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APPLICATION OF GLOBAL REGRESSION METHOD FOR CALIBRATION OF WIND TUNNEL BALANCES

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ABSTRACT

Aerodynamic forces and moments on scaled models are measured in wind tunnels by multi-component strain gauge balances whose performance and accuracy are characterized by careful calibrations. It is well recognized that calibration loads that are representative of the combined loads experienced by the balance/model in wind tunnel test conditions lead to high accuracy of the measured loads. But, in the traditional method the calibration loads are restricted to single-component and two-component loads that are unrepresentative of model loads. A recently developed method called Global Regression Method (GRM) places no such restrictions and therefore, permits application of multi-component calibration loads similar to model loads, leading to improved accuracy of measured loads. In addition, with use of GRM it is possible to substantially reduce the number of calibration loads and hence, the calibration effort by optimization of load schedule.

The GRM was recently implemented and a MATLAB software developed for balance calibration data analysis at NAL. In order to illustrate the application of GRM and use of the computer program calibrations of a 6-component internal balance were carried out with different types and number of calibration loads that included combined loads similar to model loads and the data were analyzed using GRM. The balance accuracy was assessed for a set of check loads consisting primarily of combined loads. Results showed that the accuracy when the calibration loads were similar to model loads was significantly better than that obtained using single- and two–component calibration loads. It was also found that improved accuracy could be obtained with substantially reduced number of loadings.

The paper presents an overview of the GRM and brief details of application of the GRM to calibration of multi-component balances. A description of the above calibrations and discussions of the results are included.

Key Words: Strain gauge balance, Calibration, Math model, Regression analysis

1. INTRODUCTION

Multi-component strain gauge balances are the primary instruments used for measurement of aerodynamic forces and moments on scaled models in wind tunnels. Careful calibration of a balance is necessary to characterize its performance and determine its accuracy. It is well known that apart from design and fabrication aspects, accuracy of a balance depends on the mathematical model adopted and the type of loadings applied for balance calibration (Ref.1and 2). Accuracy of the measured aerodynamic loads can therefore be expected to be highest when the calibration loads are similar to those experienced by the balance/model during wind tunnel tests, and an appropriate math model is chosen.

In the traditional method the balance is calibrated by applying only single-component and two-component loads, and 'piece-wise' curve fits to the output vs load data are obtained from each of the loading sequences. Slope of these fits are the desired calibration coefficients (Ref.3). Such 'piece-wise' curve fitting procedure, sometimes called as Cooke's method, requires that the load combinations be restricted to a maximum of two components (Ref.1). Consequently, the calibration loadings in the traditional method are also restricted to a maximum of two components, although such loadings are generally unrepresentative of actual wind tunnel test conditions.

With improvement in computational capability a new calibration data processing method that does not place any restrictions on the type of calibration loadings has been developed (Ref. 1). The method known as least square Global Regression Method (GRM) permits simultaneous loadings of any arbitrary combinations upto six components and it therefore, enables the calibration of a balance with load combinations similar to those experienced by the balance in actual wind tunnel test conditions.

It is also noted that the Ground Testing Technical Committee (GTTC) of the American Institute of Aeronautics and Astronautics (AIAA) which recently reviewed various practices adopted at major facilities in USA and Canada on calibration and use of internal balances, recommended adoption of GRM for deriving the calibration matrix (Ref.1). Following this recommendation most major wind tunnel facilities have adopted the GRM for balance calibration data anlysis (e.g. Ref. 4 and 5).

Calibration of half-model balances and other external balances are frequently made by application of loads with points of application that are laterally offset from the balance. Such loading conditions result in multi-component loading of the balance. For example, a normal force on a half-model results in combined

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loading of three components viz., normal force, pitching moment and rolling moment on the balance. Use of piece-wise curve fitting of such combined load calibration data is not possible and it is necessary to utilize the global regression method to determine the calibration coefficients of such balances.

