A&A 493, 51–54 (2009)
DOI: 10.1051/0004-6361:20078635

Astronomy & Astrophysics

Factor analysis of the long gamma-ray bursts

Z. Bagoly[1], L. Borgonovo[2], A. Mészáros[2,3], L. G. Balázs[4], and I. Horváth[5]

[1] Dept. of Physics of Complex Systems, Eötvös University, 1117 Budapest, Pázmány P. s. 1/A, Hungary
e-mail: zsolt.bagoly@elte.hu
[2] Stockholm Observatory, AlbaNova, 106 91 Stockholm, Sweden
e-mail: luis@astro.su.se
[3] Astronomical Institute of the Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic
e-mail: meszaros@mbox.cesnet.cz
[4] Konkoly Observatory, 1525 Budapest, POB 67, Hungary
e-mail: balazs@konkoly.hu
[5] Department of Physics, Bolyai Military University, 1581 Budapest, POB 15, Hungary
e-mail: horvath.istvan@zmne.hu

Received 7 September 2008 / Accepted 2 October 2008

ABSTRACT

Aims. We study statistically 197 long gamma-ray bursts, detected and measured in detail by the BATSE instrument of the Compton Gamma-Ray Observatory. In the sample, 10 variables, describing for any burst the time behavior of the spectra and other quantities, are collected.

Methods. The factor analysis method is used to find the latent random variables describing the temporal and spectral properties of GRBs.

Results. The application of this particular method to this sample indicates that five factors and the $\mathcal{R}E_{\rm pk}$ spectral variable (the ratio of peak energies in the spectrum) describe the sample satisfactorily. Both the pseudo-redshifts inferred from the variability, and the Amati-relation in its original form, are disfavored.

Key words. gamma rays: bursts – gamma rays: observations – methods: data analysis – methods: statistical

1. Introduction

Factor analysis (FA) and principal component analysis (PCA) are powerful statistical methods in data analysis. Using PCA and FA Bagoly et al. (1998) demonstrated that the 9 variables typically measured (T_{50} and T_{90} durations; P_{64}, P_{256}, and P_{1024} peak fluxes; $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$, and $\mathcal{F}_4$ fluences) for gamma-ray bursts (GRBs), observed by the BATSE instrument onboard the Compton Gamma-Ray Observatory and listed in the Current BATSE Catalog (Meegan et al. 2001), can be satisfactorily represented by 3 hidden statistical variables. Borgonovo & Björnsson (2006, hereafter BB06) studied the statistical properties of 197 long GRBs detected by BATSE. They defined 10 statistical variables describing the temporal and spectral properties of GRBs. By performing a PCA, they concluded that about 70% of the total variance in the parameters were explained by the first 3 principal components (PCs). The aim of this article is to proceed in a similar way to BB06 by using instead FA.

By solving the eigenvalue problem of the correlation (covariance) matrix, PCA transforms the observed variables into the same number of uncorrelated variables (PCs). An essential ingredient of PCA is a distinction between the "important" and "less important" variables by taking into account the magnitude of the eigenvalues of the correlation (covariance) matrix. FA assumes that the observed variables can be described by a linear combination of hidden variables given by:

$$\boldsymbol{x} = \boldsymbol{\Lambda} \boldsymbol{f} + \varepsilon, \qquad (1)$$

where $\boldsymbol{x}$ denotes an observed variable of dimension p, $\boldsymbol{\Lambda}$ is a matrix of $p \times m$ dimensions ($m < p$), f represents a hidden variable of m dimensions. The components of $\boldsymbol{\Lambda}$ are called loadings, the factor f represents scores, and ε is a noise term. We can infer $\boldsymbol{x}$ from observations while the quantities on the right-hand-side of Eq. (1) have to be computed by a suitable algorithm.

PCA expresses the $\boldsymbol{x}$ observed variable as a linear transformation of a hidden variable of the same p dimension, whose components are uncorrelated. The transformation matrix is set up from the eigenvectors of the correlation matrix of $\boldsymbol{x}$. By retaining only the first $m < p$ eigenvectors, it can be shown that, the resultant transformation matrix provides the best reproduction of $\boldsymbol{x}$ among those using only $m < p$ components. By retaining only the first $m < p$ eigenvectors, one receives a transformation matrix of dimensions $p \times m$ and an expression identical to the first term on the right side of Eq. (1). Due to this fact, the PCA is a default solution of FA in many statistical packages (e.g. SPSS[1]; for a detailed comparison of PCA and FA, see Jolliffe 2002). Although PCA is a default solution in many packages, FA has other algorithms as well. In our computations, we use the Maximum Likelihood (ML) method (for details see Jolliffe 2002).

