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38050 Povo – Trento (Italy), Via Sommarive 14
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Emilia Nunzi, Gianna Panfilo, Patrizia Tavella, Paolo Carbone, Dario Petri

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E.Nunzi*, G. Panfilo**, P.Tavella**, P.Carbone*, D.Petri***

* Università degli Studi di Perugia, Dipartimento di Ingegneria Elettronica e dell'Informazione,

Phone: ++39 075 5853634, Fax: +39 075 5853654, Email: nunzi@diei.unipg.it,

** Istituto Elettrotecnico Nazionale (IEN) "Galileo Ferraris",

Phone: ++39 011 3191235, Fax ++39 011 3191259, Email: tavella@tf.ien.it

*** Università degli Studi di Trento, Dipartimento di Informatica e Telecomunicazioni,

Phone. ++39 0461 883902, Fax ++39 0461 882093, Email: petri@dit.unitn.it.

Abstract

The length of calibration intervals of measurement instrumentations can be determined by means of several techniques. In this paper three different methods are compared for the establishment of optimal calibration intervals of atomic clocks. The first one, is based on a stochastic model, and provides the estimation of the calibration interval also in the transient situation, while the others, attain to the class of the so–called reactive methods, which determine the value of the optimal interval on the basis of the last calibration outcomes. Algorithms have been applied to experimental data and obtained results have been compared in order to determine the most effective technique. Since the analyzed reactive methods present a large transient time, a new algorithm is proposed and applied to the available data.

Keywords

Calibration Intervals, Simple Response Method, Interval Test Method, Wiener Process

I. INTRODUCTION

The metrological characteristics of test and measurement instrumentations, are ordinarily subjected to degradation over time thus requiring the periodic application of calibration procedures. Usually, the calibration process is carried out in a dedicated laboratory, it presents elevated costs and it is one of the main causes of unavailability of the instrument. By optimizing the length of the maintenance period, calibrations which are not strictly necessary are prevented and given reliability targets are achieved. Published results on this subject report on many analysis methods that can be applied for identifying the proper calibration interval length. Typically, the optimal method is derived from the combination of various techniques, depending on the quality objective, calibration and maintenance budgets, risk management criteria and potential return on investment [1].

The three approaches presented in this paper for the establishment of the optimal calibration interval of an instrument, attain to two different classes: the "parametric" ones, which imply the acquisition of a large data set and the availability of the reliability model characterizing the instrument of interest, and the "non–parametric" ones, which evaluate the duration of the next interval on the basis of the last calibration results.

Algorithms have been applied to experimental data obtained by measuring every 12 hours, from September 6–th 2001 to July 9–th 2003, the deviations between two cesium clocks, in the following referred to as Cs_1 and Cs_3, situated at *Istituto Elettrotecnico Nazionale* (IEN) "G. Ferraris" in Turin–Italy. In the next sections, after a brief description of the three procedures, results obtained by the application of the methods are presented.

II. PARAMETRIC ALGORITHM DESCRIPTION

The noise affecting the time deviation of cesium atomic clocks can be satisfactorily modeled by a Wiener process corresponding to a white frequency noise [2], whose basic parameters are the drift, μ, and the diffusion coefficient, σ, which influence directly the determination of the calibration interval. The use of this model applied to atomic clocks has been already demonstrated in [8] to be very effective for determining the calibration interval. In this paper, it is evaluated the time necessary for acquiring the minimum set of data which guarantees an accurate estimate of the interval. To this aim the process parameter has been repeatedly evaluated with an increasing number of data corresponding to longer observation intervals and the two clocks have been assumed to be calibrated at time $t = 0$. In order to evaluate effects of the uncertainty on the estimate of σ, the drift μ has been removed from the available data. The whole observation interval has been divided in a number of subintervals which present the same initial reference time and an increasing time–length equal to 1, 3, 6, 9, 12, 15, 18, 22 months, respectively. Thus, the diffusion coefficient has been estimated at the end of each time interval through the evaluation of the square root of the Allan variance, $\widehat{\sigma}_y(\cdot)$, as follows [3]:

$$\widehat{\sigma}(t) = \sqrt{t}\widehat{\sigma}_y(t) \quad \left(\mathrm{ns}/\sqrt{\mathrm{day}}\right), \tag{1}$$

where $\widehat{\sigma}_y(\cdot)$ is calculated by using the algorithms described in [4].

