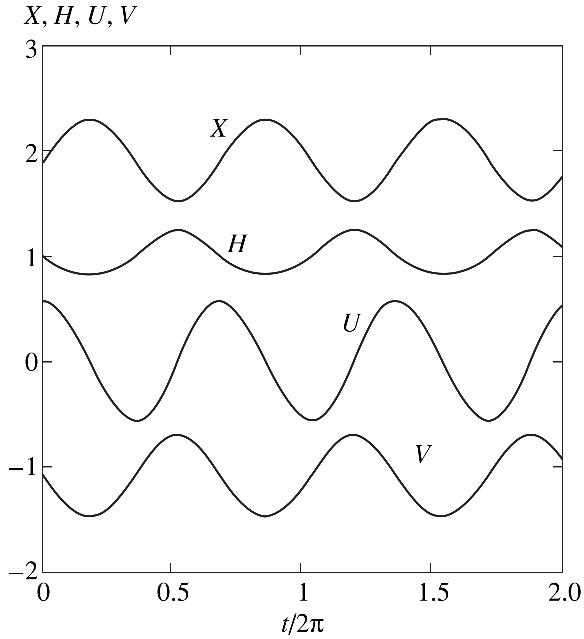

Fig. 3. Time variation of nonstationary characteristics of disturbed current (11) at $\gamma = 0.2834$. X is the position of the right-hand boundary of the front; H is the maximum depth of the frontal zone; and U and V are the transverse and longitudinal projections of the current velocity at $x = X$, respectively. The initial disturbance is the transverse current velocity corresponding to $a_0 = 0.3$, $b_0 = c_0 = d_0 = 0$.

initial disturbances in the velocity field of the geostrophic current. As follows from Fig. 2, the asymptotic formulas describe superinertial oscillations of the current velocity very accurately even if the initial disturbance of the geostrophic-current velocity is greater than 50% (Fig. 2b).

4. STRONGLY NONLINEAR FRONT OSCILLATIONS

A numerical solution of problem (14)–(18) by the Runge–Kutta method allows the consideration of nonlinear front oscillations with arbitrary amplitudes. Such an approach was employed to analyze time variations in the position of the right-hand boundary of the front $X = x_2(t)$, the maximum depth of the frontal zone $H = h(0, t)$, and the projections of the current velocity $\{U; V\} = \{u; v\}(X, t)$ on the right-hand boundary of the frontal zone. These quantities are expressed through the solution of problem (14)–(18) by the formulas

$$X = [-(1+d)/(-\gamma + c)]^{1/2}, \quad H = 1 + d, \quad U = aX, \quad V = (-2\gamma + b)X. \tag{25}$$

In addition to quantities (25), we also considered integral energy characteristics of the frontal zone such as the kinetic (K), potential (P), and total (E) mechanical energies for the entire cross section of the front, which are determined by the formulas

$$K = \int_0^X h(u^2 + v^2)dx, \quad P = \int_0^X h^2 dx, \quad E = K + P. \tag{26}$$

For hydrodynamic fields characterized by simple structure (5), integrals (26) can be calculated analytically.

For numerical analysis of the nonlinear dynamics of a frontal zone having structure (5), we used the following values of the dimensional parameters of the model: $g' = 10^{-2}$ m s^{-2}, $f = 0.7 \times 10^{-4}$ s^{-1}, $h^+ = 500$ m, and $W = 60$ km. These values are characteristic of intense oceanic fronts. In this case, $\gamma = 0.2834$.

Figure 3 shows time variations in the nonstationary characteristics of the front that are determined by formulas (25) and are due to the initial disturbance of the geostrophic-current velocity in the direction transverse to the axis of the front. The departure of the initial state of the front from a purely geostrophic current leads to the generation of undamped oscillations of fields with superinertial frequency, which is characteristic of the time variation of ocean currents on these time scales [3]. In accordance with the law of mass conservation, the increase in the front width X is accompanied by the decrease in its maximum depth H and vice versa. If the velocity U assumes positive values on the right-hand boundary of the front, the current region expands with time, whereas it contracts with time when the velocity U changes sign. The horizontal current velocity oscillates about the geostrophic value given by (11), which is demonstrated by the time dependence of V shown in Fig. 3.

