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On the Enhancement of the Reconstruction Accuracy obtained with a Multi-Source/Multi-Illumination Inverse Scattering Technique

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Abstract—This paper explores the possibility of enhancing the available information content of scattered data by means of an innovative Multi-Source strategy. The approach exploits the scattering interactions between scatterer and probing source when the investigation domain is illuminated by different (in terms of radiation patterns) illuminations. The results of a set of representative numerical simulations are shown to point out the potentialities of the inversion strategy .

***Index Terms*—Electromagnetic scattering, Microwave imaging, Multi-Source Technique, Source Diversity.**

I. Introduction

In many branches of applied sciences ([1][2]) there is the need of detecting and imaging unknown objects located in inaccessible domains by solving an inverse scattering problem.

Within such a framework, a key issue is the collection of the information content on the scenario under test in order to limit the ill-posedness of the reconstruction process for achieving a faithful reconstruction.

It is well known [3] that single illumination measurement setups are not able to provide an amount of information sufficient to obtain an accurate reconstruction of the scatterer profile. Therefore, different strategies aimed at increasing the information content of scattered data have been proposed. In general, the underlying rational of these methods is the attempt of inducing different scattering phenomena able to highlight different characteristics of the unknown objects. As an example, let us consider, the multi-view (MV) method [4] where the scatterer is illuminated from different angular directions or multi-frequency (MF) [5] techniques where different scattering contributions are excited by acting on the frequency of the probing source.

In this context, a multi-source (MS) methodology able to induce/exploit complementary/alternative scattering phenomena could represent a further mean for obtaining an enhanced quality of the reconstruction without some constraints on the investigated profile (as for MF technique). To point out these effects, this paper analyzes the scattering phenomena arising from the illumination of the investigation domain by means of probing sources with different radiation patterns.

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II. Mathematical Formulation

For sake of simplicity, let us consider a tomographic scenario where the electromagnetic properties of the investigation domain D_I are described by means of the unknown contrast function $\tau(x,y)$. To reconstruct the scatterer profile, the investigation region is sensed, according to a multi-view approach [4], with S electromagnetic sources characterized by different radiation patterns and radiating an electric field $E_{inc}^{v,s}(x,y)\hat{\mathbf{z}}$, $v=1,...,V$, $s=1,...,S$.

The scattering interactions are revealed by collecting a set of measurements of the scattered electric field in the observation domain D_o: $E_{scatt}^{v,s}(x_{m(v)},y_{m(v)})\underline{\hat{\mathbf{z}}}$; $v=1,..,V$; $m(v)=1,..,M(v)$; $s=1,..,S$.

Mathematically, the relations between scatterer and diffused field are expressed through the so-called "*Data*" equation and the "*State*" equation [6], which are usually discretized according to the Richmond's procedure [7] for the numerical solution of the inverse problem at hand. Moreover, to properly deal with the ill-posedness and the ill-conditioning [8], the problem solution has to be carefully addressed by defining a least-square solution and a suitable regularization strategy. A widely adopted technique consists in imposing a set of constraints related to inverse scattering data and/or to the available *a-priori* information in order to define a suitable cost function to be minimized. According to this idea, a new multi-source multi-view cost function is defined to fully exploit different interactions induced on the scatterer, thus enlarging the amount available of information.

As a matter of fact, the information related to a single-source (SS) multi-view experiment might be insufficient to guarantee satisfactory reconstructions. Therefore, it could be profitable to integrate a multi-source technique together with a multi-view system. Towards this end, the multi-source cost function turns out to be

$$\Phi_{MS} = \frac{\sum_{s=1}^{S}\sum_{v=1}^{V}\left\| \left[E_{scatt}^{v,s}\right] - \left[G_{ext}^{v}\right]\left[J_{eq}^{v,s}\right] \right\|^2}{\sum_{s=1}^{S}\sum_{v=1}^{V}\left\| \left[E_{scatt}^{v,s}\right] \right\|^2} + \frac{\sum_{s=1}^{S}\sum_{v=1}^{V}\left\| \left[E_{inc}^{v,s}\right] - \left[E_{tot}^{v,s}\right] + \left[G_{\mathrm{int}}^{v}\right]\left[J_{eq}^{v,s}\right] \right\|^2}{\sum_{s=1}^{S}\sum_{v=1}^{V}\left\| \left[E_{inc}^{v,s}\right] \right\|^2} \quad (1)$$

$\left[G_{ext}\right]$ and $\left[G_{\mathrm{int}}\right]$ being the free-space external and internal Green's matrices [9], respectively; $\left[J_{eq}^{v,s}\right] = \left[\tau\right]\left[E_{tot}^{v,s}\right]$.