Fig. 1 shows the values of $\widehat{\sigma}(t)$ (circled–data) estimated in each subinterval, together with the corresponding confidence interval at a 68% confidence level and proves that the σ estimates converge to an constant value, K. The relative estimation error of $\widehat{\sigma}(t)$, calculated by assuming K equal to $\widehat{\sigma}(22)$, is lower than 3% already after twelve months after the first measurement.
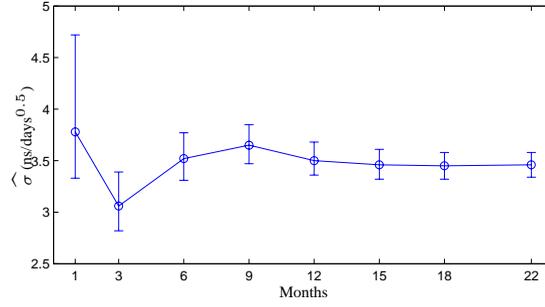
Figure 1. Estimates of the diffusion coefficient σ evaluated by employing (1) and by using an increasing number of data corresponding to the observation period reported in abscissa (circled data). For each estimated value also the confidence interval obtained by imposing a confidence level value equal to 68% is reported.
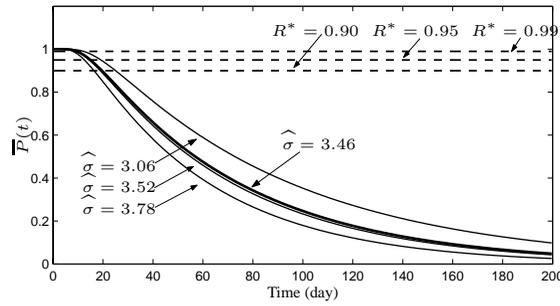
Figure 2. Probability of finding the clock deviation within the thresholds of 30 ns for different values of the σ estimates, as indicated by the corresponding labels.

Estimates of σ can then be employed for evaluating the optimal calibration interval by means of the reliability $\overline{P}(t)$, i.e. the probability that, after synchronisation, the clock error is bounded within the interval $[-k, k]$, $k > 0$, whose mathematical expression is given by [6]:

$$\overline{P}(t) = \int_{-k}^{k} \overline{p}(c;t)\mathrm{d}c = \int_{-k}^{k} \sum_{n=-\infty}^{\infty} \left(e^{\frac{4nk\mu}{\sigma^2}} p\left(4nk + c; t\right) - e^{\frac{2(3nk-c)\mu}{\sigma^2}} p\left(4nk + 2k - c; t\right) \right) \mathrm{d}c \quad (2)$$

where $\overline{p}(c;t)$ is the probability that the process arrives in c at time t without having touched the barriers with $-k < c < k$, and $p(\cdot, t)$, is the normal probability density function, with mean μt and variance $\sigma^2 t$.

Fig. 2 shows the behavior of $\overline{P}(t)$ for the Wiener process within the boundaries $k = 30$ ns

TABLE I

Calibration intervals (days) with the boundaries $k = 30$ ns for different reliability levels, R^*, calculated for each value of the σ estimate reported in Fig. 1.

Months	$\widehat{\sigma}$	Calibration interval (days)		
		$R^* = 90\%$	$R^* = 95\%$	$R^* = 99\%$
1	3.78	16.1	12.5	7.4
3	3.06	24.9	19.1	12.2
6	3.52	18.9	14.4	9.2
9	3.65	17.3	13.4	8.5
12	3.5	18.9	14.6	9.3
15	3.46	19.3	14.9	9.5
18	3.45	19.6	15	9.6
22	3.46	19.3	14.9	9.5

for four different values of the σ estimates obtained from Fig. 1, as indicated by the corresponding labels. Moreover, since the drift has been removed from available data, μ has been set equal to 0.

By identifying the abscissas corresponding to a percentage level, R^*, of $\overline{P}(t)$ equal to 0.90, 0.95 and 0.99, the optimal calibration intervals have been evaluated. Tab. I reports the length of such intervals (in days) with the boundaries $k = 30$ ns for different percentage levels, R^*, calculated for all estimates of σ reported in Fig. 1. These results show that the calibration interval estimation obtained by employing three to six months of data is already sufficiently accurate.

III. Non–parametric algorithm description

Non–parametric procedures considered in this paper are the so–called reactive methods [1]. These procedures set the duration of the calibration interval on the basis of the results of the last calibration outcomes, in accordance to predetermined algorithms which regulate the observed reliability to a given value. Because of their simplicity, the application of such methods does not require knowledge of specific statistical approaches. Clearly, obtained results are sub–optimal, since the duration of the interval between maintenances is determined only on the basis of the few last calibration outcomes. However, they are highly recommended in systems characterized

by life-time cycles much shorter than the time needed to collect statistically significant data for parameter estimation purposes. Moreover, the start-up and management costs are minimal.

Two algorithms attaining to this class and frequently used for determining the interval length are the simple response (SRM) and the interval test (ITM) methods, which are described in the next subsections. These procedures have been applied to the available record of data and results have been used for characterizing the performance of the proposed algorithms applied to atomic clock data described in Sec. II. On the basis of the obtained results a new algorithm is proposed in subsection C and estimates are compared to those obtained with the non–parametric methods.