2. The sample

We use the sample of 197 long GRBs in BB06 and the 10 variables defined there. Of the 10 variables, T_{90} and $\mathcal{F}$ were taken directly from the BATSE Catalog. The remaining 8 variables were calculated by BB06. In summary, the 10 variables are the

[1] SPSS is a registered trademark (see www.spss.com).

Table 1. Correlation matrix among the 10 variables. Except for α the decimal logarithms are taken.

Variable	$\log T_{90}$	$\log \mathcal{T}_{50}$	$\log \tau$	$\log V$	$\log \mathcal{S}_{\mathcal{F}}$	$\log \tau_{\rm lag}$	$\log \mathcal{R}E_{\rm pk}$	$\log \mathcal{F}$	$\log E_{\rm pk}$	α
$\log T_{90}$	1.00	0.78	0.58	0.18	0.09	–0.01	–0.15	0.5	0.24	–0.26
$\log \mathcal{T}_{50}$	0.78	1.00	0.87	0.51	0.25	0.09	–0.21	0.61	0.14	–0.16
$\log \tau$	0.58	0.87	1.00	0.4	0.24	0.15	–0.25	0.61	0.14	–0.12
$\log V$	0.18	0.51	0.4	1.00	0.32	–0.18	–0.37	0.33	0.08	–0.07
$\log \mathcal{S}_{\mathcal{F}}$	0.09	0.25	0.24	0.32	1.00	0.03	–0.37	0.07	–0.23	0.03
$\log \tau_{\rm lag}$	–0.01	0.09	0.15	–0.18	0.03	1.00	0.24	–0.04	–0.28	0.33
$\log \mathcal{R}E_{\rm pk}$	–0.15	–0.21	–0.25	–0.37	–0.37	0.24	1.00	–0.03	0.04	–0.01
$\log \mathcal{F}$	0.5	0.61	0.61	0.33	0.07	–0.04	–0.03	1.00	0.58	–0.2
$\log E_{\rm pk}$	0.24	0.14	0.14	0.08	–0.23	–0.28	0.04	0.58	1.00	–0.28
α	–0.26	–0.16	–0.12	–0.07	–0.03	0.33	–0.01	–0.2	–0.28	1.00

following: duration time T_{90}, emission time $\mathcal{T}_{50}$, autocorrelation function (ACF) half-width τ, variability V, emission symmetry $\mathcal{S}_{\mathcal{F}}$, cross-correlation function time lag $\tau_{\rm lag}$, the ratio of peak energies $\mathcal{R}E_{\rm pk}$, fluence $\mathcal{F}$, peak energy $E_{\rm pk}$, and low frequency spectral index α.

Since the variables have different dimensions in a similar way to BB06 we use the decimal logarithms (except for α). The correlations between the variables are indicated in Table 1. The choice of the logarithms is motivated by the fact that the distributions of most variables are well described by log-normal distributions (see the discussion of BB06).

In a similar way to BB06, we do not consider the fluence on the highest channel (>300 keV) separately, although in Bagoly et al. (1998) this variable alone was used to define a PC (factor). This choice is motivated by two reasons: first, fluences on the fourth channel often vanish or have significant errors (“the values are noisy”); second, as noted by BB06, in a sample of long-soft GRBs *only*, this quantity is less important. It is now certain that the long-soft and short-hard bursts are different phenomena (Horváth 1998; Norris et al. 2001; Horváth 2002; Balázs et al. 2003). The significance of the intermediate GRBs is unclear (Horváth et al. 2006).

3. Estimation of the number of factors

In contrast to PCA, in FA the choice of the number of hypothetical (latent) random variables (factors) is – at the beginning – a free parameter. To determine the optimal number of factors, there are no direct methods (even the notion “best number of factors” is unclear; see Jolliffe 2002).

By solving the eigenvalue problem of the correlation matrix, PCA yields PCs in descending order of the eigenvalue magnitudes. To validate a factor model, one retains the first $m < p$ PCs, which satisfactorily reproduce the original correlation matrix. In the ML method, the expected number of factors is an input parameter, and the algorithm computes the probability that the difference between the original and reproduced correlation matrix can be attributed to chance only. One stops increasing the number of factors, when this probability is already sufficiently large.