The total energy E of the frontal zone is an integral of motion of the mathematical model described by (2); the time independence of the energy is illustrated by the corresponding curve in Fig. 4. The potential (P) and kinetic (K) energies of the front obtained numerically by formulas (26) execute superinertial oscillations occurring in antiphase, which is necessary to provide the constancy of the total energy of the front.

The intensity and period of transverse oscillations of the front depend on both the parameter γ and the amplitude of the initial disturbance. Figures 5 and 6 allow the description of the dependence of the period T and the oscillation amplitude of the front boundary

$$a_X = \frac{1}{2}(\max_t X - \min_t X) \tag{27}$$

on the intensities of the initial disturbances transverse to and along the rectilinear frontal zone.

In accordance with Fig. 5, the increase in the amplitude of the velocity normal to the front leads to the decrease in the period of nonlinear oscillations, whereas disturbances in the geostrophic velocity along the axis of the front can result in both the decrease ($b_0 < 0$) and increase ($b_0 > 0$) of the oscillation period as

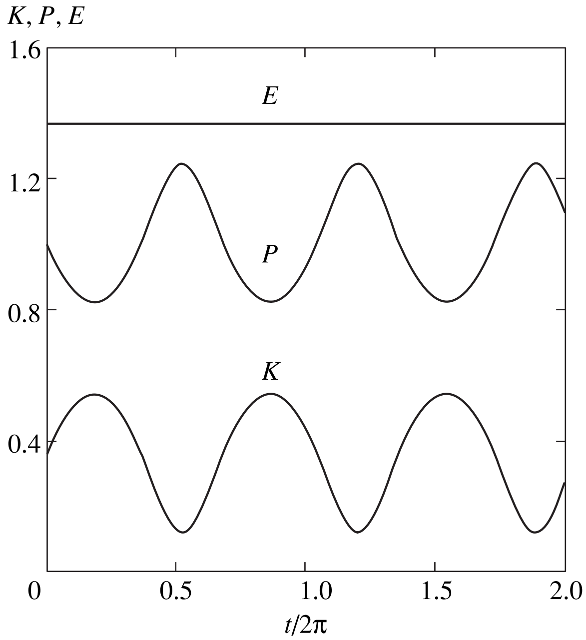

Fig. 4. Time variation of the integral (over the cross section of the front) kinetic (E_k), potential (E_p), and total (E) mechanical energies of the liquid. The initial disturbance is the same as for Fig. 3.

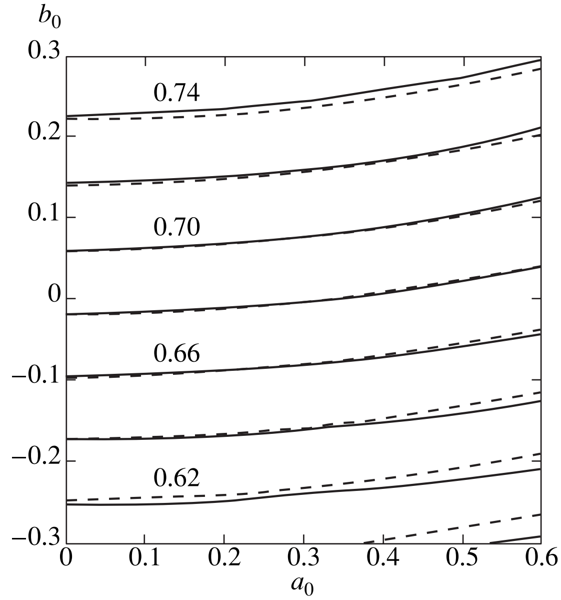

Fig. 5. Period of nonlinear oscillations of the front $T/(2\pi)$ vs. the amplitudes of the initial current-velocity disturbance transverse to (a_0) and along (b_0) the frontal zone. The solid lines correspond to an exact (numerical) value of the oscillation period, and the dashed lines, to formula (23).

compared to the linear case. This property of the oscillation period also follows from the asymptotic estimate given by (23). The dependence of the oscillation period on the amplitude characteristics of the initial disturbance is a purely nonlinear effect. Asymptotic formula (23) very accurately describes the dependence of the period of front oscillations on the amplitudes of the initial disturbances of the current velocity in the ranges $|a_0|, |b_0| \le 0.2$.

