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ALGORITHMS FOR AIRCRAFT TRIM ANALYSIS ON GROUND

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Abstract

In the analysis of aircraft dynamics for up and away flight, the general practice consists of trimming the aircraft for a specific manoeuvre at a specified altitude and mach number. In this paper conditions for quasi-steady equilibrium are motivated for aircraft dynamics in the ground roll phase. Algorithms for implementation of these conditions are also described. By use of these algorithms, the locus of trim points is obtained for a comprehensive take-off manoeuvre of a delta wing aircraft beginning with ground roll, followed by rotation and ending in lift-off. This result is compared with approximate calculations wherein the landing gear states are ignored and linear aerodynamics is assumed. The exact result is also compared with the simulation of the take-off manoeuvre in a pilot in the loop simulator.

Nomenclature

c.g.	: aircraft center of gravity
$C_{L\delta e}, C_{D\delta e}, C_{m\delta e}$	: elevator derivatives w.r.t drag, lift and pitching moment (at cg.)
D, L	: aerodynamic drag and lift forces
ELS	: **Engineer-in-the-Loop-Simulator**
F_{nose}, F_{main}	: ground reaction force on the nose and main landing gears
F_{port}, $F_{starboard}$	: ground reaction forces on the port and starboard main landing gears
GFA	: Generic Fighter Aircraft
H_E	: altitude. of the aircraft above the ground
k_{eqm}	: equivalent spring constant of the main landing gear (Fig. 6)
l_{no}, l_{mo}	: undeflected lengths of the nose and main landing gears (Fig. 6)
l_m	: deflected length of main landing gears under load R (Fig. 6)
l	: distance between main and nose wheel contact points (Fig. 6)
m	: aircraft mass
$\bar{q}$	: dynamic pressure $(=1/2\rho V^2)$
R	: normal reaction of main gear (Fig. 4)
P_B, Q_B, R_B	: x, y and z-body axis components of total rotation rate of aircraft
S	: aircraft wing area
T	: total thrust force generated by engine(s)
U_B, V_B, W_B	: x, y and z-body axis components of total velocity of aircraft
V_T	: aircraft total velocity of translation
V_{LOF}	: lift-off speed of aircraft
V_R	: rotation speed of aircraft on ground
W	: aircraft weight
x_t, z_t	: distances of thrust point along x-body and z-body axes (Fig. 5)
x_m, z_m	: dist. of main landing gear contact point in ground axes (Fig. 5)
X_E, Y_E	: position of aircraft along x and y earth fixed axes (on flat earth)

Greek Symbols

a	: angle of attack
β	: angle of sideslip
$6_e, \delta_a, \delta_r$	: surface deflections of the elevator, aileron and rudder
δ_{pla}	: power lever angle
θ_{LOF}	: lift-off pitch attitude of aircraft correspd. to lift-off speed V_{LOF}
θ_t	: angle of the thrust line with x-body axis in the xz plane
ϕ, θ, ψ	: euler angles orienting aircraft body axes to earth fixed axes
ρ	: atmospheric density

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Introduction

Trim analysis for various aircraft manoeuvres (straight and level, level turn etc.) is a standard and accepted procedure[1]. The author is not aware of similar analysis algorithms for accelerated aircraft manoeuvres on the ground such as take-off. In this paper the trim analysis methods described in reference 1 are generalised to include aircraft dynamics on the ground. This results in an exact calculation of the equilibrium points. Further, it is shown that the general practice of linearizing the equations of motion about an equilibrium point for up and away flight can be extended to the ground roll phase.

The algorithms have been implemented on a ground based simulator. In most aircraft design cycles, the flight simulator is brought in at an advanced stage where the usefulness of its results in optimising the design is limited. By introducing the flight simulator in the preliminary design stage and implementing the trim algorithms presented in this paper, the aircraft designer is fed back with more reliable information, which can be used to further optimise his design.

These trim algorithms are also very useful in initialising the aircraft to a deterministic and user specified flight condition on ground for a take-off or landing simulation. This is helpful for validating the landing gear modelling across various platforms against a given template response. Actual flight data could also be used as the template. Previously stored time histories from the same simulator are also sufficient to regenerate the response.

