Estimation of Aerodynamic Derivatives of Projectiles from Aeroballistic Range Data using Maximum Likelihood Method

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This paper describes the application of the maximum likelihood method to estimate the aerodynamic derivatives of (i) the AGARD standard ballistic model HB-2 from supersonic free flight data available in literature, and (ii) ballistic mode simulated data of a surface-to-air missile. For the AGARD data, two types of dynamic models have been used: (i) body axis coordinate system model, and (ii) hybrid type of formulation, wherein the aerodynamic forces are in wind axis arid the aerodynamic moments are represented in body axis coordinate system. For surface-to-air missile data the equations of motion are solved in fin-body axis system. The results of this analysis demonstrate the suitability arid functional adequacy of the mathematical models used, the consistency of data arid power of the parameter estimation methodology in generating aerodynamic derivatives from realistic free flight trajectories.

NOTATION

A	=	reference area
C_x, C_y, C_z	=	axial force coefficients in the body-fixed coordinate system
C_D, C_L, C_S	=	coefficients of drag, lift and normal force
C_l, C_m, C_n	=	coefficients of rolling, pitching and yawing moments
I_x, I_y, I_z	=	inertia
m	=	mass
p, q, r	=	angular velocity components about the body-fvred axes *ox, oy, oz*, respectively
$\bar{q}$	=	dynamic pressure .
V	=	flight path velocity
ox, oy, oz	=	axes in the body-fvred coordinate system
u, v, w	=	velocities in *ox, oy, oz* axes system
α, β	=	angles of attack and side slip
θ, ϕ, ψ	=	Euler angles
X_{ϕ_o}	=	vector of unknown initial conditions
P, Q	=	vectors of unknown parameters to be estimated
g_f, g_b	=	gust components in fin, body axes
T_1, T_2	=	transformation matrices from tin to body and body to inertial axes
f, b, g	=	suffixes for fin, body and gust

INTRODUCTION

Free flight testing in aerohallistic ranges with a view to extracting the aerodynamic parameters from the measured motion patterns of missile configurations have been in progress for well over three decades. But the methods used for analysis place several restrictions on the equations of motion; for example, assumptions such as linear aerodynamics, constant roll rate, small velocity variations, etc. In order to eliminate such restrictions and pave the way for complex and contemporary model configuration testing in ballistic ranges, advanced statistical methods of parameter estimation such as maximum likelihood (ML) and extended Kalman filter methods which have been successfully validated for aircraft flight data[1,2] over the past decade have been recently extended to free flight analysis problems of missile configurations also[3,4].

This paper describes the application of the maximum likelihood method to estimate the aerodynamic derivatives of (i) AGARD standard ballistic model HB-2 from supersonic free flight data available in literature, and (ii) ballistic mode generated simulated data of a surface-to-air missile (SAM). For AGARD data, axes systems used for representing the mathematical models: body axis and hybrid type formulation wherein the aerodynamic forces are in wind axis and the moments are in body axis system. For SAM data the equations of motion are solved in tin-body axis systems with appropriate transformations between fin and body axis systems. In order to be consistent with the two original data sets, the FPS and SI systems of units have been used.

MAXIMUM LIKELIHOOD METHOD

The ML method as applied to free flight trajectory of ballistic range usually begins with the mathematical model of the flight vehicle configuration whose equations of motion are formulated in general terms as

State equations : $\dot{x}(t) = f(x(t), P), x(0) = X_{\phi_o}$ (1)

Observation equations : $y(t) = h(x(t), P)$ (2)

Measurement equations : $z(k) = y(k) + v(k);\ k = 1, \ldots, N$ (3)

where $x(t)$ is $n \times 1$ state vector, $y(t)$ is $m \times 1$ observation vector, and $z(k)$ is the $(m \times 1)$ measurement

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Euler angle rates are given as

$$\dot{\varphi} = (q_b \sin\phi + r_b \cos\phi) / \cos\theta \qquad (26)$$

$$\dot{\theta} = q_b \cos\phi - r \sin\phi \qquad (27)$$

$$\dot{\phi} = p_b + \tan\theta (q_b \sin\phi + r_b \cos\phi) \qquad (28)$$

Body velocity to inertial velocity transformation is $\{\dot{x}, \dot{y}, \dot{z}\} = T_2 \{u_b, v_b, w_b\}$, where T_2 is a direction cosine matrix.

