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EXACT DESIGN SOLUTIONS FOR PHOTODIODE TRANSIMPEDANCE AMPLIFIERS BASED ON FET INPUT OP-AMPS

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Exact design solutions for photodiode transimpedance amplifiers based on FET input OP-AMPs.

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Abstract. *We calculate the transfer function of an optical receiver composed by a photodiode and a FET input operational amplifier as a current to voltage converter. According to the theory of low-pass filters, the receiver bandwidth and quality factor are here analytically evaluated, proposing design solutions and their related sensitivity to eventual parameter fluctuations. We exemplify the combined action of photodetection and filtering comparing the analytical expression with full numerical SPICE simulations.*

Keywords: *photodiodes, transimpedance amplifiers, optical receivers*.

INTRODUCTION.

Recently, the design and optimization of photo-receivers have attracted a great interest, owing to the rising demand for fiber communication networks [1]. Optical receivers usually couple a photodetection stage, where substantially the optical signal is converted into an electrical quantity, followed by an electrical filters, whose specific scope is the suppression of noise components. Filters are in general supposed to match the Heaviside distortion-free conditions, that is flat gain and linearly varying phase within the spectral bandwidth of interest. For very high bit-rates, a fourth-order electrical filter is usually required, with an half-width bandwidth of 2/3 of the bit repetition frequency [2].

Photodiodes represent very effective converters of optical signals into electric currents, with an extremely high quantum efficiency, that usually exceeds 80% [3]. The basic detection scheme for photocurrents, that are generated by such semiconductor diode junctions, should be based on very low input impedance, in order to avoid the self polarization and eventual saturation of the photodiode itself [3]. Alternatively, an inverse bias voltage can be applied to deplete the junction; however, with this second scheme, the junction inverse current may become a source of additional shot noise, leading to a reduced performance.

An additional problem is represented by bandwidth limitations, due to the unavoidable presence of the photodiode capacitance combined with other parasitic effects. In fact, the photodiode response to an optical pulse may be described as a conversion of N photons to about N*α electron-hole

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couples, being α the aforementioned quantum efficiency of the photodiode. Consequently, the photodiode capacitance C_P, that have been charged by a photocurrent up to a charge level Q, exhibits a voltage difference $V=Q/C_P$.
The transition from the optical to the electrical domain is completed, once the charge Q, that was previously stored in C_P , drops into a current-to-voltage (I/V) converter, with an input resistance R. Therefore, the lower the RC_P product results, the higher will be the signal processing speed.
For the highest speed applications, the I/V converter could be a simple resistor. The principal advantage of this choice is represented by a direct and simple relation between the frequency response and the system parameters R, C_P. However, for slowly varying signals, the conversion efficiency is mainly driven by R , with low output voltages for low values of R.

Differently, I/V active converters based on operational amplifiers (OP AMPs) can offer simultaneously high transimpedance and low input impedance, with suitable setups for photodiode applications; at the same time, the system exhibits a more complex and rich behavior [4].
In this work we calculate the input impedance and the associated transimpedance for an I/V converter composed of an operational amplifier with a Field-Effect-Transistor (FET) input stage, where the real part of its input impedance can be legitimately neglected. In a practical standpoint, FET based amplifiers exhibits lower noise figures than BJT [5]; a further additional benefit of FET stage is represented by its low input bias current [6], which in turn reflects in a low output voltage offset. We propose design solutions and numerical examples embedding a sensitivity analysis of the transfer function with respect to small fluctuations of the circuit parameters. In detail we show that the sensitivity of the quality factor Q with respect to both the OP-AMP gain and time constant may be drastically reduced satisfying, at the same time, the system requirements with commercially available devices.

THE BASIC CIRCUIT.

Figure 1 shows a basic and well known photoreceiver schematic [3]. The photodiode is here represented by a current generator I_{In} coupled with a capacitance C_P. The photocurrent is then converted into a voltage level by means of an operational amplifier with a feedback network.

