UNIVERSITY
OF TRENTO

DEPARTMENT OF INFORMATION AND COMMUNICATION TECHNOLOGY

38050 Povo – Trento (Italy), Via Sommarive 14
http://www.dit.unitn.it

ASSESSMENT OF THE RELIABILITY AND EXPLOITATION OF THE INFORMATION CONTENT OF INVERSE SCATTERING DATA THOUGH A FUZZY-LOGIC-BASED STRATEGY – PRELIMINARY RESULTS

Aronne Casagranda, Davide Franceschini, and Andrea Massa

August 2004

Technical Report DIT-04-065

Assessment of the Reliability and Exploitation of the Information Content of Inverse Scattering Data through a Fuzzy-Logic-Based Strategy - Preliminary Results

Aronne Casagranda, Davide Franceschini, and Andrea Massa

Department of Information and Communication Technology
University of Trento -Via Sommarive, 14 - 38050 Trento - ITALY
Tel.: +39 0461 882057; Fax: +39 0461 882093
E-mail: *andrea.massa@ing.unitn.it*
Web-site: *http://www.eledia.ing.unitn.it*

Assessment of the Reliability and Exploitation of the Information Content of Inverse Scattering Data through a Fuzzy-Logic-Based Strategy - Preliminary Results

Aronne Casagranda, Davide Franceschini, and Andrea Massa

Abstract

The presence of the noise in the measured data should be carefully considered in inverse scattering methodologies, because of the intrinsic ill-conditioning of the problem. To limit the effects of the noise on the retrieval procedure, this paper presents an innovative fuzzy-logic-based approach. The proposed strategy allows one to take into account the corrupted nature of the data by fully exploiting all the available information content of the measurements. Selected synthetic and experimental test cases are considered for assessing the effectiveness of the proposed approach also in comparison with a reference inverse scattering technique.

1 Introduction

When dealing with microwave imaging problems, some of the main difficulties in the reconstruction process are certainly due to the ill-posedness and to the noisy nature of the available measured data. In collecting the electromagnetic measurements, experimental and environmental noises add to the scattered signals due to the mechanical positioning of the electromagnetic field sensors or to the electromagnetic interferences in the test-site. The impact of these corrupting factors, because of the high intrinsic instability, strongly affects the accuracy of the reconstruction. Usually, such an instability is handled by looking for a regularized solution, which better fits all the available data (corrupted by the noise). The data fitting is measured by means of a cost function composed of two terms, namely the *data term* and the *state term,* which depend on the scattered field collected in the observation domain and on the incident field measured in the investigation domain, respectively. Suitable weighting parameters allows one to weight more the one or the other term, depending on the uncertainties associated with both of them.

Certainly, a direct evaluation of the reliability of inverse scattering data would be really useful. But, because of the cost and the complexity of such an estimate, it is quite hard to be obtained. To avoid such a direct estimation, but exploiting the information-content available in the noisy data, the proposed approach takes into account the presence of the noise through a strategy based on a fuzzy logic [1] system and by defining the values of the weighting parameters estimating, in an unsupervised way, the degree of reliability of the available data without any time-expensive computational operation.

This paper is organized as follows. In Section 2, the mathematical formulation of the inverse scattering problem is briefly resumed and a description of the fuzzy-logic-based approach is presented (Sect. 3). Section 4 shows a set of selected numerical and experimental results for a preliminary assessment. A final discussion with some remarks on the applicability of the approach is then reported in Section 5.

2 Mathematical Formulation

Let us refer to a two-dimensional inverse scattering problem, characterized by a tomographic scenario where a set of V plane waves $(E_v^{inc}(x,y)\hat{\mathbf{z}},\ v=1,..V)$ successively illuminates an unknown investigation domain, whose electromagnetic parameters (i.e., the object function $\tau(x,y)$) have to be determined. Without loss of generality, the background medium is assumed to be homogeneous (characterized by a dielectric permittivity ε_0) and lossless.

