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On the Orbit Determination Problem

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Various important aspects of the satellite orhit determination problem are critically discussed with a view to emphasizing the importance of a wise choice of suitable system models, coordinates sets, and estimators. We describe several aspects of the orhit determination process and also review much of the available literature on the application of Kalman-filter type algorithms to the problem of near-Earth, geosynchronous, and deep-space mission type orhit determination. Some features of on-board orbit determination are also addressed. It is believed that this review with a touch of tutorial will enable engineers and scientists to arrive at an orhit determination methodology (ODM) that will have attributes of good numerical stability, efficiency, and precision in whit estimation.

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I. INTRODUCTION

The problem of orbit determination utilizing least squares estimation techniques was addressed hy Gauss in 1795. We have come a long way from Gauss via Kalman to present-day numerically stable and accurate estimators for precision orbit determination [1–4]. It appears that the prime issue from the very beginning has been the computational speed and accuracy [1, 5]. While Gauss invented and used the method of least squares as an estimation technique. Kalman solved the problem of filtering systems with noisy measurements and process noise in dynamics in 1960 [2, 3]. The Kalman filter is regarded as an efficient computational solution of the least-squares method. It has been applied to a number of areas: power systems. aerospace navigation systems. communications, process control. and biomedical applications. Soon the researchers became aware of numerical problems associated with the Kalman filter and many related problems were studied in [6–9]. As a result new alternative filtering techniques were proposed [4, 10–171. They were claimed to be numerically more stable and efficient than the conventional Kalman filter. In the present work we discuss some of these estimators since they have a great impact on orbit determination methodology.

Since the orbit determination process (ODP) involves system models. measurements, arid estimation technique. we discuss these aspects critically and review various alternatives to illuminate their merits and demerits.

We systematically tabularize the constituents of ODP. various parameter sets (coordinate systems). and features of some orbit estimators. We also survey some of the representative application results which elucidate the aspects of stability. efficiency, and accuracy of the filtering algorithms. When appropriate we highlight some aspects related to on-board orbit determination (OBOD).

The central issue of this exposition is to take a step in the direction of evolving an ODM that has attributes of good numeric stability. efficiency. and accuracy. In our opinion this review is the first of its kind in the open literature on the orbit determination problem. Mathematics is kept to a minimum. The related aspects of spacecraft attitude estimation utilizing the Kalman filter are elegantly presented in [18, 19].

II. ORBIT DETERMINATION PROBLEM

Orbit determination is the process of obtaining values of those parameters which completely specify the motion of an orbiting body. a satellite through space, based on a set of observations of the body. The observations may be obtainable from the ground-based tracking system or from the sensors on-board the satellite. A high precision orbit determination estimator takes the measurement noise into account and determines an orbit that provides a "best fit" to the collected data (subject to dynamics of orbital motion of a satellite). The current estimate of orbital parameters must be updated periodically using tracking and ranging data. Accurate orbit prediction is necessary for determination of launch windows, planning of orbit maneuvers. and anticipating events such as eclipses.

The search and rescue satellite aided tracking (SARSAT) system [20] requires that accurate orbital information be available at all times so that an accurate position of the site of an emergency can be computed immediately [21, 22]. For a mission life of several years 3 km orbit-position error is feasible. However, the position of the satellite should be known to an accuracy of 1 km at all times during a pass, in order that this error will be a very small pan of the total error budget [22].

The communications functions of a geosynchronous satellite necessitate accurate control of orbit and attitude. To achieve the required control of a satellite's orbit. it is necessary to determine the current orbit and then plan an) required corrections. The position of a communications satellite must remain within a small tolerance band of the assigned longitude. The need for strict control arises because many groundbased receiving antennas are of the nontracking type. For Ku-band systems (14-GHz uplink. 12-GHz downlink). the tolerance bands are ± 0.05 degree in longitude and latitude [23]. To ensure that timely orbit correction can be applied for restricting the altitude error within certain limits. the altitude must be measured to a high degree of accuracy: 1 part in 10^6 [24]. The effects of orbital inclination on communications satellite system design are discussed in [25].

The navigation satellite time and ranging (NAVSTAR) system is based on satellites that continuously transmit data about their positions [26, 27]. Receiving signals from NAVSTAR satellites. ground vehicles. ships. and airplanes can determine their own positions and speeds. Ground stations nionitor the satellites and compute their orbits and clock drifts. Navigation accuracy is heavily dependent on relative satellite geometry and NAVSTAR clock accuracy [28]

III. ORBIT DETERMINATION PROCESS

Major constituents of an ODP are: system models. measurements, and estimation technique as outlined in Table I. We observe that there arc many alternatives under each major column/item of the table. For greater details on formulations of models and algorithms the cited references must be consulted.

A. System Models

By system models we mean the system description required for dynamics of orbital motion of a satellite. measurement models, Eanh's rotation. and penurbation models.

1) Dynamics: Two-body motion is the principal orbit motion. The orbits of most of the bodies in space can be described as two-body orbits to a fair degree of accuracy. Orbital elements are the parameters which completely specify the basic two-body motion of orbiting bodies. Several formulations of parameter sets are possible [21, 29, 30]. In Table II we compare commonly used parameter sets for orbit description. Also refer to Fig. 1

The classical orbital elements (C6) set employs an Eanh-centered inertial frame and six parameters to provide complete description of the orbit (see Fig. 1(a)).

Since IC-6 and U7 sets are widely used wc describe them here in some detail.

***TABLE* I**
Orbit Determination Process

Hardware	Measurements	System Models	Estimators	Other Software
Vehicle mounted sensors [38, 40, 41]	- Eanh land mark tracking data [34, 35]	Dynamic, (orbital motion of a satellite) - classical orbital elements (C6) [21] - equations of motion (IC-6) [21]	- GLS dc technique [53, 54]	Coordinate transformations [21, 30]
Sensor data acquisition system microcomputers [43, 44]	- Data of angle measurements between the moon and stars [38] - Azimuth. elevation [21] - Range. range rate [21, 22] - *Satellite-to-satellite tracking type data* [37] Mandatory data and constants	- unified state model (U7, U6) [21] - combined IC-U6 [22] - combined IC-U7 [31] Measurement models - *position space* [21] - velocity space 1321 Atmospheric models - exponential model 1331 - *Broglio's density model* [33] Earth model [21] Perturbations - zonal and Tesseral [33] - *air drag* [33]	- EKF [21, 501 -Adaptive filters [59, 61, 82] I-Adaptive filter [56, 57] - *Factorization* methods [4] - SRIF [63] - UDF [64, 65] - UDF/RTS-S [67] - $\dot{U}$-$\dot{D}$ [46] - L-D [70, 711 - SBDC technique [73] - Pugachev filter (74, 75]	IC ↔ U7 IC ↔ U6 IC ↔ topocentric Numerical integration [45 - Euler - predictor-corrector - R.K. (Gill)

Denotes alternatives or options.

TABLE II
Parameter Sets

Classical Orbital Elements	Inertial Coordinate	Velocity-Space Set U7	Velocity-Space Set U6
Six elements	Six states	Seven states	Six states
Easy to visualize	Define position and velocity	Define position and velocity in velocity space	Only one element varies rapidly
One element (η) varies rapidly	All states vary rapidly	Four position space orbits are circular in this space	Transition matrix computation is simple
Many singularities	$IC6 = \{x, y, z, v_x, v_y, v_z\}$.	Relatively free from singularities	Similar singularities as for U7
$C6 = \{a, e, i, \omega, \Omega, \eta\}$.	Requires small integration step size	$U7 = \{e_{01}, e_{02}, e_{03}, e_{04}, C, R_{f1}, R_{f2}\}$	$U6 = \{C_{v1}, C_{v2}, C_{v3}, R_{f1}, R_{f2}, \lambda\}$

In IC-6 set representation of dynamics [21, 31] we have

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ -u/r^3 x + a_x \\ -u/r^3 y + a_y \\ -u/r^3 z + a_z \end{bmatrix} \quad (1)$$

where

x, y, z	IC components of $\boldsymbol{r}$ (see Fig. 1(b))
v_x, v_y, v_z	IC components of $\boldsymbol{v}$
$\boldsymbol{r}, \boldsymbol{v}$	position and velocity vectors of an orbiting body
u	gravitational parameter for the Earth
a_x, a_y, a_z	IC components of perturbing accelerations

In this set, position and velocity determine the orbit. However, for the geometry of the orbit we need conic section parameters. But ultimately we need to compute position and velocity at each instant of processing the measurements

Another useful set for orbital dynamics is based on Altman's unified state model (USM) [21, 32, 33]. In USM both orbit and attitude dynamics of an orbiting body can he described in a unified manner. We describe only the orbital part of the model. The attractive properties of U7 (seven states) and U6 (six states) of USM are given in Table II. One of the important properties is that the parameters of this representation are free from classical singularities. However, retrograde and deep-space mission type (rectilinear) orbits cannot be represented in these velocity-space sets.