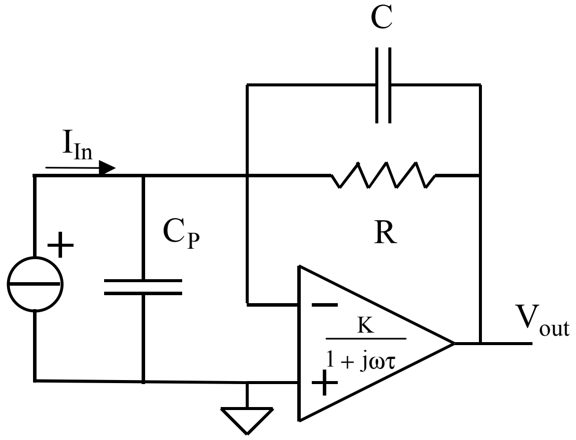

Figure 1. The equivalent circuit of the photoreceiver with a transimpedance amplifier.

A capacitance C is used to control both the system bandwidth and stability.
The current generator I_{In} stands for a photocurrent generated by an impinging light, while C_P represents the combined effect of the photodiode capacitance, the capacitance of the FET input OP-AMP and additional stray capacitances, including the eventual capacitance of the coaxial cable, whenever the cable length can be assumed short with respect to the electric signal wavelength.

CALCULATION OF THE INPUT IMPEDANCE OF THE TRANSIMPEDANCE PREAMPLIFIER.

For ease of discussion, we first calculate the input impedance, without C_P, of the following circuit:

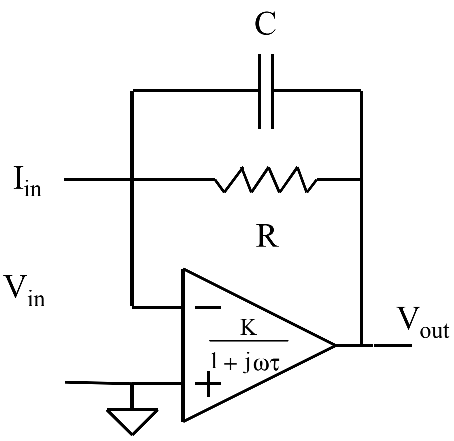

Figure 2. The simplified transimpedance amplifier.

The operational amplifier is here represented merely by a voltage gain K and a time constant τ, under the dominant pole approximation.
The input impedance of the transimpedance amplifier (TIA) can be computed easily:

$$Z_{inb} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{\dfrac{(V_{in} - V_{out})}{\left(\dfrac{R}{1+j\omega RC}\right)}} = \frac{R}{K+1} \frac{1+j\omega\tau}{\left(1+j\omega\dfrac{\tau}{K+1}\right)(1+j\omega RC)} \qquad 1)$$

We now connect in parallel to Z_{inb} the capacitance C_P. The overall impedance Z_{in} reads as:

$$Z_{in} = \frac{R}{K+1} \frac{1+j\omega\tau}{1+j\omega\left(\dfrac{\tau + R\,C_P}{K+1} + RC\right) - \omega^2\left(\dfrac{\tau R(C+C_P)}{K+1}\right)} \qquad 2)$$

For low values of frequency ω, the input impedance of the TIA reduces to R/(K+1) (see eq. 2); therefore, the eventual presence of a constant photo-current, which may lead to a potential saturation of the photodiode, is here processed by a very low impedance.

CALCULATION OF THE TRANSFER FUNCTION OF THE TRANSIMPEDANCE PREAMPLIFIER.

