Fast genetic algorithm for roundness evaluation by the minimum zone tolerance (MZT) method

M. Antonetti M. Barloscio A. Rossi M. Lanzetta

Department of Mechanical, Nuclear and Production Engineering
University of Pisa, Via Bonanno Pisano 25B, 56126 Pisa, Italy
Tel.: +39 050 2218122; fax: +39 050 2218140.

Abstract

The main goal of the minimum zone tolerance (MZT) method is to achieve the best estimation of the roundness error, but it is computationally intensive. This paper describes the application of a genetic algorithm (GA) to minimize the computation time in the evaluation of CMM roundness errors of a large cloud of sampled datapoints (0.2° equally spaced datapoints).
Computational experiments have shown that by selecting the optimal GA parameters, namely a combination of the four genetic parameters related to population size, crossover, mutation, and stop conditions, the computation time can be reduced by up to one order of magnitude, allowing real-time operation.
Optimization has been tested using seven CMM datasets, obtained from different machining features, and compared with the LSQ method.
The performance of the optimized algorithm has been validated with GA from the literature using four benchmark datasets.

Keywords: Minimum zone tolerance (MZT), roundness error, genetic algorithm, CMM

1. Introduction

According to ISO 1101 [1], "A geometrical tolerance applied to a feature defines the tolerance zone within which that feature shall be contained".
Roundness is one of the main functional product features in machining and assembly when handling, positioning and mating are involved.
The most used algorithms to determine form errors, particularly roundness errors, are:

- least-squares method (LSQ),
- maximum inscribed circle (MIC),
- minimum circumscribed circle (MCC), and
- minimum zone tolerance (MZT).

With the LSQ method, a circle is fitted to the profile using the least-squares method. The center of that circle is used to fit the smallest circumscribed and the largest inscribed circles to the profile. The radial separation between these two circles is the roundness error. The LSQ method [2] is the most popular method for evaluating the roundness error. This method has been widely used in CMMs in view of the simplicity of its application.
With the MIC and MCC methods [3], the largest inscribed and the smallest ciscumscribed circles that contain the profile are respectively found. Next the centers of these circles are used to find,

respectively, the smallest circumscribed and the largest inscribed circles. The radial separation between the two concentric circles (*substitute feature*) is the roundness error.
Unfortunately the error evaluation with the above methods is not the optimum, resulting in a possible overestimation. Therefore, although CMMs algorithms successfully reject defective parts, they may also reject parts that satisfy the design tolerances. For this reason, alternative methods able to achieve a better roundness estimate may produce economical benefits.
The MZT method consists of defining two concentric circles with minimum radial separation that contain the sampled profile. The radial separation between these two circles is the roundness error. By definition, the MZT method produces optimal as well as correct solutions, because the substitute feature has the minimum radial separation. Hence, it detects fewer out of tolerance parts compared to LSQ, MIC and MCC. ISO/TS 12181-2 [4] suggests that the MZT criterion should be applied to evaluate roundness. The drawback of the techniques based on the MZT method is that it requires the solution of a non-linear problem. The complexity and consequently the computation time of MZT algorithms is very sensitive to the number of sampled datapoints. Due to the problem complexity increase with the dataset size, suitable optimization algorithms are required.
In previous works, one of the authors developed a fast algorithms for small datasets to assess the kind of waviness deviation on the profile in order to detect critical points such as peaks and valleys [5] [6].
Among the fastest form error evaluation methods to implement, already available in the literature based on the MZT, are the steepest descent algorithm, which has been applied for roundness evaluation [7], and a two-dimensional simplex search method to evaluate several form features, including roundness [8]. However they do not guarantee a global optimal solution for non-convex problems.
Xiong [9] developed a general mathematical theory, a model and an algorithm for different kinds of profiles, including roundness where the linear programming method and exchange algorithm are used. As limaçon approximation is used to represent the circle, optimality of the solution is however not guaranteed.
A strategy based on geometric representation for minimum zone evaluation of circles and cylinders was proposed by Lai and Chen [10]. The strategy employs a non-linear transformation to convert a circle into a line and then uses a straightness evaluation schema to obtain minimum zone deviations for the feature concerned. This is an approximation strategy to minimum zone circles.
M. Wang et al. [11] presented a generalized non-linear optimization procedure based on the developed necessary and sufficient conditions to evaluate roundness error.
Y. Wang [12] proposed a general-purpose algorithm for constrained non-linear optimization problems for minimum zone evaluation of form tolerances. He used a technique similar to the sequential quadratic programming method, verifying that an optimal solution to the quadratic programming problem can be obtained in a finite number of iterations.
Samuel and Shunmugam [13] established a minimum zone limaçon based on computational geometry to evaluate roundness error; with geometric methods, exact global optimal solutions are found by exhaustively checking every local minimum candidate.
Other researchers like Chang and Lin [14] have used the Monte Carlo simulation method to evaluate roundness errors.
Another approach is based on the Voronoi diagram as described by Roy and Zhang [15]; the method yields a very accurate measurement of the roundness error, but it is computationally intensive.
As for genetic algorithms (GAs) to find the solution of the MZT problem, as proposed in this paper, Sharma et al. [16] used a standard GA for the evaluation of multiple form tolerance classes such as straightness, flatness, roundness, and cylindricity. There was no need to optimize the algorithm performance, choosing the parameters involved in the computation, because of the small dataset size (up to 100 datapoints).