Balance calibration procedures at NAL have so far followed the traditional method and efforts are now underway to bring these procedures on par with state-of-the-art and widely accepted practices in the wind tunnel community. As part of these efforts, the least squares global regression method for determination of balance calibration coefficients has recently been implemented at NAL and a computer program has been developed using MATLAB for this purpose.

To illustrate the application of the GRM and demonstrate its advantages, calibrations of a six-component internal balance were made with combined loads that were representative of typical model loads and also with traditional single-component and two-component loads. Data from these calibrations which were carried out in an Automatic Balance Calibration System at NAL were analyzed using the GRM.

The report presents details of the above work along with a brief description of the GRM with emphasis on its application to balance calibration data analysis.

2. CALIBRATION PROCESS OF A BALANCE

Before describing the application of GRM to balance calibration data a summary of calibration process is included as a background.

A balance is calibrated by applying known loads and electrical outputs from the strain gauge bridges on the balance elements are recorded. The outputs from the balance elements are related to the applied loads on the balance and this relationship is expressed through a polynomial equation called as the math model of the balance. The most general form of the equation features first order, second order and a limited number of third order load terms (Ref.1). This model, called a third order math model also accounts for the dependency of balance outputs on the sign of loads through the use of 'signed' and 'absolute' values of component loads.

The third order model expressing the output R_e from the e^{th} balance element as a function of the applied loads $\left(F_j, F_k\right)$ can be written as (adapted from Ref.1):

$$\begin{aligned} R_e = \; & a_e + \sum_{j=1}^{q} b1_{e,j}\, F_j + \sum_{j=1}^{q} b2_{e,j} \left| F_j \right| \\ & + \sum_{j=1}^{q} c1_{e,j}\, F_j^2 + \sum_{j=1}^{q} c2_{e,j}\, F_j \left| F_j \right| \\ & + \sum_{j=1}^{q} \sum_{k=j+1}^{q} c3_{e,j,k} F_j F_k + \sum_{j=1}^{q} \sum_{k=j+1}^{q} c4_{e,j,k} \left| F_j\, F_k \right| \\ & + \sum_{j=1}^{q} \sum_{k=j+1}^{q} c5_{e,j,k} F_j \left| F_k \right| + \sum_{j=1}^{q} \sum_{k=j+1}^{q} c6_{e,j,k} \left| F_j \right| F_k \\ & + \sum_{e=1}^{q} d1_{e,j}\, F_j^3 + \sum_{e=1}^{q} d2_{e,j} \left| F_j^3 \right| \qquad \text{...(1)} \end{aligned}$$

The above math model defines the balance behaviour for the following types of loads :

'linear' + and – loads
'load squared' + and – loads
'load cubed' + and – loads
'load cross product' ++, + –, – + and
– – second order load combinations

This model features a total of $2q(q+2)$ calibration coefficients for each component of a q-component balance. The third order model for a 6-component balance will lead to a 6x96 calibration matrix.

As noted earlier the traditional calibration procedure involves application of single-component loads followed by two-component loads which are applied by varying the load on one element while the load on the other is held constant. The unknown calibration coefficients are determined by making separate or piece-wise curve fits to the output vs. load term for each of the load terms of the math model (see Ref. 1-3 for more details). This procedure requires that the calibration loads be restricted to a maximum of two components, which is infrequent in most wind tunnel test conditions.

As noted earlier, the recently developed GRM does not place any such restrictions on the type of calibration loads and, in particular, this method permits simultaneous loading of any combinations of components upto a maximum of six. The GRM therefore enables the calibration loading design to be representative of the model loads in a wind tunnel. A brief description of GRM follows.

3. A BRIEF REVIEW OF GRM

Regression analysis is a widely used statistical technique for investigating and modeling the relationship between variables in various fields including engineering and physical sciences. As noted earlier, regression analysis has recently been applied for balance calibration data processing at several wind tunnel facilities. But details of application of the regression analysis to balance calibration are not available, at least in the open literature. It was therefore considered appropriate to include here a brief review of regression analysis with emphasis on its application to determination of calibration coefficients of multi-component balances. The review is based primarily on the material presented in Ref. 6 and 7.

