

# QFT holography near the horizon of Schwarzschild-like spacetimes.

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**Abstract:** It is argued that free QFT can be defined on the event horizon of a Schwarzschild-like spacetime and that that theory is unitarily and algebraically equivalent to QFT in the bulk (near the horizon). Under that unitary equivalence the bulk hidden  $SL(2, \mathbb{R})$  symmetry found in a previous work becomes manifest on the event horizon, it being induced by a group of horizon diffeomorphisms. The class of generators of that group can be enlarged to include a full Virasoro algebra of fields defined on the event horizon. These generators have a quantum representation in QFT on the event horizon and thus in the bulk.

**1. Introduction.** A number of papers has been concerned with the issue of the statistical origin of black-hole entropy. Holographic principle [1, 2, 3] arose by the idea that gravity near the horizon should be described by a low dimensional theory with a higher dimensional group of symmetry. The correspondence between quantum field theories of different dimensions was conjectured by Maldacena [4] using the machinery of string theory: There is a correspondence between quantum field theory in a, asymptotically  $AdS$ ,  $d + 1$  dimensional spacetime (the “bulk”) and a conformal theory in a  $d$  dimensional manifold (the (conformal) “boundary” at spacelike infinity). Witten [5] described the above correspondence in terms of relations of observables of the two theories. Rehren proved rigorously some holographic results for free quantum fields in a  $AdS$  background, establishing a correspondence between bulk observables and boundary observables without employing string machinery [6, 7]. To explain the correspondence one should notice that the conformal group which acts in the  $d$ -dimensional  $AdS_{d+1}$  boundary can be realized as the group of the isometries of the  $AdS_{d+1}$  bulk. In this way, the bulk-boundary correspondence has a geometric nature. The boundary of a Schwarzschild spacetime (dropping the boundary at infinity) is the event horizon of the black hole. The AdS correspondence has been used directly in Minkowski spacetime for massless particle in [8] with the help of the optical metric. Is there any bulk-boundary correspondence in a manifold containing a Schwarzschild-like black hole? Two-dimensional Rindler spacetime embedded in Minkowski spacetime approximates the

nontrivial part of the spacetime structure near a bifurcate horizon as that of a Schwarzschild black hole embedded in Kruskal spacetime. In that context, we have argued in a recent work [9] that free quantum field theory in two-dimensional Rindler space presents a “hidden”  $SL(2, \mathbb{R})$  symmetry: The theory turns out to be invariant under a unitary representation of  $SL(2, \mathbb{R})$  but such a quantum symmetry cannot be induced by the geometric background.  $SL(2, \mathbb{R})$  is the group of symmetry of the zero-dimensional conformal field theory in the sense of [10], so, as for the case of AdS spacetime, it suggests the existence of a possible correspondence between quantum field theory in Rindler space and a conformal field theory defined on its event horizon. In this letter we illustrate the basic results that can be found in a forthcoming technical paper [11] where we have shown that it is possible to build up the wanted correspondence of a free quantum theory defined in the bulk and a quantum field theory defined on the event horizon of a two dimensional Rindler space. Other involved results are that the  $SL(2, \mathbb{R})$  symmetry reveals a clear geometric meaning if it is examined on the horizon and, in that context, a whole Virasoro algebra of symmetries arises.

Some overlap with our results is present in the literature. Guido, Longo, Roberts and Verch [12] discussed in some detail the extent to which an algebraic QFT on a spacetime with a bifurcate Killing horizon induces a conformal QFT on that bifurcate Killing horizon. Along a similar theme, Schroer and Wiesbrock [13] have studied the relationship between QFTs on horizons and QFTs on the ambient spacetime. They even use the term “hidden symmetry” a sense similar as we do here and we done in [9]. In related follow-up works by Schroer [14] and by Schroer and Fassarella [15] the relation to holography and diffeomorphism covariance is also discussed.