The conditions for quasi-steady equilibrium of aircraft on the ground are derived in Section 1 of this paper. The equilibrium point of this set of equations can be obtained by standard numerical constrained optimisation techniques. The current practice in aircraft industry for rake-off analysis makes several simplifying assumptions such as linear aerodynamics and consideration of only the longitudinal degrees of freedom, resulting in simple formulae used in the conceptual design stage. An example of this approach is presented in Section 2 of this paper. As preliminary design progresses and more aerodynamic data becomes available, the designer may wish to refine his earlier calculations. This may not be an easy undertaking, the reason being the high complexity of the aerodynamic database of a typical aircraft. If the algorithm implementation is not general enough, asymmetric operating conditions on rake-off (e.g., one engine fail, cross winds etc.) cannot be tackled. Therefore, there is a need for a general trim algorithm for obtaining the flight mechanics parameters of interest, which can work with any database without assumptions.

All the simulations and trim algorithms presented in this paper have been coded on the ELS (Engineer-in-the-Loop Simulator). The ELS at the Flight Mechanics and Controls Division, NAL, is a six degree of freedom, pilot in the loop real time simulation facility. Presently the ELS has been configured to simulate a single engine, tailless delta wing fighter aircraft (henceforth called the GFA).

1. Derivation of Equilibrium Conditions

The rigid body dynamics of aircraft is governed by six degrees of freedom, namely the three translations and three rotations along the spatial co-ordinates. The resulting equations[3] can be referred in standard textbooks. The general aircraft dynamic equations which are non-linear in nature can be cast in the following implicit form[1].

$$\bar{f}(\dot{\bar{X}},\bar{X},\bar{U})=0 \tag{1}$$

where $\bar{f}$ is a vector of n scalar non-linear functions f_i. $\bar{X}$ is the state Vector $X^T=[V_T \;\; \alpha \;\; \beta \;\; \phi \;\; \theta \;\; \psi \;\; P_B \;\; Q_B \;\; R_B \;\; X_E \;\; Y_E \;\; H_E]$ and $\bar{U}$ is the input vector $U^T=[\delta_{pla} \;\; \delta_e \;\; \delta_a \;\; \delta_r]$. The equilibrium or singular point(s) of (1) satisfy the following condition.

$$\dot{\bar{X}}=0 \text{ for some given value of } \bar{U}. \tag{2}$$

Under the assumptions of constant aircraft mass, flat earth approximation and neglecting atmospheric density effects on aircraft motion, the equations of motion allow us to decouple the earth co-ordinates (X_E, Y_E, H_E) from the closed set represented by (1). Thus, we obtain set of nine first order differential equations. The state vector is now given by $X^T=[V_T \;\; \alpha \;\; \beta \;\; P_B \;\; Q_B \;\; R_B \;\; \phi \;\; \theta \;\; \psi]$. For purposes of aircraft performance, stability analysis and control law design, the aircraft motions need to be analysed for various manoeuvres (straight level flight, turning flight, pull-up , push over etc.). To obtain the commonly used equilibrium points for flight mechanics analysis one has to further reduce the state vector from nine to six, corresponding to aircraft degrees of freedom $(X^T=[V_T \;\; \alpha \;\; \beta \;\; P_B \;\; Q_B \;\; R_B])$ with constraints on either the euler angles (ϕ,θ,ψ) and/or their rates. Therefore (2) implies the following conditions to be satisfied for equilibrium.

$$\dot{V}_T,\dot{\alpha},\dot{\beta},\dot{P}_B,\dot{Q}_B,\dot{R}_B=0,\; \bar{U}=\text{constant} \tag{3}$$

subject to appropriate the constraints. The constraints applied depend upon the type of flight mode required[1] (straight and level, level turn, pull up etc.).

Based on (3), *one can* derive equilibrium **conditions** for **the various phases** *of* **the** rake-off manoeuvre (Fig. 1a). These will be discussed in **the** following sections.

1.1 Phases of Take-off Manoeuvre

For deriving **the trim** point **conditions the** manoeuvre **has been** divided into three **phases as shown** in Figure 1b. They are - **the accelerated ground** roll, the nose wheel **lift-off** point and **rotation** to lift-off pitch attitude. In what follows, these **phases are** examined separately and appropriate **trim** conditions defined.

a. Accelerated ground **roll** with thrust *set* to fixed value (usually **at max.** dry **)** : Since the aircraft is accelerating **on the** runway, all **the** six accelerations **terms** in (3) are non **zero. A** reasonable estimate *of* **the** equilibrium **state** *of* **the** aircraft is obtained if **the** V_r **equation** is ignored. For this phase *of* flight all **body** axis angular rates are close to **zero.** Further, **the bank** angle (ϕ) and **the** yaw angle (ψ) are **also** equal to zero. Thus, *we* have **the** following **conditions** to **be** satisfied at trim point.

$$\dot{\alpha},\dot{\beta},\dot{P}_s,\dot{Q}_s,\dot{R}_s \equiv 0 \qquad \textbf{(4a)}$$

$$\phi,\dot{\phi},\dot{\psi},\theta \equiv 0 \text{ and } \theta = \alpha \qquad \textbf{(4b)}$$

The *case of* asymmetries arising out of single engine failure where above assumptions are violated, is examined in the concluding **part** of **Section** 1.