The relation between dielectric properties of the investigation domain and radiated scattered fields is mathematically described through the well-known integral scattering equations [2], whose discretized counterparts are reported in the following

$$\Im_n^v\left\{\tau(x_n,y_n);\ E_v^{tot}(x_n,y_n)\right\}=E_v^{tot}(x_n,y_n)-j\frac{k_0^2}{4}\sum_{p=1}^{N}\tau(x_p,y_p)E_v^{tot}(x_p,y_p)G_{2D}^v(x_n,y_n|x_p,y_p) \tag{1}$$

where

$$G_{2D}^v(x_m,y_m|x_n,y_n)=\begin{cases}(j/2)\left[\pi k_0a_pH_1^{(2)}(k_0a_p)-2j\right] & if\ p=n\\ (j\pi k_0a_n/2)H_0^{(2)}(k_0\rho_{mn}^v)J_1(k_0a_n) & otherwise\end{cases} \tag{2}$$

and

$$\aleph_m^v\left\{\tau(x_n,y_n);\ E_v^{tot}(x_n,y_n)\right\}=j\frac{k_0^2}{4}\sum_{n=1}^{N}\tau(x_n,y_n)E_v^{tot}(x_n,y_n)G_{2D}^v(x_m,y_m|x_n,y_n) \tag{3}$$

where

$$G_{2D}^v(x_m,y_m|x_n,y_n)=(j\pi k_0a_n/2)H_0^{(2)}(k_0\rho_{mn}^v)J_1(k_0a_n) \tag{4}$$

$E_v^{tot}(x_n,y_n)$ being the unknown electric field computed in the nth discretization cell of the investigation domain ($n=1,...,N$), M being the number of measurement points, and $\rho_{mn}^v=\sqrt{\left(x_m^{(v)}-x_n\right)^2+\left(y_m^{(v)}-y_n\right)^2}$.

Through a multi-view/multi-illumination measurement system, the data of the inverse scattering problem are acquired. The scattered electric field $E_v^{scatt}(x_m, y_m)$, $m = 1, ..., M$, $v = 1, ..., V$ is collected at the measurement points equally-spaced along a circular observation domain. Moreover, the incident field $E_v^{inc}(x_n, y_n)$, $n = 1, ..., N$, $v = 1, ..., V$ is measured in the investigation domain.

The inversion procedure is aimed at retrieving the object function $\tau(x_n, y_n)$ as well as the electric field $E_v^{tot}(x_n, y_n)$ induced in the investigation domain by minimizing a suitable cost function defined according to the fuzzy-logic approach described in the following Section.

3 The Fuzzy-Logic-Based Strategy

The fuzzy-logic system operates between the data acquisition and the definition of the cost function to be minimized, by determining the values of a set of weighting coefficients (α_v^m and β_v^n, $m = 1, ..., M$, $v = 1, ..., V$, $n = 1, ..., N$) that quantifies the reliability of each sample of the measured scattered and incident field, respectively. Consequently, the arising cost function turns out to be

$$\Phi\left\{\tau(x_n, y_n); E_v^{tot}(x_n, y_n)\right\} = \frac{\sum_{v=1}^{V}\sum_{m=1}^{M}\left\{\alpha_v^m\left|E_v^{scatt}(x_m,y_m)-\aleph_m^v\left\{\tau(x_n,y_n);E_v^{tot}(x_n,y_n)\right\}\right|^2\right\}}{\sum_{v=1}^{V}\sum_{m=1}^{M}\left\{\left|E_v^{scatt}(x_m,y_m)\right|^2\right\}} + \frac{\sum_{v=1}^{V}\sum_{n=1}^{N}\left\{\beta_v^n\left|E_v^{inc}(x_n,y_n)-\Im_n^v\left\{\tau(x_n,y_n);E_v^{tot}(x_n,y_n)\right\}\right|^2\right\}}{\sum_{v=1}^{V}\sum_{n=1}^{N}\left\{\left|E_v^{inc}(x_n,y_n)\right|^2\right\}} \tag{5}$$