The factor model assumes that a linear transformation exists between the observed and the latent (factor) variables. The number of unknown parameters (i.e. $p\,(m+1)$ on the right side of Eq. (1)) are constrained by the dimension of the covariance matrix of $\boldsymbol{x}$ (i.e. $1/2\;p(p+1)$ independent parameters) and the need for factor-loading orthogonality, which provides $1/2\;m(m-1)$ free parameters (Kendall & Stuart 1973). Thus, the number m of factors can be constrained by the following inequality:

$$m \leq (2p + 1 - \sqrt{8p+1})/2, \qquad (2)$$

which provides $m \leq 6$ in our case. Since the number of factors is an integer, $m = 6$ is a maximum value in our case. Equation (2) provide the upper limit to the number of factors, although the true number remains to be estimated.

There are several further criteria that constrains the required number of factors (Jolliffe 2002, and references therein). The first additional criterion follows from the “cumulative percentage of the total variance”. Taking into account any new factor, the percentage of the variation explained by these factors should increase. Then, if one defines a cut-off percentage, the number of factors m required is given by the value factors, when the cumulative variance in percentage is already higher than this cut-off percentage. There is no exact rule about the best value of the cut-off: Jolliffe (2002) proposes to choose a value around 70–90%, and in addition, if $p \gg 1\%$, a smaller value is proposed. Hence, in our case the value around 70% seems to be a good choice. For PCA and for the correlation matrix, m can also be estimated from the eigenvalues of the PCs – PCs with eigenvalues larger than 0.7 should be retained. Using FA – instead of the PCA – one may also assume that the number of factors in general should not be larger than the number of PCs (in most cases it is even smaller) (Jolliffe 2002). The most accurate estimate of the number of factors m is therefore a combination of several criteria.

The advantage of the ML approach is that it helps to constrain the value of m, the dimension of the hidden factor variables. This is because the ML method provides a probability of the null hypothesis, i.e. that the correlation matrix of the observed variables and that reproduced by the factor solution are identical from the statistical point of view.

By performing FA on the observed variables assuming 6 factors, which is the maximum number allowed by Eq. (2), one observes the validity of the null hypotheses with only $p = 0.0191$, which implies that even the maximum allowable number of factors can’t reproduce the original correlation matrix of the observed variables satisfactorily. Table 2 shows the factor coefficients (loadings) of this solution.

By inspecting Table 2, it becomes obvious that *Factor3* and *Factor5* are dominated by only one variable ($\log \mathcal{R}E_{\rm pk}$ and α, respectively) and are hardly affected by the other variables. Therefore, it appears reasonable to exclude one of them and repeat the calculations with the remaining 9 variables. In this case, the maximum allowable number of factors is $m = 5$, which corresponds to either the null hypotheses $p = 0.11$, after excluding α, and $p = 0.273$ after excluding $\log \mathcal{R}E_{\rm pk}$. We therefore decided to exclude $\log \mathcal{R}E_{\rm pk}$, and the ML solution assuming $m = 5$ factors is given in Table 3. The cumulative variance, defined by 5 factors, is 71.9%. This fulfills the “cumulative percentage of the total variance” criterion for PCA, considering the corresponding high value of p. This also supports the choice of 5 factors.

Table 2. ML solution assuming 6 factors. In any column for the given factor the loadings are given (a larger value represents higher weight for a given variable); the sum of their squares is denoted by *SS loading*; the value *Proportion Var* defines the proportion of *SS loading* to the sum of variances of the input variables; *Cumulative Var* defines the sum of proportional variances.

Variable	*Factor1*	*Factor2*	*Factor3*	*Factor4*	*Factor5*	*Factor6*
$\log T_{90}$	0.418	0.128	–0.066	0.884	–0.133	0.017
$\log \mathcal{T}_{50}$	0.770	0.022	–0.087	0.490	–0.036	0.320
$\log \tau$	0.928	0.038	–0.158	0.198	–0.006	0.146
$\log V$	0.249	0.063	–0.225	0.043	–0.041	0.844
$\log \mathcal{S}_{\mathcal{F}}$	0.173	–0.241	–0.319	0.036	–0.042	0.252
$\log \tau_{\rm lag}$	0.246	–0.269	0.235	–0.008	0.333	–0.187
$\log \mathcal{R}E_{\rm pk}$	–0.070	0.001	0.981	–0.050	0.003	–0.159
$\log \mathcal{F}$	0.564	0.499	0.047	0.226	–0.066	0.187
$\log E_{\rm pk}$	0.108	0.974	0.054	0.074	–0.159	–0.008
α	–0.098	–0.105	–0.024	–0.106	0.981	–0.004
SS loadings	2.126	1.363	1.212	1.134	1.126	0.995
Proportion Var	0.213	0.136	0.121	0.113	0.113	0.099
Cumulative Var	0.213	0.349	0.470	0.584	0.696	0.796