Equations (4a,b) are in **the** form *of* a *constrained* minimisation **problem** with equality constraints. The Newton-Raphson algorithm is suitable for **this** problem. The following is a commonly **used** cost **function.**

$$C = \dot{\alpha}^2 + \dot{\beta}^2 + \dot{P}_s^2 + \dot{Q}_s^2 + \dot{R}_s^2 \qquad (5)$$

In order **to** satisfy each of **the** equations in (4a), *we* need **to** select **the** five control variables. The choice *of* these variables should be such that each **equation** in (4a) is strongly influenced **by** at least *one* different variable. At **the** same **time** these control variables must ensure a unique solution. **To** select such variables *we* appeal **to the** physics *of* **the** problem. In up and away flight, **the** usual control variables[1] **are** $\alpha,\beta,\delta_e,\delta_a,\delta_r$. **It can be argued that the** $\dot{\alpha}$ equation is **a strong function of** α **, the** $\dot{\beta}$ equation that of β, the $\dot{Q}_s$ thatof δ_e, $\dot{P}_s$ thatof δ_a and the $\dot{R}_s$ equationisa strong function of δ_r. In **the** *case of* aircraft resting on **both** wheels, α depends on θ (constraint (4b)), which in turn depends on **the** force and moment balance at equilibrium *(see* Fig. **2).** Also in **most** *cases* δ_e, **the** elevator angle is preset during ground roll. Therefore, *we* **need to** replace α and δ_e with some other **appropriate** control variables. The **most** obvious **choice** is **the** forces **on the nose** (Fnose) and main (Fmain) wheels respectively making $F_{main},\beta,F_{nose},\delta_a,\delta_r$ **as the** set *of* control variables. Thus *we* have to *satisfy* (4a) **subject to** constraints (4b), **for** fixed values of the mach number, elevator setting δ_e *and* throttle setting δ_{pla}.

When the aircraft reaches **rotation** *speed* V_R, **the** pilot applies back **stick** (up elevator) to ease **the** aircraft nose up **to the desii** rake-off attitude. Between the time he **starts** rotation and **the** point **at** which nose wheel lifts *off* **the ground,** a **trim** condition *can* be defined which **satisfies** (4). Therefore, *we can* use **the trim** algorithm described above to ***obtain* the locus *of* such trim points as** a **function** *of* **the stick** deflection (*or* equivalently **the** elevator deflection).

b. Rotation on both wheels till nose-wheel lift-off point : **For** determination *of* **the** exact point *of* **nose** wheel lift-off, further **conditions** need to be satisfied. **These** conditions **are as** follows.

i. **Ground** reaction force **on** nose wheel is **zero** (Fnose = **0)** **(6a)**

ii. Nose tire is just touching the ground (in undeflected state). **(6b)**

The two constraints in (6) have to be imposed in addition to those specified in **(4b). Since** Fnose is now identically **zero,** elevator **deflection** *can* be taken **as** a control variable. Thus for nose wheel-off **trim, the condition** (4a) **has** to be satisfied subject **to the** constraints specified in (4b) and ***(6).* The** control variables are now $F_{main},\beta,\delta_e,\delta_a,\delta_r$. **For a fixed mach** number **and** throttle setting δ_{pla} the user *can* determine **the** minimum elevator required to just lift **nose** wheel off the ground. In aircraft preliminary design, the designer may want **to** determine **the** rotation **speed** for a given maximum elevator deflection. In such a *case, one can* replace elevator deflection with mach number **as** a control variable. Then for a given maximum

elevator deflection the user can determine the minimum speed **at** which rotation **can be initiated.**

c. **Roll on the ground at fixed attitude with nose wheel off** : For the **case** of nose wheel **off the** ground, the **constraints** *an?* same as in **(4b)** and the **equations** to be minimised are as in (4a). Since the nose **wheel is off the** ground, the control **variables chosen are** $F_{main}, \beta, \delta_e, \delta_a, \delta_r$.

