Pulsatile flow between two coaxial porous cylinders with slip on inner cylinder for large frequency of pulsation

Elsadig A. Hamza
Sultan Qaboos University

Sunita R. Jain
Chandigarh College of Engineering

Nirmal C. Sacheti
Sultan Qaboos University

Abstract

The unsteady component of axial flow of an incompressible fluid in the region lying between the annular spaces of two coaxial cylinders has been investigated. The outer cylinder is assumed to possess uniform permeability and the inner one is a naturally permeable tube obeying Darcy's Law. The flow is maintained by a periodic pressure gradient across the annulus. Making use of an appropriate set of similarity variables, the governing partial differential equations have been transformed to a set of a system of nonlinear ordinary differential equations. The solution of the resulting equations subject to appropriate boundary conditions has been obtained using a singular perturbation technique. The frequency of pulsation has been considered to be large. The effect of the flow parameters on the unsteady component of axial flow has been discussed. The analytical results have been compared with numerical solutions.

Keywords: Pulsatile flow, Porous annulus, Darcy's law, Singular Perturbation

MSC(2000): Primary:76M45, Secondary:76S05, 76M25

1   Introduction

Pulsatile flows occur in many areas of engineering fluid dynamics like pressure surges in pipelines, cavitations in hydraulic systems, pumping of slurries, refrigeration systems, combustion mechanisms, de-watering devices and cardiovascular biomechanics. Considerable attention has been given to the study of the problems of pulsatile flow of fluids in channels of various cross-sections due to their increasing application in the analysis of blood flow and in the flows of other biological fluids.

Flow through porous medium has generated a lot of interest due to applications in many industrial operations in the areas of chemical and metallurgical engineering. In recent years flow through porous medium has gained considerable attention due to its relevance in a wide range of applications such as underground spreading of chemical waste and water movements in geothermal engineering. The problem of the flow through an annulus with porous walls is useful in the transpiration cooling, boundary layer control etc. The drilling operations of oil and gas wells involve flow in annular space. The flow of the drilling mud in the annulus between the well wall and drill pipe is extremely complex problem.

For circular pipes and tubes, Yuan & Finkelstein [1] presented asymptotic solutions in the limiting cases of small suction and both small and large injection. Their formulation depended on the cross-flow Reynolds number. Berman [2] obtained the exact solutions of the steady state laminar flow of an incompressible viscous fluid in an annulus with porous walls by assuming the constant influx through one wall equal to the efflux through the other wall. Terrill [3] explicated the laminar flow through a porous annulus. He assumed the swirl to be zero and his work includes the cases of small and large suction/ injection. Terrill [4] obtained general solution for fully developed flow in a permeable annulus. Verma & Bansal [5] discussed the solution of Navier-Stokes equations in cylindrical coordinates for flow of a viscous incompressible fluid in a porous annulus with different radial velocities at the walls. Verma & Gaur [6] described unsteady flows of an incompressible viscous fluid in a porous annulus. Verma & Gaur [7] investigated the flow of an incompressible viscous fluid in a porous annulus, under constant pressure gradient with mass transfer across the boundaries. They have discussed in detail the perturbation in the radial and axial velocities by decaying swirl. Huang [8] employed the quasilinearization technique to solve the nonlinear differential equations representing the boundary value problem of a fluid flow though an annulus with porous walls of different permeability.

Skalak & Wang [9] presented solution for axial pulsatile flow in a tube with injection/suction on the wall. They used method of matched asymptotic expansions for solving the equation of fluid flow. In a subsequent paper [10] they presented similarity solutions for pulsatile flow in a porous tube with wall suction and wall injection. The analysis centres on the effect of suction and injection on the pulsatile flow. The governing equations have been solved numerically and by the method of matched asymptotic expansions. Kanwal & Verma [11] examined the unsteady flow of a viscous fluid through an annulus when one boundary of annulus is fixed and the other boundary is subjected to a series of pulses. Singh and Rajvanshi [12], [13] presented detailed study of the viscous flow in a porous annulus assuming pressure gradient to be periodic. They obtained the solution for small mass transfer across the porous walls as well as for large values of the frequency of pulsation. Their analytical results compared well with numerical solution.

Hamza et al [14] generalized the study reported in [12] to investigate the flow of an incompressible fluid in the annular space between two coaxial cylinders, the outer one having uniform permeability and the inner one being a naturally permeable tube. The inner tube was assumed to be of small permeability and fully saturated with a viscous incompressible fluid and the governing equations were simplified using a similarity transformation [12]. The transformed set of equations were solved for small suction at the outer boundary using BJ conditions [15] on the surface of inner permeable tube. The technique of regular perturbation has been used for the solution and the results compared well with those obtained numerically. Frequency of pulsation was taken to be small in that investigation and Hamza et al [14] did not include the case of large frequency in their work.
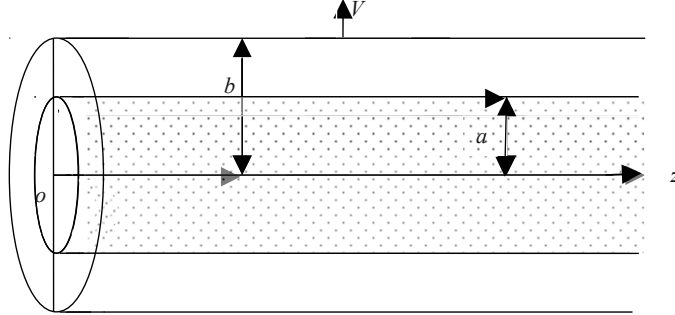
Figure 1: Geometrical configuration of the problem

In the present paper the case of large frequency of pulsation is examined for the case described by Hamza et al [14]. A singular perturbation technique is used to solve the governing equations and a comparison with numerical results shows a good agreement.