The Current to Voltage transfer function of the TIA can be completed taking into account the open loop transfer function of the OP-AMP. Consequently

$$V_{out} = -\frac{I_{in} Z_{in} K}{1 + j\omega\tau} = -I_{in} R \frac{K}{K+1} \frac{1}{1 + j\omega\left(\frac{\tau + R C_P}{K+1} + RC\right) - \omega^2\left(\frac{\tau R(C + C_P)}{K+1}\right)} = -I_{in} Z_T \qquad 3)$$

where the denominator in eq. 3 has the following structure:

$$(1 + j\omega\tau_1)(1 + j\omega\tau_2) = 1 + j\omega(\tau_1 + \tau_2) - \omega^2\tau_1\tau_2 \qquad 4)$$

With real time constants τ_1 and τ_2, Eq.3 represents a cascade of two first-order low-pass filters with corner frequencies $f_1=1/(2\pi\tau_1)$ and $f_2=1/(2\pi\tau_2)$. The two roots for the second order polynomial in eq.3 (see denominator) are:

$$\tau_{1,2} = \frac{\tau + R(C_P + C(K+1)) \pm \sqrt{(\tau + R(C_P + C(K+1)))^2 - 4(1+K)(C_P + C)R\tau}}{2(K+1)} \qquad 5)$$

for real time constants the discriminant in eq.5 satisfies :

$$(\tau + R(C_P + C(K+1)))^2 \geq 4(1+K)(C_P + C)R\tau \qquad 6)$$

A dominant pole solution can be found when the left hand member of the above inequality is much higher than the right hand one; the approximate solutions for the two time constants are :

$$\tau_1 \approx \frac{\tau + R(C_P + C(K+1))}{K+1}; \quad \tau_2 \approx 0 \qquad 7)$$

With the further constrains $C(K+1) >> C_P$ and $RC(K+1) >> \tau$, we may write $\tau_1 \approx RC$.

More generally, the representation for a second order low pass filter transfer function as:

$$W(j\omega)=\frac{1}{1+\frac{j}{\omega_0 Q}\omega-\frac{\omega^2}{\omega_0^2}} \qquad 8)$$

may be applied to the TIA transfer function; specifically we obtain the central frequency:

$$\omega_0^2=\frac{K+1}{\tau R(C+C_P)} \qquad 9)$$

that substantially qualify the low-pass filter bandwidth ; similarly from

$$Q\omega_0=\frac{1}{\left(RC+\frac{\tau+RC_P}{K+1}\right)} \qquad 10)$$

the factor of merit Q can be easily exploited in terms of the circuit parameters in the following form:

$$Q^2=\frac{\tau R(C+C_P)(K+1)}{(RC(K+1)+\tau+RC_P)^2} \qquad 11)$$

An almost flat and wide band response, under a low-passing filter shape, can be achieved by setting Q=1. Different values of Q can be assumed whenever the TIA plays the role of a first stage for a more complex filter of high order [4].

One can even solve eqs. 9, 11 putting into evidence the feedback parameters R, C so that

$$C=\frac{C_P\left[(1+K)\tau\omega_0-Q\left(1+K+\tau^2\omega_0^2\right)\right]}{Q\left[(1+K)^2+\tau^2\omega_0^2\right]-(1+K)\tau\omega_0} \qquad 12)$$

$$R=\frac{Q\left[(1+K)^2+\tau^2\omega_0^2\right]-(1+K)\tau\omega_0}{C_P K Q\tau\omega_0^2} \qquad 13)$$

We observe that the feedback capacitance is directly proportional to C_P , whereas the feedback resistance is inversely proportional to the same capacitance C_P. From Eq. 12, 13, the low frequency transimpedance is now determined as $Z_T(\omega=0)= Z_0 =RK/(K+1) \approx R$ as shown by Eq. 3.
Let us apply now the analytical approach to the design of a TIA with a predetermined low frequency transimpedance. For this application a possible excess of bandwidth could be controlled by cascading a low pass filter to the TIA. Therefore using Eq. 11, with R as a given free parameter, and ω_0 defined by Eq. 9, after straightforward calculations one can easily find out the feedback capacitance C that verifies, for Q=1, the following closed form relation:

$$\tau R(C+C_P)(K+1)=(RC(K+1)+\tau+RC_P)^2 \qquad 14)$$

After some algebra we have:

$$C_{1,2} = \frac{-2RC_P - \tau \pm \sqrt{4K\tau RC_P - 3\tau^2}}{2R(K+1)} \tag{15}$$