**2. Hidden  $SL(2, \mathbb{R})$  symmetry.** Consider a general Schwarzschild-like metric (namely a static black hole metric with bifurcate event horizon),  $ds_{\mathbf{S}}^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2d\Omega^2$ ,  $\Omega$  denoting angular coordinates. Near the horizon ( $r = r_h$ ), the nonangular part of the metric reduces to the metric of a two-dimensional Rindler wedge  $\mathbf{R}$ ,  $ds_{\mathbf{R}}^2 = -\kappa^2 y^2 dt^2 + dy^2$  with  $A'(r_h) = 2\kappa$ , and  $\kappa y^2 = 4(r - r_h)$ . Also dropping the angular coordinates, let us consider a free Klein-Gordon scalar field  $\phi$  with motion equation  $-\partial_t^2 \phi + \kappa^2 (y\partial_y y \partial_y - y^2 m^2) \phi = 0$ . To built up the one-particle Hilbert space referred to the quantization with respect to the Rindler Killing time  $t$ , any real solution  $\psi$  of the K-G equation must be decomposed in  $\partial_t$ -stationary modes as follows

$$\psi(t, y) = \int_0^{+\infty} \sum_{\alpha} \Phi_E^{(\alpha)}(t, y) \tilde{\psi}_+^{(\alpha)}(E) dE + c.c. \quad (1)$$

$E \in [0, +\infty) = \mathbb{R}^+$  is an element of the spectrum of the Rindler Hamiltonian  $H$  associated with  $\partial_t$  evolution. Concerning the index  $\alpha$  we distinguish between two cases: if  $m > 0$  there is a unique mode  $\Phi_E^{(\alpha)} = \Phi_E$  whose expression is  $\sqrt{2E \sinh(\pi E/\kappa)} / \sqrt{2\pi^2 \kappa E} e^{-iEt} K_{iE/\kappa}(my)$ . If  $m = 0$  there are two values of  $\alpha$ , corresponding to *ingoing* and *outgoing* modes,  $\Phi_E^{(in)/(out)}$  whose expression are  $e^{-iE(t \pm \ln(\kappa y)/\kappa)} / \sqrt{4\pi E}$ . If  $m > 0$  there is no energy degeneration and the one-particle Hilbert space  $\mathcal{H}$  generated by the positive frequency part of the decomposition above is isomorphic to  $L^2(\mathbb{R}^+, dE)$ . In the other case ( $m = 0$ ), twofold degeneracy implies that  $\mathcal{H} \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$ . Quantum field operators, acting in the symmetrized Fock space

$\mathcal{F}(\mathcal{H})$  and referred to the Rindler vacuum  $|0\rangle$  – that is  $|0\rangle_{in} \otimes |0\rangle_{out}$  if  $m = 0$  – read

$$\hat{\phi}(t, y) = \int_0^\infty \sum_\alpha \Phi_E^{(\alpha)}(t, y) a_{E\alpha} + \overline{\Phi_E^{(\alpha)}(t, y)} a_{E\alpha}^\dagger dE. \quad (2)$$

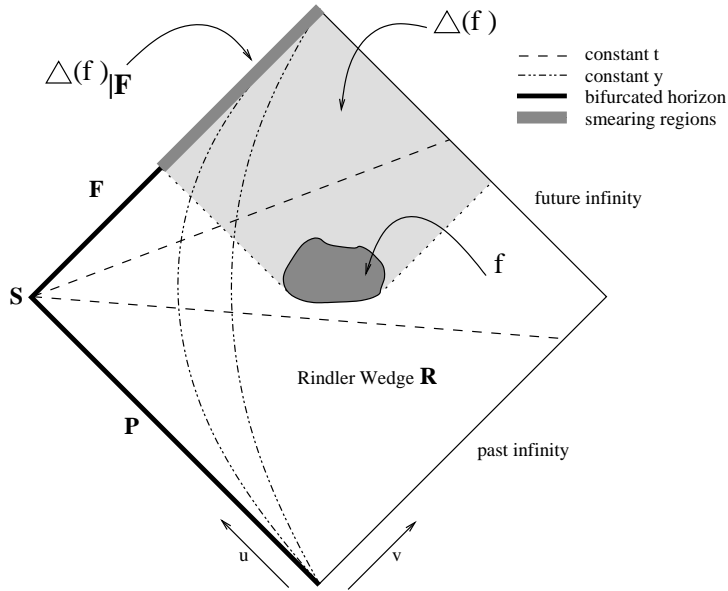
As usual, the causal propagator  $\Delta$  satisfies  $[\hat{\phi}(x), \hat{\phi}(x')] = -i\Delta(x, x')$ .