Wen et al. [17] implemented a GA in real-code, with only crossover and reproduction operators applied to the population. Thus in this case mutation operators were not used. The algorithm proposed is robust and effective, but it has only been applied to small datasets.
Another metaheuristic method to approach this problem is swarm particle optimization (SPO) [18].
MZT, despite its complexity, is a potential replacement of traditional methods in industry for the mentioned economical benefits. However for on-line inspection the computation time becomes a key factor. So not only the roundness error needs to be minimized, but also the computation time is a performance index.
Metaheuristic methods are faster with large cloud of sampled datapoints compared to those cited above, consequently they are suitable in manufacturing systems where the total computation time is an index of the system performance for on-line evaluation.
GAs differ considerably in conception from other search methods. The basic difference is that while other methods always process single points in the search space, GAs maintain a population of potential solutions, and so perform a multi-directional search.
The proposed GA has been tested on larger datasets than available in the literature (1800 sampled datapoints).
Larger datasets available by CMM scanning techniques allow reducing the form error estimation but they would require optimal GA parameters for faster convergence. On the other hand, larger datasets allow an effective tuning in order to minimize the computation time, because the GA parameters optimized on larger datasets may provide useful information about the behavior of GAs with few hundreds or less datapoints, like in standard CMM applications.
The optimal GA parameters have been experimentally evaluated on seven datasets and the performance validated on four datasets from the literature.

2. The MZT problem

In the MZT problem, the unknown are the (x,y) coordinates of the center (MZC) of the inner and outer (concentric) circles with the minimum radial separation containing all the sampled points (Figure 1). The two concentric circles are the substitute feature of the inspected profile and the difference between the inner and the outer circle is the minimum zone error (*MZE*).

[Figure 1 about here.]
Figure 1: Profile of sample n. 1 (Table 2) with 1800 equally spaced CMM datapoints and substitute features: minimum, medium, and maximum circles for current (x,y).

Given a profile $r(x,y,\theta)$, with $\theta \in (0, 2\pi]$, of a section perpendicular to the axis of a cylindrical feature, the roundness error $R(x,y)$ is defined by:

$$R(x,y) = MCC(x,y) - MIC(x,y)\,,\ (x,y) \in E_{r(x,y,\theta)} \qquad (1)$$

where $MCC(x,y)$ and $MIC(x,y)$ are the radii of the substitute feature of centre (x,y), and $E_{r(x,y,\theta)}$ is the area enclosed by $r(x,y,\theta)$:

$$MCC(x,y) = \max_{\theta\in(0,2\pi]} r(x,y,\theta) \qquad (2)$$

$$MIC(x,y) = \min_{\theta\in(0,2\pi]} r(x,y,\theta) \qquad (3)$$

As a CMM samples a finite number, n, of equally-spaced points $\theta_i = i\times\frac{2\pi}{n}$, i=1,...,n, the $MCC(x,y)$ and $MIC(x,y)$ are evaluated by:

$$MCC(x,y) = \max_{\theta_i = i\times\frac{2\pi}{n}, i=1,..,n} r(x,y,\theta_i) \tag{4}$$

$$MIC(x,y) = \min_{\theta_i = i\times\frac{2\pi}{n}, i=1,..,n} r(x,y,\theta_i) \tag{5}$$

MZE is evaluated by applying the MZT data-fitting method to solve the following optimization problem:

$$MZE = \min_{(x,y)\in E_{r(x,y,\theta)}} R(x,y) = \begin{cases} \min\left[\max_{\theta_i = i\times\frac{2\pi}{n}, i=1,...,n} r(x,y,\theta_i) - \min_{\theta_i = i\times\frac{2\pi}{n}, i=1,...,n} r(x,y,\theta_i)\right] \\ \text{subject to } (x,y)\in E_{r(x,y,\theta_i)} \end{cases} \tag{6}$$

where $E_{r(x,y,\theta_i)}$ is the convex envelopment of the n equally-spaced sampled points, i.e. the *search space.*

3. Genetic parameters

GAs were proposed for the first time by Holland [19] and constitute a class of search methods especially suited for solving complex optimization problems [20].
GAs maintain a *population of individuals* that are represented by their chromosomes, which are made of *genes*; GAs operate on the genes, which represent the inheritable properties of the individuals, by means of *genetic operators*. In each generation, the genetic operators are applied to selected individuals from the current population in order to create a new generation according to the iterative standard scheme in Figure 2 also used in this work.
A *fitness function* is assigned to an individual in order to reflect how well a solution fulfills the requirements of the given problem. The fitness function also makes it possible to evolve only good solutions.

[Figure 2 about here.]
Figure 2: Standard GA scheme.

The standard genetic operators are the following:
Selection: during this operation, a solution has a probability of being selected according to its fitness. One mechanism that allows fitness proportional selection is the *roulette wheel* procedure. Each individual has a chance to be selected, which is directly proportional to its fitness in the roulette wheel. *Tournament selection* is another reproduction mechanism. Tournament selection involves running several *tournaments* among a few individuals randomly chosen from the population. The individual with the best fitness is selected for crossover. If the tournament size is larger, weak individuals have a smaller chance of being selected. A third reproduction mechanism is *elitist selection*. With this method, the individuals are ordered on the basis of their fitness function; the best individuals produce offspring. The next generation will be composed of the best chromosomes chosen between the set of offspring and the previous population.

Crossover: new individuals are generally created as offspring of two parents. This method allows inheriting genes from parents with high fitness by the selection mechanism. One or more so-called crossover points are selected within the chromosome of each parent, at the same place in each. The parts delimited by the crossover points are then interchanged between the parents. The resulting individuals are the offspring. These kinds of crossovers are controlled by the *crossover probability*. Beyond *one point* and *multiple point crossover*, there exist more sophisticated crossover types. The so-called *knowledge-augmented* crossover operator constructs offspring from parents by making use of domain knowledge related to the given problem. Another crossover mechanism is *arithmetic crossover*, which generates offspring as a component-wise linear combination of the parents.
Mutation: a new individual is created by making modifications to one selected individual. The modifications consist of changing one or more values in the representation, or in adding/deleting parts of the representation. In GAs, mutation is a source of variability, and is applied in addition to selection and crossover. This method prevents the search to be trapped only in local solutions. The *mutation probability* controls the operator behavior.
Stop criterion: the algorithm has an iterative behavior and needs a stop condition to end the computation. Possible criteria include: overcoming a predefined threshold for the fitness function or iteration number or their combinations.

