Path Design and Receding Horizon Control for Collision Avoidance System of Cars

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Abstract: - The paper deals with path design and control realization problems of collision avoidance systems (CAS) of cars (ground vehicles). CAS emergency path design is based on the principle of elastic band with improved reaction forces for road borders and static obstacles allowing quick computation of the force equilibrium. The CAS path (reference signal) is smoothed and realized using receding horizon control (RHC). The car can be modelled by full (non-affine) or simplified (input affine) nonlinear models. The nonlinear predictive control problem is solved by using time varying linearization along appropriately chosen nominal control and state sequences, and analytical solution of the minimization of a quadratic criterion satisfying end-constraint. Differential geometric approach (DGA), known from control literature for the input affine nonlinear model, has been used for control initialization in the first horizon. For state estimation Kalman filters and measurements of two antenna GPS and Inertial Navigation System (INS) are used. A stand-alone software has been been developed using the C Compiler of MATLAB R2006a satisfying real time expectations.

Key-Words: - Collision avoidance; Ground vehicle; Lie-algebra; Receding horizon control; GPS/INS; Stand-alone software

1 Introduction

In a Collision Avoidance System (CAS) the path design has to be performed online in the presence of static and dynamic obstacles. The path information can be converted to reference signals for the low level control subsystems based differential geometry (Lie-algebra), optimal predictive control etc. Examples for static and dynamic obstacles may be the lost payload of a truck and an oncoming vehicle in the other lane. To solve the full problem sensor fusion of environmental information produced by radar, laser and computer vision systems is preferred, but this is not part of the paper. The sensory information can be used to initiate an elastic band which is the basis for path design [1].

The stability properties of the path can be divided into 5 cathegories called Characteristic Velocity Stability Indicator (CVSI) which may be interpreted as understeering, neutral steering, oversteering, high oversteering and braking away [2]. If the generation of the drivable trajectory cannot be finished within a given time limit $T_{\max}$, or if the estimated CVSI cannot be accepted because wrong stability properties, then emergency braking follows without steering. Otherwise the developed path will be performed by using an appropriate control method.

The speed of the computation of path design depends on the complexity of the situation and the number of iterations needed to find the force-equilibrium of the elastic band.

The control of cars is usually based on classical control methods (PID etc.), or advanced methods using Lie-algebra results of nonlinear control [7], optimal control [8] or predictive control [8], [9], [10]. Nonlinear control assumes known dynamic model of the car. Based on a simplified (input affine) nonlinear model Freud and Mayr [6] derived a control method for cars using differential geometric approach (DGA). Although the paper concentrates on receding horizon control (RHC), the DGA solution of Freud and Mayr is used for initialization of the control sequence in the first horizon and for comparison.

For state estimation of cars Kalman filters and measurements of two antenna GPS and Inertial Navigation System (INS) are typical approaches [11].

An important question is the quick development and test of the full software. For this purpose MATLAB and Optimization Toolbox is a useful developing environment. Stand-alone software can be generated using the C Compiler of MATLAB R2006a.

The paper is organized as follows. The elastic band model for path design and some modified potential functions are given in Section 2. The nonlinear car model and its simplified version are presented in Section 3. Control based on DGA approach is summarized in Section 4. RHC control based on linearization along the CAS path in the horizon is presented in Section 5. State estimation using GPS/INS sensory information and Kalman filters is discussed in Section 6. Results of simulation tests are reported in Section 7. Conclusions are given in Section 8.

2 Path design using elastic band

Typical approaches for path design are constrained optimization using splines [4] and the elastic band originated from Quinlan and Khatib [3]. Because of real time considerations and the presence of moving obstacle the paper prefers path design using elastic band with some modifications of the results in [1].

It is assumed that satisfactory sensory information is present about the path section and the static and dynamic obstacles which can be used to initiate an elastic band which is the basis for CAS path design.

In the actual state of the vehicle an elastic band will be started from the origin r_0 of the vehicle coordinate sytem. The elastic band consists of N springs, the connection points r_i of the springs are the nodes. The spring constant and the initial length of spring are k_i and $l_{0,i}$, respectively. The goal is to find the force equilibrium. In every iteration step the reaching time t_i will also be determined based on spline technique and the longitudinal velocity of the vehicle. If the path belonging to the force equilibrium is stabilized then the vehicle can move along the path taking into consideration the reaching time. The elastic band is characterized by the information:

$$(r_0,t_0)\xrightarrow{\cdots}(r_i,t_i)\xrightarrow{k_i,l_{0,i}}(r_{i+1},t_{i+1})\xrightarrow{\cdots}(r_N,t_N)$$

Obstacles are modelled by safety circles. The center and diameter of the safety circle O_j are r^{O_j} and d_j, respectively. If the elastic band of the vehicle reaches the safety circle of a node then new elastic bands will be generated from the existing ones so that typically more than one elastic bands are iterated in order to find force equilibrium. After reaching the force equilibrium, for each equilibrium the time-parametrized trajectory, curvature and lateral acceleration are determined based on spline technique. For the vehicle that path will be chosen for which the lateral acceleration is the smallest.

