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Stability of the feasible set in balanced transportation problems

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Abstract

In this paper we study the stability of the feasible set of a balanced transportation problem. A transportation problem is balanced when the total supply is equal to the total demand. One can easily see that when we make minor adjustments to the data (supply and demand), the resulting problem may lose the property of balance. Therefore, although the transportation problem is a particular case of linear programming, you cannot apply the familiar results of stability. For a fixed number of origins and destinations we have obtained a vector representation for any feasible solution of the transportation problem. We have used this representation to prove that the feasible set mapping is continuous. We have also proved that the extreme point set mapping is lower semi continuous.[1]

Keywords: Transportation problem, Stability, Linear programming

MSC(2000): 74PXX, 47N10

1 Introduction

The well-known Hitchcock transportation problem (TP in short) with m-sources and n-destinations can be formulated as the following linear program:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, ..., m, \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, ..., n, \tag{3}$$

$$x_{ij} \geq 0, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n, \tag{4}$$

where c_{ij} is the unit shipping cost from source i to destination j; the variable x_{ij} represents the number of units shipped from source i to destination j; a_i is the

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supply of the source i and b_j is the demand of destination j. Let us define

$$\boldsymbol{d} = (a_1, ..., a_m, b_1, ..., b_n)$$

as the supply and demand vector, wich is a vector with $m+n$ positive numbers with $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$; this equality means that transportation problem is balanced. Let $\boldsymbol{F}$ denote the solution set of the equality and inequality systems (2), (3) and (4). In this work we study the behavior of $\boldsymbol{F}$ when data suffer small changes. As you can see, from (2) and (3), $\boldsymbol{F}$ depends only on the supply and demand. Therefore, to study the stability of the feasible set $\boldsymbol{F}$, see [1], [2] and [3], we fix m and n, and we vary only the right hand side of the constraints, i.e., the vector $\boldsymbol{d}$, which we will call the parameter. Thus, our parameter space will be the set

$$\mathcal{B} = \left\{ \boldsymbol{d} \in \mathbb{R}^{m+n}_{++} \quad | \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \right\},$$

where $\mathbb{R}^{m+n}_{++} = \{\boldsymbol{x} \in \mathbb{R}^{m+n} : x_i > 0,\ i = 1, ..., m+n\}$. The set $\mathcal{B}$ is a relatively open cone of the dimension $m+n-1$ and it does not contain the origin. For instance, for $m = 1$, $n = 2$,

$$\mathcal{B} = \left\{ \boldsymbol{d} \in \mathbb{R}^{3}_{++} \quad | \quad a_1 = b_1 + b_2 \right\}.$$

When various transportation problems (or vectors $\boldsymbol{d}$) are simultaneously considered, they and their associated feasible sets are distinguished by means of subscripts: $\boldsymbol{d}_k$ and $\boldsymbol{F}_k$ respectively.

To measure the size of the adjustments we consider the Euclidean norm of $\mathbb{R}^{m+n}$ restricted to $\mathcal{B}$.

The following results can be found in [4], p. 557 : the vector $\bar{\boldsymbol{x}}$ with components

$$\bar{x}_{ij} = \frac{a_i b_j}{\sum_{i=1}^{m} a_i}$$

satisfies (2), (3) and (4), and if $\boldsymbol{x} \in \boldsymbol{F}$ then the next inequalities hold:

$$0 \leq x_{ij} \leq \min \{a_i, b_j\},$$

for each component of $\boldsymbol{x}$. Therefore $\boldsymbol{F}$ is a nonempty, convex, closed and bounded set.

The paper is organized as follows: Section 2 introduces the necessary concepts and notation and summarizes the existing theory on stability in linear semi-infinite programming, when the right hand side of the constraints is the only one variable. Section 3 gives a vector representation of a feasible solution of the TP. Finally Section 4 shows that the feasible set mapping $\boldsymbol{\mathcal{F}}$ is continuous everywhere in $\mathcal{B}$, and also shows that the extreme points set mapping is lower semicontinuous everywhere in $\mathcal{B}$.