In [9] we have found that, if  $m > 0$ ,  $\mathcal{H}$  is irreducible under a unitary representation of  $SL(2, \mathbb{R})$  generated by (self-adjoint extensions) of the operators  $iH, iD, iC$  (which enjoy the commutation relations of the Lie algebra of  $SL(2, \mathbb{R})$ ), with

$$H := E, \quad D := -i \left( \frac{1}{2} + E \frac{d}{dE} \right), \quad C := -\frac{d}{dE} E \frac{d}{dE} + \frac{(k - \frac{1}{2})^2}{E}. \quad (3)$$

$k$  can arbitrarily be fixed in  $\{1/2, 1, 3/2, \dots\}$ . See [11] for details on domains and all that. If  $m = 0$  and so  $\mathcal{H} \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$ , an analogue representation exists in each space  $L^2(\mathbb{R}^+, dE)$ . Making use of Heisenberg representation it is simply proven that the algebra generated by  $H, D, C$ , with depending-on-time coefficients, is made of constant of motions [9, 11, 10]. Thus  $SL(2, \mathbb{R})$  is a symmetry of the one-particle system (that can straightforwardly be extended to the free quantum field in Fock space). The crucial point is that the found symmetry is *hidden*: it cannot be induced by the background geometry since the Killing fields of Rindler spacetime enjoy a different Lie algebra from that of  $H, D, C$ .

**3. Fields on the horizon.** Let us to investigate the nature of the found symmetry exactly on the event horizon assuming  $\mathbf{R}$  to be naturally embedded in a Minkowski spacetime. In particular we want to investigate its geometrical nature, if any, on the event horizon.



(Rindler) *light coordinates*  $u = t - \log(\kappa y)/\kappa$ ,  $v = t + \log(\kappa y)/\kappa$  (where  $u, v \in \mathbb{R}$ ) cover the (open) Rindler space  $\mathbf{R}$ . Separately,  $v$  is well defined on the future horizon  $\mathbf{F}$ ,  $u \rightarrow +\infty$ , and  $u$  is well defined on the past horizon  $\mathbf{P}$ ,  $v \rightarrow -\infty$  (see figure). Take the wavefunction in (1) and consider the limit on the future horizon  $u \rightarrow +\infty$ . That is equivalent to restrict the wavefunction on the event horizon when it is considered as a wavefunction in Minkowski spacetime, obtaining

$$\psi(v) = \int \frac{e^{-iEv}}{\sqrt{4\pi E}} e^{i\rho_{m,\kappa}(E)} \tilde{\psi}_+(E) dE + \text{c.c.} \quad (4)$$

$e^{i\rho_{m,\kappa}(E)}$  is a pure phase (see [11] for details). In coordinate  $u \in \mathbb{R}$ , the restriction of  $\psi$  to  $\mathbf{P}$  is similar with the  $v$  replaced for  $u$  and  $\rho_{m,\kappa}(E)$  replaced by  $-\rho_{m,\kappa}(E)$ . If  $m = 0$  the restrictions to  $\mathbf{F}$  and  $\mathbf{P}$  read respectively

$$\psi(v) = \int \frac{e^{-iEv}}{\sqrt{4\pi E}} \tilde{\psi}_+^{(in)}(E) dE + \text{c.c.}, \quad \psi(u) = \int \frac{e^{-iEu}}{\sqrt{4\pi E}} \tilde{\psi}_+^{(out)}(E) dE + \text{c.c.} \quad (5)$$