In order to define the weighting coefficients, by taking into account the presence of the noise on the measurement data, the fuzzy-logic strategy is implemented according to the block-diagram shown in Figure 1. Firstly, the inverse scattering data-sample values are normalized ("*normalization*" block) and the following coefficients are then defined

$$\eta_v^m = \frac{\left|\frac{E_v^{scatt}(x_m,y_m)}{E_v^{tot}(x_m,y_m)}\right|}{max_v\left\{max_m\left|\frac{E_v^{scatt}(x_m,y_m)}{E_v^{tot}(x_m,y_m)}\right|\right\}}, \qquad \xi_v^n = \frac{\left|E_v^{inc}(x_n, y_n)\right|}{max_v\left\{max_n\left|E_v^{inc}(x_n, y_n)\right|\right\}} \qquad \begin{array}{c} v = 1, ..., V \\ m = 1, ..., M \end{array} \tag{6}$$

Then, the coefficients η_v^m and ξ_v^n are used as inputs of the fuzzy-logic system, which operates accordingly to a specific set of heuristically-defined rules $\Re$, composed by a set of *antecedents* Γ (Fig. $2(a)$) and relative *consequences* $\mathcal{C}$ (Fig. $2(b)$). The *fuzzyfier* processes the input values determining their fuzzy counterparts and by associating to each sample a gaussian membership function $\left(\varphi\left(\eta_m^v\right), \varphi\left(\xi_n^v\right)\right)$ [1] as indicated in Fig. $2(a)$. Such a process allows to activate a rule according to the degree of similarity between $\varphi(\bullet)$ and the antecedent of the rule in Γ. An activated rule is characterized by an activation value that determines the degree of truth of every consequence of $\mathcal{C}$. Finally, the *defuzzyfier* provides the reliability index (α_m^v, β_n^v) by considering the composition of the activated consequences.

The inversion process is successively completed, by determining an estimate of the unknown object function, through the iterative minimization of (5) (where each measured data contributes according to own reliability index).

4 Numerical Results

For the numerical assessment, some preliminary results from selected numerical experiments are reported.

The first example deals with the reconstruction of a square domain (λ_0 in side) partitioned in $N = 25 \times 25$ cells. In such a domain two lossless square dielectric cylinders ($\frac{\lambda_0}{5}$ in side) are present, both characterized by an object function equal to $\tau = 2.5 + j0.0$. The measurement data have been collected in $M = 10$ measurements points and the scenario has been probed with $V = 4$ illuminations. Moreover, the effects of the noise has been simulated by adding a Gaussian random noise ($SNR = 5\,dB$) to the data of the problem. The grey-scale representation of the reconstructions obtained with a reference conjugate-gradient-based strategy [3] and with the same strategy but exploiting the previously discussed fuzzy logic approach are reported in Figure 3. As can be observed, a non-negligible improvement in the profile retrieval has been obtained in terms of localization as well as of reconstruction accuracy. Accordingly, the values of the quantitative error figures, computed as in [2], turn out to be reduced and equal to $\xi_{(tot)}^{fuzzy} = 10.09$ (vs.

$\xi_{(tot)}^{standard} = 12.66$) and $\xi_{(int)}^{fuzzy} = 26.00$ (vs. $\xi_{(int)}^{standard} = 38.20$).

To further assess the effectiveness of the proposed approach, a test case of the real-dataset "Marseille" [4] has been considered. The scenario under test is constituted by an off-centered dielectric cylinder (described through an homogeneous object function $\tau = 3.0 + j0.0$), $15\,mm$ in radius. Only a working frequency $f = 4\,GHz$ has been taken into account and neither hopping nor multi-frequency procedure has been adopted.