The path design is based on the internal potentials V_i^{int} of the springs (forces at each node are given by the directional derivatives of the corresponding potential fields), the external forces $F_i^{B_q}$ of the left (B_l) and right (B_r) borders of the road, $q\in\{l,r\}$, and the external forces $F_{i,stat}^{O_j}$ and $F_{i,mov}^{O_j}$ of obstacles. For every node r_i the nearest points $r_i^{B_l}$ and $r_i^{B_r}$ along the borders will be chosen.

The forces $F_i^{B_q}$ suggested in [1] were not able to place back the elastic band in the middle of the lane without obstacles and acted in different way at left and right road borders. Similarly, $F_{i,stat}^{O_j}$ in [1] were not able to guarantee satisfactory path curvature and centripetal acceleration in case of force equilibrium. Hence our choice was:

$$V_i^{\text{int}}=\frac{1}{2}k_i(|r_{i+1}-r_i|-l_{0i})^2 \qquad (1)$$

$$V^{\text{int}}=\sum_{i=0}^{N-1}V_i^{\text{int}}=\sum_{i=0}^{N-1}\frac{1}{2}k_i(|r_{i+1}-r_i|-l_{0i})^2 \qquad (2)$$

$$F_i^{B_q}=M^B\exp[-\frac{1}{2}(|r_i-r_i^{B_q}|/\sigma^{B_q})^2]\frac{r_i-r_i^{B_q}}{|r_i-r_i^{B_q}|} \qquad (3)$$

$$\sigma^{B_q}=k^{B^q}/\sqrt{2\ln(M^B/m^B)},M^B=2,m^B=0.05$$

$$F_{i,stat}^{O_j}=k^{O_j}\frac{d^{O_j}/2}{|r_i-r^{O_j}|}\cdot\frac{r_i-r^{O_j}}{|r_i-r^{O_j}|},\ k^{O_j}=3 \qquad (4)$$

$$F_{i,mov}^{O_j}=k^{O_j}\exp\left(-\left(|r_i-r^{O_j}(t_i)|-\frac{d_j}{2}\right)^2\right)\frac{r_i-r^{O_j}(t_i)}{\left|r_i-r^{O_j}(t_i)\right|} \qquad (5)$$

Let M be the number of static and moving obstacles. Then the force equilibrium condition is:

$$F_i^{sum}=F_i^{\text{int}}+F_i^{B_l}+F_i^{B_r}+\sum_{j=1}^{M}F_{i*}^{O_j}=0 \qquad (6)$$

The equilibrium should be satisfied for each node. Introducing the notations $x=(r_1^T,r_2^T,\ldots,r_N^T)^T$ and

$$f(x)=\left((F_1^{sum})^T,(F_2^{sum})^T,\ldots,(F_N^{sum})^T\right)^T, \qquad (7)$$

the numerical problem is to solve the nonlinear sytem of equations $f(x)=0$. We used *fsolve* of the Matlab Optimization Toolbox supplementing to it beside $f(x)$ also the Jacobian (derivative) of $f(x)$.

The path design was tested for a problem similar to that in [1] (41 nodes; static obstacle of diameter 2.5m; moving obstacle of diameter 3.5m, initial distance 120m, assumed velocity 15m/s; average velocity of own car 20m/s). The computation of the force equilibrium using *fsolve* needed approximately 1s.

In the developed program first the elastic band was approximated by linear sections and the time instants belonging to the nodes were determined assuming known average velocity of the own car. Then based on the time distribution the kinematic variables have been determined by using third order polynomial approximations between the nodes. The approximation was performed using the spline technique of MATLAB (*spline, ppval, unmkpp, mkpp*). Based on the derivatives the kinematic variables v (velocity), ψ (orientation), and κ (curvature) to the CAS reference path can easily be determined.

3 Nonlinear dynamic model of the car

Two nonlinear dynamic models have been used based on the bicycle model of vehicles which differs in the approximation of trigonometrical functions [5]. Denote β the side slip angle, v_G the velocity, X,Y the position, and δ_w the steering angle, respectively. Front and rear longitudinal forces are denoted by F_{lF} and F_{lR}, cornering stiffnesses by c_F and c_R. It is useful to introduce front and rear side forces S_v and S_h according to

$$S_h = c_R(-\beta - l_R\dot{\psi}/v_G)) \tag{8}$$

$$S_v = c_F(\delta_w - \beta - l_F\dot{\psi}/v_G) \tag{9}$$

The front longitudinal force is assumed to be $F_{lF} = 0$. The air resistance disturbance $T(x)$ is not considered during control. Control input and state are $u = (S_v, F_{lR})^T$ and $x = (\beta, \psi, \dot{\psi}, v_G, X, Y)^T$, respectively.