In *case of* **asymmetries** arising out *of* engine *or* single **control surface failures,** the **aileron deflection 6, can** be set to **a fixed** value, while F_{main} *can* **be split into two** forces (*one* each for either of the **main** gear struts) F_{port} and $F_{starboard}$. Also the **bank** angle (ϕ) must **be reset** every cycle depending **on the** values *of* the three landing **gear** farces F_{nose}, F_{port} and $F_{starboard}$. This concludes the derivation of the trim algorithms for the **complete p o d** roll **phase.** In the next section **approximate** equations for **trim on ground are** derived.

2 Approximate Equations for Aircraft Trim on the Ground

The approximate equations for **aircraft** aim **on the** ground can be derived by considering the simplified pitch plane **equations only** (Fig. 3). The force **and** moment balance **equations are as follows.**

Horizontal force balance

$$T\cos(\theta-\theta_t)-D-\mu R=m\ddot{x} \qquad (7)$$

Vertical force balance

$$T\sin(\theta-\theta_t)+L+R=W \qquad (8)$$

Pitching moment balance

$$T(z_t\cos\theta_t-x_t\sin\theta_t)+M-R(x_m+\mu z_m)=0 \qquad (9)$$

The **lift, drag** and pitching moment terms *can* be expanded using the **aerodynamic** derivative **formulation. There** are three equations and an *equal* number *of* **unknowns (θ, R and 6,) provided forward speed,** forward **acceleration** and the thrust **are fixed. Unfortunately, (7) above** is **poorly conditioned** with respect to the pitch attitude θ. The remaining **two** equations (8) and (9) *can* be solved **simultaneously for** *Rand* 6,. **provided** θ is known. Using **(8)** and **(9),** an **estimate of** the **elevator** required to hold **a** given pitch attitude while rolling on the ground at **a** given **speed and thrust** setting *can* be computed. This **computation** is sensitive to the values **of** aerodynamic derivatives used, (particularly $C_{m_{\delta e}}$) as well as the distances (x_m and z_m).

In order to calculate the minimum elevator required to just lit the nose wheel **off the ground,** *one* more relation between **the aircraft attitude** and R or δ_e **is needed. This** can be obtained **from the** geometry in Figure **4.**

$$\theta=\tan^{-1}\left(\frac{l_{no}-l_n}{l}\right) \qquad (10)$$

If the strut deflection vs. force characteristics is **linear** over the range of **interest,** l_n is related to l_{no} and **normal reaction** force R in a **direct** way as follows.

$$l_n=l_{no}-\frac{R}{2k_{eqn}} \qquad (11)$$

The factor of two in equation (11) **accounts** *for* the **presence of** two main gears on the **aircraft. The minimum** elevator required to lift the **nose** wheel off the ground, main wheel **reaction force** and pitch **attitude at** nose wheel lift-off point **can now** be found using **equations (8),** (9) **(10)** and **(11) after a few** iterations. The results **of** this **method are compared** with **the** exact values in Section **4.** In **the** next **section** implementation issues for the exact **formulation** *of* **Section 1 are** discussed.

3. Implementation of the Trim Algorithm

The aim constraints explained in **section 1** can be implemented with minor **modifications** to the up and away **trim** algorithms[1]. **For aircraft on ground,** the farce and moment **contributions** *arise* from *tbe* following **sources -** aerodynamic **forces** and **moments,** propulsive forces and **moments, gravitational** forces and **ground reaction** forces and **moments** (due **to** undercarriages). **The** contributions **from** aerodynamics, propulsion, and **gravitational** forces and **moments are** already **computed** io the up and **away trim algorithms.** The flowchart **for** calculating **the** forces and **moments** due to the landing **gear** model in **the** aircraft **body axis** *is* **presented** in Figure 5. **This portion** *of* the **code** is common to **all** the algorithms described above. **Hence** it **can be** coded as a separate module.

4. Comparison of Results

The values obtained by the **approximate method outlined in Section 2 are** compared **with** *tbe* 'exact' values computed by the corresponding **trim** algorithm described in Section 1.1c in Table 1 for the GFA. It is seen that for all the values in Table 1, the error is within ldeg.

The results of the computations of the minimum elevator point using equations of Section 2 are compared with those obtained from the trim algorithm described in Section 1.1b in Table 2 for various configurations of the GFA. Again the agreement is good for the predicted elevator deflection (within 1.5deg) and the pitch attitude at nose wheel lift-off (within ldeg). The normal reaction force is predicted with a fair degree of accuracy (within 6% of total weight). This is due to the approximation involved in representing the main landing gear by an equivalent spring (Fig. 4).