2 Preliminaries

In this section we present some well-known definitions of continuity of set-valued mappings and a characterization of lower semicontinuity that will be used in the proof of the main result.

Definition 1. *([1] p. 128) Let us have a set-valued mapping $P : X \to Y$, between two topological spaces. We say that P is lower (upper) semicontinuous, according to Berge, at the point $x_0 \in X$, if for every open set W such that*

$$W \cap P(x_0) \neq \emptyset \qquad (W \supset P(x_0))$$

there exists a neighborhood $V \ni x_0$ such that for every $x \in V$, the following holds:

$$W \cap P(x) \neq \emptyset \qquad (W \supset P(x)).$$

Also we say that P is lower (upper) semicontinuous on X, if P is lower (upper) semicontinuous at every $x \in X$. We say that P is continuous, according to Berge, at the point $x_0 \in X$, if it is both lower and upper semicontinuous at $x_0 \in X$. Finally we say that P is continuous on X if it is continuous at every $x \in X$.

We will use the following characterization of the lower semicontinuity of a set valued mapping.

Lemma 1. *(see Lemma 1.1 [6]). Let us have a set-valued mapping $P : X \to Y$, between two metric spaces. Then P is lower semicontinuous at the point $x_0 \in X$ if and only if for every $y_0 \in P(x_0)$ and for each sequence $\{x_n\}_{n\geq 1}$ converging to x_0, there exists a subsequence $\{y_{n_m}\}_{m\geq 1}$ such that $y_{n_m} \in P(x_{n_m})$, m=1,2,..., and $\{y_{n_m}\}_{m\geq 1}$ converges to y_0.*

We also use the results concerning the stability of the solution set of a linear semi-infinite system represented by σ. That is,

$$\sigma := \{\boldsymbol{a}_t'\boldsymbol{x} \leq b_t : t \in T\},$$

where $\boldsymbol{a}_t$ and $\boldsymbol{x}$ are vectors in $\mathbb{R}^n$; $\boldsymbol{a}_t'$ represents the corresponding transposed vector, b_t is a scalar, and the nonempty index set T is a Hausdorff compact space, possibly infinite. This class of systems are considered in [6] where the right hand side of the inequalities is the only one variable, so the parameter space will be the set of continuous functions $C(T)$, endowed with the Tchebycheff norm. Let $\boldsymbol{P}$ denote the solution set of σ. Then we can define the feasible set mapping $\boldsymbol{\mathcal{P}} : C(T) \to \mathbb{R}^n$ that assigns to each $\boldsymbol{b} \in C(T)$ the feasible set $\boldsymbol{P}$ of the system σ defined by $\boldsymbol{b}$. Next we recall the well-known Slater condition:

Definition 2. *([1] p. 128) We say that σ satisfies the Slater condition if there exists a $\boldsymbol{x} \in \mathbb{R}^n$ such that $\boldsymbol{a}_t'\boldsymbol{x} < b_t$, for every $t \in T$.*

Note that if T is a finite set, then a Slater point is in fact an interior point of the feasible set of the system σ.
We study the behavior of the feasible set $\boldsymbol{F}$ of the TP by mean of the continuity, according to Berge, of the feasible set mapping $\boldsymbol{\mathcal{F}} : \mathcal{B} \to \mathbb{R}^{mn}$ that assigns to each $\boldsymbol{d} \in \mathcal{B}$ the feasible set $\boldsymbol{F}$ of the TP defined by $\boldsymbol{d}$. In ordinary linear programming (OLP) and in linear semi-infinite programming (LSIP), as can be seen in Theorem 1 and Theorem 4.6 of [5], there is a characterization of the lower semicontinuity of the feasible set mapping with the Slater condition.

Theorem 1. *(see Theorem 2.1 [6]) Let $\boldsymbol{b} \in C(T)$. Then the following two statements are equivalent:*

1. *The feasible set mapping $\boldsymbol{\mathcal{P}}$ is lower semicontinuous at $\boldsymbol{b}$.*
2. *The system σ defined by $\boldsymbol{b}$ satisfies the Slater condition.*

However, in the feasible set of a TP there is not a Slater point. Furthermore, it is no difficult to see that, in the general theory of set mappings, those properties are not equivalent.

