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What Mathematicians' Claims Mean: In Defense of Hermeneutic Fictionalism[1]

Abstract. Hermeneutic fictionalism about mathematics maintains that mathematics is not committed to the existence of abstract objects such as numbers. Mathematical sentences are true, but they should not be construed literally. Numbers are just fictions in terms of which we can conveniently describe things which exist. The paper defends Stephen Yablo's hermeneutic fictionalism against an objection proposed by John Burgess and Gideon Rosen. The objection, directed against all forms of nominalism, goes as follows. Nominalism can take either a hermeneutic form and claim that mathematics, when rightly understood, is not committed to the existence of abstract objects, or a revolutionary form and claim that mathematics is to be understood literally but is false. The hermeneutic version is said to be untenable because there is no philosophically unbiased linguistic argument to show that mathematics should not be understood literally. Against this I argue that it is wrong to demand that hermeneutic fictionalism should be established solely on the basis of linguistic evidence. In addition, there are reasons to think that hermeneutic fictionalism cannot even be defeated by linguistic arguments alone.

Fictionalism is a general term for approaches which analyze a particular discourse or a particular idiom in terms of fictions. Take, for example, the sentence 'The average star has 2.4 planets'. Given the logical form of sentences involving definite descriptions, this sentence seems to assert that there is one and only one object which is the average star. But there is no such object, so the sentence is false. How come, then, that we find it true? The fictionalist says that in using this sentence we engage in a sort of game. We pretend that there is such an object and use this pretense to express a truth, namely, that if divide the number of planets with the number of stars we get 2.4.

Fictionalism can be pursued in a hermeneutic and in a revolutionary spirit.[2] Hermeneutic fictionalism seeks to uncover how the given discourse or idiom is in fact understood, i.e. to bring to the fore the meaning which has been there all along. The example just used is an instance of hermeneutic fictionalism. It does not tell us that we should stop believing in the existence of an average star, for we have never believed that. It tells us that instead of looking for a novel construal of the logical form of the sentence which would make it literally true, we should accept that it has the logical form it seems to have and it is not literally true.[3] Revolutionary fictionalism, in contrast, claims to reveal that what we took to be real is in fact a piece of fiction. It opens our eyes to the fact that we were wrong, and calls on us to change our commitments. Such is Field's attempt to counter Quine's and Putnam's indispensability argument, according to which we cannot but accept that the abstract objects of mathematics exist, because physics cannot do without them.[4] He attempts to show that physics can be pursued without numbers, so we do not have to put up with their existence.[5]

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[2] The hermeneutic-revolutionary distinction was introduced in (Burgess 2008a) and is first applied to fictionalism by (Stanley 2001).

[3] For a criticism of the fictionalist analysis of 'the average' example see (Stanley 2001, 54-58). For a response see (Yablo 2001, 93-96.)

[4] (Quine 1980a, 1980b, 1981a, 1981b), (Putnam 1979a, 1979b).

[5] (Field 1980)

Stephen Yablo advocates hermeneutic fictionalism with respect to mathematics, and his theory has many attractions. It is nominalistic, so it can avoid the epistemological problem raised by Benacerraf. (A note of clarification: by 'nominalism' I mean the rejection of abstract objects and not the rejection of universals; nominalism so conceived is compatible with *in re* realism about universals.) In addition, it promises to explain why mathematics is necessary, how we can know it a priori, why we feel that mathematics is absolute in the sense that there cannot be an alternative arithmetic or set theory, why mathematics can be applied to the physical world, and many other things, including certain features of mathematical language. I will not elaborate on these, I will simply assume that it can deliver what it promises. In this paper I attempt to defend hermeneutic fictionalism against an objection first formulated by John Burgess, which he repeated several times, sometimes together with Gideon Rosen. I will start by a brief sketch of the account, which certainly will not do justice to its full complexity. Then I respond to the objection in two steps. Burgess and Rosen claim that the fate of hermeneutic fictionalism should be decided solely on the basis of empirical linguistic evidence. I argue first that the supportive evidence may come from philosophical considerations as well. Then I suggest, somewhat tentatively, that linguistic evidence alone might not even be sufficient for refutation.