Adopting notations commonly used in regression analysis the functional relation between the response y of a system to a set of independent variables x_j, $j = 1, 2 \cdots\cdots k$ is expressed by the following equation:

$$y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon \quad \ldots (2)$$

where ε is the difference between the observed and fitted values of y, and x_j s are called as predictors or regressors. The above equation is also called the regression model of the system under consideration. The model represented by eqn. (2) is called a multiple linear regression model with k regressors and the coefficients β_j, $j = 0, 1, \cdots k$ are called the regression coefficients. The term linear is used because eqn. (2) is a linear function in unknown parameters β_j, while the regressor x_j can take any form (such as x^3, $\sin x$ etc.). The term multiple refers to the fact that the equation involves more than one regressor.

In order to find the unknown coefficients of eqn. (2) measurements of the response y are made for n different sets of predictors x_j. When the number of measurements is greater than the number of unknown coefficients, i.e. $n > k+1$ the method of least squares can be used to estimate the regression coefficients as noted below.

The set of response values y for different sets of predictor values x_j can be written in matrix notation as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad \ldots (3)$$

where $\mathbf{y}$ is a $n \times 1$ vector of response values, $\mathbf{X}$ is a $n \times p$ matrix of the levels of regressor variables, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown regression coefficients and $\boldsymbol{\varepsilon}$ is a $n \times 1$ vector of random errors, where $p = k+1$.

Using the least squares method matrix of estimated regression coefficients can be obtained as (see Ref. 6 for derivations) :

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'}\ \mathbf{X})^{-1}\ \mathbf{X'}\ \mathbf{y} \quad \ldots (4)$$

provided that the inverse matrix $(\mathbf{X'}\ \mathbf{X})^{-1}$ exists. The vector of fitted values of response corresponding to the measured values i.e. the fitted regression model is given by:

$$\hat{\mathbf{y}} = \mathbf{X}\ \hat{\boldsymbol{\beta}} = \mathbf{X}(\mathbf{X'}\ \mathbf{X})^{-1}\ \mathbf{X'}\ \mathbf{y} \quad \ldots (5)$$

As part of regression analysis an assessment of the fitted regression model is made to ascertain the adequacy and quality of fit through statistical procedures called as 'hypothesis testing' and 'tests for significance' and using various quality metrics. Some of the commonly used quality metrics are : standard error of regression $\hat{\sigma}$, coefficient of variation *CV*, coefficient of multiple determination R^2, t-statistic and variance inflation factor *VIF* (Ref. 6 and 7). Estimates of standard error and confidence interval widths of the estimated regression coefficients are also used to assess the quality of fitted math model. The above metrics are obtained using certain statistical quantities that are computed using the measured and fitted response data and the estimated regression coefficients. These statistical quantities are usually summarized in an analysis of variance (*ANOVA*) Table, a format of which is shown in Table.1

Table 1. Analysis of variance (*ANOVA*) for significance of regression

Source of variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o
Regression	SS_R	k	MS_R	$\frac{MS_R}{MS_{Res}}$
Residual	SS_{Res}	$n-k-1$	MS_{Res}	
Total	SS_T	$n-1$		

Equations for the various quantities noted in the *ANOVA* Table and the quality metrics are presented in Ref. 6.

4. APPLICATION OF GRM TO BALANCE CALIBRATION DATA ANALYSIS

Regression analysis methods described in the previous section are applied to determine the calibration coefficients of multi-component balances. For this purpose the polynomial equation representing the balance math model (such as eqn (1)) is expressed in the format used in regression analysis, eqn (3). In order to do this, the various types of loads

F_j, $|F_j|$, F_j^2, F_jF_k .... $|F_j^3|$ in the polynomial equation are rewritten as load terms g_j, $j = 1, 2 .... k$, and the coefficients a_e, $b1_e$, $b2_e$, ... $d2_e$ are written as $C_{e,o}$, $C_{e,j}$, $j = 1, 2 .... k$ respectively.