The USM is based on the orbit description in the velocity-space as against the conventional position-space. The circular, elliptic, parabolic, and hyperbolic orbits (of position-space) transform into circular velocity-hodographs in the velocity-space (Fig. 1).

The four-parameter set of Euler parameters (e_{01}, e_{02}, e_{03}, e_{04}) and the parametric variables (C, R_{f1}, R_{f2}) together define the orbital states in the instantaneous orbital plane [21]. Consequently we have

$$\frac{d}{dt}\begin{bmatrix} e_{01} \\ e_{02} \\ e_{03} \\ e_{04} \end{bmatrix} = 1/2 \begin{bmatrix} 0 & \omega_3 & 0 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & 0 \\ 0 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & 0 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} e_{01} \\ e_{02} \\ e_{03} \\ e_{04} \end{bmatrix} \quad (2)$$

and

$$\frac{d}{dt}\begin{bmatrix} C \\ R_{f1} \\ R_{f2} \end{bmatrix} = \begin{bmatrix} 0 & -p & 0 \\ \cos\lambda & -(1+p)\sin\lambda & \frac{-v R_{f2}}{V_{e2}} \\ \sin\lambda & (1+p)\cos A & \frac{v R_{f1}}{V_{e2}} \end{bmatrix} \begin{bmatrix} a_{e1} \\ a_{e2} \\ a_{e3} \end{bmatrix} \quad (3)$$

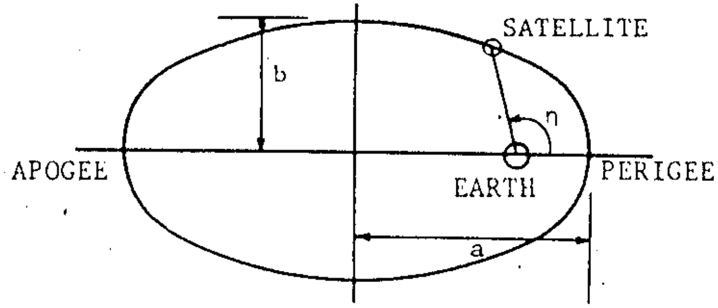
where

C	u/h (u is defined earlier)
h	specific angular momentum
R	eC
e	eccentricity of the orbit
R_{f1}, R_{f2}	components of vector $\boldsymbol{R}$ ($\lvert\boldsymbol{R}\rvert = R$; $R_{f3} = 0$) (see [21, fig. 2.3])
e_{0i}	the four Euler parameters describing the rotation from the inertial frame to the velocity-orbital frame

and $e_{01}^2 + e_{02}^2 + e_{03}^2 + e_{04}^2 = 1$. The associated auxiliary equations are given in the Appendix. The a_{ei} are the velocity-space components of the perturbing acceleration. The operational details for these are given in [33].

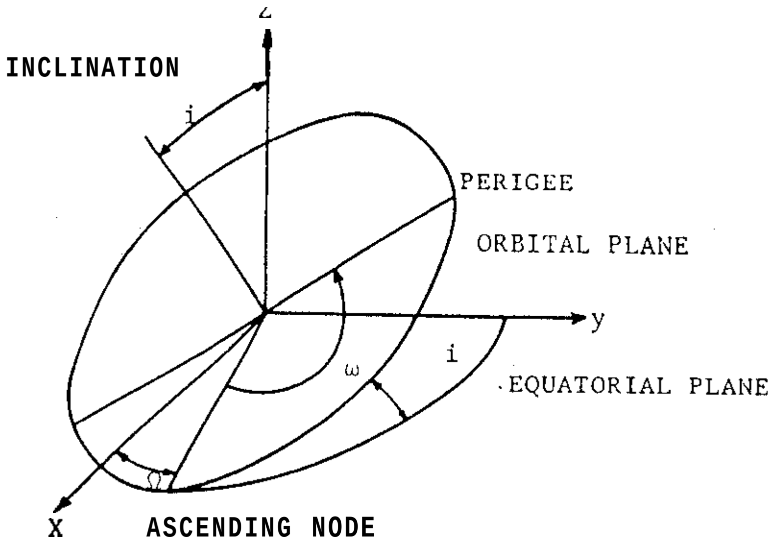
It would often be advantageous to combine two parameter sets in order to derive certain advantages from

a = semi-major axis
b = semi-minor axis
e = eccentricity
i = inclination
ω = argument of perifocus
Ω = argument of ascending node
η = true anamoly

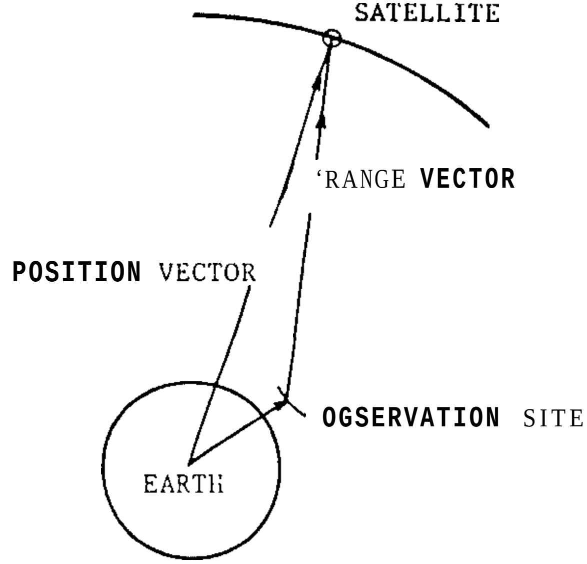


$C6 = \{a, e, i, \omega, \Omega, \eta\}$

(a)



(b)

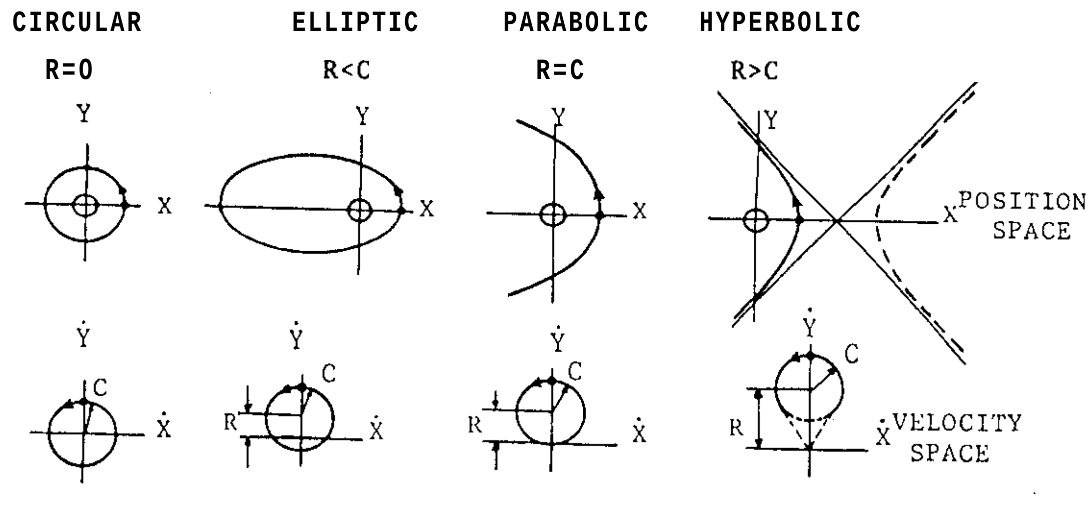


$IC6 = \{x, y, z, v^x, v^y, v\}$

x,y,z components of position vector

v_x, v_y, v_z components of velocity vector

(c)



(d)

Fig. 1. (a) Shape and size of orbit. (b) Classical re!. (c) Inertial coordinate set. (d) Position and velocity space orbits.

both. One such combination is IC-6 set for filter and U6 (USM set with 6 elements) for integration (time-evolution of state dynamics). The former permits slightly better use of computer core and the latter is efficient from the speed and accuracy point of view [22]. We use the IC-6 and U7 combination for similar and other reasons for SARSAT satellite orbit determination [31].