The results of the trim algorithm for the GFA are plotted in Figure 6 in the pitch attitude vs. elevator deflection plane for various speeds of rotation (200kmph, 220kmph and 240kmph). The initial decreasing segment of each curve corresponds to accelerated ground roll on all wheels. The lowest point of each curve corresponds to the nose wheel lift off point. The increasing segment of each curve corresponds to the rotation upto θ_{ur} (which for this aircraft is 13deg) after nose wheel lift-off. The maximum up elevator deflection required reflects the static stability (for a given speed) that must be overcome in order to lift the nose wheel off the ground.

In Figure 8 a typical simulation result of a take-off run is presented. The GFA is actually unstable in pitch in the low mach number range. It has been stabilised by the use of pitch rate and normal acceleration feedback. It is clear that the nose wheel (EV04) leaves the ground just as the elevator crosses the minimum elevator required point. The rotation was initiated by the pilot at about 210kmph with a back stick (PSTICK) of roughly 75% of full back stick deflection (42mm). The plots in the second column of Figure 8 show that take-off occurred at about 10sec (EV03).

In Figure 7. the results of the simulation (as presented in Figure 8) are plotted with the results of trim analysis (Fig. 6) in the θ vs. elevator deflection δ_e plane. The simulation results are quite close to the trim calculations, since, the pilot is attempting to execute a smooth rotation manoeuvre and therefore, he needs to apply only an incremental elevator deflection over and above that which is already required to overcome the static stability (Fig. 6). For a non-zero pitch rate, more elevator deflection over and above that shown in the trim plots of Figure 6 is required. Exactly how much more will obviously depend upon the control effectiveness of the aircraft in pitch and its inertia.

Conclusions

An extension of the equilibrium trim analysis to the ground roll phase has been proposed. Its applicability has been demonstrated by comparison with six degree of freedom simulation of a typical fighter aircraft. It is argued that due to the nature of the take-off rotation manoeuvre, the proposed trim strategy is equally applicable to all class of aircraft and for all types of control mechanisms (conventional or fly-by-wire). The comparison also shows that linearisation of the equations of motion in the take-off phase is meaningful. An approximate method for the computation of the parameters of interest to the aircraft design engineer is also presented. This method can be used in the preliminary design stage and gives reasonably accurate results. The advantages of the approximate method is that it relies on a minimum of information about the aircraft and landing gear parameters. It is noted that a simulator platform is the ideal environment for the analysis and application of the trim algorithms presented in this paper.

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Table 1. Exact and Predicted Elevator as Function of Pitch Attitude

Speed = 200Kmph				Speed = 220Kmph				Speed = 240Kmph			
θ	δe	δe*	error	θ	δe	δe*	error	θ	δe	δe*	error
1.00	0	-	-	0.99	0	-	-	0.94	0	-	-
1.57	-5	-	-	1.67	-5	-	-	1.82	-5	-	-
2.50	-10	-	-	3.08	-10	-	-	3.26	-10	-	-
3.40	-15	-	-	3.48	-15	-	-	3.50	-13.6	-13.9	-0.3
3.60	-20.0	-	-	3.56	-16.6	-16.7	-0.1	5.00	-11.4	-11.4	+0.0
3.62	-20.6	-20.3	+0.3	5.0	-14.3	-14.0	+0.3	10.0	-5.7	-5.7	+0.0
5.0	-18.1	-17.4	+0.7	10.0	-7.0	-7.4	-0.4	13.0	-4.4	-3.9	+0.5
10.0	-9.6	-9.5	+0.1	15.0	-4.7	-4.0	+0.7				
15.0	-5.7	-5.2	+0.5								

Table 2. Exact **and Predicted Parameters** at Nose **Wheel Lift-off Point**

CONFIGURATION	Pitch attitude at nose wheel lift-off (deg)		Elevator deflection at nose wheel lift-off (deg)		Main-wheel reaction force at nose wheel lift-off (% of total weight)	
	θ	θ*	δe	δe*	R	R*
CONFIG 1 (200kmph)	3.80	3.06	-18.6	-19.6	55.1	58.0
CONFIG 1 (220kmph)	3.74	3.02	-14.9	-16.0	53.3	57.0
CONFIG 1 (240kmph)	3.67	2.98	-12.1	-13.3	51.3	56.1
CONFIG 2 (200kmph)	3.74	3.02	-17.6	-18.8	54.7	57.7
CONFIG 2 (220kmph)	3.68	2.99	-14.1	-15.4	52.9	56.8
CONFIG 2 (240kmph)	3.62	2.95	-11.5	-12.8	50.9	55.9
CONFIG 3 (200kmph)	3.62	3.03	-20.6	-21.7	58.3	61.0
CONFIG 3 (220kmph)	3.56	2.99	-16.6	-17.8	56.3	60.0
CONFIG 3 (240kmph)	3.50	2.95	-13.6	-14.8	54.0	58.9

* Indicates the values estimated from approximate equations of *section 2*.