3.1 GA for roundness error evaluation

We use a GA in order to apply the MZT data-fitting algorithm [16] [17] [21].
A GA to solve the optimization problem (6) for a two-dimensional search space entails a population of chromosomes made of pairs of coordinates (their genes). These genes represent respectively the x and y coordinates of the center of the substitute feature of radii $MCC(x,y)$ and $MIC(x,y)$, in other words a set of possible candidates of the MZC for the optimization problem (Figure 1). For three-dimensional features such as spheres and cylinders, a third coordinate (z), and consequently a third gene, is required.
A fitness function defines the goodness of each chromosome (x,y) as solution of (6). As shown by (1), the fitness function is represented by the roundness error $R(x,y)$.
By applying genetic operators, the data-fitting algorithm evolves from generations of 2D coordinates implicitly exploring the search space, until a stop condition establishes the convergence to the MZC.

4. The proposed GA to assess the MZT to large datasets

GAs are able to provide a (local) optimum in all cases. However, to improve their convergence, particularly with large datasets, the genetic parameters need to be tuned.
In this work we describe the configuration of the genetic parameters in order to optimize the computation time for an evaluation error target.
The GAs performance are influenced by the following four main parameters as detailed in Table 1:

- population size (*pop*),
- probability of crossover (*pc*),
- probability of mutation (*pm*), and
- *stability condition* (N).

To stop the algorithm the stability condition with the mechanism described in Table 1 is proposed because the computation time is the main focus of this work. The selection of N is a compromise between two opposites criteria too small a value leads to an early stop of the evaluated generations with a non-optimal solution; too big a value guarantees the optimal solution, but causes an unacceptable increase in the computation time.

Table 1: Genetic operators, their parameters and mechanisms

Genetic operator	Parameter	Remark
-	*pop*	population size
selection	-	elitist selection
crossover	*pc*	one point crossover of the *pc*×*pop* parents' genes (i.e. coordinates) at each generation
mutation	*pm*	*pm*×*pop* individuals are modified by changing one gene (i.e. coordinate) with a random value
stop criterion	*N*	the algorithm computes *N* generations after the last best roundness error evaluated rounded off to the fourth decimal digit (0.1 μm)

The total computation time can be considered as an index of the algorithm performance for a given workstation.
GA was implemented on Intel® Core™2 Duo CPU P8400 @ 2.26 GHz Workstation, using C++ 5.0.

4.1 Large datasets

Different materials (steel, aluminum, and marble), geometrical features and processes (drilling and internal turning) have been considered (Table 2).

Table 2: Experimental set

Dataset	Diameter [mm]	Process	Material
1	10	drilling	aluminum alloy
2	10	milling	steel
3	14	turning	steel
4	17	drilling	marble
5	23	turning	steel
6	25	turning	aluminum alloy
7	39	turning	steel

For each dataset, the profile was acquired by scanning: a cloud of 1800 equally spaced points on the circumference (two tenths of a degree each) was sampled during the scan. The sampled profile of dataset n. 1 is also shown in Figure 1.

4.2 Explorative phase

The genetic parameters have been preliminarily explored over the range pointed out by the literature (Table 3) on the datasets of Table 2:

- *pop*, 4 values: 200 – 300 – 400 – 500; this range is higher than proposed in the literature because of the higher complexity coming from the larger datasets;
- *pc*, 14 equally-spaced values between 0.01 and 0.9;
- *pm*, 14 equally-spaced values between 0.001 and 0.04;
- *N* was conservatively set AS HIGH AS to 150.

Table 3: GA parameters from the literature

Genetic parameter	Values from [16]	Range from [21]**	Optimal values from [21]**
pop	30	10 – 50	30
pc	0.6	0.8	0.8
pm	0.033	0.06, 0.1	0.06
generation number	50	180 – 4000	4000

** for cylindricity.

Values of *pop*, *pc* and *pm* greater than respectively 500, 0.9 and 0.04 have also been explored and ignored because produced no significant improvement.

4.3 Computational experiments

The results of computation experiments are shown in Figure 3 with the total computation time as a function of *pop*, *pc* and *pm*. The four graphs are parameterized on *pop* and represent the contour maps of time in the two-dimensional space *pc*-*pm*.

[Figure 3 about here.]

Figure 3: GA parameters optimization: average computation time on the datasets of Table 2 as a function of *pc* and *pm* for four different values of *pop*.

Figure 3 is the result of 784 runs of the GA by changing the parameters of *pop*, *pc* and *pm* as designed.

It can be noticed that the computation time increases with *pop* in the examined range. For this reason we have shifted the optimization range below 100.

On the other hand, lower values of the population size do not provide sufficient accuracy in the detection of the roundness error.

[Figure 4 about here.]

Figure 4: GA parameters exploration: minimum roundness error R^* on dataset n. 1 of Table 2 as a function of *pc* and *pm* for 2 different values of *pop* upon an area of promising configuration (from Figure 3) of *pc* and *pm*.

As an example, $R^* = 0.0239$ mm (as also reported in Table 5) is the minimum roundness error of dataset n. 1 of Table 2 and it has been used as the target error in the explorative search shown in

Figure 4. It has been noticed that R^* cannot be reached with *pop* = 50 for most *pc* and *pm* values as shown from the black area in Figure 4. For *pop* = 70 instead the white coverage is almost filling the examined range of *pc* and *pm*. Consequently *pop* = 70 is the lower bound of the optimization phase discussed in the next paragraph.