Since the multi-source technique does not depend on the minimization algorithm, (1) is minimized by means of a deterministic optimizer [10]. Moreover, to reduce the occurrence of false solutions, the reconstruction approach has been also integrated into a multi-resolution meta-heuristic (IMSA) [10], which allows an iterative synthetic zoom of the region-of-interest (RoI) thus reducing the ratio between data and unknowns.

III. Numerical Analysis

In this section, some representative numerical experiments are analyzed to highlight the effectiveness of the proposed method based on a source diversity technique in imaging unknown scatterer thanks to its capability to exploit the enhanced information content of scattered data.

Let us consider the dielectric profile shown in Fig. 1 centered at $x_c^{ref} = -0.392\lambda_0$, $y_c^{ref} = 0.374\lambda_0$ in an investigation domain $L_{D_I} = 3.0\lambda_0$ in side and characterized by the following parameters: $H_{ext} = 1.05\lambda_0$, (height of the object), $t_{arm} = 0.15\lambda_0$ (thickness of the arms), $d_{arm} = 0.3\lambda_0$ (distance between arms), and $\tau = 2.0$.

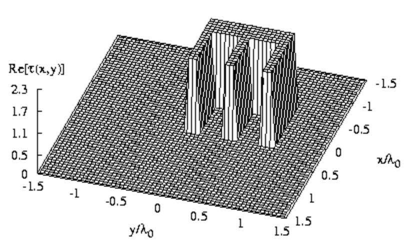


Fig. 1. Actual dielectric profile.

When the scattering scenario is illuminated by a plane wave ($s = 1$) that impinges from $\theta_{inc}^{v}\big|_{v=1} = 0^o$ on D_I, the equivalent current turns out to be as in Figs. 2(*a*)-2(*b*). A different distribution is induced when $\theta_{inc}^{v}\big|_{v=2} = 90^o$ [Fig. 2(*c*)-2(*d*)] and non-negligible differences can be observed.

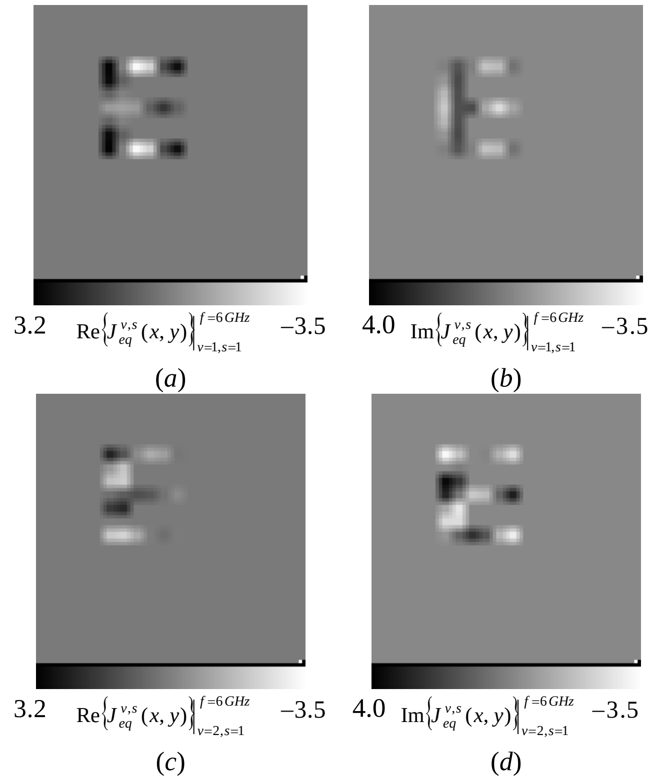


Fig. 2. Equivalent current distribution induced in the investigation domain by a plane illumination ($s = 1$) at $f = 6GHz$: Real (*a*) and Imaginary part (*b*) when ($v = 1$). Real (*c*) and Imaginary part (*d*) when ($v = 2$).

Such a behavior gives an indication that different scattering interactions are excited in a multi-view system and thus complementary information can be extracted from the corresponding scattered fields [Fig. 3].

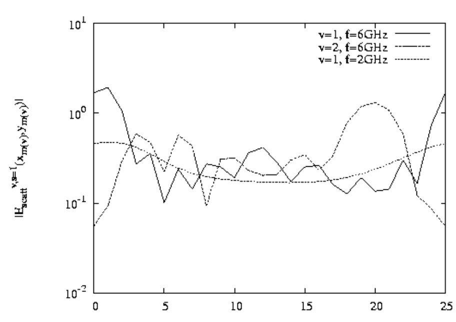


Fig. 3. Amplitude of the scattered field collected in D_O under plane wave ($s = 1$) illumination for different views and frequencies.