With these new symbols the balance math model can be written as :

$$R_{e,i} = C_{e,o} + \sum_{j=1}^{k} C_{e,j}\, g_{j,i} \quad i = 1, 2 ... n \qquad \text{… (6)}$$

where

$R_{e,i}$ = output in e^{th} element due to i^{th} loading L_i

$C_{e,j}$ = calibration coefficient of e^{th} element associated with j^{th} load term g_j

$g_{j,i}$ = value of load term j due to i^{th} loading L_i

L_i = i^{th} loading

In general, each applied load denoted by L_i will cause loading of any one of the balance elements (single-component load) or simultaneous loading of two or more components (i.e. multi-component or combined loading). The load terms or regressors $g_{j,i}$, $j = 1, 2 .... k$ resulting from each such loading L_i are computed from the functional relationships between g_j and component loads F_1, F_2 ... F_q. These functional relationships are obtained from the chosen math model.

Eqn. (6) represents the multiple linear regression model of the balance and the sets of measured values of balance outputs $R_{e,i}$ and the known values of load terms $g_{j,i}$ are utilized in a regression analysis to determine the unknown coefficients $C_{e,j}$ as noted below.

Eqn. (6) is written in matrix format as :

$$\underset{(1,n)}{\mathbf{R_e}} = \underset{(1,p)}{\mathbf{C_E}}\ \underset{(p,n)}{\mathbf{G_N}} \qquad \text{… (7)}$$

where $\mathbf{R_e}$ is a the vector of outputs of the e^{th} element due to loadings L_i, $i = 1, 2 ... n$, and is given by :

$$\mathbf{R_e} = [R_{e,1} \quad R_{e,2} \ ....R_{e,n}]$$

$\mathbf{C_E}$ is the extended vector of calibration coefficients of the e^{th} element and is given by :

$$\mathbf{C_E} = [C_{e,0} \quad C_{e,1} \quad C_{e,2} \ ....C_{e,j}.........C_{e,k}]$$

$\mathbf{G_N}$ is the extended load matrix consisting of values of load terms g_j for each of the loadings L_i, $i = 1, 2 ... n$, and is written as :

$$\mathbf{G_N} = \begin{bmatrix} 1 & 1 & . & . & 1 & . & 1 \\ g_{1,1} & g_{1,2} & \cdot & \cdot & g_{1,i} & \cdot & g_{1,n} \\ g_{2,1} & g_{2,2} & \cdot & \cdot & g_{2,i} & \cdot & g_{2,n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ g_{k,1} & g_{k,2} & \cdot & \cdot & g_{k,i} & \cdot & g_{k,n} \end{bmatrix}$$

The matrix equation (7) representing the math model of the balance is in row format (which is customary in balance calibration data analysis), and this is converted to the column matrix format commonly adopted in regression analysis by taking the transpose of both sides of the eqn. (7) and including the error vector the math model of the balance is written as :

$$\mathbf{R_e'} = \mathbf{G_N'}\ \mathbf{C_E'} + \boldsymbol{\varepsilon} \qquad \text{… (8)}$$

where ' indicates transposed quantities.

Comparing eqn. (8) with eqn. (3) the following equivalence of the terms used in balance calibration field with those used in regression analysis can be written:

vector of response $\mathbf{y}$ ≡ vector of balance outputs $\mathbf{R_e'}$

matrix of predictors $\mathbf{X}$ ≡ matrix of load terms $\mathbf{G_N'}$

vector of estimated regression coefficients $\hat{\boldsymbol{\beta}}$ ≡ vector of calibration coefficients $\mathbf{C_E'}$.