2) Measurement Model: The measurement model is defined by the type of measurements to be made. It includes the specification of the coordinates of the observation sites, Earth model, and certain coordinate transformation. The conventional position-space or regularized velocity-space interpretations can be used for this model description.

The position-space measurement model [21] is described here. The coordinates of the Earth-based observation site are given by

$$R_T = \begin{bmatrix} X_c \cos\theta \\ X_c \sin\theta \\ Y_c \end{bmatrix} \tag{4}$$

where

$$X_c = \left[\frac{a_e}{\sqrt{(1 - e_e^2 \sin^2\Phi_T)}} + H_T\right] \cos\Phi_T \tag{5}$$

$$Y_c = \left[\frac{a_e(1 - e_e^2)}{\sqrt{(1 - e_e^2 \sin^2\Phi_T)}} + H_T\right] \sin\Phi_T \tag{6}$$

- a_e semimajor axis of the ellipsoid of Earth
- e_e its eccentricity
- H_T height of the site
- Φ_T the geodetic latitude of the site
- λ_T the longitude of the site
- θ the local sidereal time, $0 \le \theta \le 2\pi$ (see Appendix).

Then vector $\boldsymbol{\rho}$ from the site to the orbiting body has inertial frame components (see Fig. 1(b)).

$$\boldsymbol{\rho} = \boldsymbol{r} - \boldsymbol{R}_T. \tag{7}$$

The components of $\boldsymbol{\rho}$ vector referred to the topocentric-coordinate frame at the site are given by

$$\boldsymbol{\rho}_T = E_T\,\boldsymbol{\rho} = [\rho_{T1}, \rho_{T2}, \rho_{T3}] \quad \text{(see Appendix for } E_T\text{)}. \tag{8}$$

The observables azimuth and elevation are defined in terms of these components as follows:

$$\mathrm{Az} = \tan^{-1} \rho_{T1}/\rho_{T2} \tag{9}$$

$$\mathrm{El} = \tan^{-1} \rho_{T3}/\sqrt{(\rho_{T1}^2 + \rho_{T2}^2)}. \tag{10}$$

The range ρ can be defined in terms of inertial frame components

$$\rho = |\boldsymbol{\rho}| = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}. \tag{11}$$

Often position-space observations can be transformed into velocity-space maps [32]. These transformations and the state maps can be used for development of the orbit observation matrix used with the unified state matrix in an estimator. For details on this type of representation see [32]. In [32] it is shown that in bispherical coordinates the field of observation maps for a ground-based tracking system site is a degenerate form of the general field of observation maps for a satellite-based tracking site. Such a set can be exploited for OBOD/ODP.

The rotation of Earth should be accounted for by a suitable model. If real observations are used then the effects of precision and mutation should be included to ensure that an accurate estimate is obtained. An excellent account of USM model and various related aspects for U7 and U6 is given is [21].

3) Atmospheric Models and Perturbations: The dynamical equations must include the influence of perturbations in order to accurately model the physical system. The perturbing accelerations due to atmospheric drag and wind forces are defined for the USM in [33, Table III]. The perturbing accelerations due to the zonal and tesseral harmonics are defined for the USM in [33, Table IV].

It is interesting to note that if the USM is used for time propagation of the states (orbital states), then these accelerations can be easily incorporated in the dynamical equations and these states are then transformed to, say, IC-6 states to be used by the estimator. Since these perturbing accelerations contribute to the complexity of the dynamical equation, the accelerations are generally excluded from the relinearization process. This simplifies the computation of the transition matrix.

B. Measurements

The choice of the kind or type of measurements [bo] should be used depends on the problem at hand. For autonomy of navigation the onboard measurements obtainable from star-tracker, horizon tracker, or multispectral/landmark trackers can be used [34–36].

A system that utilizes a landmark tracker as the navigation sensor for autonomous orbit navigation is considered in [34]. Such a device has the capability of measuring the direction of the vector connecting a satellite and an Earth-fixed landmark of known location. A set of angles are measured at a fixed sampling rate whenever a landmark is within the satellite's field of view.

In [35] the possibility of acquiring onboard the satellite, all the measurement information required for estimation of both spacecraft's attitude and orbital ephemeris, is examined in detail. This is generally achieved by using an Earth-observing multispectral scanner, a star tracker and a set of strapdown gyros. The information from these sensors can be transmitted to the ground and processed by an estimation technique. Alternatively a microcomputer onboard the satellite can process this information utilizing a computationally efficient, numerically stable, and accurate filtering algorithm. Subsequently the estimated orbital states can be transmitted to the ground station for mission data processing.

In [37] the use of satellite-to-satellite (STS) tracking data is considered for orbit determination. An STS tracking system makes measurements of such parameters as range, range rate, angles. and direction cosines to a spacecraft relative to a given tracking station. In two-way tracking a signal is transmitted from a well surveyed ground station to a spacecraft transponder which frequently translates the signal for retransmission directly back to the ground station or, as in the case of STS tracking, to another spacecraft. With a single synchronous relay satellite. an STS tracking system is capable of observing a near-Earth satellite during almost half of every orbit. Similar coverage of a satellite in a high inclination orbit would be difficult to obtain with a ground based system: SARSAT system provides a promising application of STS tracking system. Yet another kind of autonomous system with data of angle measurements between the moon and stars is considered in [38].

Conventional data type is a set of azimuth, elevation. range. and range-rate observables obtainable from a groundbased observation site of known coordinates. The angle measurements are with respect to a topocentric coordinate system located at the observation site. Range and range-rate can be specified in IC reference frame. The ranging data provide the means of establishing satellite position. The uncertainty is about 0.015 km rms. Azimuth and elevation angles cannot be measured with similar accuracy. Previous studies have shown that angle data supplemented with range data achieve acceptable accuracy in position estimation.

Occasionally various observables can be combined from different ground stations in order to achieve desired accuracies in position and velocity of a satellite. This choice should be balanced against the increased computational burden. In [39] the use of combined spacecraft-based optical observations and Earth-based radiometric observations to achieve accurate orbit determination of Voyager during the Jupiter encounter approach phase is considered. The combination of both data types from Voyager (I and II) encounters results in a Galilean satellite ephemeris significantly improved over that available to voyager from Earth-based observations.

Other mandatory data and constants include: coordinates of selected stars, landmark and ground stations. Earth model and geopotential constants for autonomous/nonautonomous orbit determination. It may be noted that uncertainties in these coordinates and parameters may reflect as additional modeling errors. These may be accounted for by estimating them as the additional states.

C. Hardware

Vehicle Mounted Sensor Systems (VMSS): The equipment complex for an autonomous satellite navigation system consists mainly of sensors for observing navigation quantities, and space-borne computer (SBC) to process prestored and sensors data.

In this section we briefly describe some of the VMSSs that can be used for attitude/orbit determination. References [38, 41] contain an excellent account of two such systems.