Deterministic (gust) input excitation transformations are $\{u_{gb}, v_{gb}, w_{gb}\} = T_2^T \{$ wind $u = 0$, wind v, wind $w\}$

and

$\{u_{gf}, v_{gf}, w_{gf}\} = T_1^T \{u_{gb}, v_{gb}, w_{gb}\}$.

Angle of attack and side slip are given as

$a = \tan^{-1} \{(w_f - w_{gf}) / (u_f - u_{gf})\}$;

$\beta = \tan^{-1} ((v_f - v_{gf}) / (u_f - u_{gf}))$

with

$\tan\alpha_T = \sqrt{(v_f - v_{gf})^2 + (w_f - w_{gf})^2} / (u_f - u_{gf})$

and

$V = \sqrt{(u_f - u_{gf})^2 + (v_f - v_{gf})^2 + (w_f - w_{gf})^2}$

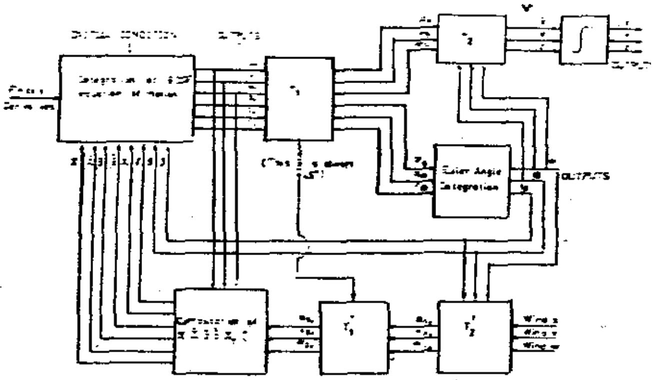

Fig 3 Block diagram of transformations / variables used for simulation / estimation

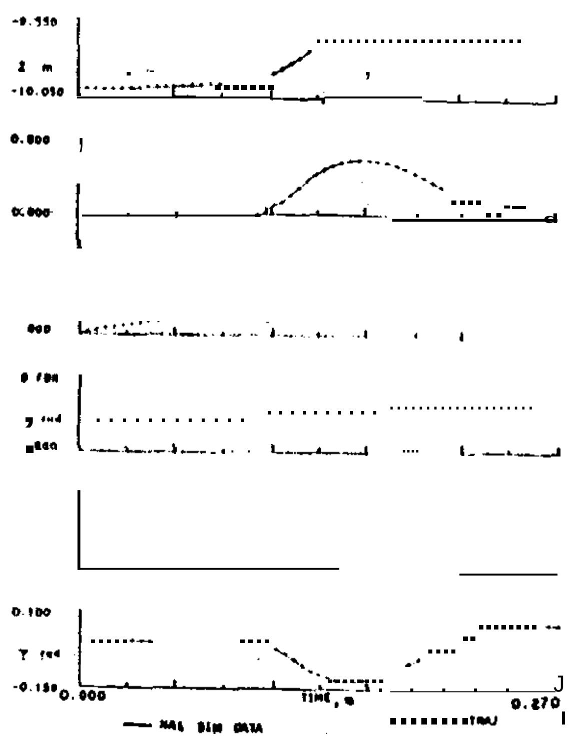

Fig 4 Time history match

These transformation equations are given for the sake of completion. For clarity the various transformations and operations involved in modelling are shown in Fig 3. The **SAM trajectory was used as data input to the MLE software** for estimation of aerodynamic derivatives as incorporated in this fin-body model. The estimation was started with initial values of some of the derivatives as 20% off from the original derivative values. The estimates were almost close to the original reference values. The results of time history match between SAM trajectory and ML predicted data are shown in Fig 4. The purpose of this exercise was to evaluate feasibility of fin-body type of modelling in analysis of ballistic type data. The various results from these three types of models seem consistent with the data sets used except for some differences.