Using the above equivalence relations in eqn. (4) and after carrying out simplifications, the matrix of calibration coefficients is obtained as:

$$\mathbf{C_E} = \mathbf{R_e}\,\mathbf{G_N'}\left(\mathbf{G_N}\,\mathbf{G_N'}\right)^{\mathbf{-1}} \qquad \dots (9)$$

The fitted regression model of the balance is given by:

$$\hat{\mathbf{R}}_{\mathbf{e}} = \mathbf{R_e}\,\mathbf{G_N'}\,(\mathbf{G_N}\,\mathbf{G_N'})^{\mathbf{-1}}\,\mathbf{G_N} \qquad \dots (10)$$

An assessment of the adequacy and quality of the above fitted regression model of the balance can be made using various quality metrics noted in section 3.

In order to implement the above GRM concepts for balance calibration, a MATLAB software was developed. The software computes the calibration matrix of a *q*-component balance (*q* can be between 1 and 6) for the chosen math model. In addition, the program computes the quality metrics used to assess the adequacy and quality of the fitted regression model, noted in section 3. Computed quality metrics and other data can also be utilized to carry out further analysis to choose the most appropriate math model or reduce the number of terms in the model by deleting insignificant terms.

Although as per the current practice the choice is either second order or third order math model with or without considering the asymmetric behaviour of the balance, the program can also handle higher order math models that include higher powers of component loads and load cross product terms with more than two combinations of component loads.

5. ESTIMATION OF BALANCE ACCURACY

Accuracy of a balance is estimated by comparing 'back-calculated loads' with actual applied loads on the balance. The 'back-calculated loads' are the loads computed utilizing the calibration matrix generated from regression analysis and the recorded balance outputs for the applied loads. In general such back calculation of loads are made for all the calibration loads and also for a set of check loads that are not identical to the calibration loads.

The difference ΔF_e between the back-calculated load and the actual applied load in a balance element *e* is the residual load error in the computed load F_e. A standard deviation of the residual load errors in the e^{th} balance element is computed from the following equation :

$$\sigma_e = \left[\frac{1}{n-1} \sum_{i=1}^{n} \Delta F_{e,i}^2 \right]^{1/2} \qquad \dots (11)$$

where $\Delta F_{e,i}$ is the residual error of i^{th} load in the e^{th} balance element and *n* is the total number of applied loads.

The above standard deviation normalized with respect to its rated load is regarded as a measure of the accuracy of the balance element, i.e.,

$$\text{Accuracy of } e^{th} \text{balance element} = \frac{\sigma_e}{F_{e\ Rated}} \times 100\ \%$$

Balance element accuracies are computed separately for the calibration loads and the check loads.

6. CALIBRATIONS OF A 6-COMPONENT BALANCE

To illustrate the application of the GRM and use of the computer program developed for this purpose and also to bring out the advantages of the GRM, calibrations of a 6-component internal balance

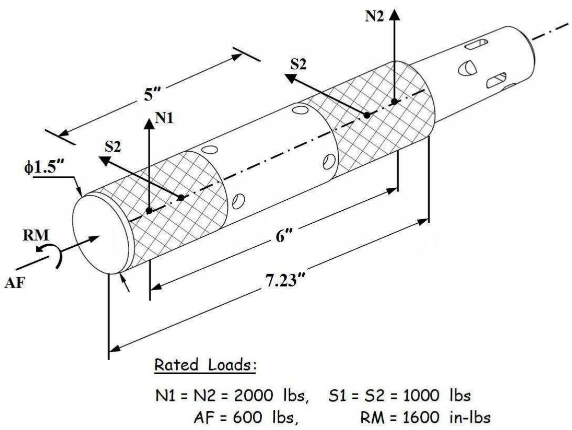


Figure 1. 1.5" diameter floating frame balance

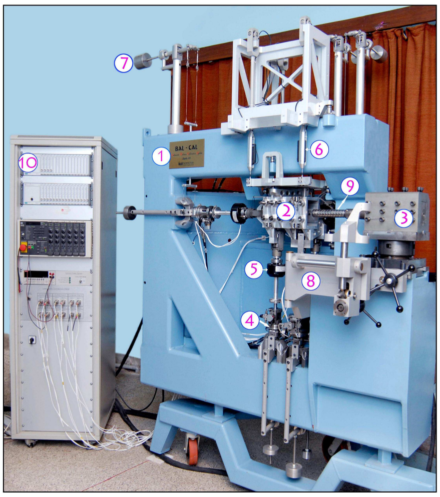


Figure 2. Automatic Balance Calibration System

were carried out and the calibration data were analyzed using the program. Details of these calibrations and some results are presented below.