The space-sextant system (SSS) uses angle measurements between the moon and second magnitude (or brighter) stars in a Kalman navigation filter (KNF) to estimate vehicle position and velocity. The SSS is a lightweight, low power, high performance, onboard navigation system. It has a very high level of autonomy. It provides an accuracy of 0.241 km within 24 hours of commencement of navigation and achieves about 1.8 km ($\approx$ 1 nm) accuracy within 10 minutes [38, 41]. It is claimed [41] that the space-sextant can be used also as a sensor to allow spacecraft altitude detemiination to an accuracy of 1 arcsec. It requires data storage for lunar ephemeris and terrain height compensation if ultimate accuracy is required. Design concepts of the operational space sextant are discussed in [38].

The passive ranging interferometer system (PRAIS) [41] also uses a KNF. but it measures angles to existing radiometric landmarks and signal time of arrival from some of these landmarks with special characteristics. It is a completely strapped down sensor which only requires small antennas to be mounted on the Earth-pointing face of the spacecraft. The PRAIS navigation system uses simultaneous measurements to three landmark radars. Pseudorange (PRA) and interferometer landmark tracker (ILT) measurements are employed to each landmark. The ILT measurements quickly reduce navigation errors while the PAR measurements contribute to accurate steady state performance. The PRAIS provides potential accuracy on most orbits. up to about 0.015 km position error. It has no moving parts.

Electronics and ***Space-borne*** *Computers:* Autonomous systems require electronics and computers for acquisition and preprocessing of measurements and implementation of Kalman filter type algorithms. In [38] some features of the required electronics are described. In most of such systems the electronics package is an integral part of the entire system.

In what follows, we briefly state the requirements and characteristics of space-borne computers that may be utilized for implementation of the filtering algorithm.

Spacecraft computers have evolved from simple. hard-wired sequencers. to complex general-purpose machines with large memories and redundant processors. The reference [42] provides an excellent account of the SBCs used for control. The performance of SBCs measured by such factors as improved speed and larger memory has improved more than a factor of 1000 in the last 12 years. The cost. weight, and power to perform a given function are decreasing significantly. As missions become more complex and as spacecrafts operate for deep-space missions. spacecrafts must be capable of greater autonomy. Some of these functions are: attitude/orbit determination, image processing, feature extraction, data compression.

The recent paper [43] is a state of the art survey on SBCs. It compares space division (SD) space qualified computers CDC 469, DELCO M362S, GEDEC PDP-11 (bit slice), LITTON 4516E, RCA SCP-234, and Rockwell DF-224 on the basis of seven criteria: throughput, memory, I/O capability, power, reliability, technology, and radiation hardness. It also gives an account of six candidate SBCs: ATAC-16S, HTP, IBM, RCA SCP-050, Rockwell IDF-224, and TELEDYNE MECA 43, that have been or are being considered for various SD programs. The process of selecting a satellite computer system is explored in [44]. It describes the programmatic requirement considerations and also sheds light on new developments that will affect future SBC systems.

D. Other Software

The ODP/OBOD require coordinate transformation and numerical integration routines. If IC-6 set is used for filter and U6 or U7 for the integration, then naturally two-way coordinate transformation is required at every instant of observation data processing. If coordinate set other than IC-6 is used, then transformation to IC-6 is required in order to obtain position and velocity of the spacecraft. Details of such transformation equations/matrices are given in [21, 30].

Numerical integration of dynamical equations is required for calculating time-propagation of states. This is generally accomplished by Runge-Kutta methods [45]. Other alternatives used are Euler and predictor-corrector methods. In [46] the authors study, among other things, the effect of numerical integration step-size on the accuracy of (U-D) algorithms (to be described later) in orbit determination. For such algorithms, which numerical integration methods are most efficiently applied to the U-D differential equations [47] remains to be studied.

IV. ORBIT ESTIMATORS

Now we are ready to state the orbit estimation problem.

Let

$$\dot{x}(t) = f(x(t), t) + n(t) \quad (12)$$

$$z(t) = h(x(t), t) + v(t). \quad (13)$$

be the system and measurement models associated with particular orbiting spacecraft and data tracking system.

Here x is the vector of n-states of the chosen coordinate system and other augmented parameters; z is the vector of observables; f, h are known, nonlinear functional relationships; and n, v are process and measurement noise vectors.

Given these models, some a priori information on $x(0)$, n, and v and the (noisy!) measurements, we require the filtered estimates of x, the orbital states.

The position and velocity components of the satellite are obtained from the orbital state estimates by suitable transformations [21]. This is not required if the states are in the IC-6 parameter set.

We next describe important features of some of the commonly used estimation techniques to obtain the filtered or smoothed estimates of the orbital states of a satellite.

The references [10, 48–50] give lucid and detailed exposition of various types of estimators for linear and nonlinear problems. However, [4] is the only book form account of many numerically stable and accurate filtering algorithms [51]. It deals with square root information filter (SRIF), UD filters (UDF), and smoothers in great detail and gives algorithms and partial FORTRAN codings. Golub and Van Loan [52] is a very recent treatise wherein basic factorization techniques can be found.

In Table III we present salient features of some of the commonly used orbit estimators.

For details of derivations and other theoretical as well as numerical properties the cited references (Table I, column 4) may be consulted. We briefly describe some properties and problems associated with these discrete-time estimators relevant to orbit-determination applications.

A. GLSDC Technique

The Gaussian least squares differential correction (GLSDC) technique is used to iteratively estimate the initial state [53]. It provides successive approximations for the initial condition estimate:

$$x(t_0) = x(t_0) + \Delta x \quad (14)$$

where $\Delta x = (A^T W A)^{-1} A^T W \, \Delta z$

$\Delta z = \tilde{z} - z_c$

and

- A matrix of partial derivatives of observed functions with respect to the states
- W weighting matrix
- $\tilde{z}, z_c$ actual and predicted measurements (vector).

Numerical problems would arise if initial $x(t_0)$ states are too far from the minimizing x or the matrix $A^T W A$ is ill conditioned. Being batch mode, it needs all the measurement data before it can be used to estimate $x(0)$. Hence it cannot be used for real-time filtering. Once the initial state is accurately estimated, any good numerical integration scheme can be used to predict the state-time history over the entire measurement-data span. In order to obtain accurate estimates this technique might require many iterations.

TABLE III
Estimators for OD

Estimator	Estimation	Mode	Convergence/Computational Aspects	Applications
Gaussian least squares differential correction technique (GLSDC)	Initial state	Batch iterative	Not fail-safe use better numerical algorithm to calculate Δx	To many nonlinear estimation problems [53]
Extended Kalman filter (EKF)	State parameter	Recursive. Iterations done to improve accuracy	Prone to divergence P may become nonpositive definite-use of patch-up techniques to prevent divergence Computationally efficient	Numerous For attitude/orbit estimation [10, 19, 21, 22, 73]
Adaptive filters (AF)	State-parameter/Q, R, modeling errors	Recursive	Improvement over EKF computational burden increased over EKF (estimates Q, R, modeling error adaptively to prevent divergence)	Orbit determination/reentry trajectory [10, 56, 57]
SRIF/SRCF	State parameter	Sequential	Definite improvement over EKF. Computationally not burdensome uses square root of information matrix $\Lambda = P^{-1}$ or covariance matrix P	*Apollo Lunar missions Mariner 9 Mars orbiter Mariner 10 Venus-Mercury space probe Aircraft navigation
UD filter (UDF)	State parameter	Recursive Iterations can be made to improve accuracy	Similar to SRIF except that it uses U-D factors of $P(=UDU^T)$ Does not involve square roots Computationally efficient Suitable for onboard applications	*Viking Mars Voyager Jupiter spacecraft FPS-aided aircraft navigation Seasat altimetry calibration Aircraft and missile tracking
UDF/RTS smoother	State parameter	Forward pass-UD filter Backward pass-RTS or RTS-B smoother [64] Does not need measurement data for this pass Nonreal time	No convergence problems (Backward sweep) smoothing may improve accuracy Computationally not burdensome	Yet to appear
$\dot{U}$-$\dot{D}$ (U-D differential) filter ($\dot{U}$-$\dot{D}$F)	State parameter	Based on propagation of UD factors of continuous time equations	Computationally less efficient than UDF Many properties need to be explored	GPS [46]
L-D filter (LDF)	State parameter	Centers around L-D factors and exploits block rectangular structure of H matrix	Computationally more efficient than conventional LDF —	[71]
Sliding batch differential correction technique (SBDC)	State	Forgetting past observations Otherwise same as differential correction technique	Performance compares with EKF	TDRSS OROD [73]
Pugachev filter (PF)	States (nonlinear models)/unknown parameters	Recursive	Complex calculations of gains Filter simple Numerical properties need to be explored	[75]

*See Applications References of [78].