So let me start with Yablo. Quine has taught us that ontological commitment is marked by quantification. The entities whose existence we are committed to are the ones which we quantify over. Mathematics abounds with theorems which quantify over numbers, e.g. 'Any two numbers have a product'. It seems then that the truth of mathematical theorems implies that numbers exist. Yablo claims that quantifying over numbers incurs no such commitment just as by asserting that 'The average star has 2.4 planets', we do not incur commitment to the existence of the average star. But how can we quantify over numbers and yet abstain from ontological commitment?

Here is how. Number words have a use which is ontologically innocent, namely when they occur as devices of numerical quantification, like in 'There are twelve apostles'. Here the number word can be resolved into the standard devices or first order predicate logic with identity.[6] Starting from this innocent use we can get to quantification over numbers which is just as innocent by adopting a rule, which licenses the expression of the content of sentences involving numerical quantification in terms of quantification over numbers. Stated in a preliminary form, the rule says: if there are *n F*s, imagine there is a thing *n* which is identical with the number of *F*s. Using **S** as notation to be read 'imagine/suppose that *S*', the rule can be written as follows:

($N_{preliminary}$) if $\exists_n x\ (Fx)$, then *there is a thing *n* (*n* = the number of *F*s)*[7]

F is a predicate applicable to ordinary objects, and in the antecedent, we have a simple numerical quantification that does not assume the existence of numbers as objects. In the consequent we have quantification over numbers, but the quantification is ontologically innocent, since it occurs in the scope of the 'imagine that' operator. When we merely imagine that something exists, we are not committed to its existence. What the rule says is not that whenever a specifiable real world condition obtains, there exists a given number; it says that whenever a certain real world condition obtains we are allowed to engage in a game of make-belief and pretend that a given number exists.

This rule, however, will not quite do, because it does not allow us to assign numbers to numbers, like when we say 'The number of even primes equals 1'. 'Even' and 'prime' are

[6] There are *n F*s can be defined recursively as follows: $\exists_0 x\ Fx =_{df} \forall x\ (Fx \supset x \neq x)$, and $\exists_{n+1} x\ Fx =_{df} \exists y\ (Fy\ \&\ \exists_n x\ (Fx\ \&\ x \neq y))$.

[7] The following account is based primarily on (Yablo 2002).

predicates applicable to numbers, not to ordinary objects, so they cannot occur in the antecedent of the rule. We need to liberalize the rule and allow such predicates in the antecedent. But if we deny that numbers exist, we must also deny that the properties even and prime are instantiated. However, if we may imagine that numbers exist, we may also imagine that these properties are instantiated. This gives us a clue as to how the rule should be amended:

(N) if *$\exists_n x$ (Fx)*, then *there is a thing n (n = the number of Fs)*

This rule says that if you imagine that there are n Fs, where F may be a property of ordinary objects or numbers, you may also imagine that there is an object which is the number of Fs. This rule includes the preliminary one as a special case: if the antecedent of ($N_{preliminary}$) is satisfied, i.e. if there are indeed a certain number of ordinary objects which are F, you are certainly entitled to imagine that. [8]

But why is it worth pretending that numbers exist? Because of the expressive power the quantificational idiom brings. Without this idiom, it would not be possible, for example, to formulate the laws of physics. Instead of Newton's second law, we could only formulate a huge conjunction with conjuncts of the form 'if a force F is exerted on a body with the mass M, it produces acceleration A'. But we would need an infinite number of conjuncts. Worse, since the magnitudes in question can take real numbers as values, the number of conjuncts should have to be uncountably infinite. If we are allowed to quantify over numbers, we can simply say, 'For all real numbers F, M and A, if F = the force acting on a body with the mass = M, and A = the acceleration produced, then $F = M \times A$'.