2   Governing equations and boundary conditions

A fully developed incompressible laminar flow in the region bounded by two long coaxial cylinders of radii a and b $(a < b)$ respectively is considered. The following assumptions are made:

(i) the flow is axisymmetric,

(ii) there is no swirl velocity,

(iii) the cylinders are sufficiently long as compared to the diameter, so that end-effects can be neglected,

(iv) a periodic pressure gradient is imposed across the annulus,

(v) the inner cylinder is made of naturally permeable material.

Keeping in view the above configuration, it is assumed that the volume of the fluid entering the annulus and the pressure gradient maintain the laminar flow. The cylindrical polar coordinate system (r, θ, z) has been used to write the governing equations. -axis is taken along the common axis of cylinders as depicted in Fig. 1

Let u and v be the velocity components of the fluid in the positive directions of r and z respectively. Navier - Stokes equations in the annular free fluid region $(a < r < b)$ are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) \qquad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{2}$$

The equation of continuity is

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

In the above equations t, p, ρ and ν denote time, pressure, fluid density and the kinematic viscosity respectively. The flow through the inner cylinder follows Darcy's law. The boundary conditions assume that there is no-slip on the inner boundary of the outer cylinder. The governing equations for the flow through inner cylinder consisting of naturally permeable material with U and W as radial and axial velocity components are

$$U = 0, \quad W = -\frac{\kappa}{\mu}\frac{\partial P}{\partial z} \tag{4}$$

where κ and μ are the permeability of the porous medium and the coefficient of viscosity of the fluid respectively and P is fluid pressure in the inner cylinder.

The boundary conditions on the outer cylinder are

$$u = V, \quad w = 0, \quad \text{at} \quad r = b \tag{5}$$

V being the constant suction velocity at the surface of the outer cylinder.

The slip condition on the surface of the inner cylinder is prescribed by the modified B-J condition, applicable at a curved surface as given in Hamza et al [14];

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = \gamma(w - W) \tag{6}$$

where $\gamma = \alpha^*/\sqrt{\kappa}$, α^* is an empirical dimensionless constant dependent upon the nature of the porous material. In addition the continuity of normal velocity on the interface of the inner permeable material and the free fluid region has been assumed. The radial velocity has been taken as a function of the radial coordinate r only. It can thus be written as

$$u = \frac{1}{r}F(r) \tag{7}$$

where $F(r)$ is an arbitrary function to be determined.

Using (7) in equation of continuity ([3]) we have

$$w = -\frac{z}{r}F'(r) + \phi(r,t) \tag{8}$$

where $\phi(r,t)$ is an arbitrary function of r and t and prime denotes differentiation with respect to r . Substituting (7) and (8) in (1) and integrating with respect to r we get

$$-\frac{p}{\rho} = \frac{1}{2}\left(\frac{F'}{r}\right)^2 - \nu\frac{F}{r} + p_1(z,t) \tag{9}$$

Taking into account pulsatile nature of flow, $p_1(z,t)$ and $\phi(r,t)$ are assumed in the form

$$p_1(z,t) = L_1 z^2 - L_2 z + L_3 + Re\left[(L_4 z + L_5)e^{\iota\omega t}\right] \tag{10}$$

$$\phi(r,t) = G(r) + Re\left[H(r)e^{\iota\omega t}\right] \tag{11}$$

where $G(r)$ and $H(r)$ are arbitrary functions, ω is the frequency of pulsation of pressure gradient and Re means "Real part of". The following non- dimensional variables are introduced

$$r = b\sqrt{n},\ F(r) = bVf(\eta),\ G(r) = bG(\eta),\ H(r) = \left(\frac{L_4 b^2}{4\nu}\right)h(\eta) \tag{12}$$

Using equations (7) to (12) in equation (2) and equating coefficients of like terms on both sides yields the following

$$\eta f''' + f'' + \frac{1}{2}R(f'^2 - ff'') = \beta \tag{13}$$

$$\eta g'' + g' + \frac{1}{2}R(gf' - fg') = d \tag{14}$$

$$\eta h'' + h' - \iota\alpha^2 h + \frac{1}{2}R(hf' - fh') = -1 \tag{15}$$

where the set of additional non-dimensional parameters are defined as

$$\beta = -\frac{Rb^2 L_1}{4V^2},\ d = \frac{L_2 b^2}{4\nu V^2},\ \alpha^2 = \frac{\omega b^2}{4\nu},\ R = \text{Reynold number} = \frac{bV}{\nu} \tag{16}$$