5. GA optimization

As a consequence of the explorative phase, the GA optimization has been carried out in the range 70 – 100 of *pop* on the datasets of Table 2.
The results of computational experiments for the GA optimization are shown in Figure 5.
As for Figure 3, Figure 5 is the result of 784 runs of the GA by changing the parameters of *pop*, *pc* and *pm* as designed. Each run has a similar profile to the one displayed in the left inset for *pop*=70, *pc*=0.7 and *pm*=0.07.

[Figure 5 about here.]
Figure 5: GA parameters optimization: average computation time on the datasets of Table 2 as a function of *pc* and *pm* for four different values of *pop* and convergence of the average roundness error for the selected parameters (left inset).

5.1 Results and discussion

From the observation of Figure 5 it can be noticed that varying the *pop* value, there is always the same optimal region for *pc* and *pm*, and the same occurred in Figure 3. This region is placed as specified in Table 4.

Table 4: Optimal range of GA parameters from computational experiments and selected value used in benchmark tests

Parameter	Optimal range	Selected values
pop	70 – 80	70
pc	0.7 – 0.8	0.7
pm	0.001 – 0.010	0.007
generation number	40 – 130	–
N	–	30

With regard to *pop*, the computation time increased with this parameter, so the optimal value is the lowest one: *pop* ≥ 70. Smaller optimality areas are also present (brighter areas of Figure 5).
By comparing the performance in Figure 3 and Figure 5 it is clear that an incorrect or random choice of the parameters can lead to an increase of the computation time by one order of magnitude, from 70-80 s to 6-9 s.
The optimal range in Table 4 can be compared to values from the literature (Table 3) although we have worked with large datasets: in particular, it can be noticed that *pop* increases from 30 (50 for cylindricity) to 70. It has also been shown that a lower *pop* (e.g. 50 in Figure 4) does not allow sufficient accuracy to the GA.

The *pop* increase from 30 to 70 seems a consequence of the larger dataset.
The mutation probability *pm* also shows a significant change, three times lower and over.
The reduction of *pc* or *pm* can be considered as a beneficial effect, because it reduces the effect of the parameter itself and the "entropy" of the algorithm.
A lower *pc* or *pm* reduces the random search and enforces the solution search in fewer areas with higher success probability. Lower crossover and mutation is then beneficial for the convergence speed with user defined accuracy because fewer search areas are involved.
It is then speculated that the dataset size increase is beneficial for the GA stability. If a larger dataset implies a lower *pm* (like in the current case) and a lower *pm* provides higher stability to the algorithm, it could be concluded that with larger datasets the search is less random and driven by the most promising areas.
This looks reasonable because of the higher amount of information involved.

[Figure 6 about here.]
Figure 6: Convergence of MZT for the seven datasets of Table 2 with the selected values of the optimal parameters in Table 4.

Figure 6 shows the convergence of the algorithm with the selected values of the optimal parameters in Table 4 on the seven datasets.
The behavior of the convergence curve changes for the different datasets, because of their different nature. However already after 10 generations, their average represents the general trend within a range of 0.8 μm. From Figure 6 it can also be noticed that the major improvements on $R(x,y)$ are concentrated in the first few tenths of generations, particularly before 100.
As far as the stop criterion is concerned, it has been observed that the speed of convergence of the roundness error is lower than 0.1 μm per 16 generations. By increasing N over 16 the computation time increases without significant accuracy improvement nevertheless N can be conservatively set to 30 as reported in Table 4.
For applications where the accuracy matters more than the computation time (e.g. prototypical versus mass production) N can be increased accordingly.

6. Benchmark tests

In Table 5 the algorithm with the selected values of the optimal parameters in Table 4 has been compared with the LS method. The computation time in Table 5 is related to the stability condition of N=30 hence the total amount of processed generations are between 40 and 130 (Table 4).

Table 5: Test phase. Comparison between GA and LSE on the seven datasets of Table 2. Computation time for LSE is approximately zero for all datasets.

Datasets	*LSE* [mm]	*MZE* [mm]	t [s]
1	0.0242	0.0239	3.2
2	0.0299	0.0262	3.4
3	0.0292	0.0273	2.1
4	0.0668	0.0666	1.2
5	0.0725	0.0667	13.4
6	0.0934	0.0925	2.9
7	0.0423	0.0391	1.3

From Table 5 it can be noticed that by paying a low computation time, the roundness error estimation can be reduced by up to 6 µm.

The proposed algorithm has been compared with four datasets from the literature as detailed in Table 6. Unfortunately no large datasets are available (thousands datapoints versus less than hundred).

Table 6**:** Comparison with other datasets and optimization algorithms from the literature.

Data	**Methods**	***MZE* [mm]**	**t [s]**
Benchmark n. 1	Reference [3]	0.0085	-
39 datapoints	Proposed GA	0.0085	0.05
	GA [17]	0.0085	0.28*
	EGA [17]	0.0106	0.75*
Benchmark n. 2	Reference [22]	0.9574	0.02*
100 datapoints	Proposed GA	0.9649	0.11
	GA [17]	0.9574	0.40*
	EGA [17]	1.0866	1.05*
Benchmark n. 3	Reference [7]	0.0382	-
24 datapoints	Proposed GA	0.0382	0.09
	GA [17]	0.0382	0.25*
	EGA [17]	0.0405	0.69*
Benchmark n. 4	Reference [7]	0.0293	-
25 datapoints	Proposed GA	0.0295	0.05

* not comparable because no processor information is declared.