B. Extended Kalman Filter

The filter is generally given in two algorithms.

Measurement update:

$$\hat{x}_i = \bar{x}_i + K_i(z_i - \hat{z}_i), \quad \text{(states)} \tag{15}$$

$$K_i = P_i H_i^T (H_i P_i H_i^T + R_i)^{-1}, \quad \text{(gain)} \tag{16}$$

$$P_i = P_i - K_i (P_i H_i^T)^T, \quad \text{(covariance)} \tag{17}$$

$$\hat{z}_i = h(x_i, i), \quad \text{(predicted measurements).} \tag{18}$$

Time update:

$$\bar{x}_{i+1} = \hat{x}_i + \int_{t_i}^{t_{i+1}} f(x(t|t_i), t)\,dt, \quad \text{(state integration)} \tag{19}$$

$$P_{i+1} = \Phi_i P_i \Phi_i^T + B_i Q_i B_i^T, \quad \text{(covariance)} \tag{20}$$

where

$\tilde{x}_i = x(t_i/t_{i-1})$	the one step predicted estimate of **x**,
$\hat{x}_i = x(t_i/t_i)$	the filter estimate
$\tilde{P}_i, \hat{P}_i$	predicted and filter estimate error covariance matrices
H_i	measurement matrix ($H = \partial h/\partial \mathbf{x}^T$)
Q_i, R_i	process noise and measurement noise covariance matrices
Φ_i	state transition matrix ($\Phi = e^{F(t_i - t_{i-1})}$, $F = \partial f/\partial \mathbf{x}^T$)
i	discrete-time index.

The filter obtains estimates of the states recursively and can be used in real-time mode.

In off-line recursive iterative (ORI) mode the filter starts processing data after all the measurements have been collected. Then starting from the final state estimate the filter is run backward in time to process measurement data (or to predict the states) up to initial time. This can be called forward-backward filtering (FBF). For nonlinear problems this mode is used to improve the estimates' accuracy. It is not clear whether this mode will always converge to the same estimates as obtained by the batch least squares formulation.

Although the extended Kalman filter (EKF) has been applied to numerous aerospace problems, the phenomenon of divergence associated with its implementation has caused a great deal of concern. When errors in the model build up over a period of time and cause a significant degradation in the accuracy of the estimate, divergence is said to occur.

Divergence may occur due to [4]:

1) the use of incorrect a priori statistics and unmodeled parameters (P, Q, R chosen incorrectly, incomplete knowledge of system models)
2) the presence of nonlinearities when linear models are used
3) the effect of computer roundoff due to finite word-length implementation.

The effects of type 1) are analyzed in [6–9]. For problems of type 2) iterated EKF, second-order filter [55], and J-adaptive filter [57] have been proposed. The problems of type 3) are elegantly addressed in [4].

In [58] a convergence analysis of the EKF is given for the combined parameter and state estimation problem for linear systems with unknown parameters.

C. Adaptive Filters

Adaptive estimation may be loosely defined as an estimation or filtering process which can adapt to any environment by appropriately changing or estimating the system model while extracting all available information from the data. An accurate model is a solution to the problem of divergence due to type 2). The model errors covariance matrix Q can be determined so as to produce consistency between residuals and their statistics. Real-time feedback is thus provided from the residuals to the filter gain. In [59] Mehra gives detailed exposition of several adaptive filtering techniques. In [57] the J-adaptive estimator is given. To the right-hand side of the differential, or difference, equation representing system model is added a low frequency random forcing function $u(t)$ representing the model errors:

$$\dot{x} = f(x,t) + G\,u(t) \tag{21}$$

where G $u(t)$ represents the model errors.

The J-adaptive sequential estimator tracks this function as well as the system state, thus adapting to any observable model or environmental variations. It must be noted here that the adaptive filtering would increase the computational burden.

Recently an adaptive robustizing approach to Kalman filtering has been considered in [60]. A robust Kalman filter based on the m-interval polynomial approximation (MIPA) method for unknown non-Gaussian noise is proposed. The MIPA Kalman filter is shown to be computationally feasible, more efficient, and robust as compared with other non-Gaussian filters. A recent account on adaptive filtering is found in [61].

D. Factorization Methods

These techniques [4] mainly address the problem of divergence due to effect of type 3), mentioned earlier. The effects of numerical errors are generally manifested in computed covariance matrices that fail to retain nonnegativity. There have been in use several methods to improve accuracy and to maintain nonnegativity and symmetry of the computer covariances, e.g., periodic testing and resetting of the diagonal and the off-diagonal elements [21].

An alternative to such patching techniques is to modify or replace the algorithm by one that is numerically reliable. Such alternative algorithms involve square-rooting or factorizing the covariance or information matrices. The algorithms exhibit improved numerical accuracies and reliability as compared to the adhoc methods [4]. The factorization implicitly preserves the symmetry and guarantees nonnegative eigenvalues of the covariance matrix. The improved behavior of the new algorithm can be seen as follows. Suppose the computation involves the numbers ranging from 10^{-N} to 10^{N}. Due to square-root type factorization this range will become $10^{-N/2}$ to $10^{N/2}$. As a result of this, the new algorithm implemented in 1-precision would obtain accuracies that are obtainable with 2-precision implementation of the conventional Kalman filter [4].

The references [17, 62] survey and compare discrete square-root filters on the basis of operations count. We briefly describe principles of some such filters.

SRIF: Consider a linear system corrupted by unit covariance noise [63]

$$z = Ax + v; \qquad A(m,n) \tag{22}$$

where

$E(v) = 0, E(vv^T) = I$ (identity), and $m \geq n$

If $\bar{x}$ and $\bar{P}_x$ are the a priori estimate and covariances, then

$$\bar{P}_x = \bar{R}_x^{-1} \bar{R}_x^{-T} \tag{23}$$

and

$$\bar{z}_x \triangleq \bar{R}_x \bar{x} = R_x x + \bar{v}. \tag{24}$$

A computationally accurate method of solving (22) **is** to construct an orthogonal transformation $V, (VV^T = I)$, such that

$$VA = \begin{bmatrix} \hat{R}_x \\ 0 \end{bmatrix} \begin{matrix} \}^{n} \\ \}_{m-n} \end{matrix} \tag{25}$$

where R_x is upper triangular. and nonsingular when $\boldsymbol{A}$ has full rank. n.

Also,

$$V[A|z] = \begin{bmatrix} \hat{R}_x & \hat{z}_x \\ 0 & z' \end{bmatrix}. \tag{26}$$

The importance of the array $[\hat{R}_x, \hat{z}_x]$ is that one can construct the minimum variance estimate of $x, \hat{x}$ and its covariance $\hat{P}_x$ from it. Finally. we have (details omitted):

$$\hat{x} = \hat{R}_x^{-1} \hat{z}_x \tag{27}$$

$$\hat{P}_x = \hat{R}_x^{-1} \hat{R}_x^{-T}. \tag{28}$$

The pair (R, z) is called the information array. and $z = Rx + v$ is called the data equation. The complete description of SRIF is given in [4].

UDF: The UDF centers around factorizing and updating the factors of covariance matrix P:

$$P = UDU^T \tag{29}$$

where U is unit upper triangular matrix. and D is the diagonal matrix.