It is exactly because of the expressive power of quantification over numbers that Quine believes that mathematical objects are indispensible for physics. Whereas Field accepts that the quantificational idiom yields ontological commitment, and tries to show that we can achieve the same expressive power without quantifying over numbers, Yablo maintains that we may quantify over numbers and yet avoid commitment. We simply pretend that there are mathematical entities. He points out that the use of fictions for purposes of representation is very common. For instance, you may describe a certain bodily feel of nervousness by saying 'There are butterflies in my stomach'. Of course, you do not believe that there are. But if there were, you think that would feel in this way. So you call us to imagine a fictitious state of affairs in order to describe a state of affairs which is real. Indeed, this is the way in which metaphors usually work. Metaphors, read literally, are typically false, but they call us to imagine something. If the call is accepted, the features of what is imagined point us to certain features of reality. One may describe the location of the city of Crotone saying 'It is on the arch of the Italian boot'.[9] Italy is not a boot, but if you are willing to pretend that it is, the sentence tells us where the city is to be found. It is because mathematics shares this feature of figurative speech that Yablo prefers to call his approach 'figuralism'.

We have seen that real contents of sentences of applied mathematics are states of affairs which include nothing mathematical. But what about pure mathematics? What is, for instance '3 + 5 = 8' about if not about numbers? Yablo shows how sentences of pure mathematics can be recast in the ontologically innocent idiom of numerical quantification. The basic idea is to use rule (N) backwards. What the previous sentence really says is something like this: 'If there are exactly three Fs and there are exactly five Gs, and no F is a G, then there are exactly eight objects which are Fs or Gs'. This is a logical truth. Yablo goes on to show how to reconstruct all sentences of arithmetic, including the ones which quantify over numbers, as

[8] Once (N) is in place, we can have infinitely many numbers even if there are only finitely many ordinary objects. 0 is the number of things not identical to themselves, n is the number of numbers smaller than n.
[9] The example from (Walton 1993) 40-41, whose work is a major source of inspiration for fictionalism.

logical truths, and he does the same for set theory. You can already see how Yablo can explain why mathematics is necessary and how it can be known a priori.

This should suffice to give us a flavor of Yablo's approach. Let us now see why Burgess and Rosen believe that an account along these lines is untenable. The objection is not directed specifically against fictionalism but against nominalism in general. The nominalist denies the existence of abstract objects, so he does not accept that the mathematical sentences apparently asserting the existence of such objects are literally true. At this point, he has two options. To admit that these sentences are true and deny that they are understood literally, or to admit that they are understood literally and deny that they are true. The former is the hermeneutic, the latter is the revolutionary position. Burgess and Rosen argue that both are untenable. The hermeneutic position fails because it is not supported by *scientific* evidence. The revolutionary position fails because there are no sound scientific reasons to challenge the truth of mathematics or to replace current mathematics with a nominalistic alternative such as Field's or Chihara's. I emphasize "scientific", because Burgess and Rosen are of the conviction that purely philosophical considerations can never take precedence over scientific reasoning. For example, epistemological worries about how we can acquire knowledge of the abstract entities of mathematics are not sufficient to discredit mathematicians' claims to knowledge, and *a fortiori*, the truths of mathematics.[10] I grant this.

Nonetheless—and now I am starting with the response—when it comes to arguing against the hermeneutic approach, the point that purely philosophical considerations cannot trump scientific ones is replaced by something stronger, namely that philosophical considerations are simply irrelevant and carry no weight at all. They write "no nominalists favoring such a reconstrual have ever published their suggestions in a linguistics journal with evidence such as a linguist without ulterior ontological motives might accept".[11] At another place Burgess briefly responds to those criticisms which allege that nominalists can have a third alternative in addition to hermeneutics and revolution.