Discarding the phase it is possible to consider the following real “field on the future Horizon”:

$$\varphi(v) = \int_{\mathbb{R}^+} \frac{e^{-iEv}}{\sqrt{4\pi E}} \tilde{\varphi}_+(E) dE + \int_{\mathbb{R}^+} \frac{e^{+iEv}}{\sqrt{4\pi E}} \overline{\tilde{\psi}_+(E)} dE \quad (6)$$

as the basic object in defining a quantum field theory on the future event horizon. The same can be done for the past event horizon. The one-particle Hilbert space  $\mathcal{H}_{\mathbf{F}}$  is defined as the space generated by positive frequency parts  $\tilde{\psi}_+(E)$  and turns out to be isomorphic to  $L^2(\mathbb{R}^+, dE)$  once again. The field operator reads, on the symmetrized Fock space  $\mathcal{F}(\mathcal{H}_{\mathbf{F}})$  with vacuum  $|0\rangle_{\mathbf{F}}$ ,

$$\hat{\phi}_{\mathbf{F}}(v) = \int_0^\infty \frac{e^{-iEv}}{\sqrt{4\pi E}} a_E + \frac{e^{iEv}}{\sqrt{4\pi E}} a_E^\dagger dE. \quad (7)$$

The causal propagator  $\Delta_{\mathbf{F}}$  is defined by imposing  $[\hat{\phi}(v), \hat{\phi}(v')] = -i\Delta_{\mathbf{F}}(v, v')$  and it takes the form  $(1/4)\text{sign}(v - v')$ . In spite of the absence of any motion equation the essential features of free quantum field theory are preserved by that definition as proven in [11]. Degeneracy of the metric on the horizon prevents from smearing field operators by functions due to the ill-definiteness of the induced volume measure. However, employing the symplectic approach [16], a well-defined smearing-procedure is that of field operators and exact 1-forms  $\eta = df$  where  $f = f(v)$  vanishes fast as  $v \mapsto \pm\infty$ . The integration of forms does not need any measure. In other words for a real exact 1-form  $\eta$  as said above

$$\hat{\phi}_{\mathbf{F}}(\eta) = \int_0^\infty \frac{dE}{\sqrt{4\pi E}} \left( \int_{\mathbb{R}} e^{-iEv} \eta(v) \right) a_E + \left( \int_{\mathbb{R}} e^{iEv} \eta(v) \right) a_E^\dagger \quad (8)$$

is well defined and diffeomorphism invariant. In a suitable domain the map  $\eta(v) \mapsto \Delta_{\mathbf{F}}(\eta) = \frac{1}{4} \int_{\mathbb{R}} \text{sign}(v - v') \eta(v')$  defines a one-to-one correspondence between exact one-forms and

horizon wavefunctions of the form (1) and  $\eta = 2d\psi_\eta$ . Finally, similarly to usual quantum field theory [16], it holds

$$[\hat{\phi}_{\mathbf{F}}(\eta), \hat{\phi}_{\mathbf{F}}(\eta')] = -i\Delta_{\mathbf{F}}(\eta, \eta') = \int_{\mathbf{F}} \psi_{\eta'} d\psi_\eta - \psi_\eta d\psi_{\eta'}.$$

The last term define a diffeomorphism-invariant symplectic form on horizon wavefunctions.

**4. Unitary and algebraic holographic theorems.** It is possible to prove the existence of a unitary equivalence between the theory in the bulk and that on the horizon in the sense we are going to describe. Consider the case  $m > 0$  and the future horizon  $\mathbf{F}$ .