Table 3. ML solution assuming 5 factors after removing the $\log \mathcal{R}E_{\rm pk}$ variable. Testing the hypothesis that 5 factors are sufficient resulted $p = 0.273$.

Variable	*Factor1*	*Factor2*	*Factor3*	*Factor4*	*Factor5*
$\log T_{90}$	0.875	0.009	0.088	–0.152	–0.051
$\log \mathcal{T}_{50}$	0.895	0.353	0.039	0.026	0.236
$\log \tau$	0.704	0.277	0.090	0.095	0.592
$\log V$	0.176	0.973	0.091	–0.098	0.016
$\log \mathcal{S}_{\mathcal{F}}$	0.133	0.320	–0.244	–0.020	0.141
$\log \tau_{\rm lag}$	0.110	–0.144	–0.175	0.490	0.141
$\log \mathcal{F}$	0.528	0.183	0.520	–0.068	0.245
$\log E_{\rm pk}$	0.146	–0.060	0.947	–0.272	–0.005
α	–0.191	0.038	–0.053	0.730	–0.100
SS loadings	2.459	1.309	1.285	0.895	0.519
Proportion Var	0.273	0.145	0.143	0.099	0.058
Cumulative Var	0.273	0.419	0.561	0.661	0.719

We have proven that $m = 5$ factors are sufficient. To prove that it is essential, we also performed the ML analysis with $m = 4$ factors. This calculation resulted only $p = 0.0044$ that 4 factors are sufficient. One can therefore conclude that $m = 5$ factors are necessary and sufficient for describing the observed variables.

4. Results and discussion of FA

The first factor is constrained by T_{90}, $\mathcal{T}_{50}$, τ and $\mathcal{F}$, i.e. the first factor is determined mainly by the temporal properties. Hence measures $\mathcal{T}_{50}$ and T_{90} are the preferred length indicators over τ.

The second factor is dominated by V. However, according to Ramirez-Ruiz & Fenimore (2000), Reichart et al. (2001), and Guidorzi et al. (2005), the variability should be correlated with the luminosities of GRBs, and hence to the fluence. No significant connection is, however, inferred by the second factor raising queries about the redshift estimations derived from variability.

The third factor is mainly driven by $E_{\rm pk}$. It is interesting that the peak energy in the spectra appears to dominate the third factor so significantly. It emphasizes that the spectrum itself is an important quantity (an expected result), and, in the spectrum $E_{\rm pk}$ itself, is a significant descriptor (an unexpected result). In addition, the loading of $\mathcal{F}$ is also important to the third factor. All this has a remarkable impact on the Amati-relation.

The Amati-relation (Amati et al. 2002) proposes that there should be a linear connection between $\log E_{\rm pk;intr}$ and $\log E_{\rm iso}$, where $E_{\rm iso}$ is the emitted energy under the assumption of isotropic emission, $E_{\rm pk;intr} = (1 + z)E_{\rm pk}$ is the intrinsic peak energy, and z is the redshift. This relation, which follows from the relation $E_{\rm pk;intr} \propto E_{\rm iso}^x$ found by Amati et al. (2002) from the analysis of twelve bright long GRBs with well-measured redshifts. The most probable value of x was around $x = 0.5$. Thus, the Amati-relation – in its original form – claims that a direct linear connection exists *only* between $\log E_{\rm pk;intr}$ and $\log E_{\rm iso}$. We note that the Amati-relation was predicted even earlier by the strong correlation between $\log \mathcal{F}$ and $\log E_{\rm pk}$ (Lloyd et al. 2000). The importance of the Amati-relation is straightforward: if it holds, then it is possible to determine the redshift of the given long burst from the value of $E_{\rm pk}$ *alone*, because $E_{\rm pk}$ defines $E_{\rm iso}$ independently of $\mathcal{F}$. Then, by applying standard cosmology, we can calculate from the known $E_{\rm iso}$ and $\mathcal{F}$ values the redshift (e.g. Mészáros & Mészáros 1995).