> [I]t is sometimes said that a nominalist interpretation represents "the best way to make sense of" what mathematicians say. I see in this formulation not a third alternative, but simply an equivocation, between "the empirical hypothesis about what mathematicians mean that best agrees with the evidence" (hermeneutic) and "the construction that can be put on mathematicians' words that would best reconcile them with certain philosophical principles or prejudices" (revolutionary).[12]

What these remarks indicate is that the evidence for a nominalist interpretation of mathematics, such as Yablo's, should be purely empirical and should not rely on philosophical considerations. This is actually how Burgess and Rosen proceed when they take up Yablo's position. [13] They systematically ignore the philosophical benefits Yablo's account may bring, and focus on the evidence from linguistic behavior. E.g. Yablo claims that the ease with which we pass from ontological innocent number talk to the quantificational formula, that we do not demand a proof existence, suggests that the latter idiom does not carry ontological commitment either. Or: if the Oracle mentioned in Burgess' and Rosen's book,[14] who knows exactly what exists, would proclaim that only concrete objects exist, mathematicians would not renounce their existence claims. I do not want to discuss Yablo's linguistic arguments and Burgess' and Rosen's rejoinders. Suffice it to say that I do not find

[10] (Burgess and Rosen 2005, 520-523.)
[11] (Burgess and Rosen 2005, 525.)
[12] (Burgess 2008b, 51).
[13] (Burgess and Rosen 2005, 528-534.)
[14] (Burgess and Rosen 1997, 3.)

the rejoinders convincing, and I will later argue that a knockdown linguistic counterargument might not be that easy to formulate.

What I contend is that in assessing the case for hermeneutic fictionalism, it is wrong to disregard philosophical considerations.[15] I do not base this on the intrinsic importance of philosophy but on two facts about interpretation. First fact: interpretation—be it the interpretation of a text, of the behavior of a person, of a set social practices—is aimed at making sense, i.e. showing how the various parts hang together, how they cohere. The pursuit of coherence is checked against the empirical facts. Here is an example. Before the elections, a politician promises not to raise taxes, he comes to power, then raises them. There are several ways this may make sense. One: he believed he would not have to raise taxes and later found, to his dismay, that he was mistaken. Two: he knew all too well that he could not avoid raising taxes and calculated that the loss of credibility would be acceptable price for the increase of popularity the false promise would bring. Three: something in between; he was not certain, but he hoped he would not have to and took a calculated risk. Which is right? Empirical evidence decides. We have to find out what information he had about the state of the economy, how well he understood the information he had, what his advisors said, how often he kept his earlier promises, etc. And there are also several ways the story does not make sense (or at least does not make sense without further assumptions). One: he believed he would not have to raise taxes, and indeed he did not have to, still he raised them just for the fun of it. Two: he made a sincere promise and intended to keep it, just did not realize the legislation he passed was about tax raises. So an interpretation can fail in two ways: by conflicting with the empirical evidence and by violating the demand for coherence.

Second fact: judging whether or how much certain patterns are coherent draws heavily on the interpreter's own beliefs. This element of subjectivity is ineliminable, because there is no universal manual for identifying coherent patterns. The closest we have to such a manual is logic, but in matters of interpretation, logic might not have the last word. An interpretation which involves the attribution of inconsistency, might, on the whole, be better than one which involves the attribution a very far-fetched idea which happens to restore consistency. And to tell whether an idea is indeed far-fetched one has to rely on his own beliefs. Let me illustrate the same fact with the earlier example. Suppose you are thinking black and white. Then you will think that our politician either made a sincere promise but was unlucky, or he lied, and there are no other options. If you do think that, then, of course, you are a lousy interpreter. A good understanding of the field, human psychology and politics in this case, is necessary for a good interpretation. So the element of subjectivity does not imply arbitrariness.

How does this all bear on hermeneutic fictionalism? A philosopher, whose purpose is to interpret mathematics as a cognitive enterprise, wants to find out how various things in and around mathematics hang together. In deciding whether certain ideas cohere, he cannot but rely on what he believes. Suppose he believes that knowledge presupposes some kind of causal access. In that case, he would find it difficult to conceive how the Platonist account of mathematics, according to which mathematics provides literally true descriptions of abstract objects, which are not located in space-time and which are causally inert, may rationally cohere with the fact we do have mathematical knowledge. Or he may wonder how

[15] If I succeed, I shall have also disposed of Mark Balaguer's objection. In Balaguer's taxonomy there is no room for hermeneutic fictionalism. He defines fictionalism as the view that mathematical sentences should be taken at face value and are false. Yablo believes that mathematical sentences are true, so he is what Balaguer calls a paraphrase nominalist. Paraphrase nominalism is wrong because the empirical evidence suggests that mathematicians understand mathematical sentences literally and not according to the nominalist paraphrase. To me, this sounds like the same complaint as the one raised by Burgess and Rosen. (Balaguer 2008), (Balaguer 2009, 152, 158).

mathematics, alleged to describe causally inert objects, can benefit physics, which provides causal explanations.