In order to improve the accuracy of the proposed reliability estimators, the unique available record of data has been subjected to a random re–sampling mechanism, which gives M new records of data, representing samples of M different atomic clocks, each of length equal to a needed value N. Such a technique, widely used in *bootstrap*–based algorithms, is justified by the characteristic of the Wiener process, which ensures that the clock random error increments are independent and identically distributed [7].

A. *Simple response method*

The SRM requires that the n–th calibration interval be increased by an amount equal to $a > 0$ if the instrument results to be in–tolerance, and decreased by an amount equal to $b > 0$, otherwise. By defining with I_n the duration of the n-th maintenance interval, the next calibration interval length is calculated as follows:

$$I_{n+1} = I_n \left(1 + a\right) y_n + I_n \left(1 - b\right) \overline{y}_n, \quad n = 0, 1, ... \tag{3}$$
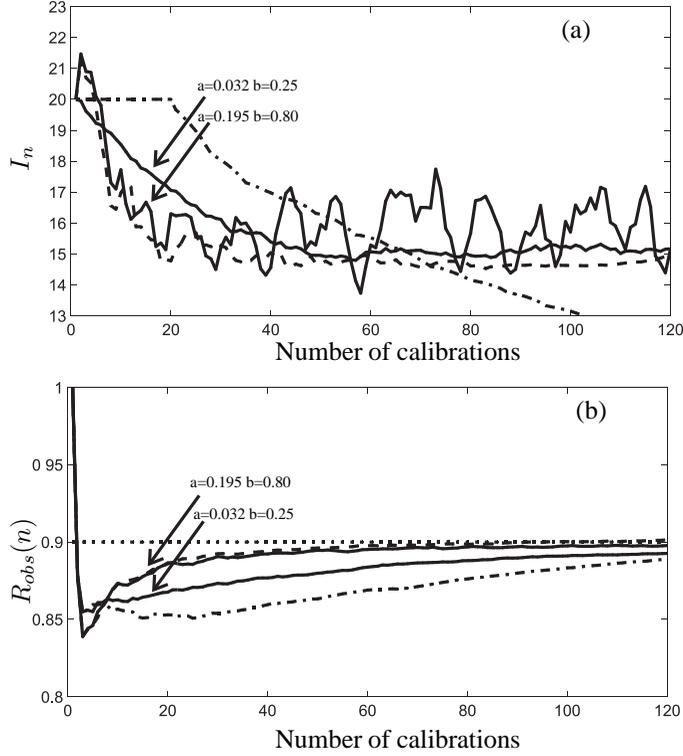
Figure 3. Maintenance interval I_n (a) and observed reliability $R_{obs}(n)$ (b), averaged on 100 records of data, obtained by applying the SRM (solid lines), the ITM (dash–dotted line) and the MSRM (dashed line) methods with a target reliability $R^* = 90\%$ and an initial calibration interval value equal to 20 days. The ITM method has been applied by considering $\alpha = 5\%$, $a = 0.05$ and $b = 0.1$.

where y_n is a boolean variable which is equal to 1 if the instrument has been found in–tolerance at the end of the n–th maintenance interval, and is equal to 0 otherwise, and where $y_n + \overline{y}_n = 1$. By indicating with g the number of in–tolerance events after n maintenance operations, i.e. $g = \sum_{i=0}^{n} y_i$, the observed reliability is equal to $R_{obs}(n) = g/n$. After a transient time, the observed reliability reaches a constant asymptotic value R^*, that is the method reliability target. R^* is largely independent on the initial interval length I_0, but it depends only on parameters a and b through the following relationship [5]:

$$R^* = \frac{\log\left(1-b\right)}{\log\left(\frac{1-b}{1+a}\right)}. \tag{4}$$

Fig. 3 shows with solid lines the behavior of the maintenance interval (a) and of the observed reliability (b), averaged on 100 records of data obtained by processing the atomic clock data and calculated by applying the SRM method with a target reliability $R^* = 90\%$ and an initial interval length equal to 20 days. The two solid lines refer to different values of a and b, as indicated by the corresponding labels. This figure shows that both couples of a and b guarantee the target reliability value, and that a large value of b implies a high standard deviation of the maintenance interval estimator and a short transient of the observed reliability, as clearly demonstrated in [5].