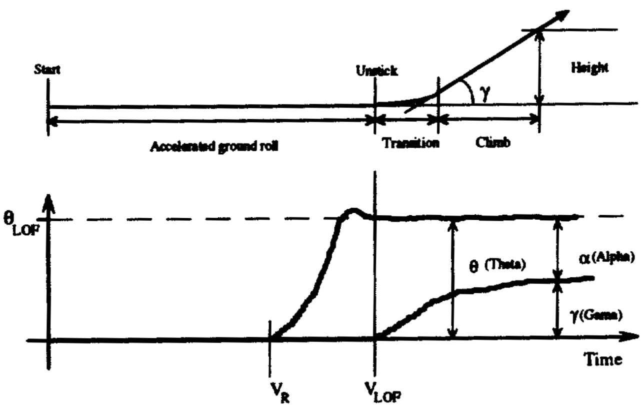

Figure 1a. Typical Pitch Attitude and Flight Path Angle Time Histories for Take-off

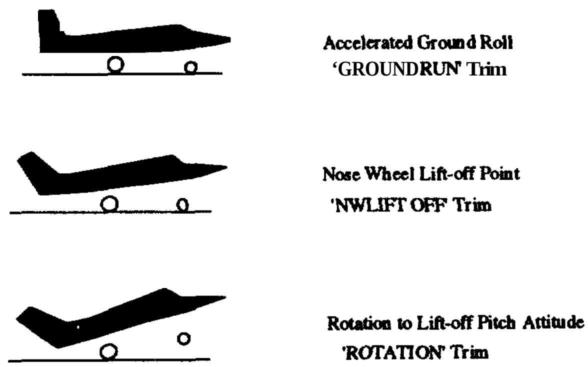

Figure lb. **The Different Phases** of a Take-off Manoeuvre

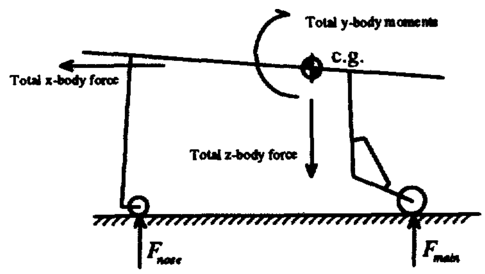

Figure 2. **Longitudinal** Forces and Moments on the Aircraft *on* Ground

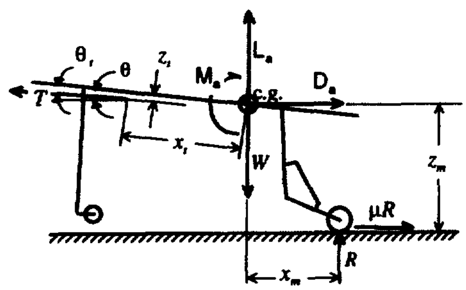

Figure 3. Longitudinal Force and Moment Balance (nose wheel-off)