If this is right, and the philosopher's interpretation of mathematics is a genuine interpretative enterprise, it cannot make do without reliance on the philosopher's own convictions. So Burgess and Rosen are wrong when they demand that the interpretation of mathematics is to be based purely on empirical evidence, and should be free of philosophical considerations. Interpretation is never based purely on empirical evidence. It is in the business of uncovering coherence, rational connections between parts—and whether the parts are indeed rationally connected, is not something that can be empirically determined. The objection rests on a misunderstanding of what interpretation involves.

I want to emphasize that the above view of interpretation does not mean that philosophers are entitled to read into mathematics whatever philosophical views they happen to have. In order to see that, it is worth taking a look at how the empirical evidence and the interpreter's convictions interact in the course of interpretation. Suppose a historian is writing a book on Kepler. The dates when Kepler's books were published can be determined empirically. Once again, empirical evidence shows that the astronomical theory of *Harmonice Mundi*, which includes what we now call Kepler's laws, is superior to the astronomical theory in his first book, *Mysterium Cosmographicum*. But it is not empirical evidence which says that it was extremely odd of Kepler to republish his first book two years after the publication *Harmonice Mundi*. This judgment draws on the historian's own understanding that science aims primarily at empirically accurate theories. Given this understanding, the publication of an empirically inferior theory just does not make sense. The historian needs to find a coherent pattern which Kepler's actions fit. He may, for example suggest, that Kepler does not share the current view that empirical accuracy has exclusive importance. Kepler was a Platonist and held that that the world should exhibit an impressive mathematical order. Now *Mysterium Cosmographicum* is superior to *Harmonice Mundi* in terms of mathematical order. (Its leading idea is that orbits are circular and their distances are regulated by the five platonic solids: a platonic solid circumscribed around the orbit a planet closer to the Sun is inscribed in the orbit of the planet farther from the Sun.) Now, the historian who proceeds like this does not simply impute his own beliefs to Kepler, since he admits that Kepler's vision of science is different from his own. But he does not put his own beliefs aside either. After all, it is in terms of a belief he shares with Kepler that he makes sense of Kepler's actions, namely that it is right to publish what one believes to be good science. It is in the light of this conviction that Kepler's actions turn out in a way rational. So the way to conceive the role of the interpreter's own convictions is this. The interpreter's convictions provide ways in which what is interpreted can be construed as exhibiting coherence. The role of empirical evidence is to determine which ones of these coherent patterns are, in fact, exhibited.

I have been arguing so far that Burgess and Rosen cannot rule that evidence from philosophical considerations is inadmissible. This may strengthen the case for hermeneutic fictionalism. Now I want to go further and suggest that it is not entirely clear that hermeneutic fictionalism can be refuted at all solely by non-philosophical considerations. Suppose we consider only arguments from the mathematicians' linguistic behavior and in the interpretation of what mathematicians say and write we consciously abstain from relying on philosophical considerations. I will consider two scenarios in which the result of such non-philosophical arguments is apparently unfavorable to hermeneutic fictionalism and claim that these scenarios do not suffice to refute hermeneutic fictionalism.

The first scenario is that we find that mathematicians do not believe that mathematical objects are fictions because they do not have beliefs about their ontological status. For instance, an empirical survey shows that the overwhelming majority of mathematicians say

that they have not thought much about this question, they are not particularly interested in it, or claim to be ignorant about it, or are ready to adopt any position recommended to them; and the minority which displays interest consist of two groups. Members of the one have views which are vague, ambiguous, inconsistent or otherwise unsatisfactory. Members of the other minority group are very sophisticated but cannot agree among themselves. If this were the case, the hermeneutic fictionalist would have to choose carefully the way in which he formulates his position. In particular, he should make it very clear that he is not offering a psychological description of what mathematicians think. He should possibly avoid talking about mathematicians' beliefs, or explain that what he calls beliefs are the views which make best sense of what mathematicians do rather than the dispositional mental states they have. Or he should prefer to talk about mathematics and mathematical practice rather than of mathematicians.