B. Interval test method

The interval test method uses more than one calibration outcome for deciding, by means of a statistical test, if the chosen interval complies with the predetermined reliability target R^*. To this aim, a confidence interval $[R_L, R_U]$ that comprises the observed reliability, $R_{obs}(n)$, with a given significance level $(1 - \alpha)$, is determined. Since each time a calibration process is carried out there are only two possible outcomes, i.e. the instrument is in–tolerance or not, [1] suggests to consider $R_{obs}(n)$ as a binomially distributed random variable and to calculate its lower (R_L) and upper (R_U) significance limits by solving the following expressions with respect to R_L and R_U [6]:

$$\Pr\left\{R_{obs}(n) \leq \frac{g}{n}\right\} \triangleq \sum_{i=0}^{g} \frac{n!}{i!(n-i)!} R_U^i (1 - R_U)^{n-i} = \frac{\alpha}{2} \tag{5}$$

$$\Pr\left\{R_{obs}(n) \geq \frac{g}{n}\right\} \triangleq \sum_{i=g}^{n} \frac{n!}{i!(n-i)!} R_L^i (1 - R_L)^{n-i} = \frac{\alpha}{2} \tag{6}$$

where g is again the number of in–tolerances after n maintenance operations. Then, the calibration interval is changed only if R^* is not included in $[R_L, R_U]$. In particular, if $R^* < R_L$ or $R^* > R_U$, the next calibration interval is lengthened or shortened, respectively.

Published results report an interval change mechanism based on the so-called extrapolation and interpolation methods [1]. However, results obtained using available data have shown that ITM is sensitive to design parameters and that the application of such a criterion to data coming from atomic clocks induces a significant error on the estimated calibration interval. In fact, the ITM assumes an exponential reliability model for the available data, while the method presented in Sec. II validates the hypothesis of a Wiener process. For these reasons, the interval change mechanism employed in this paper is the SRM algorithm described in the previous subsection, whose application does not involve any restrictive hypothesis on the data reliability model.

The dash–dotted line in Fig. 3 shows the results obtained by applying the ITM to the available record of data with $\alpha = 5\%$, a=0.005 and b=0.1, and by using the same initial condition and target reliability used for the SRM. The figure shows that this method requires a high number of calibrations in order to achieve the desired reliability level and presents a longer transient time with respect to the SRM algorithm, thus reducing the effectiveness of the method. This is due to the particular application considered in this paper. In fact, each time a maintenance operation is needed, the index n is incremented by 1 and the index g may be incremented by 1 also. Since values of n and g employed in consecutive maintenance operations are close to each other, also intervals $[R_L, R_U]$ result to be similar. Accordingly, R^* is out of $[R_L, R_U]$ for many consecutive calibration operations, thus requiring many succeeding increments or decrements of the maintenance interval length. As a consequence, I_n converges to a specific value by employing a high number of calibrations which is not acceptable in practical cases. In fact, the transient of I_n shown in Fig. 3(a) ends within 500 calibrations, i.e. 20 years. Many simulations have shown that the a and b parameters values do not affect significantly the performance of the algorithm.

C. Modified simple response method

In order to improve the transient time of the observed reliability while reducing the variance of the estimated interval length, changes have been applied to the SRM method. In particular, since an increasing b reduces the transient period of $R_{obs}(n)$ and gives a large value of the variance of the estimated interval length, the modified SRM (MSRM) method proposes an interval change mechanism with the values of a and b varying during each maintenance operation. In particular, a high value of b is needed during the firsts calibrations, in order to achieve rapidly the target reliability value. Then, b is decreased for reducing the variance of the estimated interval length. For each value of b, the corresponding value of a is calculated by means of (4) for guaranteeing the target reliability value.

Fig. 3 shows with dashed lines results obtained by applying the MSRM to the record of available data. In particular, the value of b has been decreased from 0.80 to 0.20, and a has been evaluated for guaranteeing $R^* = 90\%$.

Estimates of the calibration interval obtained with parametric and non–parametric algorithms are equal to 19 and 15 days, respectively. Such a difference is due to the distribution of the I_n values estimated with the non–parametric methods, which has been verified to be non symmetric, thus implying a biased estimate of the interval.

IV. CONCLUSIONS

In this paper, the performance of four algorithms for the estimation of the optimal calibration interval of atomic clocks have been analyzed. The first one is based on the experimental verifica-

tion that the noise affecting such instruments can be modeled as a Wiener process corresponding to a white frequency noise and by using an appropriate stochastic model. Conversely, the SRM and the ITM methods estimate the next interval on the basis of the last calibration results. These methods usually require a lower number of measurements in order to evaluate the optimal calibration interval for a given reliability target. However, since atomic clocks are extremely accurate, the number of calibrations needed for obtaining an accurate estimate of the optimal interval result to be higher than the number of measures required by the parametric algorithm.

In order to reduce the transient time needed for the estimates, a new algorithm has been proposed in this paper. In particular, the interval change mechanism of the SRM method has been modified, on the basis of simulations and theoretical results [5], for guaranteeing a quick convergence and a low estimator variance.

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