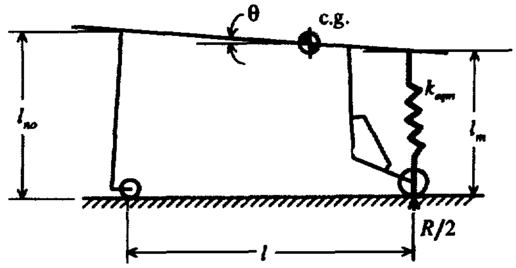

Figure 4. **Equivalent Spring Representation** *of* **Main Landing** Gear

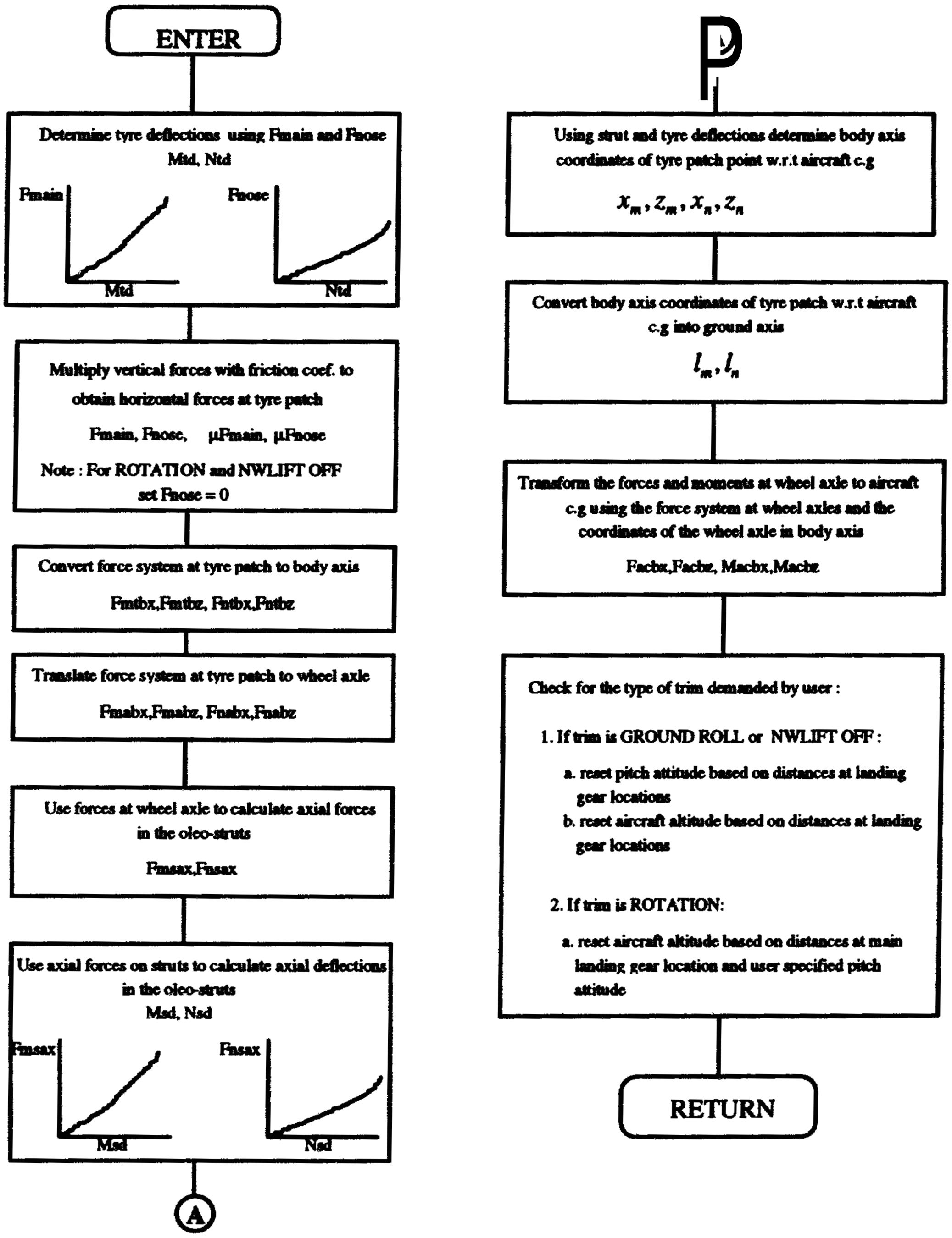

Figure 5. Flow Chart of Trim Constraints **Routine**

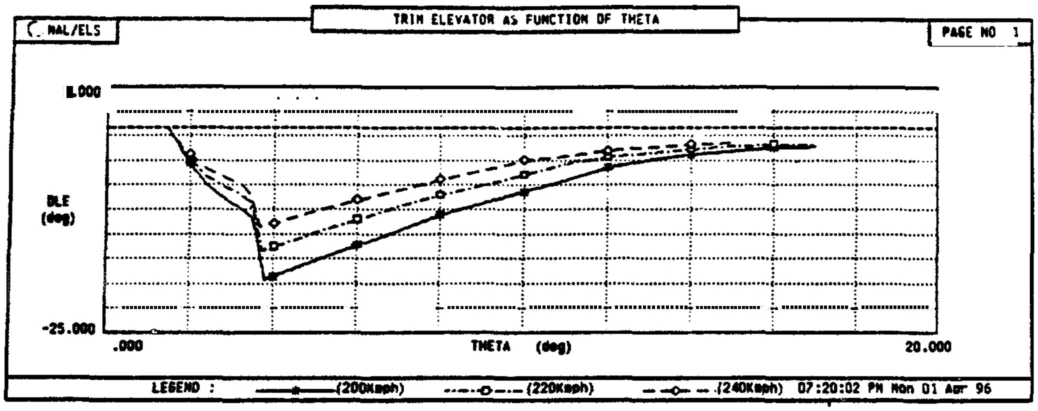