This would not be an ad hoc maneuver. Interpretations often involve elements which are not meant to be psychologically faithful.[16] As a first example, take some current interpretations of Descartes which allege that ideas are to be understood as intentional contents. Viewed as a psychological statement ,this would involve some distortion, because Descartes did not possess the concept of intentional content. Today's concept of intentional content is informed by the tradition of Brentano, Husserl, Frege and Chisholm, which emerged only much later. Instead, advocates of this interpretation should be viewed as claiming that understanding Descartes's concept as intentional content is consistent with what Descartes actually says, and sheds light on how various elements of Descartes's thought hang together. For a more dramatic example, take the interpretation of potlatch as a means of maintaining hierarchical relations between clans or villages. Surely, when the Indians of the Pacific Northwest gather to give away and often destroy vast amount of goods, they do not think of this as a way of reinforcing their social status. It is not just that they do not possess the concepts of social science. Even if they did, the social scientists' explanation, which is thoroughly secular, would not occur to them, because in their eyes, potlatch has a religious character.

It is important to see that interpretations which fail in terms of psychological faithfulness may be fully legitimate as interpretations—they are not abnormal or deviant. Interpretation is in the business of making sense, displaying how things rationally cohere. Now sometimes we cannot capture coherence in terms of the actual beliefs of the people we interpret. Descartes's concept of idea is not sufficiently clear to make the coherence of his thought transparent. The people practicing potlatch explain this custom in terms of following the law. But we believe that laws must serve some purpose, so we need a rationale, and the people do not provide one. If an interpreter finds that coherence cannot be captured by psychologically faithful descriptions, he forgoes psychological faithfulness. Similarly, if hermeneutic fictionalism succeeds in making sense of mathematics and its use in physics, it should not be faulted on grounds that it does not represents mathematicians' beliefs.

Let us move over to the second scenario. Here, the interpretation of the linguistic behavior of mathematicians—which relinquishes philosophical considerations—makes it clear that mathematicians reject fictionalism. Imagine, it turns out, they are all Platonists.

Notice that this would not automatically refute fictionalism. It might be the case that the fictionalists are right, and the mathematicians are wrong. This is Mark Balaguer's favored

[16] For the purposes of discussion, I assume two things. First, that facts about beliefs are as "hard" as any physical fact. Second, that people are not mistaken about their beliefs. Giving up either assumptions would give me more room to maneuver but would also invite several objections.

response to the Burgess-Rosen argument.[17] He claims that revolutionary fictionalism, which accepts that mathematical statements are understood literally and are false is tenable. It would be admissible to overrule mathematicians' judgments concerning the ontological status of mathematical entities, for two reasons. First, such a decision would be of little significance for mathematical practice. Second, mathematicians' professional expertise, which a philosopher cannot question, does not extend to the issues of ontological status.

However, hermeneutic fictionalism holds that mathematical statements are true, but are not understood literally, and it is hermeneutic fictionalism I wish to defend. There are two forms the defense can take. One is to reevaluate the mathematician's alleged commitment to Platonism. Suppose mathematicians explain why they take mathematical sentences literally true in the following way. "Look, we know how to tell metaphors from literal speech. We speak literally when we use the words as we ordinarily do. Now the word 'Sun' normally refers to a hot ball of gas. When Romeo calls Juliet the Sun, he cannot be talking literally, since he cannot possibly believe that Juliet is a hot ball of gas. But as opposed to the word 'Sun', mathematical terms do not have an established use with which our use could be contrasted. So we are talking literally." In response to this, the hermeneutic fictionalist may point out that certain expressions are inherently metaphorical in the sense that they do not have literal uses. Take the word 'Vulcan' introduced in Star Trek. If you call someone who always behaves in a cool, emotionally detached and highly logical fashion a Vulcan, you do not mean that he comes from a humanoid race which evolved on the planet Vulcan, since you know all too well that he does not. Or if you describe someone prone to emotional and illogical behavior as not being a Vulcan, you do not mean to assert that he does not from that race. And even if you call Captain Spock a Vulcan, you do not mean in all seriousness that there is an individual bearing this name who comes from the planet Vulcan. This example is meant to illustrate that when mathematicians confess to Platonism, that may be due to the fact that they misconstrue 'literal' or construe it in a way that differs from the hermeneutic fictionalist's intention.[18]