3.1 Full nonlinear model of ground vehicles

Ground vehicles can usually be modelled with satisfactory precision by the state equations

$$\begin{aligned}
\dot{\beta} = -\dot{\psi} + \frac{1}{m_v v_G}\{&F_{lF}\sin(\delta_W - \beta) - F_{lR}\sin(\beta) \\
&+c_F(\delta_W - \beta - \frac{l_F\dot{\psi}}{v_G})\cos(\delta_W - \beta) \\
&+c_R(-\beta + \frac{l_R\dot{\psi}}{v_G})\cos(\beta)\}
\end{aligned}$$

$$\begin{aligned}
\ddot{\psi} = \frac{1}{I_{zz}}\{&l_F F_{lF}\sin(\delta_W) \\
&+l_F c_F(\delta_W - \beta - \frac{l_F\dot{\psi}}{v_G})\cos(\delta_W) \\
&-l_R c_R(-\beta + \frac{l_R\dot{\psi}}{v_G})\}
\end{aligned}$$

$$\begin{aligned}
\dot{v}_G = \frac{1}{m_v}\{&F_{lF}\cos(\delta_W - \beta) + F_{lR}\cos(\beta) \\
&-c_F(\delta_W - \beta - \frac{l_F\dot{\psi}}{v_G})\sin(\delta_W - \beta) \\
&+c_R(-\beta + \frac{l_R\dot{\psi}}{v_G})\sin(\beta)\}
\end{aligned} \tag{10}$$

$$\dot{X} = v_G\cos(\psi + \beta)$$

$$\dot{Y} = v_G\sin(\psi + \beta)$$

3.2 Approximated nonlinear vehicle model

Using first order Taylor approximation except the position X,Y the following input affine nonlinear model can be derived:

$$\dot{x} = \begin{pmatrix} -x_3 + S_h/(m_v x_4) \\ x_3 \\ -S_h l_R / I_{zz} \\ 0 \\ x_4 C_{12} \\ x_4 S_{12} \end{pmatrix} + \begin{bmatrix} 1/(m_v x_4) & -x_1/(m_v x_4) \\ 0 & 0 \\ l_F / I_{zz} & 0 \\ 0 & 1/m_v \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

$$\dot{x} = A(x) + B(x)u, \quad y = \begin{pmatrix} x_5 \\ x_6 \end{pmatrix} = C(x) \tag{11}$$

4 Nonlinear output feedback

Freund and Mayr [6] had shown that the approximated (input affine) nonlinear car model has differential orders $d_1 = d_2 = 2$ and hence it can be controlled by the control law

$$u = S^{-1}(x)\left\{-C^*(x) + \Lambda w - M^*(x)\right\} \tag{12}$$

$$S^{-1}(x) = m_v \begin{bmatrix} C_{12}x_1 - S_{12} & S_{12}x_1 + C_{12} \\ C_{12} & S_{12} \end{bmatrix}_{2\times 2}$$

$$C^*(x) = \begin{bmatrix} -\frac{1}{m_v}[T(S_{12}x_1 + C_{12}) + S_{12}S_h] \\ \frac{1}{m_v}[T(C_{12}x_1 - S_{12}) + C_{12}S_h] \end{bmatrix}_{2\times 1}$$

$$M^*(x) = \begin{bmatrix} \alpha_{01}x_5 + \alpha_{11}x_4C_{12} \\ \alpha_{02}x_6 + \alpha_{12}x_4S_{12} \end{bmatrix}_{2\times 1}$$

$\Lambda = diag(\lambda_1, \lambda_2)$, $S_{12} = \sin(x_1 + x_2)$, etc.

By introducing the notations

$$\begin{aligned}\bar{y}_1 &= \lambda_1 w_1 - \alpha_{01} x_5 - \alpha_{11} x_4 C_{12} \\ \bar{y}_2 &= \lambda_2 w_2 - \alpha_{02} x_6 - \alpha_{12} x_4 S_{12}\end{aligned} \tag{13}$$

the closed loop system can be written as

$$\begin{aligned}u_1 &= -S_h + [(C_{12}x_1 - S_{12})\bar{y}_1 + (S_{12}x_1 + C_{12})\bar{y}_2]m_v \\ u_2 &= [C_{12}\bar{y}_1 + S_{12}\bar{y}_2]m_v\end{aligned}$$

$$\begin{aligned}\ddot{y}_1 = \bar{y}_1 &= \lambda_1 w_1 - \alpha_{01} x_5 - \alpha_{11} x_4 C_{12} \\ &= \lambda_1 w_1 - \alpha_{01} y_1 - \alpha_{11} \dot{y}_1 \\ \ddot{y}_2 = \bar{y}_2 &= \lambda_2 w_2 - \alpha_{02} x_6 - \alpha_{12} x_4 S_{12} \\ &= \lambda_2 w_2 - \alpha_{02} y_2 - \alpha_{12} \dot{y}_2\end{aligned} \tag{14}$$