CONCLUSION

A preliminary investigation of extracting non-dimensional stability derivatives from supersonic free flight data measurements in an aeroballitic range using the maximum likelihood method has been successfully demonstrated. Analysis and estimation using various trajectories (AGARD, SAM) have validated the utility of various types of mathematical models including fin-body transformations. Thus the parameter estimation methodology has been established for the kind of data that could arise out of aeroballistic range experiments being presently conducted in the country and similar or related realistic data of a missile or a projectile.

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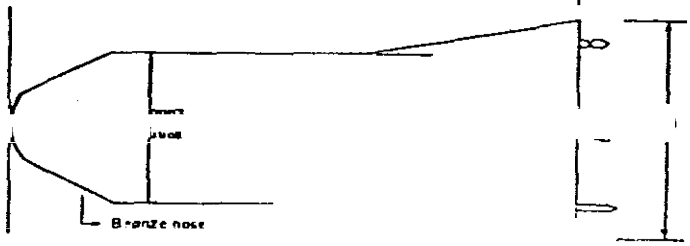
vector sampled at N discrete points. The $m \times 1$ sequence of measurement noise $\nu(k)$ is assumed to be gaussian with zero mean and covariance matrix of R. Based on the ballistic output time history data at N discrete points, the parameter

Mass, M slug	Length, in	Dia, d in	I_{cg} in	I_x, slug-ft^2	, slug-ft^2
0.0225	5.125	1.25	2.438	3.5E-05	6.2E-04



C_m / I_y I_x r / I_y

history model with many

q r

p

β

p

C C β C

$\bar{q}A$

not uniformly sampled in time, a cubic spline interpolation[3] routine was utilized to curve fit the experimental motions and generate adequate number of uniform time-spaced data samples to be used in the ML estimation programme. The α, β trajectories were reconstructed using the formulae $\alpha = \theta + \tan^{-1}(dz/dx)$ and $\beta = -\varphi + \tan^{-1}(dy/dx)$. Fig 2 illustrates the matching between the AGARD experimental trajectory and the ML predicted trajectory based on the models described elsewhere. The close match is obvious. Some of the important aerodynamic derivative estimates along with the percentage standard deviations are summarised in Table1. For the sake of comparison, the NASA estimates[6] of the longitudinal derivatives obtained for the same set of flight data are also shown in the table. The results indicate that the MLE method, in addition to providing very satisfactory and acceptable estimates for the longitudinal derivatives, also simultaneously determines the lateral derivatives, which was not the case for the NASA technique. The NASA technique was based on least squares approach to estimate C_D from x, t data in iterative manner and gaussian least squares differential technique to further extract other derivatives from the range data.

Table 1 RESULTS OF AGARD MODEL DATA ANALYSIS

Derivatives	NASA Estimates	ML Estimates V-α-β Model	ML Estimates Body Axis Model
C_D	1.239	1.207 (0.12)	1.190 (0.15)
C_{L_α}	3.590	3.155 (0.59)	4.353 (0.38)
C_{L_q}	24.400	49.524 (13.6)	48.065 (12.13)
C_{m_α}	–1.259	–1.277 (0.21)	–1.277 (0.20)
C_{m_q}	–83.000	–73.700 (1.26)	–72.680 (1.23)
$C_{S\beta}$	—	–2.700 (2.96)	–4.002 (1.00)
C_{l_p}	—	–40.730 (27.8)	–22.670 (38.3)
$C_{n\beta}$	—	1.277 (0.48)	1.284 (0.44)
C_{n_r}	—	–90790 (2 78)	–92 780 (2 56)

(% standard deviation)

The absence of roll angle, ϕ, trajectory information is probably the cause for fairly high uncertainty noticed in the estimation of the C_{l_p} derivative. The axi-symmetricity of the AGARD model configuration is established by almost identical values (with proper signs) obtained in both pitch and yaw planes, demonstrating the functional adequacy of the mathematical models used. The differences in estimates across the models may be attributed to different degree of information processing in two non-linear models. Since the data set and the estimation procedures are the same for these two types of mathematical models, the differences in some of the estimated parameters could arise from the degree of parameter identifiability governed by these models and their associated transformations.