A comparison between the reconstructed profiles with and without the fuzzy logic (but considering the iterative multi-scaling procedure described in [2]) is shown in Figure 4. As can be observed, the fuzzy-logic-based approach allows one to obtain a more detailed representation of the profile under test. However, it should be pointed out that, since the measurements have been carried out in a controlled-environment (therefore with a greater $SINR$ than that of the first synthetic test case), the improvement in the reconstruction accuracy does not turn out so large pointing out the effectiveness of the approach especially in heavy noisy conditions.

5 Conclusions

In this paper, an innovative inverse scattering approach based on a fuzzy logic strategy has been presented. The approach allows a simple and effective estimate of the reliability of collected noisy data. Such an information is introduced into the cost function through suitable regularization coefficients determined by the fuzzy-logic system. A set of selected numerical test cases have been considered in order to preliminary show the effectiveness of the proposed approach and to point out its advantages over standard inversion approaches in dealing with inversion data seriously affected by high levels of noise.

References

[1] L. A. Zadeh, ”Fuzzy Sets,” *Information and Control*, vol. 8, pp. 338-353, 1965.

[2] S. Caorsi, M. Donelli, D. Franceschini, and A. Massa, "A new methodology based on an iterative multi-scaling for microwave imaging," *IEEE Trans. Microwave Theory Tech.*, vol. 51, pp. 1162-1173, 2003.

[3] R. V. Kohn and A. McKenney, “Numerical implementation of a variational method for electrical impedance tomography,” *Inverse Problems*, vol. 6, pp. 389-414, 1990.

[4] K. Belkebir and M. Saillard, Special section: "Testing Inversion Algorithms against Experimental Data," *Inverse Problems*, vol. 17, pp. 1565-1702, 2001.

Figures Captions

- **Figure 1.** Block diagram of the fuzzy system.
- **Figure 2.** Fuzzy rules: (a) *antecedents* and (b) relative *consequences*.
- **Figure 3.** Actual profile (a). Reconstructed profile obtained with (b) the Standard Technique and (c) with the Fuzzy Strategy (please note that the black pixel in the upper left corner is used for reference).
- **Figure 4.** Real dataset "Marseille" [4] - "dielTM_dec8f.exp". Reconstructed profile obtained with (a) the Standard Technique and (b) the Fuzzy Strategy.