In above equations prime denotes derivative with respect to η. The parameter α characterizes the frequency of pulsation. The equations (13) and (14) depict the steady flow while the equation (15) corresponds to the unsteady component of the flow. In terms of similarity functions, the modified boundary conditions are given by

$$f(1) = 1, \quad f'(1) = 0, \quad g(1) = 0, \quad h(1) = 0 \tag{17}$$

$$f''(\eta_0) - \frac{1}{\lambda}f'(\eta_0) + \frac{\beta}{\lambda}C^* = 0 \tag{18}$$

$$g'(\eta_0) - \frac{1}{\lambda}g(\eta_0) - \frac{d}{\lambda}C^* = 0 \tag{19}$$

$$h'(\eta_0) - \frac{1}{\lambda}h(\eta_0) - \frac{1}{\lambda}C^* = 0 \tag{20}$$

$$f(\eta_0) = 0 \tag{21}$$

$$\lambda = \frac{2a}{\gamma b^2}, \quad C^* = \frac{4\kappa}{b^2}, \quad \eta_0 = \frac{a^2}{b^2}$$

3 Solution of unsteady component

We now proceed to solve the set of coupled nonlinear ODEs (13) to (15), subject to the boundary conditions (16) to (20). Following Hamza et al [14] a particular solution of equation (14) is taken as

$$g(\eta) = -\frac{L_2 f'(\eta)}{bL_1} \tag{22}$$

It is required to determine the solution of equations (13) and (15) for large frequency of pulsation. We further assume

$$\epsilon = \frac{R}{2} \tag{23}$$

where ϵ is small.

Substituting (23) in equations (13) and (15) we get

$$\eta f''' + f'' + \epsilon\left(f'^2 - ff''\right) = \beta \tag{24}$$

$$\eta h'' + h' - \iota\alpha^2 h + \epsilon(hf' - h'f) = -1 \tag{25}$$

and the boundary conditions reduce to

$$\lambda f''(\eta) - f'(\eta) + \beta C^* = 0, \quad \lambda h'(\eta) - h(\eta) + C^* = 0 \quad \text{at} \quad \eta = \eta_0 \tag{26}$$

$$f(\eta) = 0, \quad h(\eta) = 0 \quad \text{at} \quad \eta = 1 \tag{27}$$

Equations (24) and (25) subject to boundary conditions (26) and (27) were solved by Hamza et al [14] for small ? by using a regular perturbation method.

Following Hamza et al [14], we express $h(\eta)$ and the non dimensional parameter β appearing in equations (24) and (25) as a power series in ϵ in the following form

$$h(\eta) = \epsilon h_1(\eta) + O\left(\epsilon^2\right) \tag{28}$$
$$\beta = \beta_0 + \epsilon\beta_1 + O\left(\epsilon^2\right) \tag{29}$$

Equations (25) and (28) yield the equations for $h_0(\eta)$ and $h_1(\eta)$ as under

$$\eta h_0'' + h_0' - \iota\alpha^2 h_0 = -1 \tag{30}$$

$$\eta h_1'' + h_1' - \iota\alpha^2 h_1 = f_0 h_0' - f_0' h_0 \tag{31}$$

$h_0(\eta)$ and $h_1(\eta)$ as follows have been evaluated from (30) and (31) for small values of α^2 using regular perturbation technique in [14]. The corresponding boundary conditions are

$$C_1^* h_0' - C_2^* h_0 + 1 = 0, \quad \lambda h_1' - h_1 = 0 \quad \text{at} \quad \eta = \eta_0 \tag{32}$$

$$h_0 = 0, \quad h_1 = 0 \quad \text{at} \quad \eta = 1 \tag{33}$$

where $C_1^* = \frac{\lambda}{C^*}$ and $C_2^* = \frac{1}{C^*}$

In the present paper the solution of equations (30) and (31) is obtained for large values of frequency of pulsation ($\alpha^2 >> 1$). Under the above condition the equations belong to the category of singular perturbations. The equations are solved by the method of matched asymptotic expansions. Following Nayfeh [16] the flow field is divided into an "inner" region near each boundary separated by an "outer" region. The boundary conditions on the unsteady component h_0 and h_1 are satisfied by the "inner" solutions, hence these are not imposed on the "outer" solution.

In order to find "outer" solution, we represent $h_0(\eta)$ and $h_1(\eta)$ in this region by h_0^0 and h_1^0 respectively. The solution of equations (30) and (31) is sought in the forms

$$h_0^0 = \frac{1}{\alpha} s_1 + \frac{1}{\alpha^2} s_2 + \frac{1}{\alpha^3} s_3 + \frac{1}{\alpha^4} s_4 + \dots \tag{34}$$

$$h_1^0 = \frac{1}{\alpha} S_1 + \frac{1}{\alpha^2} S_2 + \frac{1}{\alpha^3} S_3 + \frac{1}{\alpha^4} S_4 + \cdots \tag{35}$$

Equations (30), (31), (34) and (35) give

$$h_0^0 = -\frac{\iota}{\alpha^2} \tag{36}$$

$$h_1^0 = \frac{1}{\lambda^4} \left[-A_2 - A_3 \left(1 + \ln \eta \right) + \beta_0 \eta \right] \tag{37}$$

where A_2 and A_3 are recorded in [14]. For finding an "inner" solution near the inner cylinder, let $h_0(\eta)$ and $h_1(\eta)$ in this region be represented by h_0^ι and h_1^ι respectively. In order to make the terms containing frequency parameter α and the terms independent of α comparable in this region, a stretching variable defined by

$$\zeta_1 = \sqrt{\iota} \alpha (\eta - \eta_0) \tag{38}$$

where $\iota = \sqrt{-1}$ is introduced.