The balance is of floating-frame type and strain gauged to measure 5 forces viz. N1, N2, S1, S2 and AF and one moment i.e. RM. Figure 1 shows the major dimensions, sign conventions of the load components and the rated loads of the balance. The balance was calibrated in an Automatic Balance Calibration System (ABCS) at NAL (Figure 2). The ABCS is of non-re-positioning type featuring 6 servo-controlled hydraulic actuators for load application and the loads are measured by 0.02% accuracy load cells. Six high resolution optical sensors incorporated in the ABCS measure the balance deflections to an accuracy of 1 micron. More details of the ABCS are found in Ref. 8.

The three calibrations differed in the type and total number of calibration loads as noted below :

(i) Cal 1 : a 'standard' calibration featuring 935 loads consisting of 84 single-component loads and 851 two-component loads

(ii) Cal 2 : the loads were similar to those in (i), but, the total number of loads was 300 consisting of 43 single-component and 257 two-component loads, and

(iii) Cal 3 : a 'combined load' calibration featuring 300 loads consisting of 35 three-component loads, 37 four-component loads, 38 six-component loads in addition to 30 single-component and 161 two-component loads. The magnitude of the three-component and six-component loads were similar to those on a typical model at subsonic / transonic speeds in the NAL 1.2m tunnel.

Calibration data from the above calibrations were analyzed using the GRM and three different third-order regression models and corresponding 6x96 calibration matrices were obtained. Assessment of the fitted regression model was also made using the various quality metrics noted in section 3 for the three calibration models.

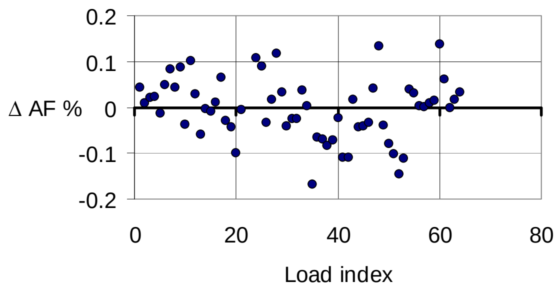
R^2 value which is often used as a measure of the overall success of the regression model was > 0.997 for all three calibration models (R^2= 1 implies a perfect fit). Overall quality of the three fits was therefore judged excellent. A typical *ANOVA* table and values of some of the quality metrics are shown in Table 2.

Table 2. *ANOVA* table for AF element

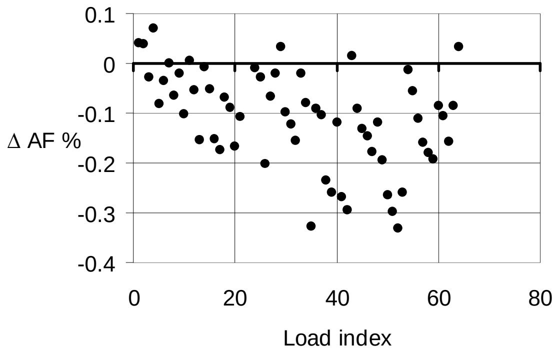
Source	Deg of Freedom	Sum of Squares	Mean Square	F
Regression Residual Total	96 838 934	101.01 1.49E-03 101.01	1.05 1.7E-6	5.8E+05

Element	Std Error of Regression	Coeff of Variation	R^2 Statistic
AF	1.78E-06	0.134	0.999