But suppose no such maneuver is possible. Mathematicians happen to be very sophisticated in matters of linguistics, they do not misconstrue hermeneutic fictionalism, but they reject it in full knowledge of what it involves. That alone would still not be enough to refute hermeneutic fictionalism. When defending revolutionary fictionalism, Balaguer considers the idea that his revolutionism might not concern mathematics at all.[19] He envisages a version of Platonism which runs as follows. Mathematical facts are compounded of two sorts of facts: ontologically neutral facts about the correctness of mathematical sentences construed in fictionalist terms, and platonic facts to the effect that the abstract objects mathematical sentences seem to describe exist, which make it the case that the sentences which are correct in the fictionalist terms are actually true. It is only these platonic facts which on the fictionalist view do not obtain. Balaguer wonders if the platonic facts are mathematical facts at all. If not, the fictionalism he proposes would amount to a revolution in philosophy rather than mathematics. He admits that he does not know how to show that the alleged platonic facts are not mathematical in nature, and neither do I.

I believe, however, that the hermeneutic fictionalist can make a similar move and is in a position to argue for it. Suppose that if we take into account philosophical considerations and no others, fictionalism scores better than other alternatives. This should be granted for the sake of argument, since if fictionalism fails on philosophical grounds, it fails, and there is no

[17] (Balaguer 2009, 153-157.); he believes though that there might also be a way to reject the hermeneutic-revolutionary distinction, 157-161.
[18] For a more inclusive discussion see (Yablo 2000, 221-224.)
[19] (Balaguer 2009, 156.)

point in trying to show that it can be maintained in the in face of its rejection by mathematicians. Then from the hermeneutic fictionalist's point of view, the situation looks as follows. Certain things mathematicians say, e.g. 'For every prime number there is a larger one', are true, even though not in a literal sense. Other things they say, e.g. 'Numbers are abstract objects and they do exist' are false in the literal sense. (If mathematicians did not intend these sentences in the literal sense, they would not be contradicting the fictionalist.) For sentences in the first group, they have arguments, which are virtually impossible to resist, and these arguments apply a small group of very special methods, such as deduction from axioms. Arguments for the sentences in the second group are not based on these special methods, and they can and should be resisted. Add to these certain behavioral or, if you wish, sociological facts. The professional training mathematicians receive prepares them to deal with the first group. The scholarly journals they publish in are devoted to the first group. One may gain recognition as a great mathematician only by establishing claims in the first group. Those who are exclusively concerned with the second group are typically not regarded as mathematicians, and the list may be continued. All in all, we find that the distinction between the two groups of sentences is not a local phenomenon but is manifested in many ways. Given the significance this distinction seems to have, an interpretation of mathematical practice has to account for it. And the easiest way to account for it is to say that sentences in the first group are the only ones that genuinely belong to mathematics. If this is right, then the mathematicians' uniform commitment to Platonism envisaged in this second scenario does not provide much of an argument against hermeneutic fictionalism, because this commitment falls outside territory of mathematics.

Let me summarize. I argued that Burgess and Rosen are wrong when they demand that hermeneutic fictionalism should be established purely by linguistic considerations. This argument was based on the nature of interpretation. I also raised doubts whether hermeneutic fictionalism can be defeated purely by linguistic considerations. I did that by considering two scenarios which might have seemed to support decisive linguistic objections. This latter argument was not meant to be conclusive. Perhaps one may develop a very well motivated account of fictional talk and use this to show that hermeneutic fictionalism is untenable.

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