Consider the dependence of the oscillation amplitude of the front boundary (27) on the intensity of the initial disturbance of the velocity field. The solution of the linearized problem corresponding to (14)–(18) is obtained analytically. At $a = a_0$, $b = b_0$, and $c_0 = d_0 = 0$, we find that

$$c = -3\gamma d,$$
$$d = \omega^{-2}[-b_0 + \sqrt{b_0^2 + \omega^2 a_0^2}\sin(\omega t + \varphi)], \tag{28}$$

where φ is an arbitrary constant. For small c and d, the first of formulas (25) assumes the form

$$X \approx \frac{1}{\sqrt{\gamma}}\left(1 + \frac{1}{2}d + \frac{c}{2\gamma}\right). \tag{29}$$

Using formulas (27)–(29), we obtain the amplitude of linear oscillations of the front boundary

$$a_X = \frac{1}{\sqrt{\gamma}\omega^2}\sqrt{\omega^2 a_0^2 + b_0^2}. \tag{30}$$

The contours of a_X in the (a_0, b_0) plane are ellipses elongated along the b_0 axis.

The calculation results presented in Fig. 6 indicate that the oscillation amplitude increases as the intensity

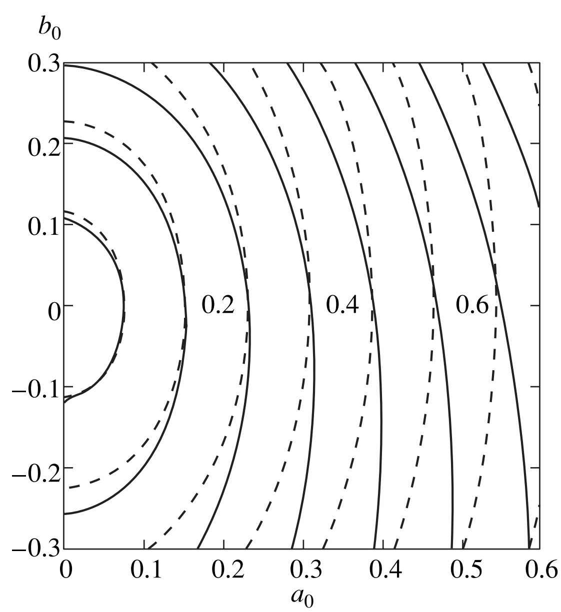

Fig. 6. Amplitude a_X of nonlinear oscillations of the frontal-zone boundary vs. the amplitudes of the initial current-velocity disturbance transverse to (a_0) and along (b_0) the front. The solid lines correspond to exact values of the amplitude of nonlinear oscillations, and the dashed lines, to formula (30) for the linear case.