A real and positive value for C imposes the following constraint to the system parameters

$$\left(4K\tau RC_P - 3\tau^2\right) \geq \left(2RC_P + \tau\right)^2 \tag{16}$$

that gives:

$$K \geq \frac{RC_P}{\tau} + \frac{\tau}{RC_P} + 1 \tag{17}$$

More generally, the solutions for the feedback capacitance C, under the hypothesis of a given Q is

$$C_{1,2} = \frac{\tau - 2Q^2\left(RC_P + \tau\right) \pm \sqrt{4\tau Q^2\left(KRC_P - \tau\right) + \tau^2}}{2RQ^2(K+1)} \tag{18}$$

and for obtaining at least one positive solution for C we must satisfy :

$$4\tau Q^2\left(KRC_P - \tau\right) + \tau^2 \geq \left(\tau - 2Q^2\left(RC_P + \tau\right)\right)^2 \tag{19}$$

In practical situations however, the capacitance C includes also the parasitic effects in R; this fact in turn translates into a more selective constraint for $C > C_0 > 0$ (instead of $C > 0$).
Similarly equations 9 and 11. may be solved under the additional requirement of a given DC transimpedance: $Z_0 = R\,K/(K+1)$.
The feedback resistance R, the feedback capacitance C and the photodiode capacitance C_P can be calculated in terms of filter requirements (ω_0, Q) , OP-AMP gain K and dominant pole time constant τ. Photodiodes with a junction capacitance lower or equal than the obtained result must be chosen, and the exact value required by the solution can be obtained with a parallel capacitor. Consequently we have:

$$C_P = \frac{1}{Z_0}\left(\frac{\tau}{1+K} + \frac{1+K}{\tau\omega_0^2} - \frac{1}{Q\omega_0}\right) \tag{20}$$

$$C = \frac{1}{Z_0}\left(\frac{1}{Q\omega_0} - \frac{\tau}{1+K} - \frac{1}{\tau\omega_0^2}\right) \tag{21}$$

$$R = Z_0\left(1 + \frac{1}{K}\right) \tag{22}$$

SENSITIVITY ANALYSIS

For the optimisation of photoreceiver electronics, the low pass filter intrinsically associated to a TIA could be a stage of a N-pole Bessel [2,4] filter usually required for the minimisation of error probability in digital data links.
The sensitivities of Q and ω_0 to K and τ of the operational amplifier are now evaluated, being these last two parameters potentially affected by a non negligible manufacturing spread.
With the expression of Q^2 of Eq. 11 we have:

$$\frac{\partial Q^2}{\partial K}=-\frac{R\tau(C+C_P)[RC(1+K)-\tau-RC_P]}{[R(C+C_P+CK)+\tau]^3}=2Q\frac{\partial Q}{\partial K} \qquad 23)$$

$$\frac{\partial Q^2}{\partial \tau}=\frac{R(C+C_P)(1+K)[R(C+C_P+CK)-\tau]}{[R(C+C_P+CK)+\tau]^3}=2Q\frac{\partial Q}{\partial \tau} \qquad 24)$$

similarly, dealing with the expression of ω_0^2 in Eq. 9 we have:

$$\frac{\partial \omega_0^2}{\partial K}=\frac{1}{R\tau(C+C_P)}=2\omega_0\frac{\partial \omega_0}{\partial K} \qquad 25)$$

$$\frac{\partial \omega_0^2}{\partial \tau}=-\frac{1+K}{R\tau^2(C+C_P)}=2\omega_0\frac{\partial \omega_0}{\partial \tau} \qquad 26)$$

more strikingly, observe that the sensitivities of Q to K and τ , which are stipulated by eqs. 23-24, can be reduced to zero if $RC(1+K)-\tau-RC_P=0$ or $R(C+C_P+CK)-\tau=0$ respectively.
The consistent set of equations represented by Eqs. 9, 10, 22, admit the presence of a further constraint. Indeed one can optimize the circuit parameters C, R, C_P, for a given values of Q, ω_0, Z_0, being, at the same time, one of the two sensitivities for Q (with respect to τ or K fluctuations) equal to zero. The additional requirement may set the OP-AMP time constant to a specific value. Differently one can solve the problem with respect to K, but in a practical standpoint, the time constant τ can be easily managed by adding a further capacitance. Solving with respect to C, R, C_P, τ and imposing $\partial Q/\partial\tau=0$

