# ON THE MIRROR POINT DISTRIBUTIONS IN THE MAGNETOSPHERE

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THE charged particle motion in the Van Allen radiation belts is composed of a gyration about a line of force, a uniform motion along this field line and a longitudinal drift around the earth. In a treatment due to Alfvén,[1] this spiralling particle is regarded as an elementary magnetic dipole and it has been shown that under certain conditions, the magnetic moment of this dipole remains an adiabatic invariant of the motion.

The constancy of the magnetic moment leads to the expression

$$\frac{\sin^2\alpha}{H} = \text{Constant} \tag{1}$$

where $\alpha$ and H stand for the pitch angle of the particle and the intensity of the magnetic field where the particle is incident. If a charged particle crosses the equator at an angle $\alpha_e$, where the magnetic field intensity is $H_e$, equation (1) means that the particle gets reflected at a point, known as the mirror point, where the field intensity $H_M$ satisfies the equation

$$\frac{\sin^2\alpha_e}{H_e} = \frac{1}{H_M}. \tag{2}$$

If the earth's magnetic field is treated as that of a dipole, the lines of force and the field intensity have the form

$$r = \rho \sin^2\theta$$

$$H = \frac{M}{r^3}(1 + 3\cos^2\theta)^{\frac{1}{2}}. \tag{3}$$

where M is the magnetic moment of the earth's dipole, $\theta$ the colatitude, and $\rho$ is a constant. Equation (2) then reduces to

$$\frac{\sin^6\theta_M}{(1 + 3\cos^2\theta_M)^{\frac{1}{2}}} = \sin^2\alpha_e. \tag{4}$$

Here $\theta_M$ is the colatitude corresponding to $H_M$, at the point where the particle is reflected.

A table of numerical estimations, relating $\theta_M$ and $\alpha_e$ has been given by Hamilin *et al.*[2]

In this paper, we have considered the main field of the earth expressed as a spherical harmonic summation and the effect of the non-dipole terms on the mirror points is investigated.

In terms of the potential function, the main geomagnetic field is expressed as

$$V = a \sum_n \sum_m \left(\frac{a}{r}\right)^{n+1} P_n^m(\cos\theta)\{g_n^m \cos m\lambda + h_n^m \sin m\lambda\} \tag{5}$$

where $p_n^m(\cos\theta)$ is the normalized associated Legendre polynomial and $g_n^m$ and $h_n^m$ are the Gauss-Schmidt coefficients.[3] In our discussion, we have used the values of $g_n^m$ and $h_n^m$ as given by Kern and Vestine.[3] The radial, polar and the azimuthal components of the geomagnetic field are respectively given by

$$H_r = -\frac{\partial V}{\partial r}, \quad H_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta},$$

$$H_\lambda = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \lambda}. \tag{6}$$

Since $dr/H_r = r\,d\theta/H_\theta$, it follows that the equation to a line of force, in general, has the form

$$r = \rho \,\text{Exp.} \int_{90}^{\theta} \frac{H_r}{H_\theta}\, d\theta. \tag{7}$$

Our method consisted in solving, numerically, the equation (2) in conjunction with equations (5), (6) and (7) to obtain corresponding values of $\theta_M$ and $\alpha_e$. Numerical evaluations have been carried out on a Ferranti Sirius digital computer. In the formula for the geomagnetic field terms upto $n = 4$, $m = 4$ are retained. As a first approximation $r = \rho \sin^2\theta$ (dipole case) has been employed in the integrand of equation (7) and keeping $\lambda$ a constant in the expression of the magnetic field, $r$ is obtained at different $\theta_M$. Keeping $\lambda$ a constant is tantamount to considering the path of the charged particle on a plane $\lambda$ = constant. Further, the value of $r$ at $\theta = 90°$ has been regarded as the average of $r$ at $\theta = 89°$ and $r$ at $\theta = 91°$. From the table of values of $\theta_M$ and $\alpha_e$ thus obtained, $\theta'_M$s are re-evaluated at some desired values of $\alpha_e$ by means of inverse interpolation. The distribution of mirror points with respect to the longitude is shown schematically in Fig. 1. While the continuous curves

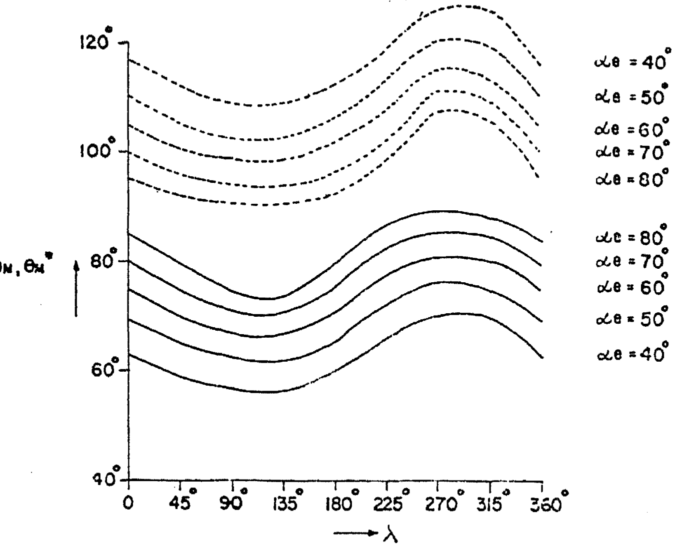


FIG. 1.

relate to the Northern hemisphere, the dotted curves pertain to the Southern hemisphere. In Table I, we have given numerical values of $\theta_M$ and $\theta_{M*}$ at some longitudes. It can be noted that the mirror point has a strong dependence on the longitude. Also, $\theta_M + \theta_{M*}$ is not equal to 180°, unlike the dipole case. The curves indicate that the distribution of $\theta_M$ with respect to the longitude has a similar pattern for all values of $a_e$.