Using (38) in equations (30) and (31) we get

$$\left(\sqrt{\iota}\zeta_1 + \iota\alpha\eta_0 \right) (h_0^\iota)'' + \sqrt{\iota} (h_0^\iota)' - \iota\alpha h_0^\iota = -\frac{1}{\alpha} \tag{39}$$

$$\left(\sqrt{\iota}\zeta_1 + \iota\alpha\eta_0 \right) (h_1^\iota)'' + \sqrt{\iota} (h_1^\iota)' - \iota\alpha h_1^\iota + \frac{h_0^\iota f_0'}{\alpha} - \sqrt{\iota} (h_0^\iota) f_0 = 0 \tag{40}$$

The boundary conditions for the equations (39) and (40) are

$$C_1^* \sqrt{\iota}\alpha (h_0^\iota)' - C_2{}^* h_0^\iota + 1 = 0, \quad \lambda (h_1^\iota)' - h_1^\iota = 0 \quad \text{at} \quad \zeta_1 = 0 \tag{41}$$

where prime denotes differentiation with respect to ζ_1 . h_0^ι and h_1^ι are assumed to be of the form

$$h_0^\iota = \frac{1}{\alpha}k_1 + \frac{1}{\alpha^2}k_2 + \frac{1}{\alpha^3}k_3 + \frac{1}{\alpha^4}k_4 + \cdots \tag{42}$$

$$h_1^\iota = \frac{1}{\alpha}\bar{k}_1 + \frac{1}{\alpha^2}\bar{k}_2 + \frac{1}{\alpha^3}\bar{k}_3 + \frac{1}{\alpha^4}\bar{k}_4 + \cdots \tag{43}$$

Substituting equations (42) and (43) into equations (39) and (40) respectively and solving the resulting equations subject to the boundary conditions, equation (41) and subsequent matching with the "outer" solutions (36) and (37) the expressions for h_0^ι and h_1^ι are obtained in the following form

$$
\begin{aligned}
h_0^\iota &= \frac{1}{\alpha}\left[\frac{1}{\sqrt{\iota}C_1^*a_0}exp(-a_0\zeta_1)\right] \\
&\quad + \frac{1}{\alpha^2}\left[\iota\left(\frac{1}{4C_1^*}\frac{C_2^*}{C_1^*a_0^2} - \frac{a_0^2\zeta_1^2}{4C_1^*} + \frac{a_0\zeta_1}{4C_1}\right)exp(-a_0\zeta_1) - \iota\right] \\
&\quad + \frac{1}{\alpha^3}\left[\frac{\sqrt{\iota}}{C_1^*}exp(-a_0\zeta_1)\left(\frac{3}{8}a_0 + \frac{C_2^2}{2a_0C_1^*} + \frac{C_2^{*2}}{C_1^{*2}a_0^3} - \frac{C_2^*}{a_0} + \frac{a_0^5\zeta_1^4}{32}\right.\right. \\
&\quad\quad \left.\left. - \frac{3a_0^4\zeta_1^3}{16} + \frac{3a_0^2\zeta_1}{8} - \frac{C_2^*a_0\zeta_1^2}{4C_1^*} + \frac{C_2^*\zeta_1}{4C_1^*}\right)\right]
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
h_1^\iota &= \frac{1}{\alpha^2}\left[\frac{\iota M_1}{2a_0^2}(1 + a_0\zeta_1)exp(-a_0\zeta_1)\right] + \frac{1}{\alpha^3}\sqrt{\iota}exp(-a_0\zeta_1) \\
&\quad \left[\frac{M_1C_2^*}{2a_0^3C_1^*} - \frac{M_2}{2a_0^2} - \frac{M_3}{4a_0^3} - \frac{M_4}{4a_0^4} - \frac{M_2\zeta_1}{2a_0} + \frac{M_3}{4a_0^2}\left(a_0\zeta_1^2 + \zeta_1\right)\right. \\
&\quad\quad \left. - \frac{M_4}{2a_0}\left(\frac{1}{3}\zeta_1^3 + \frac{1}{2a_0}\zeta_1^2 + \frac{1}{2a_0^2}\zeta_1\right)\right]
\end{aligned}
\tag{45}
$$

where

$$a_0 = \frac{1}{\sqrt{\eta_0}}$$

$$M_1 = \frac{1}{C_1^*}\left(\frac{A_1}{\eta_0} + A_2 + A_3 \ln \eta_0 + \frac{1}{2}\beta_0\eta 0\right)$$

$$M_2 = -\frac{a_0}{C_1^*}\left[(A_2 + A_3 + A_3 \ln \eta_0 + \beta_0\eta 0) - \frac{C_2^*}{C_1^*}\left(A_1 + A_2\eta_0 + A_3\eta_0 \ln \eta_0 + \right.\right.$$

$$\left.\left.\frac{1}{2}\beta_0\eta_0\right)\right]$$

$$M_3 = M_1 a_0^2 - \frac{a_0^2}{C_1^*}(A_2 + A_3 + A_3 \ln \eta_0 + \beta_0\eta_0) + \frac{3a_0^4}{4C_1^*}\left(A_1 + A_2\eta_0 + \right.$$

$$\left. A_3\eta_0 \ln \eta_0 + \frac{1}{2}\beta_0\eta_0\right)$$

$$M_4 = -M_1\frac{a_0^3}{2} - \frac{a_0^5}{4C_1^*}\left(A_1 + A_2\eta_0 + A_3\eta_0 \ln \eta_0 + \frac{1}{2}\beta_0\eta_0\right)$$