**Theorem 1.** *There is a unitary map  $U_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \rightarrow \mathcal{F}(\mathcal{H}_{\mathbf{F}})$  such that  $U_{\mathbf{F}}|0\rangle = |0\rangle_{\mathbf{F}}$  and  $U_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta)U_{\mathbf{F}} = \hat{\phi}(f)$  for any smooth compactly supported function  $f$  used to smear the bulk field,  $\eta = 2d(\Delta(f)|_{\mathbf{F}})$ . (See figure.)*

Details on the construction of  $U_{\mathbf{F}}$  are supplied in [11], here we give only the main idea. Take  $f$  as said and consider the associated bulk wavefunction  $\psi_f = \Delta(f)$ , restrict  $\psi_f$  to  $\mathbf{F}$  obtaining a horizon wavefunction as in (4) with positive frequency part  $e^{i\rho_{m,\kappa}(E)}\tilde{\psi}_{f+}(E)$ . Then define a horizon wavefunction  $\varphi_f$  as in (6) with  $\tilde{\varphi}_+$  replaced by  $\tilde{\psi}_{f+}$ . It is clear that the map  $\psi_f \mapsto \varphi_f$  corresponds to a unitary operator from  $\mathcal{H}$  to  $\mathcal{H}_{\mathbf{F}}$ . That is, by definition  $U_{\mathbf{F}}|_{\mathcal{H}}$ . Imposing  $U_{\mathbf{F}}|0\rangle = |0\rangle_{\mathbf{F}}$ , by taking tensor products of  $U_{\mathbf{F}}|_{\mathcal{H}}$ , this map extends to a unitary map  $U_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \rightarrow \mathcal{F}(\mathcal{H}_{\mathbf{F}})$ . Finally, by direct inspection one finds that, if  $\eta = 2d\varphi_f$ , one also has  $U_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta)U_{\mathbf{F}} = \hat{\phi}(f)$ .

The same procedure can be used to define an analogous unitary operator referred to  $\mathbf{P}$ . If  $m = 0$  two unitary operators arises. One is  $V_{\mathbf{F}} : \mathcal{F}(\mathcal{H}_{in}) \rightarrow \mathcal{F}(\mathcal{H}_{\mathbf{F}})$  such that  $V_{\mathbf{F}}|0\rangle_{in} = |0\rangle_{\mathbf{F}}$  and  $V_{\mathbf{F}}^{-1}\hat{\phi}_{\mathbf{F}}(\eta_f)V_{\mathbf{F}} = \hat{\phi}_{in}(f)$ .  $\mathcal{H}_{in}$  is the bulk Hilbert space associated with the ingoing modes and  $\hat{\phi}_{in}(f)$  is the part of bulk field operator built up using only ingoing modes. The other unitary operator  $V_{\mathbf{P}} : \mathcal{F}(\mathcal{H}_{out}) \rightarrow \mathcal{F}(\mathcal{H}_{\mathbf{P}})$  plays an analogous rôle with *in* replaced for *out* and  $\mathbf{F}$  replaced for  $\mathbf{P}$  everywhere. (More generally  $V_{\mathbf{P}} \otimes V_{\mathbf{F}} : \mathcal{F}(\mathcal{H}) \rightarrow \mathcal{F}(\mathcal{H}_{\mathbf{P}}) \otimes \mathcal{F}(\mathcal{H}_{\mathbf{F}})$  define a unitary operator which transforms the vacuum states into vacuum states and field operators into field operators.) As a consequence of the cited theorem, e.g. if  $m > 0$ , one has the invariance of vacuum expectation values:

$$\mathbf{F}\langle 0|\hat{\phi}_{\mathbf{F}}(\eta_1)\cdots\hat{\phi}_{\mathbf{F}}(\eta_n)|0\rangle_{\mathbf{F}} = \langle 0|\hat{\phi}(f_1)\cdots\hat{\phi}(f_n)|0\rangle.$$