TABLE I

| $a_e = 40°$ | | | $a_e = 80°$ | | |
|---|---|---|---|---|---|
| $\lambda^0$ | $\theta_M{}^0$ | $\theta_M{}^{*0}$ | $\lambda^0$ | $\theta_M{}^0$ | $\theta_M{}^{*0}$ |
| 0 | 62·92007 | 116·6652 | 0 | 85·1737 | 94·9865 |
| 45 | 59·17703 | 112·2483 | 45 | 79·1285 | 92·1337 |
| 90 | 57·0270 | 109·4838 | 90 | 74·6924 | 91·5340 |
| 135 | 56·3157 | 109·1483 | 135 | 73·9306 | 91·3635 |
| 180 | 59·5042 | 112·1550 | 180 | 79·6675 | 92·2540 |
| 225 | 66·1607 | 118·4004 | 225 | 86·5975 | 97·7513 |
| 270 | 70·8044 | 126·1906 | 270 | 88·7151 | 108·1319 |
| 315 | 69·8519 | 126·3337 | 315 | 88·5859 | 107·1669 |

M and M* are conjugate mirror points.

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1. Alfvén, *Cosmical Electrodynamics.*
2. Hamilin, D. A., *et al.*, *J. Geophys. Res.*, 1961, **66**, 1.
3. Kern, J. W. and Vestine, E. W., *Space Science Reviews*, 1963, **2**, 136.

# SEX-RATIO IN FISH POPULATIONS AS A FUNCTION OF SEXUAL DIFFERENCE IN GROWTH RATE

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DURING a study of the biology of some fishes from the inland waters of India (Qayyum and Qasim, 1964), the sex-ratios in the population of various species were determined by taking regular monthly samples over a period of one year or more. It then appeared that in some species like *Ophicephalus punctatus*, males were in the majority, whereas in others, *Barbus stigma* and *Callichrous bimaculatus*, females outnumbered the males. Such differences in the sex-ratio remained unexplained until the growth rate of the two sexes in each species was examined.

Table I gives the total length frequencies of all the species under investigation. It can be seen from the table that in *O. punctatus*, the males attain a size larger than the females. This was further confirmed by a study of the growth rate of the fish from opercular bones and scales (Qasim and Bhatt, 1966). In *B. stigma* and *C. bimaculatus*, on the other hand, the females grow bigger than the males. Since no reliable method of age determination in these two species could be found out, this inference was drawn by a study of the length frequency distribution alone. In *B. stigma* which attains a maximum size of 13·0 cm., fishes larger than 10·0 cm. were all females and the same was true in *C. bimaculatus* where all fishes larger than 26·0 cm. were females (Table I).

It therefore seems that the preponderance of one sex in the population is because of the sexual difference in growth rate. Faster growth

TABLE I

*Length frequencies and sex-ratios in three different freshwater fish populations of India*

| Species | *Ophicephalus punctatus* | | *Barbus stigma* | | *Callichrous bimaculatus* | |
|---|---|---|---|---|---|---|
| Length groups cm. | Males | Females | Males | Females | Males | Females |
| 3·0 | .. | .. | 12 | 19 | .. | .. |
| 4·0 | .. | .. | 31 | 28 | .. | .. |
| 5·0 | 33 | 29 | 12 | 19 | .. | .. |
| 6·0 | 28 | 29 | 28 | 25 | .. | .. |
| 7·0 | 33 | 26 | 30 | 33 | 2 | 1 |
| 8·0 | 38 | 23 | 96 | 68 | 5 | 1 |
| 9·0 | 74 | 74 | 34 | 118 | 16 | 12 |
| 10·0 | 75 | 59 | 7 | 47 | 24 | 33 |
| 11·0 | 58 | 49 | .. | 21 | 35 | 56 |
| 12·0 | 53 | 44 | .. | 5 | 31 | 49 |
| 13·0 | 54 | 57 | .. | 1 | 24 | 46 |
| 14·0 | 53 | 59 | .. | .. | 29 | 50 |
| 15·0 | 55 | 57 | .. | .. | 41 | 43 |
| 16·0 | 30 | 42 | .. | .. | 27 | 37 |
| 17·0 | 32 | 25 | .. | .. | 26 | 30 |
| 18·0 | 19 | 17 | .. | .. | 19 | 24 |
| 19·0 | 18 | 20 | .. | .. | 16 | 17 |
| 20·0 | 23 | 6 | .. | .. | 13 | 21 |
| 21·0 | 17 | 6 | .. | .. | 12 | 17 |
| 22·0 | 11 | 9 | .. | .. | 11 | 24 |
| 23·0 | 21 | 6 | .. | .. | 7 | 16 |
| 24·0 | 13 | 1 | .. | .. | 5 | 14 |
| 25·0 | 14 | .. | .. | .. | 2 | 10 |
| 26·0 | 10 | .. | .. | .. | 4 | 12 |
| 27·0 | 5 | .. | .. | .. | .. | 10 |
| 28·0 | 2 | .. | .. | .. | .. | 12 |
| 29·0 | 3 | .. | .. | .. | .. | .. |
| Total .. | 772 | 638 | 250 | 384 | 349 | 535 |
| Sex-ratio | 1 : 0·82 | | 1 : 1·5 | | 1 : 1·5 | |