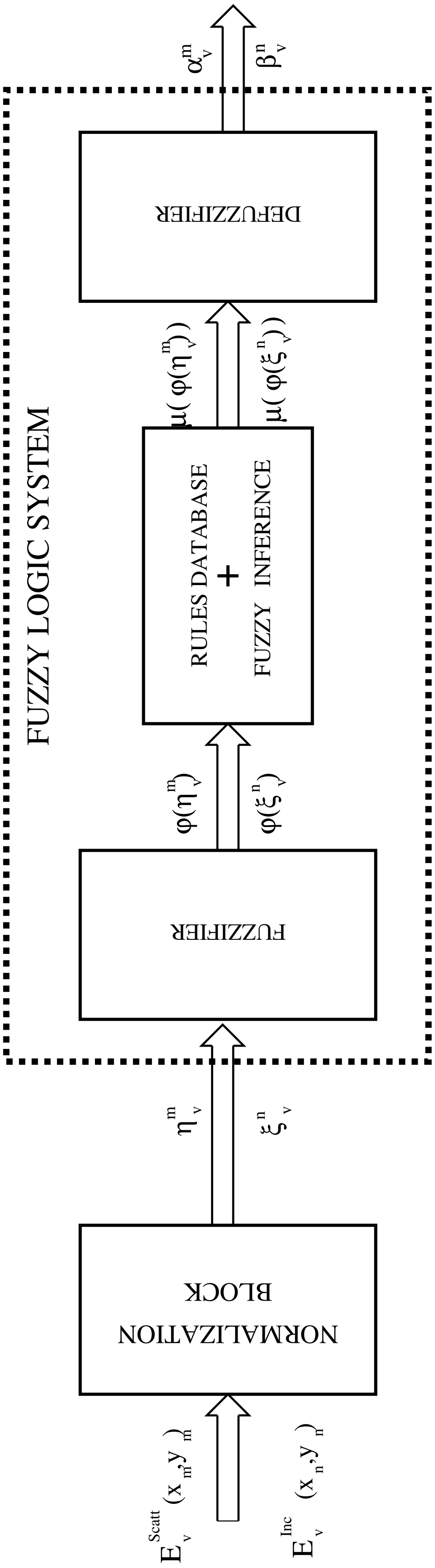
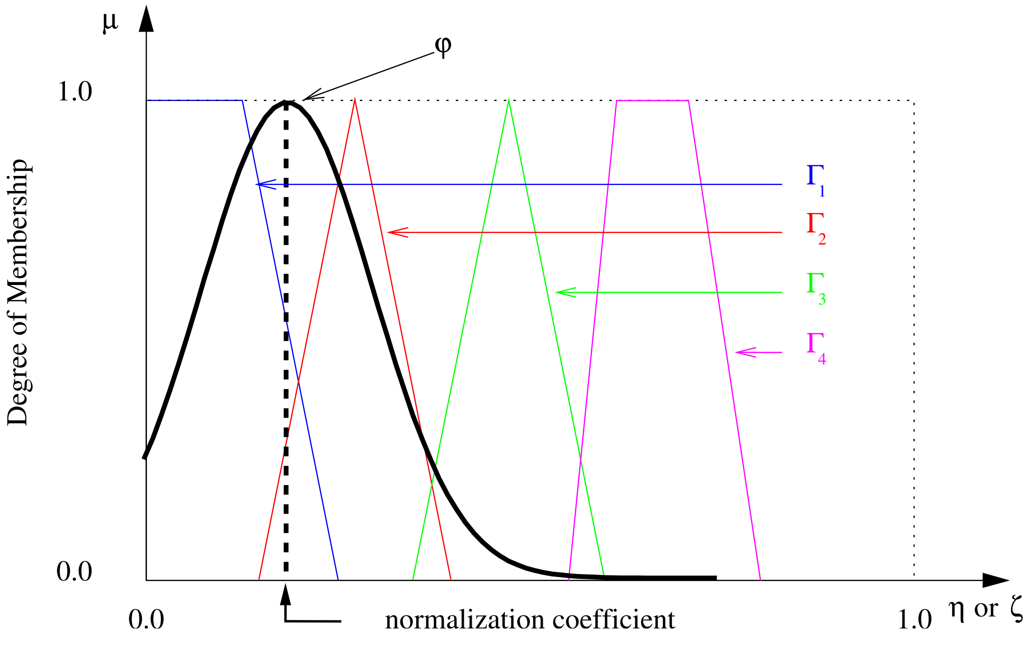