Similarly to the extent in the bulk case, one focuses on the algebra  $\mathcal{A}_{\mathbf{F}}$  of linear combinations of product of field operators  $\hat{\phi}_{\mathbf{F}}(\omega)$  varying  $\omega$  in the space of allowed complex 1-forms. We assume that  $\mathcal{A}_{\mathbf{F}}$  also contain the unit operator  $I$ . The Hermitean elements of  $\mathcal{A}_{\mathbf{F}}$  are the natural observables associated with the horizon field. From an abstract point of view the found algebra is a unital  $*$ -algebra of formal operators  $\phi_{\mathbf{F}}(\eta)$  with the additional properties  $[\phi_{\mathbf{F}}(\eta), \phi_{\mathbf{F}}(\eta')] = -i\Delta_{\mathbf{F}}(\eta, \eta')$ ,  $\phi_{\mathbf{F}}(\eta)^* = \phi_{\mathbf{F}}(\bar{\eta})$  and linearity in the form  $\eta^1$ .  $\mathcal{A}_{\mathbf{F}}$  can be studied no matter any operator representation in any Fock space. Operator representations are obtained via GNS

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<sup>1</sup>The analogous algebra of operators in the bulk fulfill the further requirement  $\phi(f) = 0$  if (and only if)  $f = Kg$ ,  $K$  being the Klein-Gordon operator. No analogous requirement makes sense for  $\mathcal{A}_{\mathbf{F}}$  since there is no equation of motion on the horizon.

theorem once an algebraic state has been fixed [16]. In the case  $m > 0$  we get the following result which is independent from any choice of vacuum state and Fock representation. The proof can be found in [11].  $\mathcal{A}_{\mathbf{R}}$  denotes the unital  $*$ -algebra of associated with the bulk field operator.

**Theorem 2.** *There is a unique injective unital  $*$ -algebras homomorphism  $\chi_{\mathbf{F}} : \mathcal{A}_{\mathbf{R}} \rightarrow \mathcal{A}_{\mathbf{F}}$  such that  $\chi_{\mathbf{F}}(\phi(f)) = \phi_{\mathbf{F}}(\eta_f)$ , where  $\eta = 2d(\Delta(f)|_{\mathbf{F}})$ . Moreover in GNS representations in the respectively associated Fock spaces  $\mathcal{F}(\mathcal{H})$ ,  $\mathcal{F}(\mathcal{H}_{\mathbf{F}})$  built up over  $|0\rangle$  and  $|0\rangle_{\mathbf{F}}$  respectively,  $\chi_{\mathbf{F}}$  has a unitary implementation and reduces to  $U_{\mathbf{F}}$ .*

Notice that, in particular  $\chi_{\mathbf{F}}$  preserves the causal propagator, in the sense that it must be  $-i\Delta(f, g) = [\phi(f), \phi(g)]I = [\phi(f), \phi(g)]\chi_{\mathbf{F}}(I) = \chi_{\mathbf{F}}([\phi(f), \phi(g)]I) = [\chi(\phi(f)), \chi(\phi(g))]$   
 $= [\phi_{\mathbf{F}}(f), \phi_{\mathbf{F}}(g)] = -i\Delta_{\mathbf{F}}(\eta_f, \eta_g)$ .

Analogous algebraic homomorphism theorems can be given for  $\mathbf{P}$  and the massless case [11].