Let $\lambda_1 = \lambda_2 := \lambda$, $\alpha_{01} = \alpha_{02} := \lambda$, $\alpha_{11} = \alpha_{12} = 2\sqrt{\lambda}$ where $\lambda > 0$, then two decoupled linear systems are arising (in aperiodic limit case) whose characteristic equation and differential equation are respectively

$$s^2 + 2\sqrt{\lambda}s + \lambda = 0$$

$$\ddot{y}_i + \alpha_{1i}\dot{y}_i + \alpha_{0i} y_i = \lambda_i w_i$$

Choosing $w_i := w_{id} + \frac{1}{\lambda}(\alpha_{1i}\dot{w}_{id} + \ddot{w}_{id})$ then the stable closed loop system will be

$$\begin{aligned}&\ddot{y}_i + \alpha_{1i}\dot{y}_i + \lambda y_i = \lambda[w_{ia} + \frac{1}{\lambda}(\alpha_{1i}\dot{w}_{ia} + \ddot{w}_{ia})] \Rightarrow \\ &(\ddot{w}_{ia} - \ddot{y}_i) + \alpha_{1i}(\dot{w}_{ia} - \dot{y}_i) + \lambda(w_{ia} - y_i) = 0\end{aligned} \tag{15}$$

where the CAS emergency path position functions $w_{1d} = X_d(t)$ and $w_{2d} = Y_d(t)$ play the role of reference signals. This DGA control method needs also the first and second derivatives of the reference signals which can be determined based on spline technique. Notice that from the control S_v and the state variables (or their estimations) the steering angle δ_w can also be computed:

$$\delta_w = \frac{S_v}{c_F} + \beta + \frac{l_F \dot{\psi}}{v_G}$$

5 Receding horizon control (RHC)

Receding horizon control [8], [9], [10] optimizes a cost function in open loop using the prediction of the system future behavior based on the dynamic model of the system, determines the future optimal control sequence within the horizon, applies the first control in closed loop to the real system, and repeats these steps for the new horizon which is the previous one shifted to the left by the sampling time T. An open question is the stability of the closed loop nonlinear system, but the chance for stability is increasing with increasing the horizon length N (time NT).

If the nonlinear dynamic model is used for prediction then the optimization is usually a nonlinear optimization problem in real time which is time-critical for quick systems. Hence linearization around the prescribed nominal trajectory may be suggested and optimization of the perturbations using quadratic cost and analytically managable end-constraint seem to be a good compromise. This is also our concept [8]. However it is a serious problem that the desired CAS emergency path can not easily be converted to the desired control which makes the linearization in the horizon more difficult (and hence nominal is not equal desired).

Denote $\{x_0, x_1, \ldots, x_N\}$ and $\{u_0, u_1, \ldots, u_{N-1}\}$ the nominal state and control sequence within the horizon, respectively, $\hat{x}_0$ the estimation of the initial state, and let $\{y_0 = Cx_0, y_1 = Cx_1, \ldots, y_N = Cx_N\}$ be the output sequence belonging to the state sequence. Let $\{y_{d0}, y_{d1}, \ldots, y_{dN}\}$ be the desired output and $\{e_0 = y_{d0} - y_0, e_1 = y_{d1} - y_1, \ldots, e_N = y_{dN} - y_N\}$ the error sequence. The nominal control sequence may be the one belonging to DGA control in case of the first horizon, or the shifted previous optimal sequence completed with one new element derived from end-constraint etc. (see Step 1 later on).

Using the perturbations $\delta x_0 = \hat{x}_0 - x_0$, $\delta x_1, \ldots, \delta x_N$, $\delta u_0, \ldots, \delta u_{N-1}$ the full or approximated nonlinear dynamic model can be linearized around the nominal sequences resulting in a linear time-varying (LTV) system $\delta x_{i+1} = A_i \delta x_i + B_i \delta u_i$. The output errors are $y_{di} - C(x_i + \delta x_i) = e_i - \delta y_i$, and the cost function J can be chosen as a quadratic function penalizing both output errors end large deviations from the nominal control:

$$J = \frac{1}{2}\sum_{i=1}^{N-1}\|e_i - \delta y_i\|^2 + \frac{1}{2}\lambda\sum_{i=0}^{N-1}\|\delta u_i\|^2 \tag{16}$$