3 Vector representation for feasible solutions of TP

To study the continuity of the feasible set mapping $\boldsymbol{\mathcal{F}}$, it is necessary a vector representation for any feasible point of the TP. We do this solving the system of equations (2) and (3) for some variables in terms of the other ones. It has not been difficult to obtain such representation in various individual cases.

Example 1. *For the case $m = 2, n = 3$, we have 2 sources and 3 destinations, and $\boldsymbol{d} = (a_1, a_2, b_1, b_2, b_3)$ satisfies the equation $a_1 + a_2 = b_1 + b_2 + b_3$. Furthermore, $\boldsymbol{x} = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) \in F$ if and only if*

$$\begin{array}{llllllll} x_{11} + x_{12} + x_{13} & & & & = a_1 \\ & & x_{21} + x_{22} + x_{23} & & = a_2 \\ x_{11} & & +x_{21} & & = b_1 \\ & x_{12} & & +x_{22} & = b_2 \\ & & x_{13} & + x_{23} & = b_3 \\ & & & x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0. \end{array} \tag{5}$$

This system has infinitely many solutions. If $x_{22} = t_{22}$ and $x_{23} = t_{23}$, we have that $\boldsymbol{x} \in F$ if and only if

$$\boldsymbol{x} = \begin{pmatrix} b_1 - a_2 + t_{22} + t_{23} \\ b_2 - t_{22} \\ b_3 - t_{23} \\ a_2 - t_{22} - t_{23} \\ t_{22} \\ t_{23} \end{pmatrix} = \begin{pmatrix} b_1 - a_2 \\ b_2 \\ b_3 \\ a_2 \\ 0 \\ 0 \end{pmatrix} + t_{22} \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t_{23} \begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{pmatrix},$$

where, because of (4), $\boldsymbol{t} = (t_{22}, t_{23})'$ *is a solution of the following inequality system:*

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} t_{22} \\ t_{23} \end{pmatrix} \leq \begin{pmatrix} b_1 - a_2 \\ b_2 \\ b_3 \\ a_2 \end{pmatrix} \tag{6}$$

$$t_{22}, t_{23} \geq 0.$$

The solution set of the inequality system (6) *is a polytope in* $\mathbb{R}^2$, *and*

$$\boldsymbol{t} = (t_{22}, t_{23}) = (\frac{a_2 b_2}{a_1 + a_2}, \frac{a_2 b_3}{a_1 + a_2})$$

is a Slater point, therefore the solution set of the inequality system (6) *has at least 3 extreme points. Also note that there is a biunivoque correspondence between the solution set of the systems* (5) *and* (6). *Moreover, there is a biunivoque correspondence between the extreme points of the systems* (5) *and* (6).

Now we will give a vector representation to any feasible point of the trasportation problem. To this end we denote by $\boldsymbol{e}_j$ the jth unit vector in $\mathbb{R}^n$. Following the last example we have obtained that, the point

$$\boldsymbol{x} = (x_{11}, x_{12}..., x_{1n}, x_{21}, x_{22}, ..., x_{2n}, ..., x_{m1}, x_{m2}, ..., x_{mn})'$$

with $x_{ij} \geq 0$ for $i = 1, 2, ..., m$ and $j = 1, ..., n$ and where each one of theses components have the form:

$$
\begin{aligned}
x_{11} &= \left(b_1 - \sum_{i=2}^{m} a_i\right) + \sum_{i=2}^{m}\sum_{j=2}^{n} t_{ij} \\
x_{12} &= b_2 - \sum_{i=2}^{m} t_{i2} \\
&\vdots \\
x_{1n} &= b_n - \sum_{i=2}^{m} t_{in} \\
x_{21} &= a_2 - \sum_{j=2}^{n} t_{2j} \\
x_{22} &= t_{22} \\
&\vdots \\
x_{2n} &= t_{2n} \\
x_{31} &= a_3 - \sum_{j=2}^{n} t_{3j} \\
x_{32} &= t_{32} \\
&\vdots \\
x_{3n} &= t_{3n} \\
&\vdots \\
x_{m1} &= a_m - \sum_{j=2}^{n} t_{mj} \\
x_{m2} &= t_{m2} \\
&\vdots \\
x_{mn} &= t_{mn}
\end{aligned}
\tag{7}
$$