For each dataset, the first values of *MZE* are optima and have been determined by the author of the original datasets with various methods. The other three methods considered are GAs, for comparison within the same class of methods.

The performance of the proposed GA are very close to those of the method proposed by the author of the respective original datasets and those available in [17], hence the GA accuracy can be

considered validated. In addition the computation time of the proposed GA is generally lower than that declared.
The proposed algorithm shows lower roundness error with benchmarks n. 1, 3 and 4. Consequently, the worst performance of the proposed GA on the benchmark n. 2 seems related to its higher roundness error (almost 1 mm). With this assumption, the proposed GA is more suitable for the inspection of more accurate manufacturing operations.
Considering the optimal result on the small datasets from [17], where the GA ran without mutation, and that the computational experiments in all the search range (*pop* = 70 – 500) detected the same optimal region for *pc* and *pm* where *pm* was in the lower bound, it can be concluded that mutation is not a fundamental operator for the examined problem.

7. Conclusion

In this paper a *fast* algorithm for *accurate evaluation* of roundness errors with large datasets according the MZT criterion has been developed.
The GA parameters optimization has been carried out on seven datasets starting from initial GA parameters taken from the literature.
Computational experiments have pointed out the following outlook:
- larger dataset require higher population size values as preliminarily conjectured;
- the literature values for the probability of crossover have not been significantly affected by the larger dataset;
- the mutation probability decreases with larger datasets because of the higher amount of information on the involved part. It is also conjectured that mutation is not a fundamental operator for the examined problem.
- a stability condition has been imposed corresponding to a convergence speed greater than 0.1 μm per 30 generations and the stop criterion determined.

Simulations have shown that the optimal genetic parameters lead to a significant decrease (one order of magnitude) of the computation time (from 70-80 s to 6-9 s).
The proposed GA with the optimal parameters has also been compared with benchmarks from the literature, necessarily smaller, showing its effectiveness.
The proposed GA can be directly applied in the shop floor to achieve the economic benefits claimed by using the scheme (Figure 2) and parameters (Table 4) described in the paper.
By reducing the computation time of the minimum zone tolerance method, the most accurate method to evaluate roundness errors can be profitably applied real-time (on-line).
The higher speed of convergence on large datasets allows the application of the MZT to CMM scanning techniques, which involve thousands of points. The optimum dataset size requires further investigation.
Future work includes the investigation of different genetic mechanisms like *roulette wheel* selection or *knowledge-augmented* crossover, neglecting mutation.

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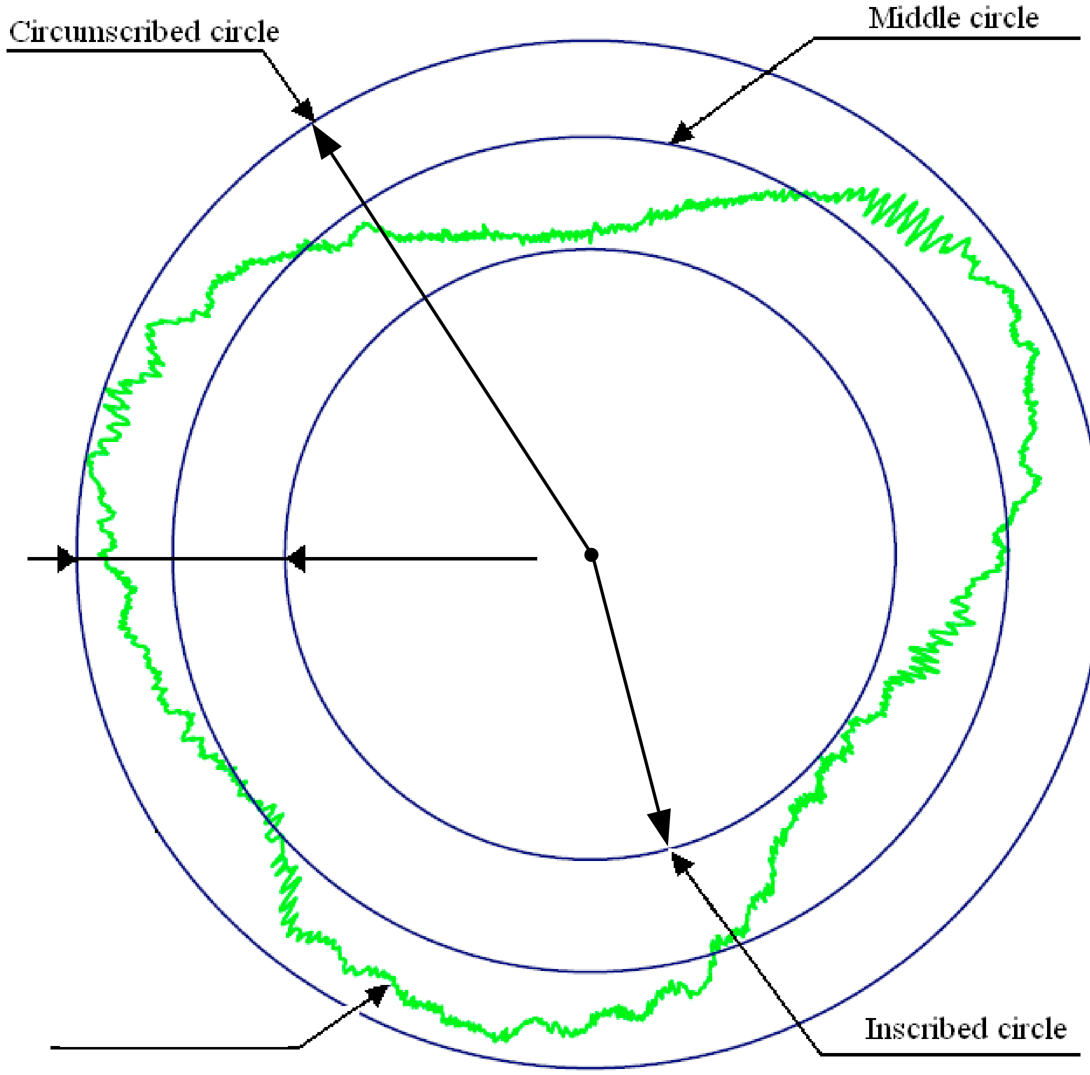

Figure 1: Profile of sample n. 1 (Table 2) with 1800 equally spaced CMM datapoints and substitute features: inner, medium, and outer circles for current (x,y).