The measurement and time updating parts of the UDF are given by [4, Theorems V.3.1 and V1.4.11. respectively. These two algorithms together with the time propagation of states constitute the UDF, which is algebraically equivalent to the Kalman filter but is numerically reliable and accurate. In [64–66] accuracy, sensitivity, stability. efficiency. error analysis, and cost are further elaborated for UDFs. The UDF does not involve arithmetic square roots and hence it is computationally more efficient than SRIF. It is recursive and seems to be most suitable for the OBOD program.

The effect of a priori statistic and poor observability on Kalman and UDFs is demonstrated by the following example [64, © 1976 IFAC].

Let

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & \epsilon \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix},$$

$$x_1(0) = 0 = \dot{x}_2(0) \tag{30}$$

where $\boldsymbol{n}$ is normalized and y_1 and y_2 are the observations to be processed. The a priori statistic P is chosen to have a large value of $\sigma^2 \boldsymbol{I}$ where $\sigma = I/\epsilon$ and $\boldsymbol{I}$ is an identity matrix. This choice reflects the lack of information about the system.

Partial results for the conventional Kalman filter and UDF are shown in the table at the bottom of this page. It is obvious from the table that the Kalman filter algorithm computes negative diagonal entries in the covariance matrix but UD factorization doesn't. For detailed comparison with other Kalman type and factorization algorithms, [64] must be consulted.

UDFIRTS Smoother: Smoothing is **a** non-real-time data processing scheme that uses all measurements between 0 and $\boldsymbol{T}$ to estimate the state of a system at a certain time t, where $0 \leq t \leq T$. **An** optimal smoother can be thought of as a suitable combination of two optimal filters. We describe fixed interval smoother, wherein the initial **and** final times 0 and T are **fixed** and the estimate $x(t/T)$ or equivalently for discrete-time $x(j/N)$, is sought.

The. UDF-RTS smoother [67] involves smoothing as a backward-pass scan to a forward-pass UD filtering. It uses UDF outputs. The backward-pass RTS recursion that generates smooth estimates and estimate error covariance is:

filter parameters	*exact values*	*conventional Kalman filter*	*UD factorization filter*
P_1 covariance	$\frac{1}{\alpha}\begin{bmatrix} 2 & -\sigma \\ -\sigma & \sigma^2+1 \end{bmatrix}$	$\begin{bmatrix} 0 & -\sigma \\ -\sigma & \sigma^2 \end{bmatrix}$	$\begin{bmatrix} 2 & -\sigma \\ -\sigma & \sigma^2 \end{bmatrix}^*$
P_2 covariance	$\frac{1}{\Delta}\begin{bmatrix} \alpha & -(1+\epsilon) \\ -(1+\epsilon) & 2+\epsilon^2 \end{bmatrix}$	$\frac{1}{\Delta r}\begin{bmatrix} -1 & +1 \\ +1 & 0 \end{bmatrix}$	$\frac{1}{\Delta r}\begin{bmatrix} 1 & -(1+\epsilon) \\ -(1+\epsilon) & 2 \end{bmatrix}^*$

*Since only the UD factors are updated, the covariances are not part of the algorithm. $\alpha = 1 + 2\epsilon^2$; $\Delta = 1 - 2\epsilon + 2\epsilon^2(2 + \epsilon^2)$; $\Delta r = 1 - 2\epsilon$.

$$x_{j/N} = x_{j/j} + G_j(x_{j+1/N} - x_{j+1/j}) \quad (31)$$

$$P_{j/N} = P_{j/j} + G_j(P_{j+1/N} - P_{j+1/j})G_j^T \quad (32)$$

where

$$G_j = P_{j/j}\,\Phi_j^T\,P_{j+1/j}^{-1},\; j = N-1, \ldots, 0 \quad (33)$$

with N the number of data points.

The variables without N are the outputs of the filter and have their usual n ing [50, 67].

This is a fixed-interval smoother that does not use measurements during a backward pass. The decomposition of the linear model dynamical equations and maximum use of rank-1 matrix modification yield a new algorithm based on RTS smoother recursion [67]. It is claimed that this new smoother is efficient, reliable, and reduces computer storage. In [31, 68] we consider the application of EKF, UDF, and these smoothers to SARSAT orbit determination.

U-D Filter: This filter allows the integration of the continuous state-error covariance differential equations in UDU^T form. This feature combined with a triangular measurement update algorithm obtains a complete square-root estimation algorithm for which square roots are avoided. The detailed formulation of the algorithm can be found in [46]. The U-D continuous-time propagation algorithm appears to be a promising solution to the continuous-time filter problem. However, incorporation of Markov colored noise and bias parameters and development of methods for directly computing U-D steady-state matrix factors remain to be studied. In this context it will be interesting to study the technical discussion that further illuminates various properties of continuous and discrete UDFs and compares them from the perspectives of accuracy and efficiency (47, 69].

L-D Filter: The measurement update algorithm for a lower unit triangular matrix L is given in [70, 71]. Also given is more efficient L-D algorithm that is superior to the conventional one when the ratio between the dimension of the state vector and the number of states directly related to the measurements is greater than one. The new algorithm takes the advantage of the block rectangular structure of

$$H = [{}^{mr}H : 0] \quad (34)$$

where ${}^{mr}H$ is an $m \times r$ submatrix of H, the measurement matrix associated with the measurement model. The measurement vector z is dependent on the first r states only. This structure is exploited to arrive at the new algorithm. It is based on separating the conventional *L-D* factors update scheme [4, 48] into a sequence of *L-D* subfactor updates. In a typical case of an inertial navigation system (INS) of 27 states with 3 measured states, the computational savings are by a factor of about .3 [71].

Yet another way of factorization is described in [72]: the matrix continued fraction expansion of a covariance matrix.

E. SBDC Technique

In a very recent paper [73] an EKF and a sliding batch differential corrector (SBDC) method are compared in terms of 1) accuracy using a nominal tracking schedule. effect of reducing the tracking schedule. 2) accuracy in the presence of anomalous or deleted passes of data, 3) effect of onboard frequency standard errors, and 4) effect of large tracking and data relay satellite system (TDRSS) ephemeris errors.

The SBDC is a variant of the conventional batch differential correcton orbit determination. The data span continues to slide forward, picking up a pass of new data and dropping off a pass of old data. A pass of data means a set of data collected within a shon time span. Only the orbital elements that were determined in a previous slide are propagated forward to serve as starting values for the next slide. Then a new solution is attempted using these data. The details of the technique may be found in [73, ref. 2–41.

F. Pugachev Filter

Pugachev considers the problem of conditionally optimal estimation and extrapolation of the state variables and of estimation of unknown parameters in nonlinear systems described by differential equations [74]. It is claimed that the general theory of conditionally optimal estimation includes all the efficient methods of nonlinear filtering. the respective filters being admissible optimal filters in the corresponding classes.

Let a stochastic system be described by the first-order difference equation [75]

$$x_{i+1} = f_i(x_i, v_i) \quad (35)$$

where f_i is a known, nonlinear function. and $\{v_i\}$ is a sequence of independent random variables with known distributions. The sequence of $\{z_i\}$ of random vectors

$$z_i = h_i(x_i, v_i) \quad (36)$$

is observed, h_i being a known function.

An optimal estimate of x_i, x_i is required. minimizing the mean square error $E(|\hat{x}_i - x_i|^2)$ in the class of functions of $x_1, \ldots, x_{i-p}$, determined by difference equations of the form

$$x_{i+p} = \mathrm{S,p} + \Upsilon_i \quad (37)$$

for all the values of δ_i and Υ_i; the integer p and the funtion ζ_i being given. and

$$\beta = \zeta_i(\hat{x}_i, \ldots \hat{x}_{i+p-1}, z_i). \quad (38)$$

The optimal values of filter gain δ_i and Υ_i are given by

$$\delta_i = L_i K_i^{-1} \quad (39)$$

$$\Upsilon_i = m_0^{(i)} - \delta_i \ell_i \quad (40)$$

where

$$L_i = E(x_{i+p} - E(x_{i+p}))\beta^T \quad (41)$$

$$K_i = E(\beta - E(\beta))\beta^T \quad (42)$$

$$m_0^{(i)} = E(x_{i+p}), \qquad \ell_i = E(\beta) \quad (43)$$

and E indicates expectation.