The state and output perturbations satisfy

$$\begin{pmatrix}\delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_{N-1} \\ \hline \delta x_N\end{pmatrix} = \begin{bmatrix} A_0 \\ A_1 A_0 \\ \vdots \\ A_{N-2}\cdots A_1 A_0 \\ \hline A_{N-1}\cdots A_1 A_0\end{bmatrix}\delta x_0 +$$

$$\left[\begin{array}{ccccc} B_0 & 0 & \cdots & 0 & 0 \\ A_1 B_0 & B_1 & \cdots & 0 & 0 \\ \vdots & \cdots & \ddots & 0 & 0 \\ A_{N-2}\cdots A_1 B_0 & A_{N-2}\cdots A_2 B_1 & \cdots & B_{N-2} & 0 \\ \hline A_{N-1}\cdots A_1 B_0 & A_{N-1}\cdots A_2 B_1 & \cdots & A_{N-1}B_{N-2} & B_{N-1}\end{array}\right]$$

$$\bullet \left(\delta u_0^T \quad \delta u_1^T \quad \cdots \quad \delta u_{N-2}^T \quad | \ \delta u_{N-1}^T\right)^T \tag{17}$$

$$\begin{pmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_{N-1} \\ \hline \delta y_N \end{pmatrix} = \begin{bmatrix} CA_0 \\ CA_1A_0 \\ \vdots \\ CA_{N-2}\cdots A_1A_0 \\ \hline CA_{N-1}\cdots A_1A_0 \end{bmatrix} \delta x_0 +$$

$$\left[\begin{array}{ccccc} CB_0 & 0 & \cdots & 0 & 0 \\ CA_1B_0 & CB_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & 0 \\ CA_{N-2}\cdots A_1B_0 & \cdots & \cdots & CB_{N-2} & 0 \\ \hline CA_{N-1}\cdots A_1B_0 & \cdots & \cdots & CA_{N-1}B_{N-2} & CB_{N-1} \end{array}\right]$$

$$\bullet \left(\delta u_0^T \quad \delta u_1^T \quad \cdots \quad \delta u_{N-2}^T \quad | \ \delta u_{N-1}^T\right)^T \tag{18}$$

or in compact form

$$\begin{pmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_{N-1} \end{pmatrix} = P_1\delta x_0 + H_1\delta U \tag{19}$$
$$\delta y_N = P_2\delta x_0 + H_2\delta U$$

where

$$P_1 = \begin{bmatrix} p_1^T \\ \vdots \\ p_{N-1}^T \end{bmatrix}_{m(N-1)\times n}, \qquad H_1 = \begin{bmatrix} h_1^T \\ \vdots \\ h_{N-1}^T \end{bmatrix}_{m(N-1)\times Nr},$$
$$P_2 = \left[p_N^T\right]_{m\times n}, \qquad H_2 = \left[h_N^T\right]_{m\times Nr}$$

and $n = \dim x,\ r = \dim u,\ m = \dim y$.

The optimization problem with end-constraint can be formulated as

$$J = \frac{1}{2}\sum_{i=1}^{N-1}\|e_i - \delta y_i\|^2 + \frac{1}{2}\lambda\sum_{i=0}^{N-1}\|\delta u_i\|^2 \to \min \tag{20}$$
$$e_N - \delta y_N = e_N - (P_2\delta x_0 + H_2\delta U) = 0$$

Using the notation in (19) the cost function can be written in the detailed form

$$\begin{aligned} J = {} & \frac{1}{2}\sum_{i-1}^{N-1} < e_i, e_i > - < (\sum_{i=1}^{N-1} p_i e_i), \delta x_0 > \\ & - < (\sum_{i=1}^{N-1} h_i e_i), \delta U > + \frac{1}{2} < (\sum_{i=1}^{N-1} p_i p_i^T)\delta x_0, \delta x_0 > \\ & + < (\sum_{i=1}^{N-1} h_i p_i^T)\delta x_0, \delta U > \\ & + \frac{1}{2} < (\sum_{i=1}^{N-1} h_i h_i^T)\delta U, \delta U > + \frac{1}{2}\lambda < \delta U, \delta U > . \end{aligned} \tag{21}$$

Since the cost function is a convex function and the constraint is linear hence the Lagrange multiplier rule is necessary and sufficient condition of the optimum. Denote μ the vector of Lagrange multipliers then

$$L = J + < \mu, e_N > - < P_2^T\mu, \delta x_0 > - < H_2^T\mu, \delta U >$$
$$\begin{aligned} 0 = \frac{dL}{d\delta U} & = -\sum_{i=1}^{N-1} h_i e_i + (\sum_{i=1}^{N-1} h_i p_i^T)\delta x_0 \\ & + (\sum_{i=1}^{N-1} h_i h_i^T)\delta U + \lambda\delta U - H_2^T\mu \\ & = -\overline{m} + H_1^T P_1\delta x_0 + (H_1^T H_1 + \lambda I)\delta U - H_2^T\mu \end{aligned} \tag{22}$$

where $\overline{m} = \sum_{i=1}^{N-1} h_i e_i$. The following notations simplify the results:

$$L_1 := H_1^T H_1 + \lambda I,\ \ L_\mu := H_2 L_1^{-1} H_2^T \tag{23}$$

Then it follows from (22) and the end-constraint

$$\delta U = L_1^{-1}(\overline{m} + H_2^T\mu - H_1^T P_1\delta x_0)$$
$$e_N - P_2\delta x_0 - H_2[L_1^{-1}(\overline{m} + H_2^T\mu - H_1^T P_1\delta x_0)] = 0$$
$$\mu = L_\mu^{-1}[e_N - H_2 L_1^{-1}\overline{m} - (P_2 - H_2 L_1^{-1} H_1^T P_1)\delta x_0]$$

and therefore

$$\begin{aligned} \delta U = L_1^{-1}\{ & H_2^T L_\mu^{-1} e_N + (I - H_2^T L_\mu^{-1} H_2 L_1^{-1})\overline{m} \\ & -[H_1^T P_1 + H_2^T L_\mu^{-1}(P_2 - H_2 L_1^{-1} H_1^T P_1]\delta x_0\} \end{aligned} \tag{24}$$

The closed loop control is $u_0 + \delta u_0$ where u_0 is the nominal control and $\delta u_0 \in R^r$ is the first element of the open loop optimal sequence δU.

In every horizon the following steps are repeated:

<u>Step 1.</u> From the initial state x_0 and the nominal control $\{u_0, u_1, \ldots, u_{N-1}\}$ the nominal state sequence $\{x_0, x_1, \ldots, x_N\}$ in the horizon is determined by using the approximated nonlinear dynamic model of the car. Here x_0 is coming from the shifted previous horizon and can differ from the estimated state $\hat{x}_0$.

The desired state sequence is the one computed from the CAS emergency path with zero slide slip angle. The output is assumed to be $y = (X, Y)^T$, hence the desired output sequence can easily be computed from the desired state sequence in the horizon using $C = [e_5\ e_6]^T$. The output sequence belonging to the nominal state sequence can also be computed using C. The error sequence is the difference between them. In case of the first horizon the nominal control sequence is computed by the DGA method and x_0 is initialized from the determined CAS path with zero side slip angle.

Step 2. The discrete time LTV model $\delta x_{i+1} = A_i \delta x_i + B_i \delta u_i$ is determined from the approximated nonlinear model $\dot{x} = f_c(x,u)$ using Euler formula and therefore $A_i := I + T\, df_c / dx|_{(x_i,u_i)}$, $B_i := T\, df_c / du|_{(x_i,u_i)}$.

Step 3. The optimal change δU of the control sequence is computed by (24) using $\delta x_0 = \hat{x}_0 - x_0$ where $\hat{x}_0$ is the estimated state. The optimal control sequence is $U := U + \delta U$. The first element u_0 of it will be applied in closed loop.

Step 4. In order to initialize the control sequence for the next horizon the optimal state sequence belonging to the initial state x_0 and the optimal control sequence $\{u_0, u_1, \ldots, u_{N-1}\}$ is determined using the approximated nonlinear dynamic model $\dot{x} = f_c(x,u)$. The result at the end of the transients is the state x_N. The unknown new u_N can be determined in three ways: i) u_N is determined by using x_N and the DGA method. ii) u_N is computed in such a way that the difference between x_{N+1} computed from the discrete time nonlinear model by Euler formula and $x_{d,N+1}$ according to the CAS path, that is $f(x_N) + G(x_N)u_N - x_{d,N+1}$, is minimized in LS-sense. iii) The last control signal is simply repeated: $u_N := u_{N-1}$.

Step 5. The nominal control sequence for the next horizon is $\{u_1, u_2, \ldots, u_N\}$ which is the augmented optimal control sequence $\{u_0, u_1, \ldots, u_N\}$ shifted by 1 to the left.

It is possible to put integrator into the controller using augmented state $\delta x_i := (\delta x_i^T, \delta u_{i-1}^T)^T$ where $\delta u_i = \delta u_{i-1} + \delta r_i$ and the change of the control δr_i has to be optimized. Putting

$$A_i := \begin{bmatrix} A_i & B_i \\ 0 & I \end{bmatrix} \text{ and } B_i := \begin{bmatrix} B_i \\ I \end{bmatrix} \quad (25)$$

the earlier results remain valid in the new variables. However in this case δR is the optimal change of the control differences and the optimal δU is the cumulative sum of δR.

6 State estimation using GPS/INS

It was assumed that the measured signals are supplied by a two antenna GPS system and an INS system containing accelerometers and gyroscopes.

The evaluation of GPS/INS signals is based on the results of Ryu and Gerdes [11]. It is assumed that the GPS/INS system software package provides high level information in form of following signals:

V_m^{GPS}: measured velocity of the car in the GPS earth coordinate system.