is a feasible solution to $m \times n$ TP. In other words, any feasible solution of the transportation problem can be expressed in vector form as follows:

$$\boldsymbol{x} = \bar{\boldsymbol{b}} + \sum_{i=2}^{m}\sum_{j=2}^{n} t_{ij}\boldsymbol{y_{ij}}, \tag{8}$$

where

$$\bar{\boldsymbol{b}} = \begin{pmatrix} \sum_{i=1}^{n} b_i e_i - \sum_{i=2}^{m} a_i e_1 \\ a_2 e_1 \\ a_3 e_1 \\ \vdots \\ a_m e_1 \end{pmatrix},$$

$$\boldsymbol{y}_{2j} = \begin{pmatrix} e_1 - e_j \\ -e_1 + ej \\ 0_n \\ \vdots \\ 0_n \end{pmatrix}, \quad j = 2, ..., n,$$

$$\boldsymbol{y}_{3j} = \begin{pmatrix} e_1 - e_j \\ 0_n \\ -e_1 + ej \\ \vdots \\ 0_n \end{pmatrix}, \quad j = 2, ..., n,$$

$$\boldsymbol{y}_{mj} = \begin{pmatrix} e_1 - e_j \\ 0_n \\ 0_n \\ \vdots \\ -e_1 + ej \end{pmatrix}, \quad j = 2, ..., n,$$

and $\mathbf{t} = (t_{22}, ..., t_{2n}, t_{32}, ..., t_{3n}, ..., t_{m2}, ..., t_{mn})'$ is a solution of the following inequality system

$$\begin{array}{lcl} -\sum_{i=2}^{m}\sum_{j=2}^{n} t_{ij} & \leq & b_1 - \sum_{i=2}^{n} a_i \\ \sum_{i=2}^{m} t_{i2} & \leq & b_2 \\ \vdots & & \\ \sum_{i=2}^{m} t_{in} & \leq & b_n \\ \sum_{j=2}^{n} t_{2j} & \leq & a_2 \\ \sum_{j=2}^{n} t_{3j} & \leq & a_3 \\ \sum_{j=2}^{n} t_{mj} & \leq & a_m \\ t_{ij} & \geq & 0 \quad \text{for} \quad i = 2, ..., m; \quad j = 2, ..., n \end{array} \tag{9}$$

The solution set of the inequality system (9) is a polytope in $\mathbb{R}^{(m-1)(n-1)}$. From (8) we get a biunivoque correspondence between the solution set of the systems (2)-(4) and (9). Also note that there is a biunivoque correspondence between the extreme points of the systems (2)-(4) and (9). Let $\boldsymbol{Z}$ denote the solution set of the inequality system (9). Then we can define a set valued mapping

$$\boldsymbol{\mathcal{Z}} : \mathcal{B} \to \mathbb{R}^{(m-1)(n-1)}$$

that assigns to each $\boldsymbol{d} \in \mathcal{B}$ the solution set $\boldsymbol{Z}$ of the inequality system (9). In the next section we will use the lower semicontinuity of $\boldsymbol{\mathcal{Z}}$ to prove the lower semicontinuity of $\boldsymbol{\mathcal{F}}$. The proof of the lower semicontinuity of $\boldsymbol{\mathcal{Z}}$ requires the next proposition:

Proposition 1. *Given $\boldsymbol{d} \in \mathcal{B}$, the point $\boldsymbol{t} \in \mathbb{R}^{(m-1)(n-1)}$ with components*

$$t_{ij} = \frac{a_i b_j}{\sum_{i=1}^m a_i}, \quad \textit{for} \quad i = 2, ..., m; \quad j = 2, ..., n,$$

is a Slater point of the $\boldsymbol{Z}$.