It is obvious that the filter gain computations are quite involved. If they are not required to be done in real-time, then it can be easily seen from (37) and (38) that the Pugachev filter is computationally very simple.

The numerical properties and computational aspects of this new filter have not yet been fully explored. The orbit determination problem. being nonlinear, offers a potential application of this filter. The filter recursion, being simple, can be easily implemented on a microcomputer. However, the filter gain computation seems to be a very complex process. The application of this filter could not be traced except in [75] where it is used for a linear problem.

Remarks: There have been many other types of estimators in use for the problem of orbit determination (OD). We chose to compare only those estimation techniques which have been widely used and/or follow a sort of evolutionary pattern. Table III reveals that the first five types of filters have been applied to numerous aerospace problems. The remaining algorithms or their applications are relatively new. These techniques offer options and opportunities for further research on their theoretical claims, numerical and computational properties, and implementation.

V. COMPUTATIONAL ASPECTS

We briefly discuss some computational aspects related to measurement data editing and filter implementation.

Data Editing

An ODM must include features of data editing. The editing may be built into the OD program or may be accomplished prior to beginning the filtering. It is common to use the current orbit estimate to form a set of measurement residuals on which to perform the editing. The measurement data is ignored if its residual is larger than some specified value or if its residual deviates from the mean residual for that measurement type by more than some number of standard deviations [76].

If measurements are available more frequently than they can be processed, then prefiltering or data compression is applied. An average of the measurements (over cycle time of filter computation) is used every cycle time. It is interesting to note that the original noise is smoothed, but an additional error due to smoothing of the signal occurs [50. ch. 8]. Many other filter implementation aspects are also discussed in [50]. More frequent data can also be handled in a two-loop implementation of the EKF [81].

Filter

If a Kalman filter type algorithm is to be used as a batch-processor then the observations are processed at the end of each satellite pass. This does not im[illegible]e real-time requirement on the SBC. However. if orbital estimates are required in real-time. say for online corrective action. then a reasonably kist processor would be required. The entire computation for estimation of states per measurement data should be done well within one sampling interval. If ohservarions are available at a very fast rate and if all of them are to be considered then the choice becomes very crucial. Computational requirements for implementation of the discrete Kalman filter are given in [77]. The time and storage requirements have been thoroughly analyzed. Logic times are also given. Using this infomiation in conjunction with the clock rate; arithmetic operation times. and instruction cycle time of a given SBC. the total computation time required for the filter to process a single (or vector) observation can be estimated. Similar calculations can be performed for factorization methods based on [4, 63].

Although the need for real-time orbit estimation may not be great. Kalman filtering and equivalently factorization methods (especially U-D filter) offer several advantages over the weighted least-squares methods for OBOD. For OBOD/SBC the major software design constraints might be processor weight. size. and power; hence computationally efficient and numerically reliable orbit estimators have much to offer.

VI. APPLICATION REVIEW

Most of the techniques discussed in Section IV have been applied to various aerospace problems: orbit determination, attitude estimation, interplanetary navigation of the Viking-Mars and Voyager-Jupiter spacecraft. GPS-aided aircraft navigation, seasat altimetry, calibration. aircraft and missile tracking. etc. [18, 21. 22. 34. 39. 56, 57. 73. 78]. Also see applications references of [78].

In Table IV we collect representative application results of some estimators for orbit determination and related problems. These results arc presented with respect to some major aspects of ODP/OBOD which we have described in the previous sections. It may be noted that the information not discussed or only implicitly mentioned in the relevant reference is denoted by a dash.

One of the earliest papers on OBOD is [34]. It presents an analysis of the accuracies obtainable with an ad hoc autonomous orbit navigation system. The analysis was made realistic by including various types of errors in initial condition, sensor, models, algorithm, and computations. Digital simulation of the actual navigation algorithms was performed with all error sources included. Authors also gave some opinions on computational aspects and SBC requirements.

Orbit Determination Results

Type of Orbit/System	Model Set	Measurement Type	Estimator	Implementation	Filter Execution Time (FET)	Numerical Problems	Accuracies Position in km (P) Velocity in cm/s (V)	Remarks
Autonomous orbit navigation Near-Earth, inclined orbit [34] (1968)	IC-6	Landmark tracking	Kalman filters	IBM 7094	15 min/1200 min run	Without Q matrix, the filter gain decays by about three orders of magnitude	P ~ 0.18	FET includes time to write on tape. Detailed analysis of effects of various error sources. 12 known landmarks are utilized. At least one landmark was visible per orbital revolution.
Joint attitude, orbit estimation 1000 km altitude, inclined orbit [35] (1975)	IC6. Orbital spacecraft 12 state model	Line of sight (LOS) to stars and known landmarks	EKF	—	—	—	P ~ 0.01 yaw ~ 15 arcsec roll pitch ~ 1 arcsec	Star measurements are major contributions to attitude accuracy. Star and landmark measurement govern the accuracy of orbital estimates.
Portion of MJS (1977) deep space mission Approach to Saturn [79] (1977)	IC 19-state model	Earth-based Doppler and range data	Conventional Kalman filter Joseph's Stab. UDF. Potter-Schmidt square root	UNIVAC 1108 27 bit (8–9 digit)1-p 60 bit (18 decdigits)2-p	1-p (seconds)2-p 29 49 45 59 38 46 63 80	1-p Kalman filters compute negative variances	All 2-p filters and U-D and Potter 1-p filters achieve similar accuracies P ~ 1.0 V ~ 5.0	U-D filter combines numerical reliability with computational efficiency. Most suitable for autonomous navigation. FET is for entire data span.
ATS-6/GEO-3 and ATS-6/NIMBUS [37] (1978)		Satellite-to-satellite tracking data	A Bayesian least squares technique	—	—	—	GEOSC-C P ~ 0.05 NIMBUS-6 P ~ 0.04	ATS-6—geostationary satellite.
GPS (12 h) [46] (1980)	IC	Pseudo range, range rate	EKF (Φ) EKF (P) U-D U-D	CDC6600	milliseconds 72.7 83.6 90.7 109.8	—	P ~ 0.147 V ~ 44	FETs are given in milliseconds per estimation cycle. Simulation of LANDSAT-D processing data from the Phase I GPS.
GPS [82] (1980)	IC user's coordinates	Pseudo range Pseudo delta range	SPWR algorithm	1-precision 32 bit	—	—	User vehicle's accuracy Altitude 0.0097 Horizontal position 0.011 Vertical velocity ~ 38 Horizontal velocity ~ 87	The SPWR filter has two loop (fast and slow) implementation and consists of an EKF algorithm with the covariances expressed in U-D factors.
SARSAT (2 h) [21] (1981)	IC, U7, U6	Azimuth, elevation, range, range rate, various combinations	EKF	Honeywell Sigma 9	—	Diagonalization technique used to prevent divergence	P ~ 0.1 V ~ 100	Multiple sites also considered.
SARSAT [22] (1981)	IC IC-U6	Range rate	Epoch-time filter EKF				P 3 V 30 P 0.5 V 50	Simulated and real data considered. EKF met the SARSAT accuracy requirement.
Near-Earth inclined orbits Landsat-4 GRO [73] (1983)	IC	TDRSS data 1-way delta range 2-way range delta range	EKF, SBDC	—	—	Two filters performed similarly Data severity will affect both equally	Landsat-4 P ~ 0.04-0.5 (9-12 h-data) GRO P ~ 0.14-34.5 (9-12 h-data)	Several combinations of data types and tracking schedules (revolutions) considered. Effect of various types of data error models is also studied.