$$R=Z_0\left(1+\frac{1}{K}\right) \qquad 27)$$

$$C=\frac{1+K-4Q^2}{2\omega_0 Q Z_0(K+1)} \qquad 28)$$

$$\tau=\frac{K+1}{2\omega_0 Q} \qquad 29)$$

$$C_P = \frac{-1+4Q^2}{2\omega_0 Q Z_0} \tag{30}$$

Similarly, if one desires that the system setup is not sensitive to weak fluctuation of the OP-AMP gain K, setting $\partial Q / \partial K = 0$

$$R = Z_0\left(1+\frac{1}{K}\right) \tag{31}$$

$$C = \frac{K}{2Q\omega_0 Z_0 (K+1)} \tag{32}$$

$$\tau = \frac{2+K \pm \sqrt{(2+K)^2 - 16Q^2(K+1)}}{4\omega_0 Q} \tag{33}$$

$$C_P = \frac{K[K \mp \sqrt{(2+K)^2 - 16Q^2(K+1)}]}{4(K+1)QZ_0\omega_0} \tag{34}$$

And again, one has to select those solutions that satisfy the physical meaning. Incidentally we observe that for large values of OP-AMP gain K (that is the case of some practical situations) the two capacities read as (considering the solution not divergent for C_P):

$$C(K \to \infty) = \frac{1}{2Q\omega_0 Z_0} \quad ; \quad C_P(K \to \infty) = \frac{-1+4Q^2}{2Q\omega_0 Z_0} \tag{35}$$

Where, for the specific choice of Q^2=1/2, and in the limit of $K \to \infty$, we have $C = C_P = 1/(\sqrt{2}\omega_0 Z_0)$

DESIGN EXAMPLES AND SPICE SIMULATIONS.

In order to exemplify the proposed method, we compare the analytical transfer function of a simple photoreceiver, with full SPICE simulations [8].
We employ the components of the "eval" library in our scheme of fig. 3.
The FET input operational amplifier is a LF411 with an open loop gain of 400000, a dominant pole time constant of 7.95 ms and an input capacitance of 14 pF.
The photodiode is simulated with a silicon diode IN4002 and an AC current generator. The zero-voltage junction capacitance of the diode is 51 pF.
We choose a feedback resistance R of 1 MΩ that, according to Eq. 3. rules the in-band transimpedance.

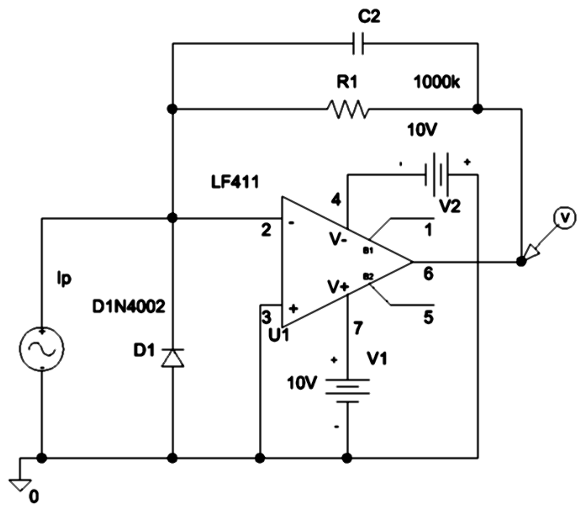

Figure 3: Photoreceiver schematics

Q has been assumed of $1/\sqrt{2}$ in order to obtain a two pole Butterworth low-pass filter transfer function [4], and from Eq. 18, a feedback capacitance of 16.1 pF results.