Proof. It is clear that

$$t_{ij} = \frac{a_i b_j}{\sum_{i=1}^m a_i} > 0, \quad \text{for} \quad i = 2, ..., m; \quad j = 2, ...n.$$

Let us see that the first inequality of (9) is satisfied strictly, i.e.,

$$\sum_{i=2}^{m} \sum_{j=2}^{n} \frac{a_i b_j}{\sum_{i=1}^m a_i} > \sum_{i=2}^{m} a_i - b_1. \tag{10}$$

Multiplying by $\sum_{i=1}^m a_i > 0$, and using the equality $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, we get that (10) is equivalent to

$$\sum_{i=2}^{m} \sum_{j=2}^{n} a_i b_j > \sum_{i=2}^{m} a_i \sum_{j=1}^{n} b_j - b_1 \sum_{i=1}^{m} a_i. \tag{11}$$

We can write (11) in an equivalent form as

$$\sum_{i=2}^{m} \sum_{j=2}^{n} a_i b_j > b_1 \sum_{i=2}^{m} a_i + \sum_{i=2}^{m} a_i \sum_{j=2}^{n} b_j - b_1 \sum_{i=2}^{m} a_i - b_1 a_1, \tag{12}$$

and finally we get that (12) is equivalent to

$$b_1 a_1 > 0.$$

Now let us prove the next inequalities of (9), for $j = 2, 3, ..., n$.

$$\sum_{i=2}^{m} t_{ij} = \sum_{i=2}^{m} \frac{a_i b_j}{\sum_{i=1}^{m} a_i} = \frac{\sum_{i=2}^{m} a_i}{\sum_{i=1}^{m} a_i} b_j < b_j \quad \text{for} \quad j = 2, 3, ..., n.$$

Finally we prove the last inequalities of (9), for $i = 2, 3, ..., m$.

$$\sum_{j=2}^{n} t_{ij} = \sum_{j=2}^{n} \frac{a_i b_j}{\sum_{j=1}^{n} b_j} = \frac{\sum_{j=2}^{n} b_j}{\sum_{j=1}^{n} b_j} a_i < a_i \quad \text{for} \quad i = 2, 3, ..., m.$$

□

So, we find that the dimension of the feasible set $\boldsymbol{F}$ is $(n-1)(m-1)$. Moreover, by Theorem 1 and Proposition 1 we have:

Corollary 1. *The set valued mapping $\mathcal{Z} : \mathcal{B} \to \mathbb{R}^{(m-1)(n-1)}$ is lower semicontinuous at every $\boldsymbol{d} \in \mathcal{B}$.*

4 Feasible and extreme point set mappings continuity

In this section we use the vector representation of any point of the set $\boldsymbol{F}$ to prove the continuity of the feasible set mapping $\boldsymbol{\mathcal{F}}$. Next we state the main result of this paper.

Theorem 2. *The feasible set mapping $\boldsymbol{\mathcal{F}} : \mathcal{B} \to \mathbb{R}^{mn}$ of the $m \times n$ transportation problem is lower semicontinuous at every $\boldsymbol{d} \in \mathcal{B}$.*

Proof. Let us assume that the mapping $\boldsymbol{\mathcal{F}}$ is not lower semicontinuous at the point $\boldsymbol{d}_0 \in \mathcal{B}$, i.e., there exists an open set $W \subset \mathbb{R}^{mn}$ such that $W \cap \boldsymbol{F}_0 \neq \emptyset$, and a sequence $\{\boldsymbol{d}_n\}_{n\geq 1} \subset \mathcal{B}$ such that $\boldsymbol{d}_n \to \boldsymbol{d}_0$, and such that

$$W \cap \boldsymbol{F}_n = \emptyset \quad \text{for every} \quad n = 1, 2, 3, ... \tag{13}$$