The validity of the Amati-relation has been a matter of intense discussion since publication. Several papers confirmed it by newer analyses (e.g. Amati 2006; Ghirlanda et al. 2007, 2008, and references therein). Cabrera et al. (2007) confirmed the existence of the $E_{\rm pk;intr}$–$E_{\rm iso}$ correlation in the rest-frame for 47 Swift GRBs. These studies considered bright long GRBs with known redshifts enabling $E_{\rm iso}$ to be determined. This causes strong selection effect in the studied samples. It is possible that this selection effect cause e.g. the entire BATSE sample to follow the Amati-relation either only in a modified version or even not at all, even though the relation holds for the truncated sample of bright GRBs (Nakar & Piran 2005; Butler et al. 2007). BB06 obtained that it is better to use $E_{\rm pk;intr} \propto E_{\rm iso}^{a_1} \tau_{\rm intr}^{b_1}$ with suitable a_1 and b_1 for the BATSE sample ($\tau_{\rm intr} = \tau/(1+z)$). Hence, if $b_1 \neq 0$, then the Amati-relation is altered. BB06 proposes, as the optimal choice, $b_1 = -0.3$. Some papers even reject the Amati-relation both in the BATSE sample (Nakar & Piran 2005) and in the Swift sample (Butler et al. 2007). The most radical solution even challenges the meaning of $E_{\mathrm{pk;intr}}$ itself in the spectra of GRBs (Ryde 2005b).

For our purposes, it is essential statistically that the correlation between $\log \mathcal{F}$ and $\log E_{\mathrm{pk}}$ does not imply that there is a linear connection *only* between $\log E_{\mathrm{iso}}$ and $\log E_{\mathrm{pk;intr}}$. BB06 also arrived at the conclusion that a relation of the form

$$\log E_{\mathrm{iso}} = a_1 \log E_{\mathrm{pk;intr}} + b_1 \log \tau_{\mathrm{intr}} + c_1 \quad (3)$$

should exist with some suitable non-zero constants a_1, b_1, and c_1. We note that $\mathcal{T}_{50}$ and τ strongly correlates with each other, i.e. in this equation either τ_{intr} or $\mathcal{T}_{50;\mathrm{intr}}$ can be used.

The factor loadings imply that $\log \mathcal{F}$ is explained basically by the first and third factors. Since in *Factor1* and *Factor3* $\log \tau$ and $\log E_{\mathrm{pk}}$ are very strong, respectively, it suggests that

$$\log E_{\mathrm{iso}} = a_2 \log E_{\mathrm{pk;intr}} + b_2 \log \tau_{\mathrm{intr}} + c_2 \log L_{\mathrm{iso}} + d \quad (4)$$

should hold with some suitable a_2, b_2, c_2, and d non-zero constants (L_{iso} is the isotropic peak luminosity). We note that a similar relation was also proposed by Firmani et al. (2006).

The correlation between $\log \mathcal{F}$ and $\log E_{\mathrm{pk}}$ is mainly determined by *Factor3*. It follows from the loadings of the first and third factors that the relationship between $\log \mathcal{F}$ and $\log E_{\mathrm{pk}}$ is as important as with the variables dominating *Factor1*. This fact disfavors a simple linear relationship *only* between $\log E_{\mathrm{pk;intr}}$ and $\log E_{\mathrm{iso}}$. The detailed study of Eq. (4) (cf. determination of a_2, b_2, c_2, d, and alternative equations) is beyond the aim of this paper. Even from this conclusion, it however follows that the Amati-relation in its original form is disfavored and some modified version proposed by BB06 is also supported here.

The fourth factor is defined by low frequency spectral index α and τ_{lag}. This implies that the direct correlation between τ_{lag} and V is negligible, and hence there is no direct support for the luminosity estimators based on these two variables (Ramirez-Ruiz & Fenimore 2000; Reichart et al. 2001; Norris 2002).

The fifth factor is dominated by τ and $\mathcal{F}$. With the first factor this demonstrates that T_{90} and $\mathcal{T}_{50}$ are not completely equivalent, although $\mathcal{T}_{50}$ characterizes a burst more closely.

In our opinion, the most remarkable result is that so few quantities are needed, i.e. that all nine quantities can be characterized by five variables. Because all of these conclusions are derived from the measured data alone, all models of GRBs must respect these expectations.