ψ_m^{GPS}: the orientation of the car in the GPS earth coordinate system

$a_{x,m}$: longitudinal acceleration of the car in the car fixed coordinate system

$a_{y,m}$: transversal acceleration of the car in the car fixed coordinate system

r_m: the angular velocity of the car in z (yaw) direction in the car fixed coordinate system

It is assumed that the first antenna is immediately above the INS system and the INS system is located in the COG, otherwise small modifications are needed. The sensors have biases which have also to be estimated. The accuracy of sensors can be characterized by $\sigma 1$ and the bias. The additive Gaussian noise is concentrated into the domain $[-3\sigma, 3\sigma]$ to which comes yet the value of the bias.

For a vehicle moving in the horizontal plane $V_m^{GPS} = (V_1^{GPS}, V_2^{GPS}, 0)^T$ is satisfied, from which the measured value of the side slip angle can be determined and from it the components of u:

$$\gamma = \mathrm{atan2}(V_2^{GPS}, V_1^{GPS}) \Rightarrow \beta^{GPS} = \gamma - \psi_m^{GPS}$$

$$u_{x,m}^{GPS} = \left\| V_m^{GPS} \right\| \cos(\beta^{GPS}) + noise$$

$$u_{y,m}^{GPS} = \left\| V_m^{GPS} \right\| \sin(\beta^{GPS}) + noise$$

The state estimation can be based on a two stage Kalman filter. The first stage estimates the angular velocity $\dot{\psi} = r$. For this purpose two methods can be suggested:

$$\begin{pmatrix} \dot{\psi} \\ \dot{r}_{bias} \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \psi \\ r_{bias} \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r_m + noise$$
$$\psi_m^{GPS} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} \psi \\ r_{bias} \end{pmatrix} + noise \quad (26a)$$

$$\frac{d}{dt} \begin{pmatrix} \psi \\ 1/s_r \\ r_{bias}/s_r \end{pmatrix} = \begin{bmatrix} 0 & r_m & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \psi \\ 1/s_r \\ r_{bias}/s_r \end{pmatrix} + noise \quad (26b)$$
$$\psi_m^{GPS} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (\psi \quad 1/s_r \quad r_{bias}/s_r)^T + noise$$

where s_r is the sensitivity of the gyroscope.

The state estimation is performed by Kalman filter, thus the continuous time models will be converted to discrete time ones:

$$A_{d1} = \begin{bmatrix} 1 & -T \\ 0 & 1 \end{bmatrix}, \; B_{d1} = \begin{bmatrix} T \\ 0 \end{bmatrix}, \; C_{d1} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (27a)$$

$$A_{d1} = I_3 + A_{c1}T,\ B_{d1} = 0_{3\times 1},\ C_{d1} = \begin{bmatrix}1 & 0 & 0\end{bmatrix} \quad (27b)$$

From the estimated values the angular velocity can be computed:

$$r = \hat{\dot{\psi}} := -\hat{r}_{bias} + r_m \quad (28a)$$

$$r = \hat{\dot{\psi}} := r_m(1/\hat{s}_r) - (\hat{r}_{bias}/\hat{s}_r) \quad (28b)$$

This value can be applied in the second Kalman filter which is based on the relation $a = \dot{u} + \omega \times u$ from which the following continuous time model arises for the estimation of the velocities:

$$\frac{d}{dt}\begin{pmatrix} u_x \\ a_{x,bias} \\ u_y \\ a_{y,bias} \end{pmatrix} = \begin{bmatrix} 0 & -1 & r & 0 \\ 0 & 0 & 0 & 0 \\ -r & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} u_x \\ a_{x,bias} \\ u_y \\ a_{y,bias} \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} a_{x,m} \\ a_{y,m} \end{pmatrix}$$
$$+ noise$$

$$\begin{pmatrix} u_{x,m}^{GPS} \\ u_{y,m}^{GPS} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} u_x \\ a_{x,bias} \\ u_y \\ a_{y,bias} \end{pmatrix} + noise \quad (29)$$

The model contains r which is varying in real time hence it is useful that the discrete time form of the model can analytically be found:

$$A_{d2} = \begin{bmatrix} C(rT) & \frac{-S(rT)}{r}/r & S(rT) & \frac{-[1-C(rT)]}{r} \\ 0 & 1 & 0 & 0 \\ -S(rT) & \frac{[1-C(rT)]}{r} & C(rT) & \frac{S(rT)}{r} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_{d2} = \begin{bmatrix} S(rT)/r & [1-C(rT)]/r \\ 0 & 0 \\ -[1-C(rT)]/r & S(rT)/r \\ 0 & 0 \end{bmatrix} \quad (30)$$

where in the system matrices C and S stand for $\cos$ and $\sin$, respectively. The estimation is performed by the second Kalman filter. From its estimated values $\hat{u}_x, \hat{u}_y$ and the output $\dot{\psi}$ of the first estimator some other state variables can be estimated:

$$\hat{v}_G = \sqrt{\hat{u}_x^2 + \hat{u}_y^2}, \qquad \hat{\beta} = \operatorname{atan2}(\hat{u}_y, \hat{u}_x) \quad (31)$$