The constants A_1, A_2 and A_3 are recorded in [14]. In order to find the "Inner" solution near the outer cylinder let us represent $h_0(\eta)$ and $h_1(\eta)$ in this region by h_0^I and h_0^I respectively. The stretching variable in the inner region near the outer cylinder is defined by

$$\zeta_2 = \sqrt{\iota}\alpha(1 - \eta) \tag{46}$$

Substituting (46) in equations (30) and (31), the following equations are obtained

$$\left(\sqrt{\iota}\alpha - \zeta_2\right)\left(h_0^I\right)'' - \left(h_0^I\right)' - \sqrt{\iota}\alpha\left(h_0^I\right) = -\frac{1}{\sqrt{\iota}\alpha} \tag{47}$$

$$\left(\sqrt{\iota}\alpha - \zeta_2\right)\left(h_1^I\right)'' - \left(h_1^I\right)' - \sqrt{\iota}\alpha(h_1^I) = -\frac{1}{\sqrt{\iota}\alpha} + \frac{1}{\sqrt{\iota}\alpha}h_0^I f_0' - (h_0^I)'f_0 = 0 \tag{48}$$

The corresponding boundary conditions for the equations (47) and (48) reduce to

$$h_0^I = 0, \quad h_1^I = 0 \quad \text{at} \quad \zeta_2 = 0 \tag{49}$$

h_0^I and h_1^I are assumed in the form

$$h_0^I = \frac{1}{\alpha}Q_1 + \frac{1}{\alpha^2}Q_2 + \frac{1}{\alpha^3}Q_3 + \cdots \tag{50}$$

$$h_1^I = \frac{1}{\alpha}\bar{Q}_1 + \frac{1}{\alpha^2}\bar{Q}_2 + \frac{1}{\alpha^3}\bar{Q}_3 + \cdots \tag{51}$$

Substituting (50) and (51) into equations (47) and (48) respectively and solving the resulting equations subjected to boundary conditions (49) and subsequent matching with the "outer" solutions (36) and (37) the expressions for h_0^I and h_1^I are obtained as

$$h_0^I = \frac{1}{\alpha^2}\left[\iota exp(-\zeta_2) - \iota\right] + \frac{1}{\alpha}\left[-\frac{\iota}{4}exp(-\zeta_2)(\zeta_2^2 - \zeta_2)\right] \tag{52}$$

$$h_1^I = \frac{1}{\alpha^3}\left[-\frac{1}{2}\sqrt{\iota}\zeta_2\left(A_1 + A_2 + \frac{1}{2}\beta_0)exp(-\zeta_2)\right)\right] \tag{53}$$

The composite solution is given by

$$h_0^c = h_0^i + h_0^I + h_0^0 - \left(h_0^i\right)^0 - \left(h_0^I\right)^0 \tag{54}$$

$$h_1^c = h_1^i + h_1^I + h_1^0 - \left(h_1^i\right)^0 - \left(h_1^I\right)^0 \tag{55}$$

where $(h_0^i)^0$ and $(h_1^i)^0$ denote the outer limits of the inner solutionsh_0^i and h_1^i as $\zeta_1 \longrightarrow \infty$ and and denote the outer limits of the inner solutions $(h_0^I)^0$ and $(h_1^I)^0$ as $\zeta_2 \longrightarrow \infty$. Equations (36), (37), (44), (45) and (52) to (55) yield the composite solutions as

$$h_0^c = \frac{N_1}{\alpha\sqrt{\iota}} + \frac{\iota N_2}{\alpha^2} + \frac{\sqrt{\iota}N_3}{\alpha^3} + \frac{\iota N_4}{\alpha^3} \tag{56}$$

$$h_1^c = \frac{\iota N_5}{\alpha^2} + \frac{\sqrt{\iota}N_6}{\alpha^3} \tag{57}$$

where

$$N_1 = \frac{1}{C_1^* a_0}exp(-a_0\zeta_1)$$

$$N_2 = \left(\frac{1}{4C_1^*} + \frac{C_2^*}{C_1^{*2}a_0^2} - \frac{a_0^2\zeta_1^2}{4C_1^*} + \frac{a_0\zeta_1}{4C_1^*}\right)exp(-a_0\zeta_1) + exp(-\zeta_2) - 1$$

$$N_3 = -\frac{1}{C_1^*}exp(-a_0\zeta_1)\left(\frac{3a_0}{8} + \frac{C_2^*}{a_0C_1^*} + \frac{C_2^{*2}}{C_1^{*2}a_0^3} - \frac{C_2^*}{a_0} + \frac{a_0^5\zeta_1^4}{32} - \right.$$