The paper by White et al. [35] addressed the problem of joint attitude and orbit estimation using stars and landmarks. Their twelve-state approach to joint attitude-orbit determination achieved acceptable filter performance. Although star measurements did not directly affect the ephemeris-components, they provided the necessary attitude accuracy needed to optimally utilize the landmark information. This is an interesting application result for OBOD.

Reference [79] compares four filtering algorithms for their performance in orbit prediction. It highlights the numerical deficiencies of the conventional and stabilized Kalman algorithms. It is important to note that accuracies of the U-D filter using single precision **(I-p)** arithmetic consistently matched the double precision results of conventional filters. The U-D filter has excellent numerical properties and **is** computationally **as** efficient as Kalman filter. It is also relatively insensitive to the variations in the a priori statistics. The authors give systematic and realistic simulation results to support their claims; actually the claims are the results of their exhaustive study of the numerical and computational aspects of these filters. They study a complete 19-state (9 dynamic and 10 bias parameters) model. scaling of the a priori state and data covariances. and reduced-dimension problems. in every case of this comprehensive study the factorization algorithms outperformed all the Kalman algorithms. Similar studies for SARSAT OD are underway.

An Aside: An interesting study of performance degradation in digitally implemented Kalman filter was presented in [80]. The causes of divergence were investigated. A software was developed for the PDP-11/45 computer system which can perform various arithmetic operations as if the processor were of any given wordlength between 1 and 62 bits. The performance analysis was dons for 4. 8. 12, and 16 bit wordlengths. The simulation results were compared with analytical prediction and the agreement was found to be good. These results should be useful in deciding what word-length processor is required for a particular application to meet accuracy specifications. Similar studies for other algorithms would shed more light on their numerical properties. Such investigations would be very valuable for OBOD projects. Further research could be done in this direction.

Vonbun et al. [37] present results of using an STS tracking system data for orbit determination. The new tracking system type is offered as an alternative to gruund-based tracking system. The results of the ATS-6/GEOS-3 and ATS-6/NIMBUS-6 STS tracking orbit determination experiments are presented. User satellite orbits were determined with accuracies comparable to what is obtainable from ground tracking systems utilizing a Bayesian least squares estimation technique with a good a priori estimate of states of a relay satellite.

Tapley and Peters [46] compare the performance of EKF. UDF. and U-D (U-D continuous) filters in terms of efficiency and accuracy. The U-D filter is offered as an alternative to UDF. Although not faster. it was claimed to yield greater accuracy than UDF. Additional study is required to test the stability and adaptability of the algorithm. The authors study the effect of step size of the modified Euler integrator on position and velocity errors. We tabulate these results only for the case of step size equal to 3 s.

Another GPS study is represented in [81]. The authors use sequential piecewise recursive [SPWR] filter that has two-loop implementation. It updates the state and error covariance at different rates. This improves the navigation. accuracy by processing all available measurements without increasing the computational load on the processor. Real-time system implementation is also discussed. Assume that four satellites (GPS-NAVSTAR) are being tracked sequentially. The filter gains for each satellite are assumed to be initialized during the signal-acquisition phase. During the steady-state operation. the receiver is making measurements on each satellite, and these measurements are incorporated one at a time in the navigation-filter fast loop. In parallel with the fast loop. the covariance is updated in real-time, and the filter gain is computed in the slow loop for the satellites in the same sequence as their measurements are incorporated into the fast loop. In all simulations and field tests presented, the algorithm performed reliably.

Reference [21] gives results of using various parameter sets for system model and measurement types in EKF to estimate orbital states of near-Earth, polar orbit satellite. Required accuracies are obtainable for all the parameter sets. However. only use of range and/or range rate data did not achieve acceptable accuracy in the satellite's position. The best data set was considered to be of observables: azimuth, elevation, and range (from ground-based site). Reference [22] deals with the similar problem but uses only range rate data and compares the performance of Epoch time filter (ETF) and EKF for simulated as well as real data. ETF did not meet the SARSAT accuracy specification: 1 km position error. However. EKF did better than the specifications. This orbit estimator was baselined for the Canadian SARSAT LUT (Local User Terminal) [22]. These two references are an excellent study on SARSAT OD.

Dunham et al. [73] tackle the problem of OBOD with TDRSS. They compare EKF and SBDC estimators for two types of orbits: a high-inclination. near-circular orbit with an altitude of 700 km (Landsat-4) and a moderately inclined. lower altitude orbit. the Gamma Ray Observatory (GRO). The TDRSS is a system of three tracking and data relay satellites to be maintained in circular. near-equatorial. geosynchronous orbits. Two tracking modes were studied for use in OBOD: one-way Doppler and two-way range and/or Doppler. The one-way Doppler measurements would be extracted onboard the user spacecraft from tracking signals originating on the ground. relayed through a TDRS. and received by the user spacecraft. The two-way data would be extracted and time tagged on the ground from the round-trip propagation of the tracking signals; the resulting data arc collected and relayed back to the user spacecraft through the command link. The conclusion of their study was that the EKF and SBDC performed similarly when similar data were given.

Scanning these results we find that in many instances the aspects of filter implementation. computation time. and numerical problems have not been discussed explicitly. That the wordlength of the digital computer on which the algorithm is implemented plays an important role in a filter's performance, is well established by the results of [79, 80]. Since these aspects assume greater significance for an OBOD project. they must be well attended to.

VII. CONCLUDING REMARKS

We have addressed the problem of OD/OBOD in the light of its constituents and features of promising alternatives. Our review of estimators and their representative application results is meant to be viewed from the common perspectives of accuracy. reliability, and efficiency. The techniques discussed span a large evolutionary frame of time and development: conventional to modern practices in system models. data types. estimators. and orbit determination. We have mainly focused our attention on merits and demerits of various alternatives offered for OD/OBOD. Our csplicit tabular approach makes this very clear. Due to space limitation we have not included details regarding models and algorithms of filters. We have highlighted the important aspects of OBOD. In particular. stability and accuracy are crucial lo OBOD implementation and performance success. The factorization methods (e.g., UDF) being numerically stable and accurate and having long since established their reliability and utility offer promising alternatives for OBOD methodology. Further research is required on the theoretical claims, numerical and computational propenies. as well as implementation aspects in respect to many contemporary alternatives being offered for OD/OBOD.

It is hoped that this exposition will help researchers evolve an ODM with good numerical properties.

APPENDIX

The auxiliary equations related to USM are given as

$$\sin \lambda = e_{03} e_{04} / \beta$$

$$\cos A = (e_{04}^2 - e_{03}^2)/\beta$$

$$V_{e1} = R_{f1} \cos A + R_{f2} \sin \lambda$$

$$V_{e2} = R_{f2} \cos \lambda - R_{f1} \sin A + C$$

$$p = \frac{C}{V_{e2}}; \quad \omega_1 = a_{e3}/V_{e2}; \quad \omega_3 = CV_{e2}^2/u$$

and

$$v = \frac{e_{01} e_{03} - e_{02} e_{04}}{\beta}, \quad \beta = e_{03}^2 + e_{04}^2.$$

The local sidereal time or the instantaneous longitude of the site is given by [30].

$$\theta = \theta_1 + \theta_2 T_C + \theta_3 T_C^2 + (t) \frac{d\theta}{dt} + \lambda_T$$

$$\theta_1 = 99°.6909833$$

$$\theta_2 = 36000°.7689$$

$$\theta_3 = 0°.000038708$$

$$T_C = \frac{\text{J.D.} - 2415020}{36525} \text{ cent.}$$

J.D. = Julian Date

$$\frac{d\theta}{dt} = 0.25068477 \text{ deg/min.}$$

The E_T matrix is given by

$$E_T = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\cos\theta \cdot \sin\Phi_T & -\sin\theta \sin\Phi_T & \cos\Phi_T \\ \cos\theta\cos\Phi_T & \sin\theta\cos\Phi_T & \sin\Phi_T \end{bmatrix}$$

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