In a similar fashion, Figs. 4(*a*)-4(*b*) show the equivalent current distribution induced by the same probing source ($s = 1$), but at another working frequency [$f = 2GHz$ instead of $f = 6GHz$ Figs. 2(*a*)-2(*b*)]. As expected, a strategy that exploits frequency diversity is able to reveal other/different "characteristics" of the same scatterer to be taken into account [5] for improving the quality of the reconstruction.

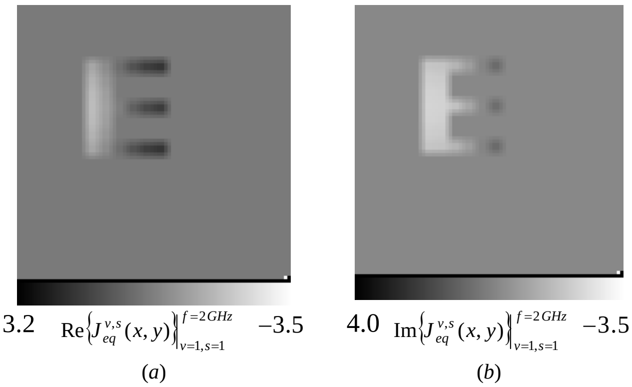


Fig. 4. Equivalent current distribution induced in the investigation domain by a plane illumination ($s=1$) at $f=2GHz$: Real (*a*) and Imaginary part (*b*).

Such an objective can also be achieved by considering a diversity strategy in which different source models are used to probe the scatterer. In this a case, the increment of information is due to the diversity of the probing source instead of that in frequency. To give some indications on such an event, let us compare the equivalent current distributions when D_I is probed by an isotropic line ($s=2$) [Figs. 5(*a*)-5(*b*)] or by plane wave source ($s=1$) [Figs. 2(*a*)-2(*b*)]. Once again, significant differences can be noticed.

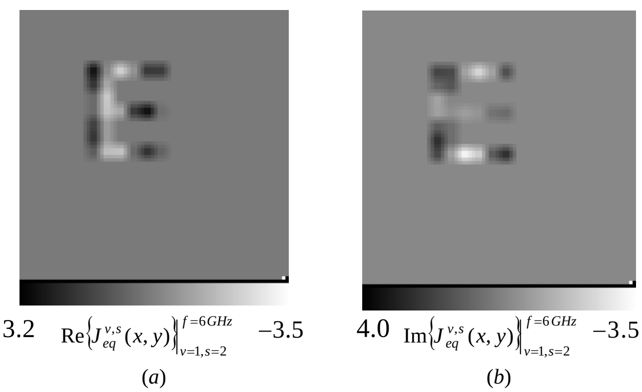


Fig. 5. Equivalent current distribution induced in the investigation domain by an isotropic line ($s=2$) at $f=6GHz$: Real (*a*) and Imaginary part (*b*).

As a consequence, different information on the unknown scatterer can be collected from the imaging system as confirmed by the amplitudes of the scattered field in D_O (Fig. 6).

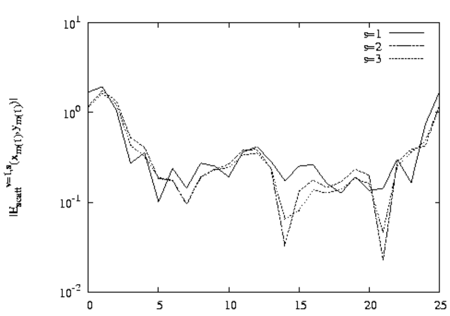


Fig. 6. Amplitude of the scattered field collected in D_O under plane wave ($s=1$), isotropic line ($s=2$), and directive line ($s=3$) illumination.

However, it should be pointed out that, even though an enhancement of the information content of scattered data takes place, suitable retrieval strategies are needed to allow an improvement in the reconstruction. To point out such a key point, let us consider the results obtained when a multi-view/multi-illumination acquisition system characterized by $M(v)=26$, $v=1,..,V$, $V=6$ sensors equally-spaced on a circular observation domain of radius $r_0=2.2\lambda_0$ is used. In the first reconstruction experiment, the process has been carried out by considering a "bare" (i.e., without multi-resolution strategies) approach and D_I has been partitioned into $N=256$ equal sub-domains.

Although the structure under test is correctly located, the "*composed*" SS approach (the reconstructed profile being the average value of the object functions retrieved with each SS illumination) [Fig. 7(*a*)] as well as the MS strategy [Fig. 7(*b*)] fail to reconstruct the correct shape of the scatterer as confirmed by the value of reconstruction error greater than 13% .