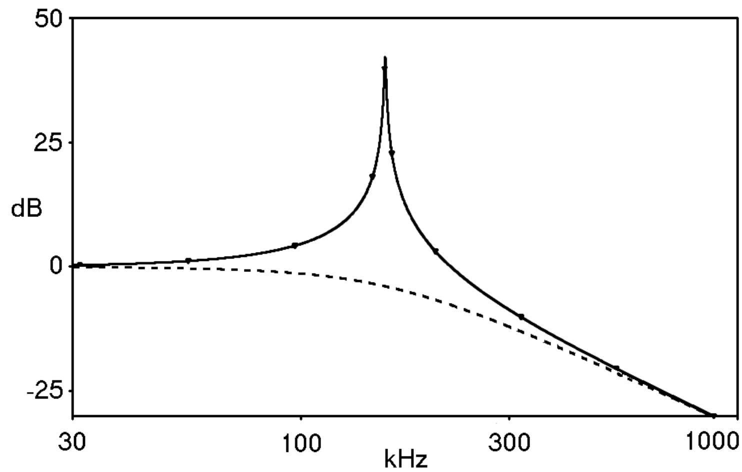

Figure 4. Frequency response of the simulated photoreceiver with no feedback capacitance (solid curve) and with 16.1 pF feedback capacitance (dashed curve). The amplitude is shown in dB, with respect to the DC transimpedance.

We exemplify in fig. 4 the frequency response of the photoreceiver using a SPICE simulation, setting C=0 (solid curve) and setting C=16.1pF (dashed curve). Observe the effect of gain peaking in absence of a capacitive feedback network; using eq.11 we can estimate Q=56.7.
Using Eq. 9 for the evaluation of the –3dB frequency, we find a value of 138.3 kHz, that can be compared to 138.6 kHz obtained from the simulation.
From eqs. 23-26, the sensitivity analysis for the present example yields :

$$\frac{\partial Q^2}{\partial K} = -1.22 \cdot 10^{-6}; \quad \frac{\partial Q^2}{\partial \tau} = 61.4\, s^{-1}; \quad \frac{\partial \omega_0^2}{\partial K} = 1.89 \cdot 10^6\, s^{-2}; \quad \frac{\partial \omega_0^2}{\partial \tau} = -9.5 \cdot 10^{13}\, s^{-3} \qquad 36)$$

Our example is also characterized by $\omega_0^2 = 7.55 \cdot 10^{11}\ s^{-2}$, $Q^2 = .5$, $K = 4 \cdot 10^5$, and $\tau = 7.95 \cdot 10^{-3}\ s$. Therefore, applying Eqs. 23-26, the relative deviation of Q and ω_0 with respect to a fluctuation of K and τ read, for the present case, as:

$$\frac{\Delta Q / Q}{\Delta K / K} = -.49; \quad \frac{\Delta Q / Q}{\Delta \tau / \tau} = .49; \quad \frac{\Delta \omega_0 / \omega_0}{\Delta K / K} = .5; \quad \frac{\Delta \omega_0 / \omega_0}{\Delta \tau / \tau} = -.5 \qquad 37)$$

We present a further numerical example of application for eqs. 31-34, choosing $\omega_0 = 300000\ s^{-1}$, and with $Z_0 = 100\ k\Omega$. The sensitivity of Q factor with respect to weak fluctuations of K ($\partial Q / \partial K$) can be set to zero by increasing the dominant pole time constant from $\tau = 7.95 \cdot 10^{-3}$ s to 0.943 s. The photodiode and feedback capacitances now read as 23.6pF. The present parameter choice gives also a very low value for $\partial Q / \partial \tau$, whereas the sensitivities of ω_0 to K and τ remains almost unchanged.

CONCLUSIONS.

In the present work, we have analytically calculated the transfer function and input impedance for a photodiode amplifier based on FET input OP-AMPS. The explicit expressions, that have been put into the context of two-pole low-pass filters, enable for satisfying easily different design requirements, under the constraints of different values for the circuit components. Additionally, we have linearized the governing equations carrying out a sensitivity analysis of filter performances with respect to the OP-AMP parameter fluctuations. We confirmed our analytical predictions with full numerical simulations of the circuit response using SPICE.

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