Let us take $\boldsymbol{x}_0 \in W \cap \boldsymbol{F}_0$. Then, using the vector representation (8), we have

$$\boldsymbol{x}_0 = \overline{\boldsymbol{b}}_0 + \sum_{i=2}^{m} \sum_{j=2}^{n} (\boldsymbol{t}_0)_{ij} \boldsymbol{y_{ij}},$$

where $(\boldsymbol{t}_0)_{ij}$ is the (ij)th component of the vector $\boldsymbol{t}_0$, for some $\boldsymbol{t}_0 \in \boldsymbol{Z}_0$, where $\boldsymbol{Z}_0$ is the solution set of the inequality system (9). Because of Corollary 1, the set valued mapping $\boldsymbol{\mathcal{Z}}$ is lower semicontinuous at $\boldsymbol{d}_0$. Then, as $\boldsymbol{d}_n \to \boldsymbol{d}_0$, by Lemma 1 there exists a subsequence $\{\boldsymbol{t}_{n_m}\}_{m\geq 1}$ such that $\boldsymbol{t}_{n_m} \in \boldsymbol{Z}_{n_m}$ and $\boldsymbol{t}_{n_m} \to \boldsymbol{t}_0$. Now let us construct a sequence $\{\boldsymbol{x}_{n_m}\}_{m\geq 1}$ as follows : for every $\boldsymbol{t}_{n_m} \in \boldsymbol{Z}_{n_m}$, using again the vector representation (8), let set

$$\boldsymbol{x}_{n_m} = \overline{\boldsymbol{b}}_{n_m} + \sum_{i=2}^{m} \sum_{j=2}^{n} (\boldsymbol{t}_{n_m})_{ij} \boldsymbol{y_{ij}}.$$

Then for every $m = 1, 2, 3, ...$, we have that $\boldsymbol{x}_{n_m}$ belongs to $\boldsymbol{F}_{n_m}$. By construction we have that $\{\boldsymbol{x}_{n_m}\}_{m\geq 1}$ converges to $\boldsymbol{x}_0$. Because of W is an open set, for m sufficiently large, $\boldsymbol{x}_{n_m} \in W$. Thus, we have reached a contradiction with (13). □

We have already seen that, for every $\boldsymbol{d} \in \boldsymbol{\mathcal{B}}$, the feasible set $\boldsymbol{F}$ of the transportation problem is closed and bounded. Since the multivalued feasible set mapping $\boldsymbol{\mathcal{F}}$ is lower semicontinuous, the following theorem is proved in a similar way to Proposition 2.1 of [6].

Theorem 3. *The feasible set mapping $\boldsymbol{\mathcal{F}}$ of the $m \times n$ transportation problem is upper semicontinuous at every $\boldsymbol{d} \in \boldsymbol{\mathcal{B}}$.*

We conclude, from Theorem 2 and Theorem 3, that the feasible set mapping $\boldsymbol{\mathcal{F}}$ of the TP is continuous at every parameter of the space $\mathcal{B}$. That is, the feasible set of any transportation problem is stable in $\mathcal{B}$.

As an application of the Theorem 2 we prove the lower semicontinuity of the extreme points set mapping $\boldsymbol{ext\mathcal{F}} : \mathcal{B} \to \mathbb{R}^{mn}_{++}$ that assigns to each $\boldsymbol{d} \in \mathcal{B}$ its extreme points set, $\boldsymbol{extF}$, of the feasible set $\boldsymbol{F}$ of the transportation problem. To prove the lower semicontinuity of the extreme points set mapping, $\boldsymbol{ext\mathcal{F}}$, we use the transmission properties theory between set mappings $\boldsymbol{\mathcal{P}} : X \to Y$, where X and Y are normed spaces. Under the assumption that for each $\boldsymbol{x} \in X$, $\boldsymbol{P(x)}$ is convex, closed and bounded, the following result can be found in [7]: If the feasible set mapping $\boldsymbol{\mathcal{P}}$ is lower semicontinuous at $\boldsymbol{x}$, then $\boldsymbol{ext\mathcal{P}}$ is lower semicontinuous at $\boldsymbol{x}$. Therefore, using this result and Theorem 2 we have the following corollary:

Corollary 2. *The extreme points set mapping $\boldsymbol{ext\mathcal{F}}$ of the $m \times n$ transportation problem is lower semicontinuous on $\mathcal{B}$.*

Finally, to prove the continuity of $\boldsymbol{ext\mathcal{F}}$ we need to prove that $\boldsymbol{ext\mathcal{F}}$ is upper semicontinuous but this is still an open problem.

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