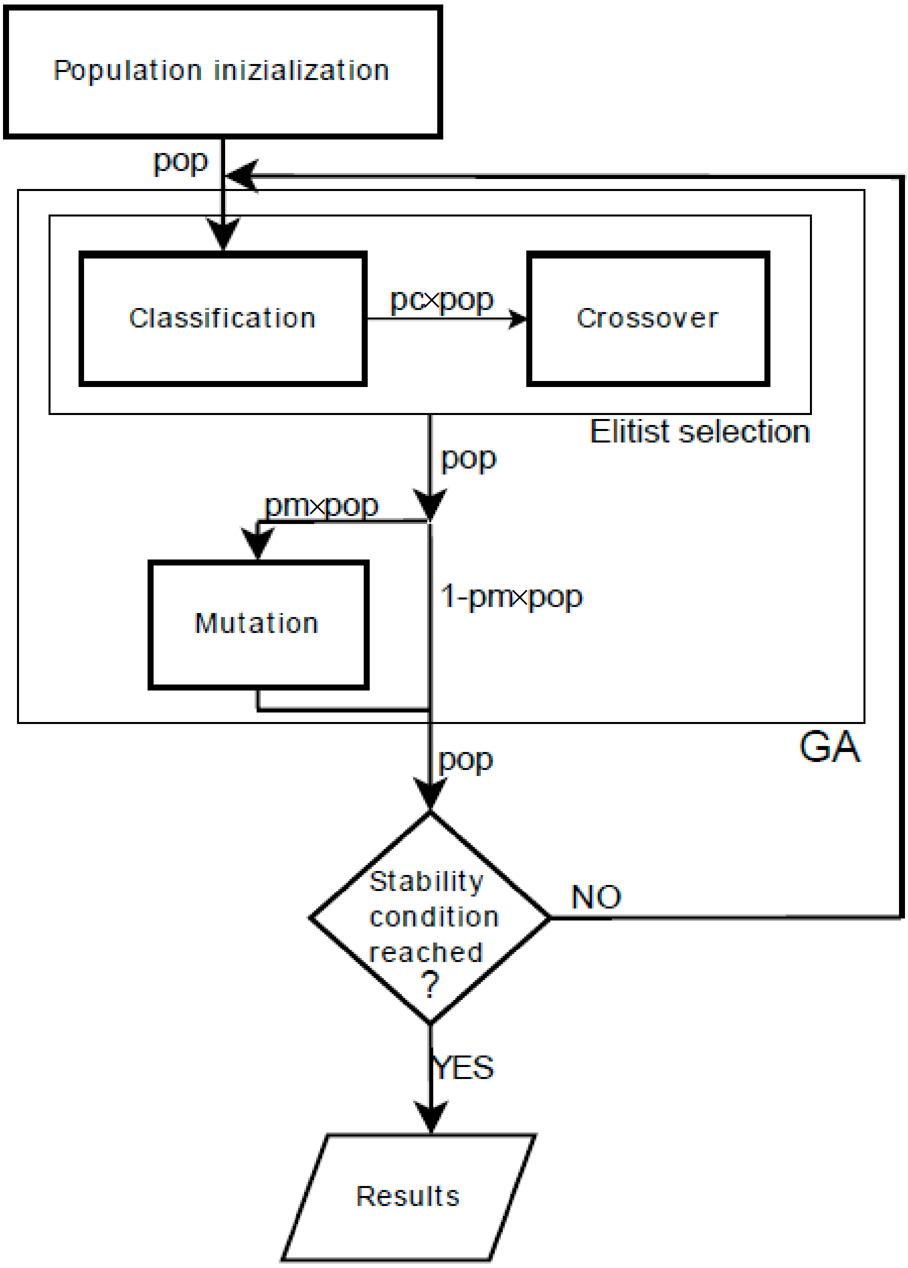

Figure 2: Standard GA scheme.