**5. The  $SL(2, \mathbb{R})$  symmetry becomes manifest on the horizon.** Consider quantum field theory on  $\mathbf{F}$ , but the same result holds concerning  $\mathbf{P}$ . In  $\mathcal{H}_{\mathbf{F}} \cong L^2(\mathbb{R}^+, dE)$  define operators  $H_{\mathbf{F}}, D_{\mathbf{F}}, C_{\mathbf{F}}$  as the right-hand side of the equation that respectively defines  $H, D, C$  in (3). Exactly as in the bulk case, operators  $iH_{\mathbf{F}}, iD_{\mathbf{F}}, iC_{\mathbf{F}}$  generate a unitary  $SL(2, \mathbb{R})$  representation  $\{\mathcal{U}_g\}_{g \in SL(2, \mathbb{R})}$ . Hence, varying  $g \in SL(2, \mathbb{R})$ ,  $U_g = (U_{\mathbf{F}}|_{\mathcal{H}})^{-1} \mathcal{U}_g U_{\mathbf{F}}|_{\mathcal{H}}$  define a representation of  $SL(2, \mathbb{R})$  for the system in the bulk. By construction  $(U_{\mathbf{F}}|_{\mathcal{H}})^{-1} H_{\mathbf{F}} U_{\mathbf{F}}|_{\mathcal{H}} = H$ . As a consequence every  $U_g$  turns out to be a  $SL(2, \mathbb{R})$  symmetry of the bulk system and the group of these symmetries is unitary equivalent to that generated by  $iH, iD, iC$ . In particular the one-parameter group associated with  $H_{\mathbf{F}}$  generates  $v$ -displacements of horizon wavefunctions which are equivalent, under the action of  $U_{\mathbf{F}}$ , to  $t$ -displacements of bulk wavefunctions. Now, it make sense to investigate the *geometrical nature* of the  $SL(2, \mathbb{R})$  representation  $\{\mathcal{U}_g\}$  that, as we said, induces, up to unitary equivalences, the original  $SL(2, \mathbb{R})$  symmetry in the bulk. In fact it is possible to prove that:

**Theorem 3.** *If  $k = 1$  in (3), the action of every  $\mathcal{U}_g$  on a state  $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$  is essentially equivalent to the action of a corresponding  $\mathbf{F}$ -diffeomorphism on the associated (by (6)) horizon wavefunction  $\varphi$ . More precisely, take a matrix  $g \in SL(2, \mathbb{R})$  and  $\varphi = \varphi(v)$  in a suitable space of horizon wavefunction (see [11]). Let  $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$  be the positive frequency part of  $\varphi$ . The wavefunction  $\varphi_g$  associated with  $\mathcal{U}_g \tilde{\varphi}_+$  reads*

$$\varphi_g(v) = \varphi\left(\frac{av+b}{cv+d}\right) - \varphi\left(\frac{b}{d}\right), \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \quad (9)$$

The term  $-\varphi(b/d)$  assures that  $\varphi_g$  vanishes as  $v \rightarrow \pm\infty$ . Notice that the added term disappears when referring to  $d\varphi$  rather than  $\varphi$ . The group of diffeomorphisms of  $\mathbf{F}$ , i.e. the real line<sup>2</sup>,

$$v \mapsto \frac{av+b}{cv+d}, \quad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{R}) \quad (10)$$

can be obtained by composition of one-parameter subgroups associated with the following three vector fields on  $\mathbf{F}$ :  $\partial_v, v\partial_v, v^2\partial_v$ . It is simply proven that the Lie brackets of those fields is a realization of the Lie algebra of  $SL(2, \mathbb{R})$ . Moreover, it turns out that [11]:

<sup>2</sup>Actually one has to consider the projective line  $\mathbf{F} \cup \{\infty\}$ .

**Theorem 4** (a) If  $k = 1$  in (3), the unitary one-parameter group generated by  $iH_{\mathbf{F}}$  is associated, through Theorem 3, to the one-group of  $\mathbf{F}$ -diffeomorphisms generated by  $\partial_v$ , (b) the unitary one-parameter group generated by  $iD_{\mathbf{F}}$  is associated to the one-parameter group of  $\mathbf{F}$ -diffeomorphisms generated by  $v\partial_v$  and (c) the unitary one-parameter group generated by  $iC_{\mathbf{F}}$  is associated to the one-group of  $\mathbf{F}$ -diffeomorphisms generated by  $v^2\partial_v$ .