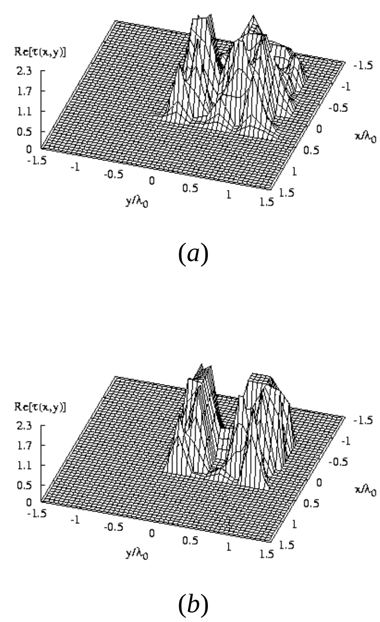
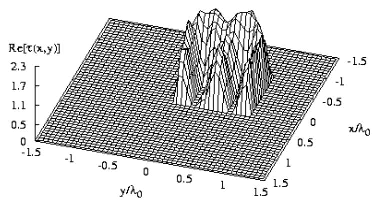
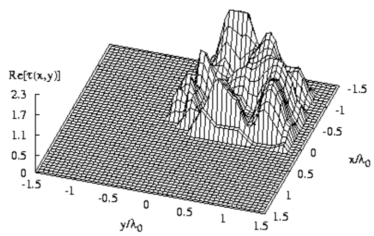


Fig.7 Reconstruction of the complex profile by means a conjugate gradient. Comparison between the mean profile achieved combining the SS approaches (*a*) versus the MS reconstruction (*b*).

This is due to an inefficient reconstruction strategy, which cannot fully exploit the information content of scattering data even though it has been increased as for the case of the MS approach. A confirmation of such a conclusion can be carried out by considering the results from the second numerical experiment where a suitable strategy (i.e., the IMSA) able to better exploit the available information is adopted. In this case, the scenario has been illuminated by $V=4$ different directions and $N=49$ partitions have been considered. Although the number of views has been reduced ($V=6 \rightarrow V=4$), the IMSA-MS strategy faithfully retrieve the actual profile as shown in Fig. 8(*a*) (with a reconstruction error of 2.4%). On the contrary, the IMSA integrated with the "combined" SS approach does not achieve a satisfactory reconstruction [Fig. 8(*b*)]



(*a*)



(*b*)

Fig.8 Reconstruction of the complex profile by means of the IMSA integrated with the MS technique (*a*) and the "combined" SS approach (*b*).

IV. Conclusions

The effectiveness of a multi-source strategy for the quantitative imaging of unknown profiles has been analyzed through selected numerical experiments. The method has demonstrated good potentialities provided that efficient algorithms able to exploit the available information are used. Further researches will be needed to quantify the additional information content of multi-source scattering data together with the definition of the source configuration able to efficiently excite independent scattering phenomena.

References

[1] J. Ch. Bolomey, *Frontiers in Industrial Process Tomography.* Engineering Foundation, 1995.

[2] E. C. Fear and M. A. Stuchly, "Microwave Detection of Breast Cancer," *IEEE Trans. Microwave Theory Tech*., vol. 48, pp. 1854-1863, 2000.

[3] O. M. Bucci and T. Isernia, "Electromagnetic inverse scattering: retrievable information and measurements strategies," *Radio Science*, vol. 32, pp. 2123-2138, 1997.

[4] S. Caorsi, G. L. Gragnani, and M. Pastorino, "An approach to microwave imaging using a multiview moment method solution for a two-dimensional infinite cylinder," *IEEE Trans. Microwave Theory Tech*., vol. 39, pp. 1062-1067, 1991.

[5] K. Belkebir, R. Kleinman, and C. Pichot, "Microwave imaging - Location and shape reconstruction from multifrequency scattering data," IEEE Trans. Microwave Theory Tech., vol. 45, pp. 469-475, 1997.

[6] D. Colton and R. Krees, *Inverse Acoustics and Electromagnetic Scattering Theory*. Berlin, Germany: Springer-Verlag, 1992.

[7] J. H. Richmond, "Scattering by a dielectric cylinder of arbitrary cross section shape," *IEEE Trans. Antennas Propagat*., vol 13, pp. 334-341, 1965

[8] A. M. Denisov, *Elements of theory of inverse problems*. Utrecht, The Netherlands: VSP, 1999.

[9] D. S. Jones, D. S., *The Theory of Electromagnetism*. Oxford, U.K.: Pergamon Press, 1964

[10] S. Caorsi, M. Donelli, D. Franceschini, and A. Massa, "A new methodology based on an iterative multi-scaling for microwave imaging," *IEEE Trans. Microwave Theory Tech*., vol. 51, pp. 1162-1173, 2003.