$$\left. \frac{3a_0^4\zeta_1}{16} - \frac{C_2^*a_0\zeta_1^2}{4C_1^*} + \frac{C_2^*\zeta_1}{4C_1^*}\right)$$

$$N_4 = -\frac{1}{4}exp(-\zeta_2)(\zeta_2^2 - \zeta_2)$$

$$N_5 = -\frac{M_1}{2a_0^2}(1 + a_0\zeta_1)exp(-a_0\zeta_1)$$

$$N_6 = \left[\frac{M_1C_2^*}{2a_0^3C_1^*} - \frac{M_2}{2a_0^2} - \frac{M_3}{4a_0^3} - \frac{M_4}{4a_0^4} - \frac{M_2\zeta_1}{2a_0} - \frac{M_3}{4a_0^2}(a_0\zeta_1^2 + \zeta_1)\right]exp(-a_0\zeta_1)$$

$$- \frac{M_4}{2a_0}\left[\frac{\zeta_1^3}{3} + \frac{\zeta_1^2}{2a_0} + \frac{\zeta_1}{2a_0^2}\right]exp(-a_0\zeta_1)$$

$$- \frac{\zeta_2}{2}\left[A_1 + A_2 + \frac{\beta_0}{2}\right]exp(-\zeta_2)$$

Equation (28) yields

$$h(\eta) = \left[\frac{1}{\alpha}\frac{N_1}{\sqrt{\iota}} + \frac{1}{\alpha^2}\iota N_2 + \frac{1}{\alpha^3}\sqrt{\iota}N_3 + \frac{1}{\alpha^3}\iota N_4\right] + \epsilon\left[\frac{1}{\alpha^2}\iota N_5 + \frac{1}{\alpha^3}\sqrt{\iota}N_6\right] \tag{58}$$
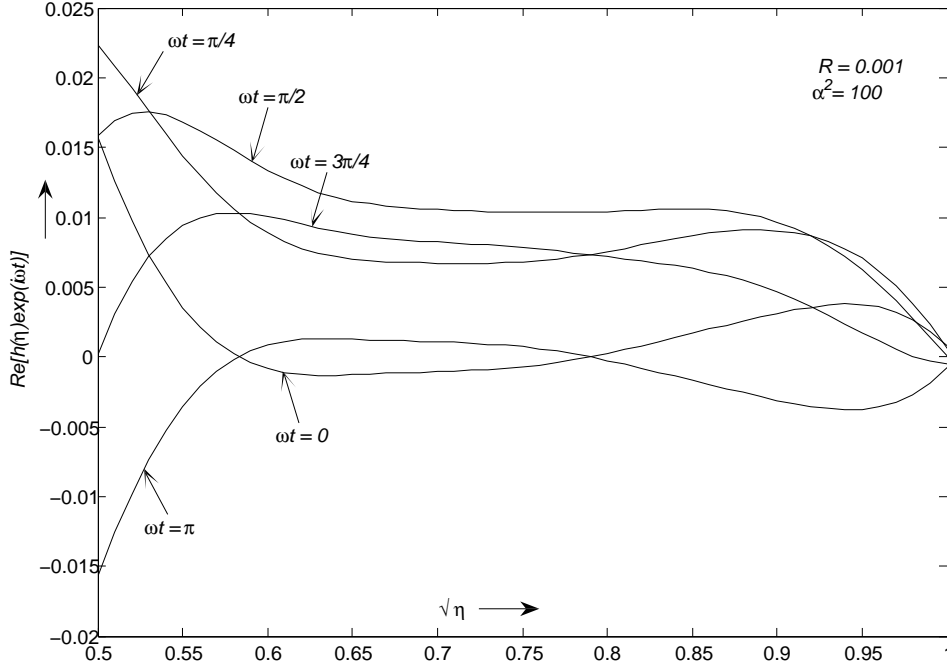
Figure 2: Unsteady axial velocity for small suction and large $\alpha^2 = 100$