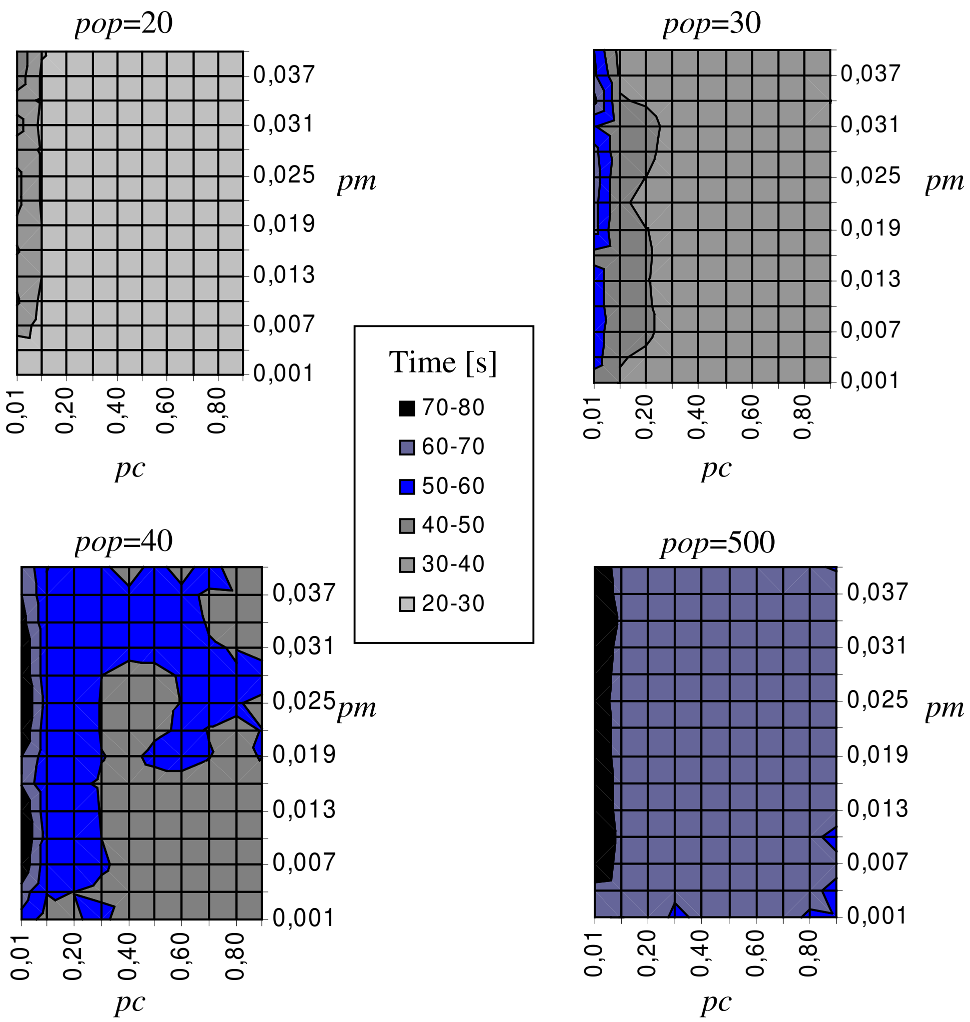

Figure 3: GA parameters optimization: average computation time on the datasets of Table 2 as a function of *pc* and *pm* for 4 different values of *pop*.

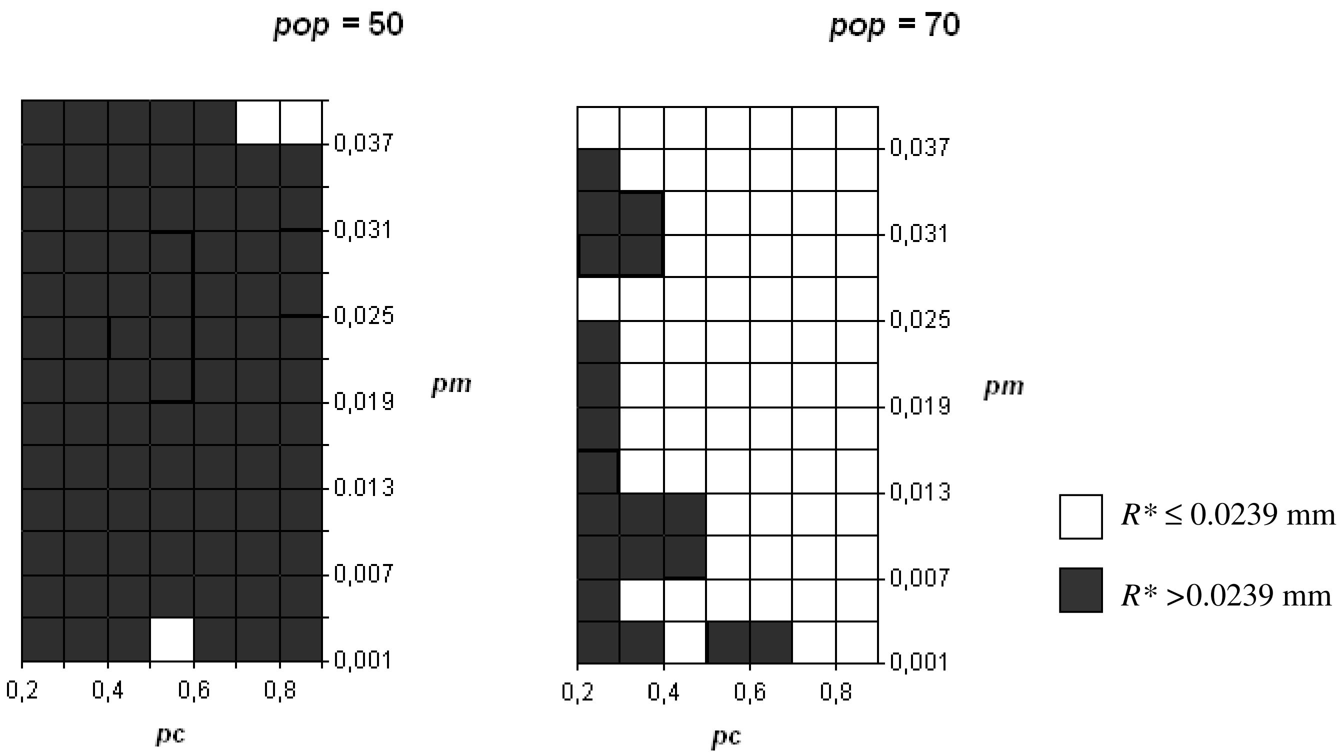

Figure 4: GA parameters exploration: minimum roundness error *R** on dataset n. 1 of Table 2 as a function of *pc* and *pm* for 2 different values of *pop* upon an area of promising configuration (from Figure 3) of *pc* and *pm*.