The number of essential variables is in accordance with BB06. They claimed that 3–5 PCs should be used, and we constrained the number of important quantities to be 5.

5. Conclusions

The results of the paper may be summarized as follows.

- No more than 5 factors should be introduced. This essential lowering of the significant variables is the key result of this paper.
- The structure of factors is similar to the PCs of BB06. The number of important quantities is more accurately defined here.
- The first factor is dependent mainly on the temporal variables, and quantities $\mathcal{T}_{50}$ and T_{90} are the preferred length indicators.
- The second factor is dominated by the variability.
- The connection of E_{pk} in the third factor with other quantities, and the structure of the first three factors cast some doubts about the Amati-relation in its original form.
- The α and τ_{lag} parameter values in fourth factor give no direct support for the luminosity estimators.
- The fifth factor demonstrates that T_{90} and $\mathcal{T}_{50}$ are not completely equivalent.

Acknowledgements. Thanks are due for the valuable discussions to Claes-Ingvar Björnsson, Stefan Larsson, Peter Mészáros, Felix Ryde, Péter Veres, and the anonymous referee. This study was supported by the Hungarian OTKA grant No. T48870, by a Bolyai Scholarship (I.H.), by a Research Program MSM0021620860 of the Ministry of Education of Czech Republic, by a GAUK grant No. 46307, and by a grant from the Swedish Wenner-Gren Foundations (A.M.).

References

Amati, L. 2006, MNRAS, 372, 233
Amati, L., Frontera, F., Tavani, M., et al. 2002, A&A, 390, 81
Bagoly, Z., Mészáros, A., Horváth, I., Balázs, L. G., & Mészáros, P. 1998, ApJ, 498, 342
Balázs, L. G., Bagoly, Z., Horváth, I., Mészáros, A., & Mészáros, P. 2003, A&A, 138, 417
Borgonovo, L., & Björnsson, C.-I. 2006, ApJ, 652, 1423 (BB06)
Butler, N. R., Kocevski, D., Bloom, J. S., & Curtis, J. L. 2007, ApJ, 671, 656
Cabrera, J. I., Firmani, C., Avila-Reese, V., et al. 2007, MNRAS, 382, 342
Firmani, C., Ghisellini, G., Avila-Reese, V., & Ghirlanda, G. 2006, MNRAS, 370, 185
Ghirlanda, G., Nava, L., Ghisellini, G., & Firmani, C. 2007, A&A, 466, 127
Ghirlanda, G., Nava, L., Ghisellini, G., Firmani, C., & Cabrera, J. I. 2008, MNRAS, 387, 319
Guidorzi, C., Frontera, F., Montanari, E., et al. 2005, MNRAS, 363, 315
Horváth, I. 1998, ApJ, 508, 757
Horváth, I. 2002, A&A, 392, 791
Horváth, I., Balázs, L. G., Bagoly, Z., Ryde, F., & Mészáros, A. 2006, A&A, 447, 23
Jolliffe, I. T. 2002, Principal Component Analysis, Ch. 7, second edition (New York: Springer)
Kendall, M. G., & Stuart, A. 1973, The Advanced Theory of Statistics (London, High Wycombe: Charles Griffin & Co. Ltd.)
Lloyd, N. M., Petrosian, V., & Mallozzi, R. S. 2000, ApJ, 534, 227
Meegan, C. A., et al. 2001, Current BATSE Gamma-Ray Burst Catalog, http://gammaray.msfc.nasa.gov/batse/grb/catalog
Mészáros, P., & Mészáros, A. 1995, ApJ, 449, 9
Nakar, E., & Piran, T. 2005, MNRAS, 36, L73
Norris, J. P. 2002, ApJ, 579, 386
Norris, J. P., Scargle, J. D., & Bonnell, J. T. 2001, in Gamma-Ray Bursts in the Afterglow Era, Proc. Int. Workshop held in Rome, Italy, ed. E. Costa et al., ESO Astrophysics Symp. (Berlin: Springer), 40
Ramirez-Ruiz, E., & Fenimore, E. E. 2000, ApJ, 539, 712
Reichart, D. E., Lamb, D. Q., Fenimore, E. E., et al. 2001, ApJ, 552, 57
Ryde, F. 2005a, A&A, 429, 869
Ryde, F. 2005b, ApJ, 625, L95