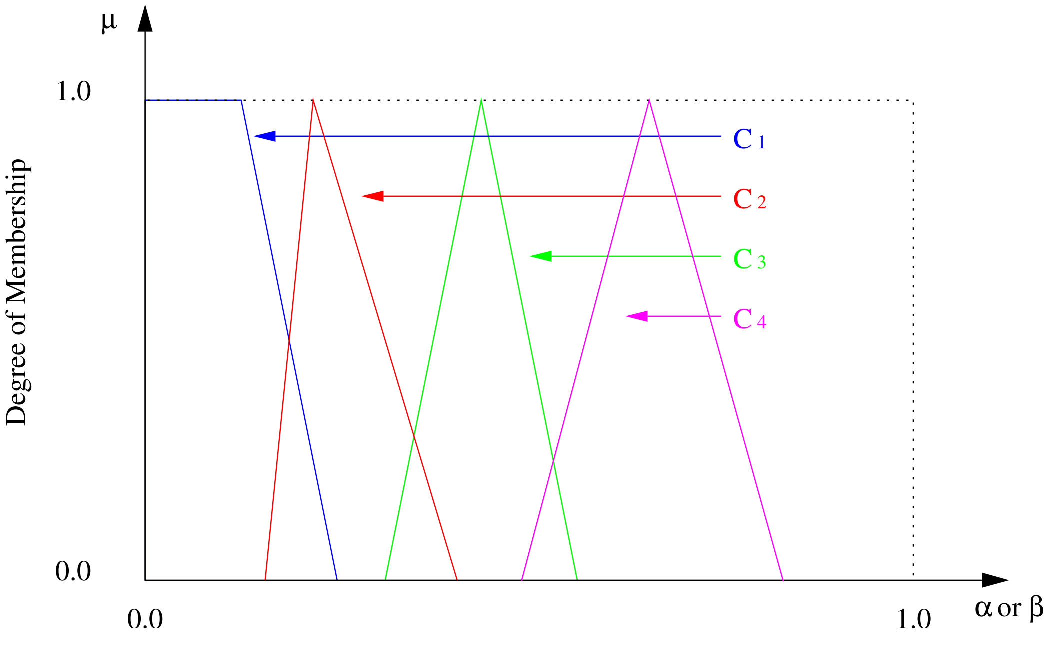