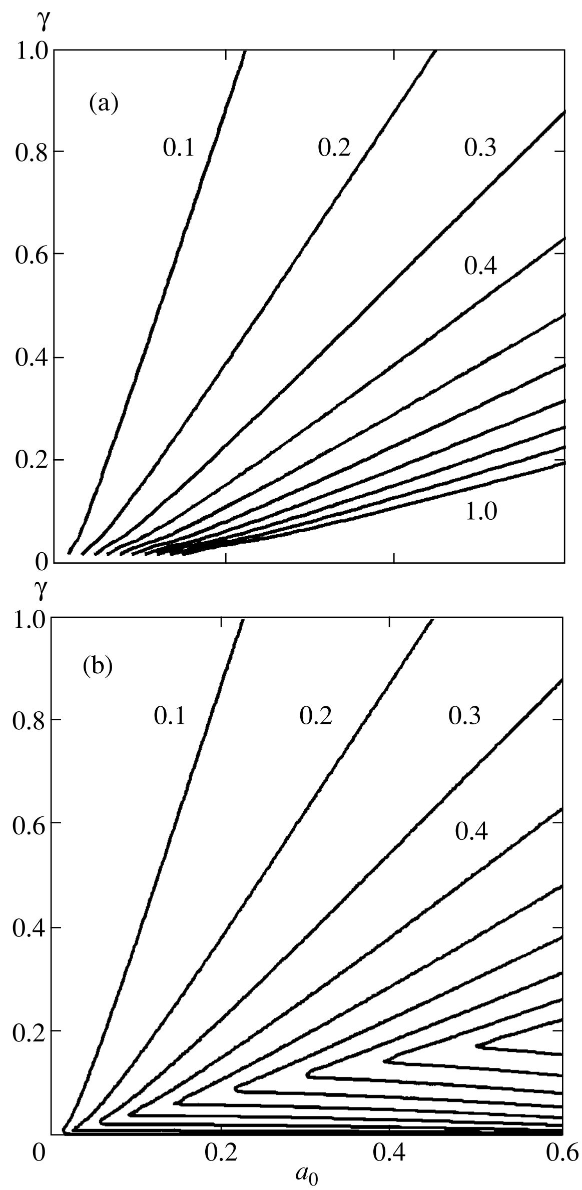

Fig. 7. (a) Oscillations amplitudes of the front boundary and (b) velocity-vector magnitude at the front boundary $x = X$ vs. the amplitude of the initial current-velocity disturbance transverse to the front ($a_0 > 0$, $b_0 = 0$) and the parameter γ.

of any disturbance in the geostrophic-current velocity increases. The amplitude was calculated by a formula similar to (27). For small disturbances of the front, when $|a_0|, |b_0| \leq 0.1$, the linearized model of transverse motions of the front can be used. It describes most accurately oscillations caused by the initial disturbances of the velocity in the direction transverse to the axis of the frontal zone. Nonlinearity causes the oscillation amplitude of the front to increase in comparison with the case of linear oscillations if $b_0 < 0$ and to decrease if $b_0 > 0$.

Figure 7a demonstrates that the intensity of oscillations of the front boundary decreases as the geostrophic-current horizontal shear, which is proportional to the main parameter of the model γ, increases. At the same time, even though the intensity of oscillations of the current velocity increases with the amplitude of the initial disturbance, the dependence of the dimensionless oscillation amplitude of the current velocity on the parameter γ is not monotonic. Current-velocity oscillations are relatively weak under small or large background-current shears (Fig. 7b). In the (a_0, γ) plane, the oscillation amplitude of the velocity reaches its maximum values along a certain curve $\gamma = \gamma(a_0)$ corresponding to the angular points of amplitude contours in Fig. 7b.

5. CONCLUSIONS

Nonlinear transverse oscillations of the near-surface (warm) frontal zone of a finite width have been studied within the framework of the reduced-gravity model of the ocean. The analysis is restricted to the planar case. The oscillations are caused by initial departures of hydrodynamic fields within the front from geostrophic equilibrium.

The approximate model used for ocean dynamics suggests that the ocean is stationary beyond the frontal zone, whose lower boundary can nevertheless deform. For this reason, the energy of the initial disturbance of the frontal zone is not emitted and the oscillations of the fields in it are undamped as in the case discussed above. In models allowing for the wave transfer of the energy of the initial disturbance into the liquid surrounding the front, a transition to a new geostrophic regime occurs [5, 6, 20, 21]. This regime decays slowly if the system is dissipative [22].

If the current-velocity vector depends linearly on the horizontal coordinate within the frontal zone and the depth of the front is distributed by a quadratic law, then the initial nonlinear problem formulated in partial derivatives is equivalent to a system of four ordinary differential equations in time. This system has periodic solutions, which was found in [11] from analysis of a mathematically equivalent problem of oscillations in a near-bottom lens of cold water. For small-amplitude oscillations of the front, a perturbation method with respect to two small parameters was used, which made it possible to obtain an approximate solution of the problem in a weakly nonlinear approximation of the second order. Front oscillations of an arbitrary amplitude were studied numerically solving the system of equations by the Runge–Kutta method.

It has been shown that hydrodynamic-field oscillations are always superinertial. Nonlinearity can lead to both an increase and a decrease in the frequency of front oscillations in comparison with the linear case. The oscillations are the most intense if they are caused by initial disturbances of the geostrophic-current velocity that are directed along the front's axis. A weakly nonlinear solution of the problem describes front oscillations fairly accurately even if the initial disturbances of the current velocity within the frontal zone are strong and reach 50% of the magnitude of the geostrophic-current velocity. Nonlinearity can lead to both an increase and a decrease in the amplitude of field oscillations in comparison with linear oscillations about the geostrophic regime of motion. The intensity of oscillations of the front boundary decreases as the horizontal shear of the background current increases. The dependence of the oscillation amplitude of the current-velocity magnitude on the background-current shear is not monotonic. Oscillations of the frontal-current velocity are relatively small under small or large background-current shears.

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