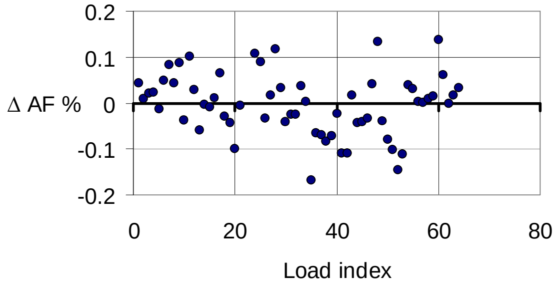
For determining the balance accuracy for these calibrations a set of 64 check loads consisting of 12 single-component loads, 28 four-component loads and 24 six-component loads were applied and balance outputs were recorded. The four-component and six-component loads were generally similar to the loads



a) Cal. 1



b) Cal. 2



c) Cal. 3

Figure 3. Comparison of typical residual load error distributions

on a typical model during actual wind tunnel tests in the NAL 1.2m tunnel, and magnitude of the check loads were different from those of the calibration loads. Balance outputs due to the check loads were processed using the computed calibration matrix for back-calculation of check loads for each of the three calibrations. These back-calculated loads were compared with known check loads, and residual load errors and balance accuracies were computed as described in section 5 for the three calibrations.

Figure 3 shows a comparison of a typical residual load error distribution for the three calibrations. Magnitudes of the load errors are, in general, lowest for Cal 3 (which includes combined loadings similar to check loads) compared to those for

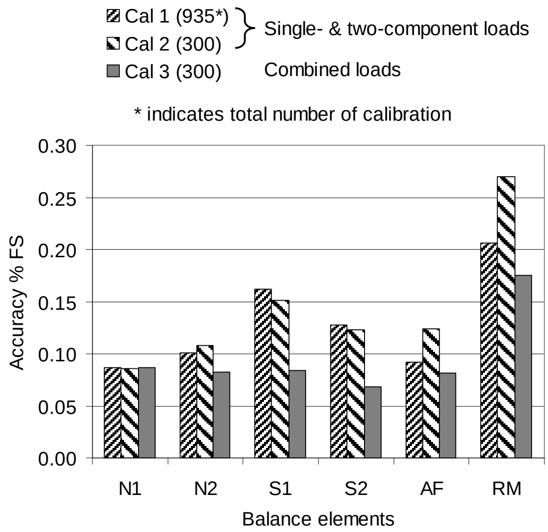


Figure 4. Comparison of balance element accuracies for calibrations with different types and number of calibration loads

Cal 1 and Cal 2 (which feature only single-component and two-component calibration loads).

Figure 4 shows the accuracy of the six elements of the balance for the three calibrations. It is seen that the balance element accuracies for Cal 3 were substantially better than those for the other two calibrations. As noted earlier, application of such combined loads during calibration is permissible only when the data analysis is made using GRM. It is also seen that these accuracy improvements have been obtained with a substantially reduced number of calibration loadings. The improvement in balance accuracy and the use of reduced number of calibration loads thus bring out the advantages of the GRM for balance calibration. However, it is to be noted that application of combined loads similar to model loads on the balance is practical only with automatic loading machines such as the ABCS.

7. CONCLUSIONS

The traditional method for calibration of multi-component balances restricts the load combinations in the calibration load design to a maximum of two components, which are generally unrepresentative of actual wind tunnel test conditions. The Global Regression Method does not place any such restrictions and permits multi-component calibration loads with load combinations of more than two components that are representative of wind tunnel loads. It has been shown that calibration with such multi-component loads along with use of GRM leads to significant improvements in balance accuracy. It has also been shown that such improvements in accuracy can be obtained with a substantially reduced number of calibration loads and a consequent reduction of calibration effort.

Use of GRM also enables adoption of higher order math models (higher than the third order used currently) including load cross product terms with more than two load combinations for balance calibrations. Inclusion of load product terms featuring load combinations similar to those on a model under wind tunnel test conditions may lead to further improvements in accuracy of measured aerodynamic forces and moments.

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