Figure 1 - A. Massa *et al.*, "Assessment of the Reliability ..."

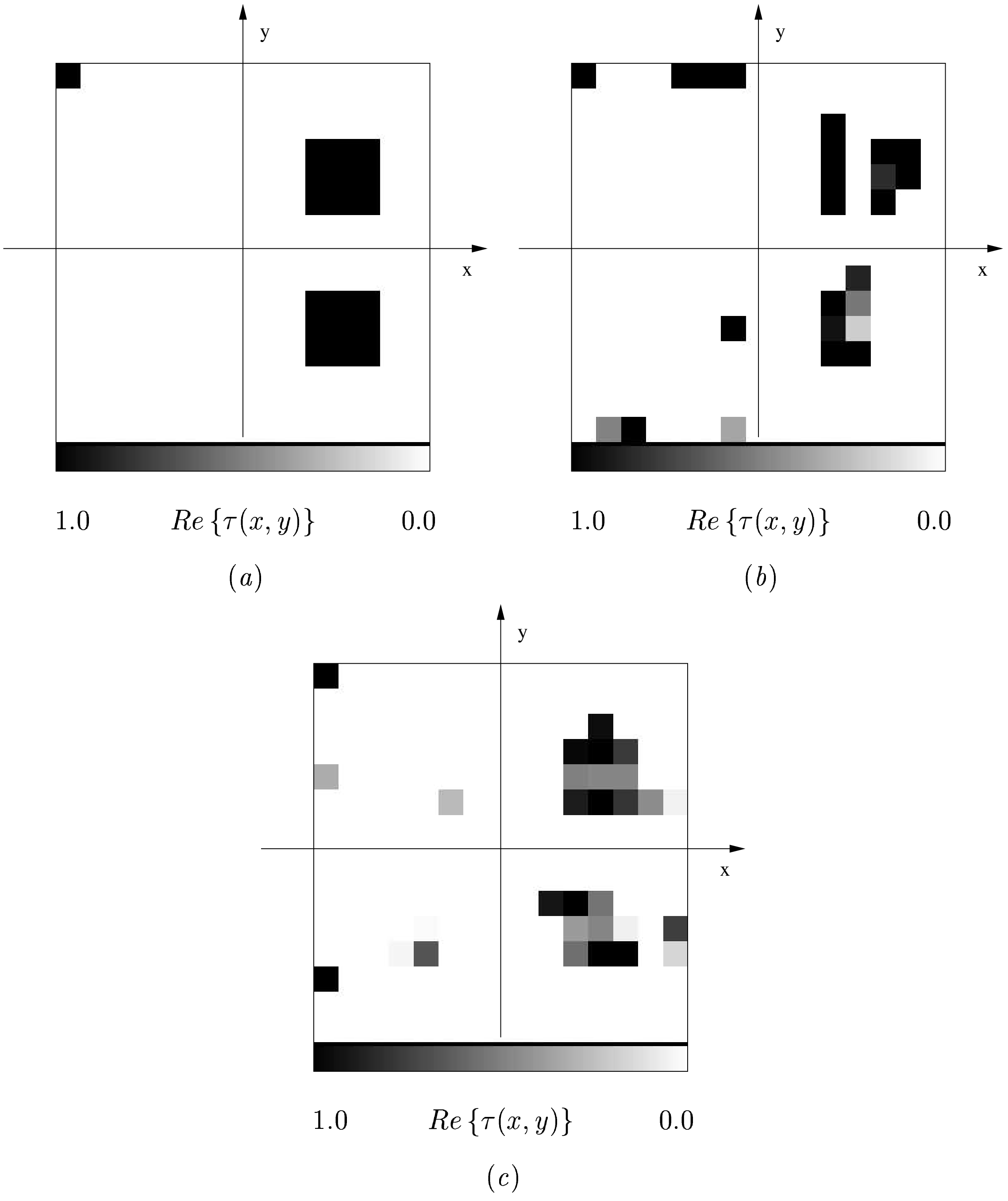


(*a*)



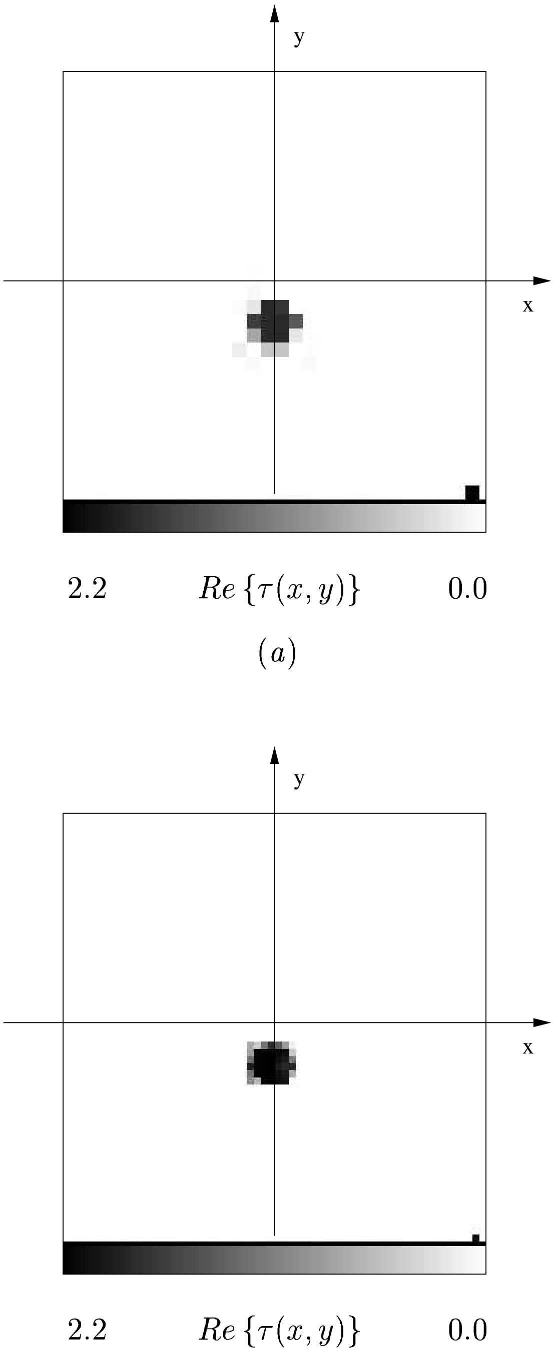
(*b*)

Figure 2 - A. Massa *et al.*, “Assessment of the Reliability ...”



(a) (b) (c)

Figure 3 - A. Massa *et al.*, "Assessment of the Reliability ..."



(a)

(b)

Figure 4 - A. Massa *et al.*, "Assessment of the Reliability ..."