4 Discussion

Fig. 2 depicts the unsteady component of axial velocity for $R = 0.001, \alpha^2 = 100, C^* = 0.05$ and $\lambda = 0.1$ at different times. It can be observed from the graph that the regions of maximum magnitude are situated near the boundaries. With the passage of time the maxima merge together giving rise to only one maximum at a certain instant. With further increase in time, different maxima show up again indicating the predominance of the frequency parameter effects near the boundaries. Fig. 3 shows the unsteady component of axial velocity for $R = 0.001, \alpha^2 = 144, C^* = 0.05$ and $\lambda = 0.1$ at different times. It shows the same qualitative pattern as shown in Fig. 3 except that with increase in the value of α^2 there is decrease in the magnitude of unsteady component of axial velocity near the inner boundary. Fig. 4 shows the unsteady component of axial velocity for $R = 0.001, \alpha^2 = 225, C^* = 0.05$ and $\lambda = 0.1$ at different times. It shows the same qualitative pattern as shown in Fig. 3 and Fig. 4 except that with further increase in the value of α^2 there is further decrease in the magnitude of unsteady component of axial velocity near the inner boundary. With the passage of time the maxima merge together giving rise to only one maximum at a certain instant. The critical points in the axial velocity profiles are closer to the lnner boundary. From the present study it has been found that large α^2 is conducive to the boundary layer growth. Fig. 5 depicts that with increase in$R$there is an
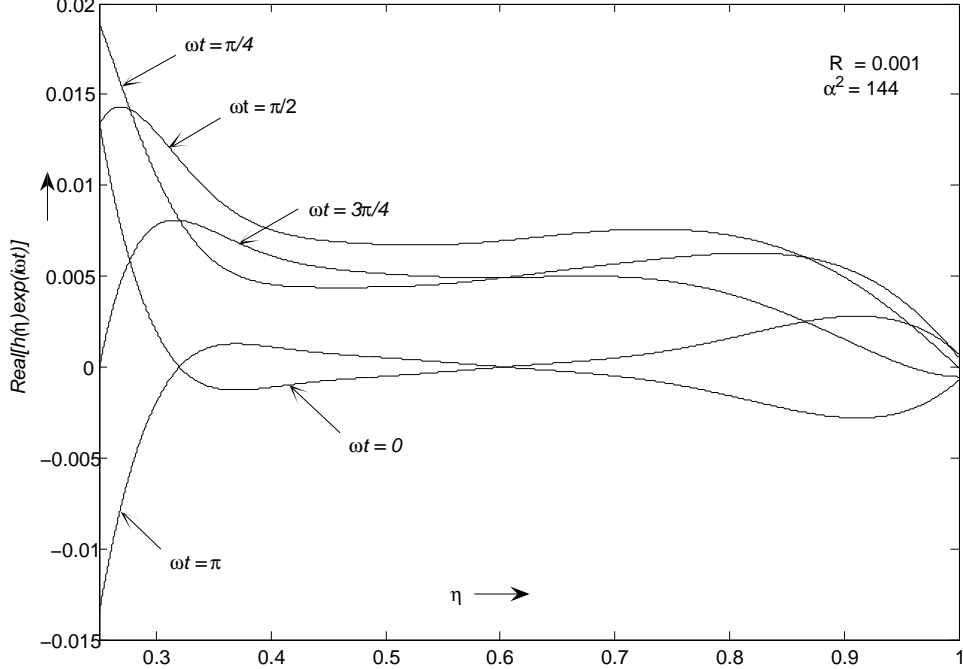
Figure 3: Unsteady axial velocity for small suction and $\alpha^2 = 144$

increase in the magnitude of unsteady component of axial velocity near the inner boundary. Fig. 6 shows that with increase in the value of λ there is further increase in the magnitude of unsteady component of axial velocity near the inner boundary. The investigation shows that the large value of frequency parameter has a marked effect on pulsatile flow in a porous annulus with small suction.