Since the state variables X, Y of the position are not supported by absolute measurements hence their values were determined by numerical integration:

$$\hat{X} := \hat{X} + T\hat{v}_G \cos(\hat{\psi} + \hat{\beta})$$
$$\hat{Y} := \hat{Y} + T\hat{v}_G \sin(\hat{\psi} + \hat{\beta}) \quad (32)$$

Kalman filters were implemented in the following form (Q and R are the covariances of the system and measurement noises, respectively):

Time update:

$$x_-(t+1) = A_d x_+(t) + B_d u(t)$$
$$P_-(t+1) = A_d P_+(t) A_d^T + Q \quad (33a)$$

Measurement update:

$$x_+(t) = x_-(t) + K[y(t) - Cx_-(t)]$$
$$K = P_-(t)C^T[CP_-(t)C^T + R]^{-1} \quad (33b)$$
$$P_+(t) = [I - KC]P_-(t)$$

The measurement sampling times were $T_{INS} = T = 0.01s$ (100Hz), $T_{GPS,vel} = 0.1s$ (10Hz), $T_{GPS,att} = 0.2s$ (5Hz).

7 Simulation results using RHC

Fig. 1-3 show the RHC control transients (error, controller output, states) realizing the determined CAS emergency path based on the modified elastic band technique and using state estimation by Kalman filters and GPS/INS sensory information. Fig. 1 contains the error transients, Fig. 2 the control signals and Fig. 3 the simulated and estimated states, all for the predictive control method with extra integrator in the controller.

The nominal control sequence for the first horizon has been generated by DGA nonlinear output feedback. In the figures all signals are in SI units (m, rad, N, s, m/s etc.). Horizon length and sampling time were $N = 10$ and $T = 0.01s$, respectively.

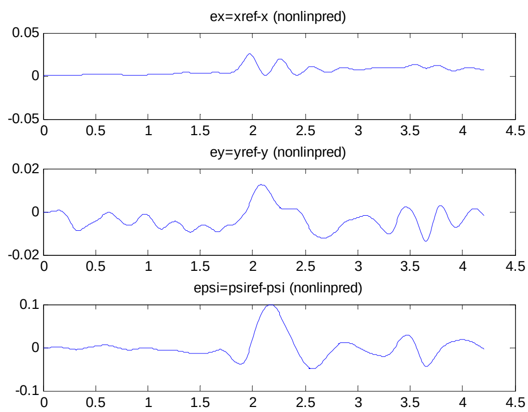

Fig. 1. Position and orientation errors using RHC.

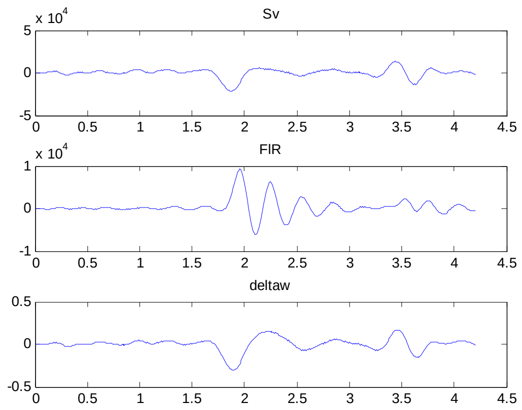

Fig. 2. Control signals using RHC

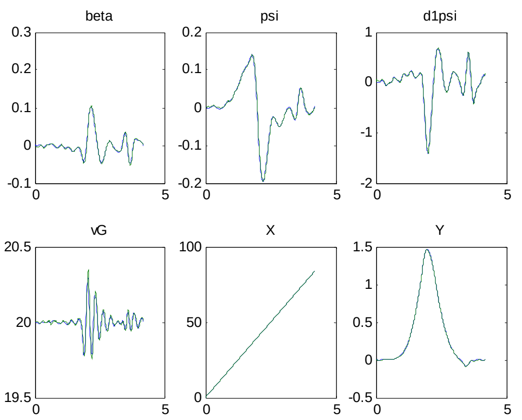

Fig. 3. Simulated and estimated states during RHC

8 Conclusion

In the paper methods have been presented for improved CAS emergency path design, nonlinear dynamic modelling of cars, control using nonlinear output feedback and nonlinear predictive control, and state estimation based on GPS/INS sensory information. The nonlinear predictive control problem is solved using time varying linearization along appropriately chosen nominal control and state sequences, and analytical solution of the minimization of the cost function satisfying end-constraint. RHC control transients show significant improvement with respect to DGA control.

The developed algorithms have been implemented in MATLAB R2006a using the Optimization Toolbox. Thorough software validity test was performed for CAS path design, state estimation, DGA and RHC control. The MATLAB software has been converted to stand-alone C program using the MATLAB C compiler. The transients in Fig. 1-3 were generated by using the stand-alone C version. The real time expectations are satisfied for 0.01s sampling time and $N=10$ horizon length.

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