ANALYSIS OF SAM BALLISTIC DATA

The 6-DOF trajectory of a model configuration of a surface-to-air missile (SAM) in a simulated ballistic mode was taken for analysis. The trajectory contained time histories of $u_f, v_f, w_f, p_f, q_f, r_f, \varphi, \theta, \phi$ and x, y, z data. The model fitted to this data is given below:

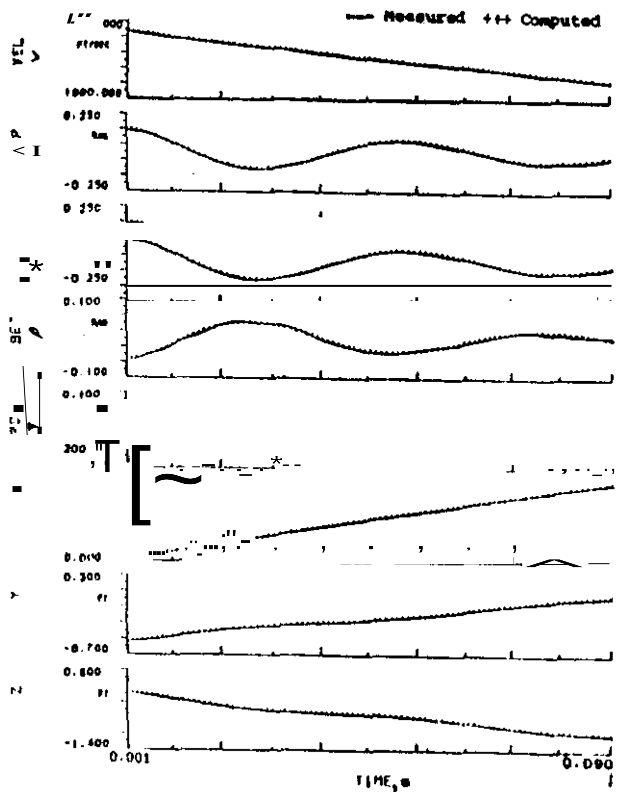


Fig 2 Free flight time histories

$$\dot{u}_f = r_f v_f - q_f w_f - g \sin\theta - \rho A V^2 \cos^2 \alpha_T C_{D_{\phi o}} / (2m) \quad (20)$$

$$\dot{v}_f = p_f w_f - r_f u_f + g/\sqrt{2} (\sin\phi \cos\theta + \cos\phi \cos\theta) + \rho A V^2 \{C_{y_\beta} \beta + C_{y_r} r_f d/(2V)\} / (2m) + \rho A V^2 C_{y_{\dot\beta}} \dot\beta d (4mV) \quad (21)$$

$$\dot{w}_f = q_f u_f - p_f v_f + g / \sqrt{2} (\cos\phi \cos\theta - \sin\phi \cos\theta) + \rho A V^2 \{C_{z_\alpha} \alpha + C_{z_q} q_f d / (2V)\} / (2m) + \rho A V^2 C_{z_{\dot\alpha}} \dot\alpha d/(4mV) \quad (22)$$

$$\dot{p}_f = \rho A d V^2 C_{l_p} p_f d / (4 I_x V) \quad (23)$$

$$\dot{q}_f = \rho A d V^2 \{C_{m_\alpha} \alpha + C_{m_q} q_f d / (2V)\} / (2 I_x) + (I_z - I_x) p_f r_f / I_y + \rho A d V^2 C_{m_{\dot\alpha}} \dot\alpha d/(4 I_z V) \quad (24)$$

$$\dot{r}_f = \rho A d V^2 \{C_{n_\beta} \beta + C_{n_r} r_f d/(2V)\} / (2 I_z) + (I_x - I_y) p_f q_f / I_z + \rho A d V^2 C_{n_{\dot\beta}} \dot\beta d / (4 I_z V) \quad (25)$$

The fin-body transformation equations are given as $\{p_b, q_b, r_b\} = T_1 \{p_f, q_f, r_f\}$

and

$\{u_b, v_b, w_b\} = T_1 \{u_f, v_f, w_f\}$, where the matrix T_1 is given by

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\phi \end{bmatrix}$$

and for the present case $\Phi = 45°$.