5   Numerical solution and comparison

Solution was also obtained numerically for large value of α^2 and compared with the values obtained from equations (58). The boundary value problem was solved by a shooting method with the initial estimates for the unknown boundary conditions at $\eta = \eta_0$ and $\eta = 1$ being the results from the regular perturbation case presented in [14]. Subsequently the results for large values of ϵ and α^2 were obtained by progressively increasing ϵ and α^2 by small values. The missing boundary conditions were estimated by extrapolating the previously computed values. These numerical values are compared with the results from the singular perturbation solution in Table 1.
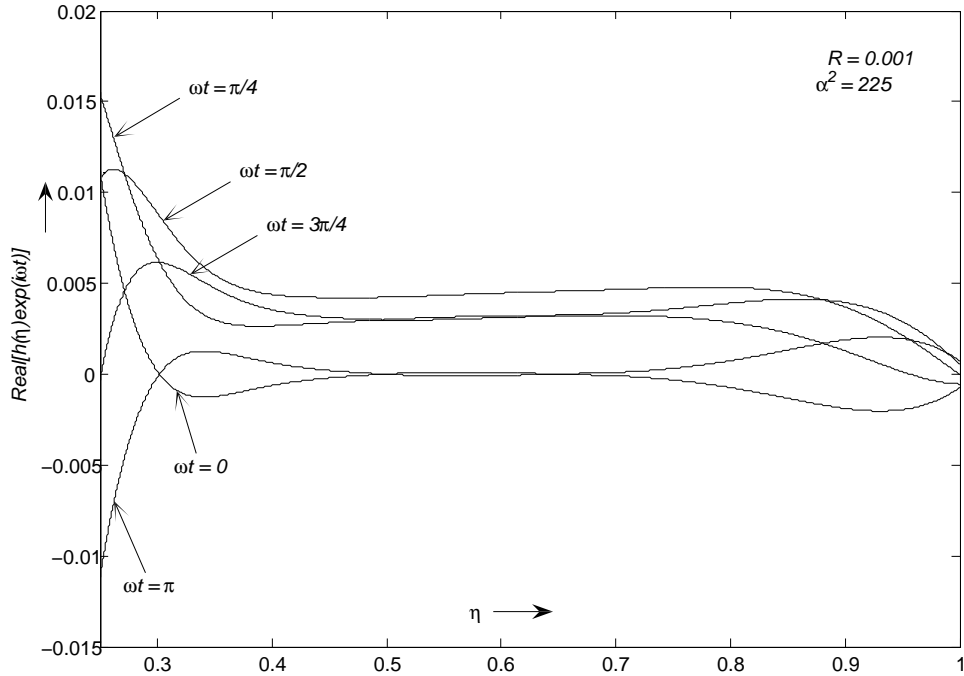
Figure 4: Unsteady axial velocity for small suction and $\alpha^2 = 225$
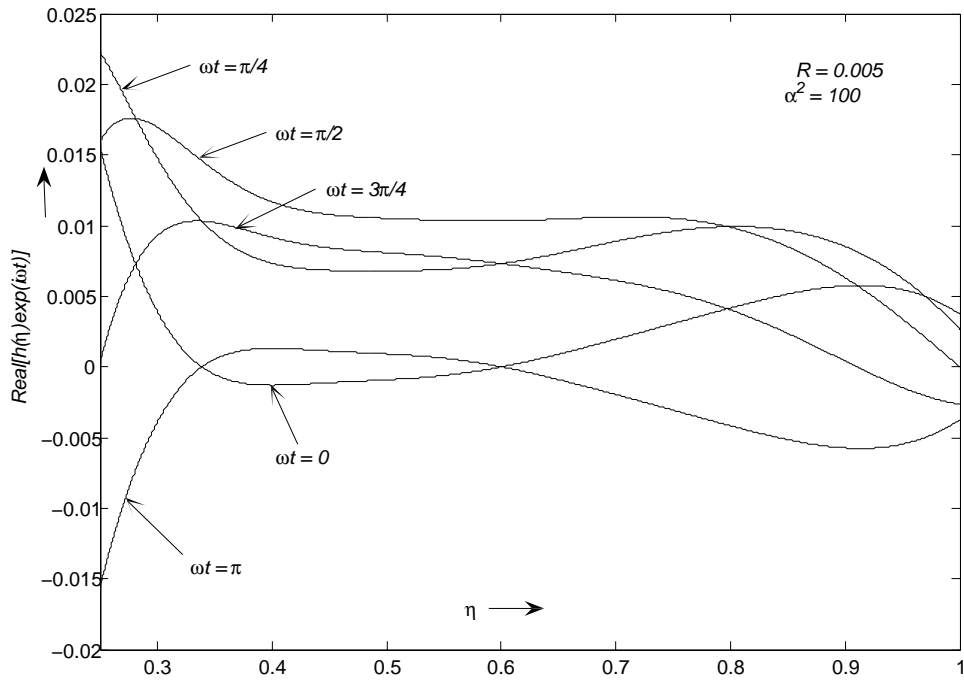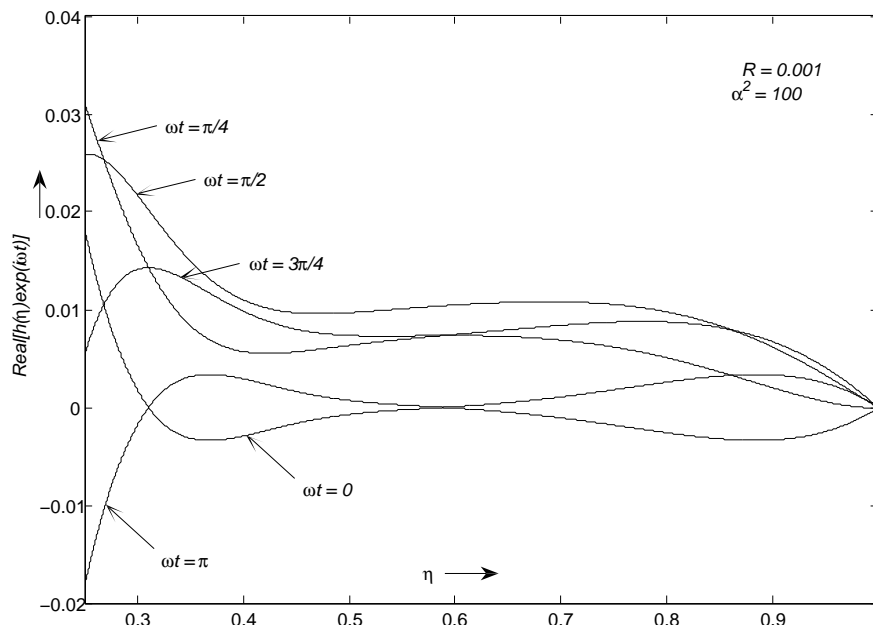

Figure 5: Unsteady axial velocity for $R = 0.005$ and $\alpha^2 = 100$

Figure 6: Unsteady axial velocity for small suction $\lambda = 0.4$ and $\alpha^2 = 100$

η	0.40	0.50	0.60	0.70	0.80	0.90
$|h|$ = Perturbation	0.0118	0.0105	0.0103	0.0107	0.0102	0.0074
$|h|$ = Numerical	0.0120	0.0100	0.0102	0.0106	0.0101	0.0071

Table 1: Comparison of Perturbation and Numerical Solutions for $R = 0.001, \alpha^2 = 100$, $C^* = 0.05$

Acknowledgements

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Authors' address

Elsadig A. Hamza — Department of Mathemathics and Statistics, Sultan Qaboos University, Moscat–Oman

Sunita R. Jain — Chandigarh College of Engineering, Chandigarh, India
e-mail: sunitarjain@yahoo.com

Nirmal C. Sacheti — Department of Mathemathics and Statistics, Sultan Qaboos University, Moscat–Oman
e-mail: nirmal@squ.edu.om