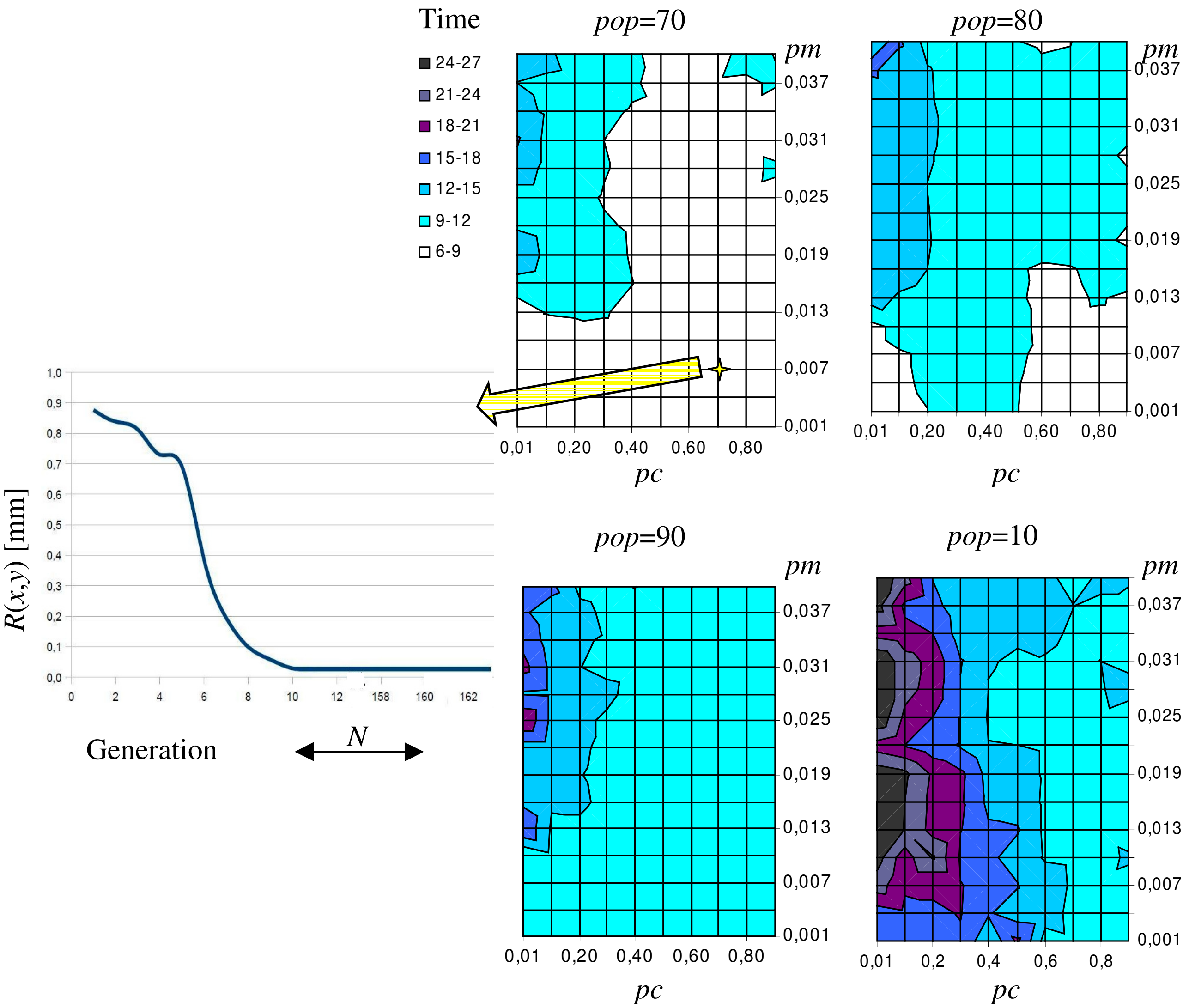

Figure 5: GA parameters optimization on the datasets of Table 2: average computation time as a function of *pc* and *pm* for 4 different values of *pop* and convergence of the average roundness error for the selected parameters.

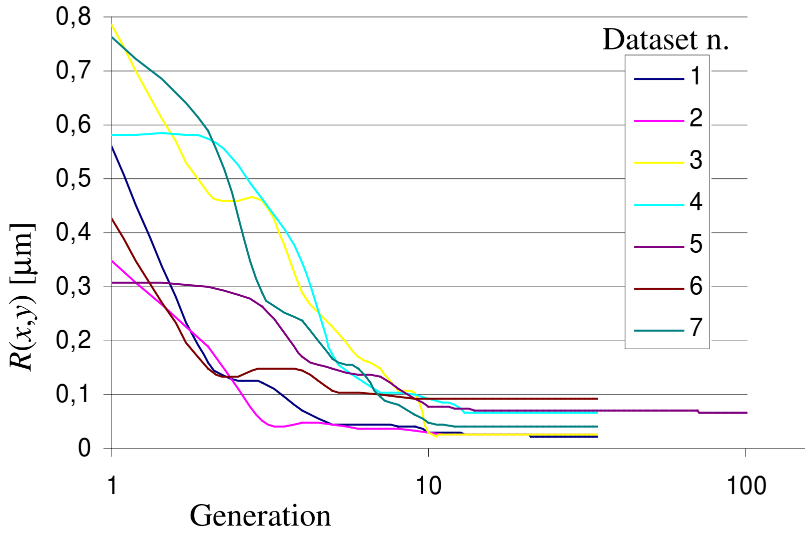

Figure 6: Convergence of MZT for the 7 datasets of Table 2 with the optimal parameters in Table 4.