Figure 6. Trim Elevator Vs. Pitch Attitude for Various Speeds

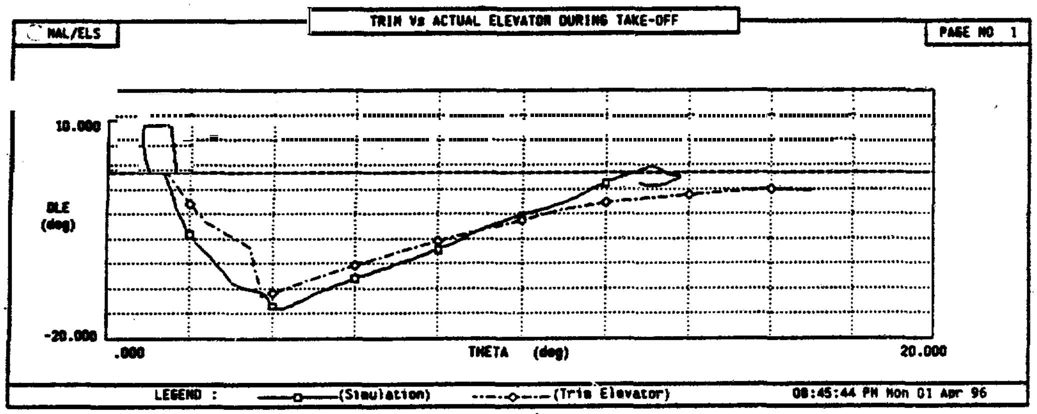

Figure 7. Trim Elevator Required Against Actual Deflection in Simulation

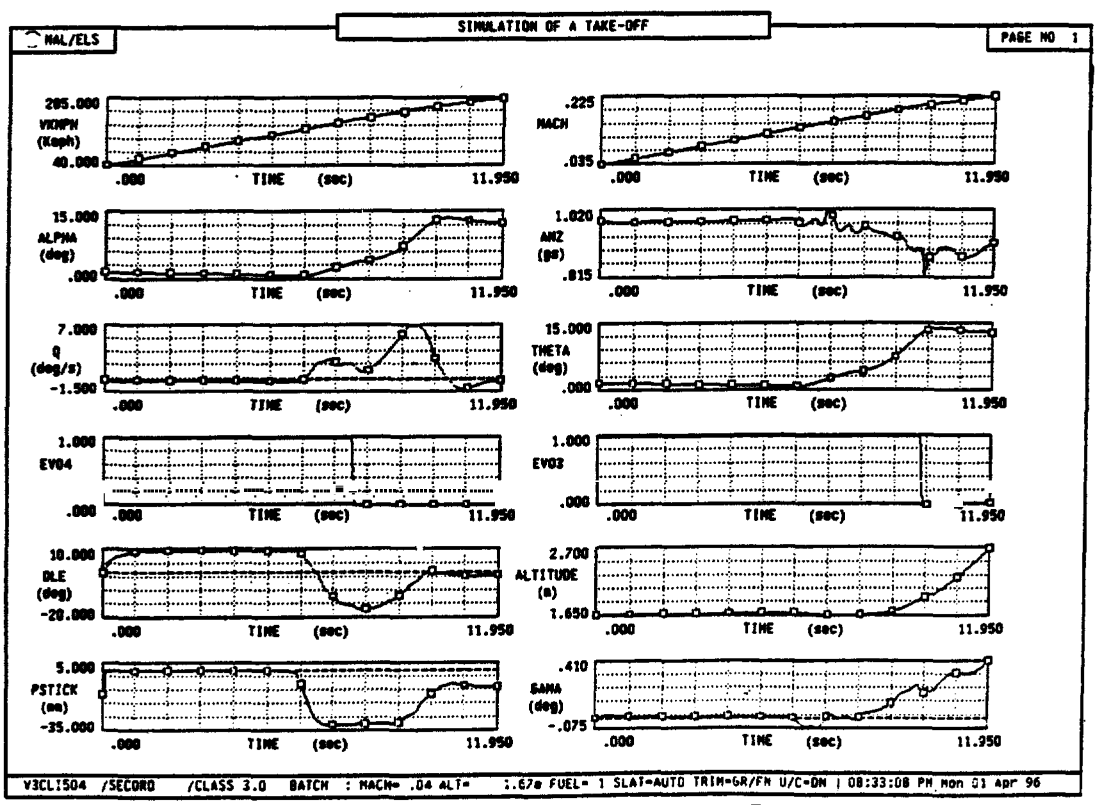

Figure 8. Real Time Simulation Responses for a Take-off