**6. Appearance of Virasoro algebra.** The bulk  $SL(2, \mathbb{R})$ -symmetry is manifest when examined on the event horizon, in the sense that it is induced by the geometry. The Lie algebra generated by vector fields  $\partial_v, v\partial_v, v^2\partial_v$  play a crucial rôle in proving this fact. That algebra can be extended to include all the class of fields defined on the event horizon  $\{\mathcal{L}_n\}_{n \in \mathbb{Z}}$  with  $\mathcal{L}_n = v^{n+1}\partial_v$ . It is interesting to notice that these fields enjoy Virasoro commutation relations without central charge,  $\{\mathcal{L}_n, \mathcal{L}_m\} = (n - m)\mathcal{L}_{n+m}$ . A natural question arises:

*Is it possible to give a quantum representation of these generators in the sense of Theorem 4?*

At least formally, the answer is positive. Indeed, by employing Theorem 4 one finds out that the infinitesimal action of the one parameter group of diffeomorphisms generated by  $\mathcal{L}_n$  on a horizon wavefunction  $\varphi = \varphi(v)$  is equivalent to the action of an anti-Hermitian operator  $L_n$  on the positive frequency part  $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$ .  $L_n$  is defined as, respectively for  $n \geq -1$  and  $n < -1$ ,

$$(L_n \tilde{\varphi}_+)(E) := i^{n+2} \sqrt{E} \frac{d^{n+1}}{dE^{n+1}} \sqrt{E} \tilde{\varphi}_+(E), \quad (11)$$

$$(L_n \tilde{\varphi}_+)(E) := -i^{-(n+2)} \sqrt{E} \int_0^E dE_1 \int_0^{E_1} dE_2 \cdots \int_0^{E_{-(n+2)}} dE_{-(n+1)} \sqrt{E_{-(n+1)}} \tilde{\varphi}_+(E_{-(n+1)}). \quad (12)$$

Those operators are at least anti-Hermitian on suitable domains and enjoy Virasoro commutation rules  $[L_n, L_m] = (n - m) L_{n+m}$ .

**7. Four dimensional case.** Up to now we have investigated only the two dimensional spacetimes, but it is possible to extend our results to a four dimensional case which better approximates the Schwarzschild extent. For this purpose consider the near-horizon approximation of a Schwarzschild-like spacetime without discarding the angular variables  $\theta, \phi$ , so that  $ds^2 = -\kappa^2 y^2 dt^2 + dy^2 + r_h^2 d\Omega^2$ . Every field takes an angular part described by the usual spherical harmonics  $Y_m^l(\theta, \phi)$ . QFT in the bulk involves the one-particle Hilbert space  $\oplus_{l=0}^{\infty} \mathbb{C}^{2l+1} \otimes \mathcal{H}_l$  with  $\mathcal{H}_l \cong L^2(\mathbb{R}^+, dE)$  if  $l > 0$ ,  $\mathbb{C}^{2l+1}$  being the space at fixed total angular momentum  $l$  and  $\mathcal{H}_0 \cong L^2(\mathbb{R}^+, dE)$  in the massive case but  $\mathcal{H}_0 \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$  in the massless case. For wavefunctions with components in a fixed space  $\mathbb{C}^{2l+1} \otimes L^2(\mathbb{R}^+, dE)$  Klein-Gordon equation reduces to the two-dimensional one with a positive contribution to the mass depending on  $l$ . Quantum field theory can be constructed on the future horizon  $\mathbf{F} \cong \mathbb{S}^2 \times \mathbb{R}$ . The appropriate causal propagator reads

$$\Delta_{\mathbf{F}}(x, x') = (1/4) \text{sign}(v - v') \delta(\theta - \theta') \delta(\phi - \phi') \sqrt{g_{\mathbb{S}^2}(\theta, \phi)}.$$

The horizon field operator  $\hat{\phi}_{\mathbf{F}}$  has to be smeared with 3-forms as  $df(v, \theta, \phi) \wedge d\theta \wedge d\phi$  and Theorems 1 and 2, at least in the massive case, can be restated as they stand for the two-dimensional

case. Theorems 3 and 4 hold true at fixed angular variables.

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